Optimizing the integrated economic production quantity for a stochastically deteriorating production system under condition-based maintenance

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***Abstract*—This paper proposes a new integrated economic production quantity (EPQ) and condition-based maintenance (CBM) model for a stochastically deteriorating production system. Inspections are performed periodically to measure the real time degradation. The system fails (out-of-control) whenever its degradation is beyond a critical threshold level. In the out-of-control state, a proportion of nonconforming items are produced. To assess the degradation of the system and to increase the production of conforming items, preventive maintenance (PM) actions are carried out. An integrated EPQ and CBM optimization model that minimizes the total expected cost rate over an infinite time horizon is developed. The objective is to determine a joint optimal EPQ and PM strategy minimizing the sum of inspection/maintenance and setup costs, cost of nonconforming items in addition to inventory holding cost. Numerical experiments are provided to illustrate the proposed approach.**

Production lot-sizing; EPQ; Condition-based maintenance; Stochastic process; Optimization.

I. Introduction

In the production and inventory management setting, the economic production quantity (EPQ) model has been extensively investigated and extended under relaxation of various assumptions initially made in the basic EPQ model [1]. Many research investigations have been made to integrate production quantity, maintenance and quality issues in a single model where their interrelations are explicitly accounted for.

Ben-Daya and Makhdoum [2] investigated the effects of various PM policies on the joint optimization of the EPQ and the economic design control charts. In [3], the author developed an integrated model for the joint determination of EPQ, quality and PM level for a process with a general deterioration distribution and increasing failure rate. The durations of inspection intervals are chosen to make the integrated hazard rate function equal over each inspection interval [3]. The reader is referred to [4] for further details on the EPQ approaches published before 2001. Wang [5] proposed an integrated EPQ model with rework activity in addition to imperfect preventive maintenance including minimal repair. Chen [6] investigated an integrated EPQ model with inspection, rework and preventive maintenance with error. The correct execution of preventive maintenance reduces the system failure rate, whereas a preventive maintenance error shifts the system to an out-of-control state with a certain probability. Chen and Lo [7], and Wang [8], and more recently, Lia [9] studied an integrated EPQ model for an imperfect production system producing items that are sold with a warranty. An age based preventive maintenance has also been used along with the EPQ model to jointly determine the production lot size and PM schedule for randomly deteriorating production system producing both conforming and nonconforming items (see [10], [11] and the references therein).

From this literature review, one can conclude that the maintenance policies integrated into the EPQ model are based on the age of the production system and the statistical information from the system's lifetimes. As a result, the changes in the system reliability caused by how it is being used is not accounted for. As pointed out in [12], the main drawback of the lifetimes distributions is that only the aging process is accounted for when evaluating whether a system is functioning or not. However, many real life production systems suffer damage and deteriorate with both age and usage. There is therefore a need to develop new integrated EPQ models where the production system failures are explicitly related to both age and usage. In the literature, deterioration process of a system is generally modeled as a time-dependent stochastic process. For example, the random deterioration rate, Markov, Wiener, Gamma and Inverse Gaussian processes are commonly used approaches that are particularly good for their mathematical properties and clear physical interpretations [13], [14], [15]. When the deterioration of the production system reaches a failure threshold, maintenance actions should be performed on the system. If the system's degradation is appropriately modeled and measurable, maintenance actions can be carried out on the basis of the observed degradation data before the system enters the out-of-control state. This paper aims to develop a cost-effective joint EPQ and degradation-based model.

The remainder of the article is organized as follows. Section II describes the production system and presents the scope of the problem under consideration. Section III presents the development of the mathematical formulation of the problem. A solution procedure is proposed in Section IV along with several numerical experiments and the discussion of their results. Conclusions are drawn and future extensions discussed in the last section.

II. System description and problem definition

Our paper develops a novel joint EPQ and degradation-based maintenance model and investigates its contribution to the reduction of non-conforming items produced. The production system produces a single product type at any given time to meet a constant and continuous customer demand rate d. The system is subjected to stochastic degradation which is defined by a measurable scalar time-dependent random variable $X(t)$ which can take linear or nonlinear forms. A dormant failure occurs whenever the accumulated degradation reaches a critical threshold X_f, which can be specified according to either economical or safety reasons. The dormant failure of the system is revealed only through periodic inspection. The system is preventively maintained (PM) when the degradation level reaches a predetermined threshold X_p. The dormant failure causes the production system to go out-of control with a fraction non-conforming rate α. These non-conforming items add non-quality costs to the producer due to additional expenses for rework or economic losses for poor service. An inventory of good products is built up when the production system is in-control. When the degradation level is found to have exceeded X_p or X_f the production is stopped for the preventive and corrective maintenance operations to be carried out. Demand is then satisfied from the built-up inventory. Production resumes as soon as the inventory is completely depleted (Figure 1).

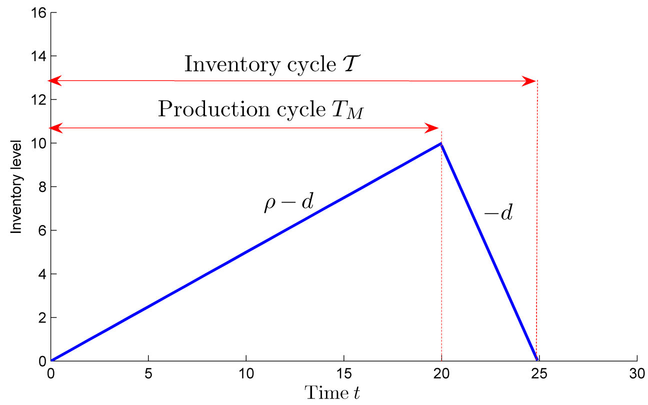

Fig. 1. Inventory cycle of the EPQ model with inventory build-up and depletion.

Our objective is to determine an optimal joint EPQ and PM strategy to minimize the sum of inspection, maintenance, setup, inventory, non-quality costs. An evaluation method is provided to optimize the expected cost rate function.

The following assumptions are considered in this paper:

1) The system deteriorates with a monotonically increasing stochastic degradation process. The damages at each time increment are cumulative. Hence, the degradation is increasing meaning that the system worsens with time due to aging and accumulated wear or damage.
2) The system is assumed to sojourn either in *out-of-control* or in *in-control* states.
3) In the in-control state, the system is producing only conforming items, while in the out-of-control state, the production system has a fraction non-conforming rate α.
4) Shortages are not allowed.
5) Failures are revealed only through error-free inspections (non-self announcing failures).
6) Inspections are carried out periodically until one or both of the thresholds are crossed. The inspection periodicity is a decision variable. Inspection duration is negligible.
7) PM is performed at a cost C_p when the degradation is beyond the threshold X_p, which is a decision variable.
8) Corrective maintenance (CM) is performed at a cost C_c when the degradation is beyond the threshold X_f. Furthermore, $C_c >> C_p$.
9) After either a PM or CM, the system becomes "*as good as new*" and is restored to the *in-control* state. Maintenance durations are negligible.
10) Ample inspection and maintenance resources are always available.

The continuous degradation of the production system is assumed to follow a stationary Gamma process. The Gamma process is appropriate for characterizing monotonically accumulating gradual damage over time [16], [14]. It has therefore been extensively used in condition-based maintenance optimization problems [17], [18]. An excellent survey dealing with the application of the Gamma process in maintenance modeling and optimization can be found in the seminal paper by van Noortwijk [14]. The proposed approach is general enough to accommodate any other degradation process.

The Gamma degradation process of the production system is a time-dependent stochastic process $\{X(t) : t \geq 0\}$ with the following characteristics:

1) $Pr\{X(0) = 0\} = 1$,

2) $X(t)$ has independent increments,

3) For all $0 \leq s < t$, the random variable $\Delta X(s,t) = X(t) - X(s)$ follows a Gamma distribution whose pdf $f(s,t,x)$ and cdf $F(s,t,x)$ are defined for all $x \geq 0$ as:

$$f(s,t,x) = \frac{x^{[(t-s)\gamma-1]}}{\Gamma[(t-s)\gamma]\,\eta^{(t-s)\gamma}} \exp\left(\frac{-x}{\eta}\right), \qquad (1)$$

and

$$\begin{aligned} F(s,t,x) &= \Pr(\Delta X(s,t)) < x) \qquad (2)\\ &= \int_0^x f(s,t,y)\mathrm{d}y, \\ F(s,t,x) &= \frac{\Gamma\left[(t-s)\gamma, \left(\frac{x}{\eta}\right)\right]}{\Gamma[(t-s)\gamma]}, \end{aligned}$$

where $(t-s)\gamma$ and η are the shape and scale parameters, respectively. The function $\Gamma(x) = \int_0^\infty u^{x-1}e^{-u}\mathrm{d}u$ is the Gamma and $\Gamma(\alpha,x) = \int_0^x u^{\alpha-1}\exp(-u)\mathrm{d}u$ is the lower incomplete Gamma function defined for $\alpha > 0$ and $x \geq 0$. From time 0 up to time t, the expected degradation is $\mathbb{E}[X(t)] = \eta\gamma t$ and its variance is $\mathbb{V}ar[X(t)] = \eta^2\gamma t$.

The lifetimes of the system are represented by the random variable $T_f = \inf\{t : X(t) \geq X_f\}$ given as the first passage time when the degradation exceeds the failure threshold X_f. Its cdf $G_f(t) = \Pr\{T_f \leq t\}$ is computed as:

$$\begin{aligned} G(t) &= \Pr\{X(t) > X_f\} \\ &= 1 - F(0,t,X_f) \\ G(t) &= \overline{F}(0,t,X_f), \qquad (3) \end{aligned}$$

where $\overline{F}(0,\tau,X_p - x) = 1 - F(0,\tau,X_p - x)$. The pdf $g(t) = \frac{\partial G(t)}{\partial t}$ corresponding to the system' lifetimes T_f is given as:

$$g(t) = \frac{\gamma}{\Gamma(\gamma t)} \int_{\frac{X_f}{\eta}}^{\infty} [\ln(u) - \Psi(\gamma t)]\, u^{\gamma t-1} \exp(-u)\mathrm{d}u, \qquad (4)$$

where $\Psi(u)$ is the digamma function defined as the logarithmic derivative of the Gamma function:

$$\Psi(u) = \frac{\mathrm{d}\ln(\Gamma(u))}{\mathrm{d}u}.$$

The production process starts initially with a new production system. The system degrades while producing items. When the system degradation is lower that the failure threshold X_f, the system is said to be in an *in-control* state, it is in an out-of-control state whenever its corresponding degradation exceeds the failure threshold X_f. The sojourn time in the in-control state is modeled by the random variable T_f whose cdf is given by Equation (3). While in its in-control state, items produced satisfy the quality requirements. However, in the out-of-control state, the system continues to produce but only a percentage

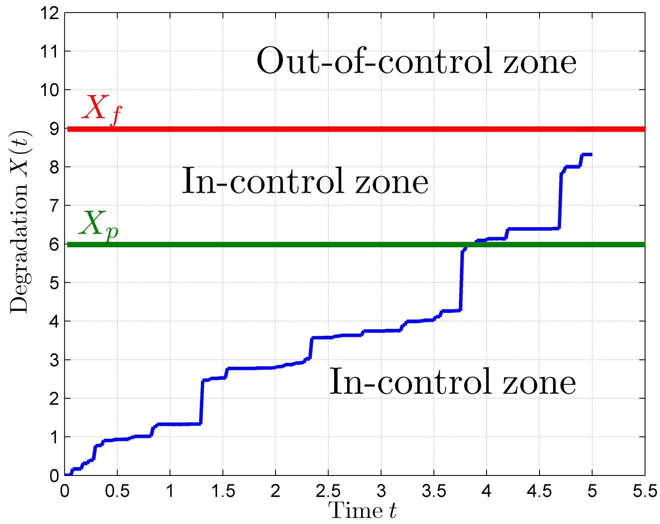

Fig. 2. Production system's degradation path and its operating zones.

$(1-\alpha)$ of the products are acceptable. An example of a possible degradation path of the production system is shown in Figure (2) where the degradation is governed by a Gamma process with a shape parameter $\gamma = 2.5$ and a scale parameter $\eta = 0.5$. The figure shows also the system's failure threshold X_f separating the two possible zones where the system is either in the in-control or in the out-of-control state.

To assess the production system's degradation and to improve production quality, the production system is subjected to an inspection and maintenance strategy described as follows. The system is periodically inspected at times $k\tau$ $(k = 1, 2, \ldots)$ where τ is the inspection interval which is a decision variable. Failures of the system are revealed only through inspections (i.e. failures are not self-announcing). During an inspection, the magnitude of the system's degradation is measured at cost C_I. If the degradation level measured is found to be less than X_p, no maintenance action is performed and the system's condition remains as it was just before the inspection. If the degradation value exceeds the failure threshold X_f, a corrective maintenance (CM) is carried out at cost C_c. If the degradation level is higher than a given degradation level X_p (see Figure 2), then a preventive maintenance (PM) action is performed. The degradation level X_p is a decision variable and $X_p \leq X_f$. It should be noted that the choice of the PM threshold X_p greatly impacts the performance of the production system. Indeed, if X_p is chosen to be close to the failure threshold X_f, then the probability of the system shifting into the out -of-control state increases significantly. Conversely, for low values of X_p, the probability of failure is reduced and the residual life of the production system is increased. Low values of X_p may reduce the risk of producing bad items but they increase the maintenance cost by generating more PM actions. There is therefore a need to find the optimal tradeoffs between the production, inspection and maintenance costs through the optimization of the economic production quantity Q, the threshold X_p and the inspection interval τ.

III. The integrated EPQ optimization model

The objective of the optimization model is to find the optimal values of the decision variables, namely the inspection interval τ and the preventive maintenance threshold X_p, which minimize the expected total cost per unit of time $C(\tau, X_p)$ over an infinite time horizon. We have a regenerative process starting and ending at the instants of complete depletion of the inventory, which follows either a preventive or a corrective replacement (see Figure 1). It follows from the theory of renewal reward processes that the long-run expected total cost per unit of time $C(\tau, X_p)$ is the average total cost $\mathbb{E}[\mathcal{C}]$ in a cycle divided by the average length $\mathbb{E}[\mathcal{T}]$ of that cycle:

$$C(\tau, X_p) = \frac{\mathbb{E}[\mathcal{C}]}{\mathbb{E}[\mathcal{T}]}. \tag{5}$$

The average total cost $\mathbb{E}(\mathcal{C})$ in a cycle is defined as the sum of the setup cost S_c, the expected inventory holding cost H_c, the expected maintenance cost M_c, and the cost of producing nonconforming items NC_c. In what follows, these costs are fully defined and discussed as well as the average length of the inventory cycle. The expected inventory cycle length $\mathbb{E}[\mathcal{T}]$ is the sum of the expected maintenance (production) cycle and the expected time required for inventory depletion (Figure 1). To compute these costs, let us first evaluate the expected inventory cycle length $\mathbb{E}[\mathcal{T}]$.

A. The expected inventory cycle length

According to Figure (1), the expected inventory cycle length $\mathbb{E}(\mathcal{T})$ is computed as:

$$\mathbb{E}[\mathcal{T}] = \frac{\rho}{d}\mathbb{E}[T_M], \tag{6}$$

where ρ is the production rate, $\mathbb{E}[T_M]$ is the expected maintenance (production run) cycle which ends either by a preventive or corrective replacement. The following lemma computes the value of $\mathbb{E}[T_M]$.

Lemma 1: The expected production cycle $\mathbb{E}[T_M]$ is computed as:

$$\mathbb{E}[T_M] = \sum_{i=1}^{\infty} i\tau \int_0^{X_p} f(0,(i-1)\tau,x)\overline{F}(0,\tau,X_p-x)\mathrm{d}x, \tag{7}$$

Proof: Comes straightforward from the fact that the probability of a maintenance action being performed after the i^{th} inspection is equal to the probability that the accumulated system's degradation in the interval $[0,(i-1)\tau]$ is lower than the threshold X_p and the degradation $X(i\tau)$ measured at time $i\tau$ is greater than or equal to the PM threshold X_p. This probability is computed as:

$$\Pr\{X((i-1)\tau) < X_p; X(i\tau) > X_p\} = \int_0^{X_p} f(0,(i-1)\tau,x)\overline{F}(0,\tau,X_p-x)\mathrm{d}x.$$

■

B. The expected inventory holding cost

According to Figure (1) together with the result of Lemma (1), the expected inventory holding cost $\mathbb{E}[H_c]$ is computed as:

$$\mathbb{E}[H_c] = C_h \frac{\rho(\rho-d)(\mathbb{E}[T_M])^2}{2d} \tag{8}$$

where C_h is the unit holding cost per unit of time.

C. The expected inspection and maintenance cost

According to the maintenance policy adopted, the production system is periodically inspected. Whenever either a PM or a CM is performed, the production systems becomes as good as new. The following lemma gives the expected maintenance cost of a maintenance cycle.

Lemma 2: The expected maintenance cost $\mathbb{E}[M_c]$ during the maintenance cycle T_M is:

$$\mathbb{E}[M_c] = \sum_{i=1}^{\infty} iC_I \int_0^{X_p} f(0,(i-1)\tau,x)\overline{F}(0,\tau,X_p-x)\mathrm{d}x + \sum_{i=1}^{\infty} \int_0^{X_p} f(0,(i-1)\tau,x)\left(C_p\overline{F}(0,\tau,X_p-x) + (C_c-C_p)\overline{F}(0,\tau,X_f-x)\right)\mathrm{d}x. \tag{9}$$

Proof: Let us consider two stochastic events E_i^p and E_i^c. The event E_i^p occurs when a PM is to be performed after the i^{th} inspection, while the event E_i^c occurs when the CM is to be carried out following the the i^{th} inspection. The occurrence probability of the event E_i^p is computed as:

$$\Pr\{E_i^p\} = \Pr\{X((i-1)\tau) < X_p; X_p \le X(i\tau) < X_f\}, \tag{10}$$

It follows that the probability to perform a PM after the i^{th} inspection is:

$$\Pr\{E_i^p\} = \int_0^{X_p} f(0,(i-1)\tau,x)\left(\int_{X_p-x}^{X_f-x} f(0,\tau,y)\mathrm{d}y\right)\mathrm{d}x \tag{11}$$

Similarly, the occurrence probability of the event E_i^c is computed as:

$$\Pr\{E_i^c\} = \Pr\{X((i-1)\tau) < X_p; X(i\tau) > X_f\} \tag{12}$$

We then have that the probability to perform a CM after the i^{th} inspection is:

$$\Pr\{E_i^c\} = \int_0^{X_p} f(0,(i-1)\tau,x)\overline{F}(0,\tau,X_f-x)\mathrm{d}x. \tag{13}$$

If the maintenance cycle is equal to $i\tau$, it follows that from Equations (11) and (13), the resulting expected inspection and maintenance cost is:

$$(C_p + iC_I)\Pr\{E_i^p\} + (C_c + iC_I)\Pr\{E_i^c\}.$$

Therefore the expected total maintenance cost can be written as

$$\sum_{i=1}^{\infty}(C_p + iC_I)\Pr\{E_i^p\} + (C_c + iC_I)\Pr\{E_i^c\}.$$

After basic algebraic operations, the result of the lemma is directly obtained. ■

D. The expected cost of producing nonconforming items

It is assumed that a percentage α of nonconforming items are produced during the period where the production system sojourns in its out-of-control state, i.e. during the time period where the system's degradation is greater than the failure threshold X_f. The expected total cost $\mathbb{E}[NC_c]$ induced by producing such nonconforming items is given by the following lemma.

Lemma 3: The expected total cost $\mathbb{E}[NC_c]$ corresponding to nonconforming items is:

$$\mathbb{E}[NC_c] = C_{nc}\alpha\rho\sum_{i=1}^{\infty}\left(\int_{(i-1)\tau}^{i\tau}(i\tau - t)g(t)\mathrm{d}t\right)\times \left(\int_0^{X_p} f(0,(i-1)\tau,x)\overline{F}(0,\tau,X_f - x)\mathrm{d}x\right), \quad (14)$$

where $g(t)$ is the pdf of the system lifetimes in Eq. (4).

Proof: The proof is obtained by computing the conditional expectation by conditioning on the event E_i^c which represents the case where the production run cycle is ended by a CM.

$$\Pr\{E_i^c\} = \int_0^{X_p} f(0,(i-1)\tau,x)\overline{F}(0,\tau,X_f - x)\mathrm{d}x.$$

We then have:

$$\begin{aligned}\mathbb{E}[NC_c] &= C_{nc}\alpha\rho\sum_{i=1}^{\infty}\mathbb{E}[NC_c|_{E_i^c}]\Pr\{E_i^c\}\\ &= C_{nc}\alpha\rho\sum_{i=1}^{\infty}\left(\int_{(i-1)\tau}^{i\tau}\mathbb{E}[NC_c|_{T_f}]\mathrm{d}G(t)\right)\Pr\{E_i^c\}\\ &= C_{nc}\alpha\rho\sum_{i=1}^{\infty}\left(\int_{(i-1)\tau}^{i\tau}\alpha\rho(i\tau - t)\mathrm{d}G(t)\right)\Pr\{E_i^c\}.\end{aligned} \quad (15)$$

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From the above results, the optimization problem considered is to find the decision variables defining the optimal joint values of the inspection time period τ and the PM threshold X_p, which minimize the total expected cost rate:

$$C(\tau, X_p) = \frac{Sc + \mathbb{E}[H_c] + \mathbb{E}[M_c] + \mathbb{E}[NC_c]}{\mathbb{E}[\mathcal{T}]}, \quad (16)$$

where we recall here that Sc, $\mathbb{E}[H_c]$, $\mathbb{E}[M_c]$, $\mathbb{E}[NC_c]$ and $\mathbb{E}[\mathcal{T}]$ are, respectively, the setup cost, the expected holding cost, the expected inspection and maintenance costs, the expected cost of producing nonconforming items, and the expected inventory cycle length. Unfortunately, the optimal solutions that minimize Equation (16) are in general difficult to obtain analytically and proof of global convexity would be intractable. Therefore, a numerical method is needed to solve this optimization problem. A numerical procedure is developed based on the fix-and-optimize method to minimize Equation (16).

IV. Numerical examples

In this section, we investigate the problem of solving the integrated EPQ and maintenance for a production system whose random degradation is governed by a stationary gamma stochastic process $\{X(t) : t \geq 0\}$. The latter is characterized by its scale and shape parameters set, respectively, to $\eta = 0.8$ and $\gamma = 1.15$. The production system fails whenever its degradation reaches the failure threshold $X_f = 4$. The demand and production rates are $d = 50$ and $\rho = 100$. Costs corresponding to setup, unit non-conforming product, PM, CM and inspection are set, respectively, to $S = 150$, $C_{nc} = 400$, $C_p = 60$, $C_c = 100$, $C_I = 0.5$, and the holding cost is set to $C_h = 0.5$. Data used within this experiment are arbitrary and considered for illustration purposes, and they are assumed to be given, if any, in appropriate time and monetary units.

Let us recall that the objective is to determine simultaneously the optimal values of the the decisions variables, namely the inspection period τ and the preventive degradation threshold X_p.

To show how important the value of the preventive threshold X_p is, let us first assume that X_p is an input parameter set by the decision-maker rather than being a decision variable. The objective of the manufacturer is then reduced to finding the optimal value of the inspection period τ. In the case where $X_p = 2.5$, the optimal solution suggests to perform inspections at a period $\tau = 0.5$. This solution induces an expected total cost rate $C(\tau, X_p) = C(1.1, 2.5) = 71.94$. If we consider the extreme case where the PM is ignored ($X_p = X_f$), in this case the optimal inspection period is $\tau = 0.6$ which induces an expected total cost rate evaluated to $C(0.6, 4) = 82.50$. This simple example clearly illustrates the balancing role played by the PM threshold in the joint EPQ optimization problem.

Now, if the integrated EPQ model is solved jointly for both decision variables τ and X_p, the optimal solution suggests to perform inspection at period $\tau = 1.4$ while PM are carried out whenever the degradation threshold reaches the level $X_p = 1.55$ (see Figure 3). This inspection/maintenance policy induces an expected total cost rate of $C(1.4, 1.55) = 70.89$.

V. Conclusion

In this paper, we developed a new integrated model for the joint optimization of EPQ and preventive maintenance for a stocastically deteriorating production system. Unlike the existing approaches, our model uses condition-based

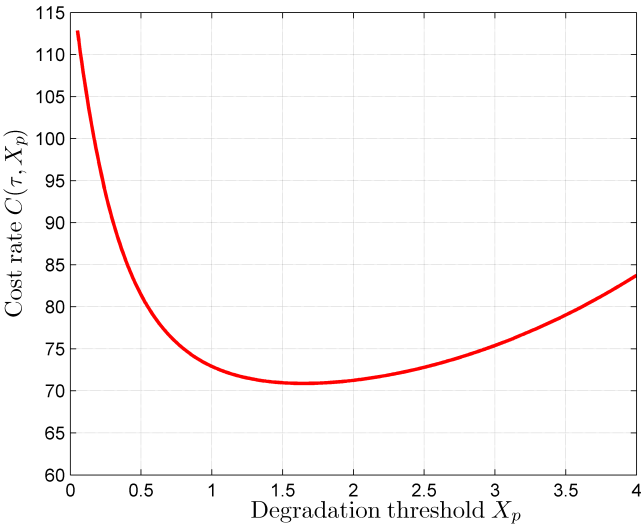

Fig. 3. Total expected cost rate versus the degradation threshold X_p: case of $\tau^* = 1.4$.

maintenance decision making. The degradation of the system is modeled as a stationary Gamma process. However the proposed approach is still general enough to encompass other kind of degradation processes. Inspections are carried out periodically to monitor the system degradation. A PM is performed whenever the degradation exceeds a threshold which is a decision variable. A CM is performed whenever a specified critical threshold is reached. A numerical example was provided to illustrate the proposed approach. Effects of both inspection period and PM degradation threshold is shown to play an important role on the joint EPQ and maintenance decision making.

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