A Generalized Flow Network for Freight Car Dispatching

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Abstract

In the freight car dispatching problem empty freight cars have to be assigned to known demands respecting a given time horizon and certain constraints. The goal is to minimize the resulting transportation costs. One of the constraints is that customers can specify the type of cars they want. It is possible, however, that cars of certain types can be substituted by other cars either in a 1-to-1 fashion or at different exchange rates. We show that these substitutions make the dispatching problem NP-complete. We model the dispatching problem as a generalized integral minimum cost flow problem on a specific directed graph. We show that in our setting its linear relaxation is half-integral. Using rounding techniques, the LP-relaxation can be transformed to a dispatching with small constraint violation at the same cost, or, under additional assumptions, to a 4-approximation. In practice, both ideas are combined to a heuristic approach without further assumptions. We conclude with computational results for this heuristic on application data provided by DB Schenker Rail Deutschland AG in context of a joint R & D project together with the Technical University of Kaiserslautern.

Keywords: transportation, logistics, dispatching, generalized flow, complexity, approximation, heuristic

1. Introduction

The general concern of a cargo railway company is to transport goods between different customer sites. For this, empty freight cars have to be brought to the initial location to get loaded and have to be collected at their destination after unloading. The transport of different goods imposes requirements on the freight cars (e.g. open or closed, bulk cargo or coil transport), which are therefore distinguished into different car types. However, a demand can be satisfied by cars of several types, as specified by allowed substitutions.

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If the freight cars are owned by the railway company, apart from the transport of loaded freight cars, the company also has to manage the rental of its freight car stock. Due to the established work flow, the allocation of empty freight cars is treated as a separate logistic *dispatching problem*: assign the available empty freight cars to given customer demands (or storages) with minimal total transport cost. Besides various (technical or marketing oriented) side constraints, a valid dispatching for a certain time interval has to respect a given freight train schedule and a set of allowed car type substitutions. A freight car can substitute a demand of a different type either in a 1-to-1 fashion or at different 'exchange rates', which make the dispatching problem NP-complete.

In the remainder of the introduction, we formally define our dispatching problem (DP) and prove that it is NP-complete in Section 2. In Section 3 we model (DP) as a generalized integral minimum cost flow problem and show that their respective (optimal) solutions are equivalent. The equivalence implies the NP-completeness of the generalized integral minimal cost flow problem on restricted networks. We show that we obtain half-integral solutions in polynomial time for instances relevant in our application by transforming the associated generalized network to a classical network. An early result ([4]) shows that a generalized flow network can be transformed to a classical flow network if and only if its node-arc-incidence matrix is not of full rank. Yet, it is not obvious if this condition either holds for all generalized networks which model instances of (DP) or how to characterize instances for which it is true. Section 4 presents two approximation algorithms based on the half-integral solution and Section 5 concludes with computational results on application data for a heuristic combining ideas from both approximation algorithms.

A *supply* $s_i \in S$ of empty freight cars is given by a tuple (l_i, t_i, c_i, n_i), where $l_i \in \mathbb{N}$ specifies its location, $t_i \in \mathbb{N}$ its type, $c_i \in \mathbb{N}$ the actual availability time and $n_i \in \mathbb{N}$ the number of supplied cars with the former three attributes. A *demand* $d_j = (l_j, t_j, c_j, n_j) \in D$ is defined analogously. We assume that demands always exceed supplies and supply can be fully disposed. This is realistic with regard to active disposition into operative storage, as is provided by the applied model. A set of allowed *substitutions* $\mathcal{S}$ contains tuples $\sigma_{sd} = (n_s t_s, n_d t_d)$, such that $n_s \in \mathbb{N}$ cars of type t_s satisfy a demand of $n_d \in \mathbb{N}$ cars of type t_d. We assume that for each pair of types there is at most one such substitution. Hence we can define a *relative valency* $v(t_s, t_d) \in \mathbb{Q}$ for type t_s with respect to type t_d as $v(t_s, t_d) = \frac{n_d}{n_s}$ (the 'exchange rate'). If for some car type t_s all its relative valencies are independent of t_d, then $v(t_s)$ denotes its *total valency*. A supply s_i is *allowed* for a demand d_j if there exists a $\sigma_{sd} \in \mathcal{S}$ with $t_s = t_i$ and $t_d = t_j$. We denote it by σ_{ij}. Furthermore, a timetable $\mathcal{T}$ is a set of tuples $\theta_{ab} = (l_a, c_a, l_b, c_a, r_{ab})$ representing direct or composed freight train connections between locations l_a and l_b. The connection θ_{ab} starts with a train, which is composed in l_a at time c_a and ends with a train being dissolved in l_b at time c_b. The cost for using connection θ_{ab} is denoted by $r_{ab} \in \mathbb{N}$. A supply s_i is *in time* for a demand d_j if there exists a $\theta_{ab} \in \mathcal{T}$ with $l_a = l_i$, $l_b = l_j$, $c_a \geq c_i$ and $c_b \leq c_j$. We denote such a connection with minimal c_a by θ_{ij}. A supply s_i *matches* a demand d_j if it is *in time* and *allowed* for the respective demand.

An instance I of (DP) is given by a tuple $I = (S, D, \mathcal{S}, \mathcal{T})$. It is called *homogeneous*, if $n_s = n_d$ for all $\sigma_{sd} \in \mathcal{S}$ and *heterogeneous* otherwise. An instance in which all car types t_i have a total valency is called *total*. The allowed substitution occurring in the practical applications by DB Schenker Rail Deutschland AG only contains tuples σ_{sd} with $n_s = n_d = 1$ or $n_s = 2$ and $n_d = 1$. We call such instances 2-*heterogeneous* in the following.

With $\delta_{ij} = (s_i, d_j, n_{ij})$ we denote a *disposition* of n_{ij} cars of supply s_i to demand d_j with cost $c(\delta_{ij}) = n_{ij} r_{ij}$. A *dispatching* $\mathcal{D} = \{\delta_{ij}\}$ is a set of dispositions and its cost $c(\mathcal{D})$ is the sum of the individual disposition costs.

Let $I = (S, D, \mathcal{S}, \mathcal{T})$ be a DP instance, $\mathcal{D}$ be a dispatching and $\delta_{ij} = (s_i, d_j, n_{ij}) \in \mathcal{D}$ a disposition.

We call a disposition $\delta_{ij} = (s_i, d_j, n_{ij})$ *LP-feasible* if $s_i \in S$, $d_j \in D$, s_i matches d_j, $n_{ij} \in \mathbb{R}$, $n_{ij} \leq n_i$ and $v(t_i, t_j) \cdot n_{ij} \leq n_j$. A dispatching $\mathcal{D}\{\delta_{ij}\}$ is *LP-feasible* if all the dispositions are LP-feasible and:

$$\sum\nolimits_{\delta_{ij} \in \mathcal{D}} n_{ij} = n_i \qquad \text{for all } s_i \in S \tag{1}$$

$$\sum\nolimits_{\delta_{ij} \in \mathcal{D}} v(t_i, t_j) \cdot n_{ij} \leq n_j \qquad \text{for all } d_j \in D. \tag{2}$$

An LP-feasible dispatching with minimal transportation costs is *LP-optimal*. A disposition (dispatching) is *feasible* if it is LP-feasible and all n_{ij} are integral. A feasible dispatching with minimal cost is optimal and a solution for (DP).

2. Complexity of the Dispatching Problem

As usual, we consider the decision version of the dispatching problem (DDP): given an instance I of the dispatching problem and a number $c_I \in \mathbb{N}$, is there a dispatching $\mathcal{D}^*(I)$ with cost $c(\mathcal{D}^*)$ smaller or equal to c_I? We prove that (DDP) is NP-complete by a reduction of a variant of the satisfiability problem [3]. For this, we consider the following SAT-type problem, which was shown to be NP-complete in [7]:

> [3V2L3SAT] Given a Boolean 3-SAT formula α in which each variable occurs at most three times and each literal occurs at most two times. Decide whether α is satisfiable.

Let $\alpha = C_1 \wedge \cdots \wedge C_n$ with $C_i = l_{i1} \vee l_{i2} \vee l_{i3}$ and $l_{ij} \in \{x_k, \neg x_k | 1 \leq k \leq m\} \cup \{0\}$ be a 3V2L3SAT-formula with m variables in n clauses. We construct a 2-heterogeneous instance $I_\alpha = \{S, D, \mathcal{S}, \mathcal{T}\}$ such that the supply of I_α can completely be dispatched at cost $c_I = 0$ if and only if α is satisfiable.

The set S consists of two sets S_c and S_v with supplies corresponding to clauses and variables in α. For each clause c_i, S_c contains one supply s_i and for each variable x_k, S_v contains three supplies s_k^1, s_k^2, s_k^3 defined as follows:

$$S_c = \{s_i = (l_i, t_1, 0, 1) : 1 \leq i \leq n\}$$

$$S_v = \{s_k^1 = (l_k^1, t_1, 0, 2) : 1 \leq k \leq m\} \cup \{s_k^2 = (l_k^2, t_2, 0, 1) : 1 \leq k \leq m\}$$
$$\cup \{s_k^3 = (l_k^3, t_1, 0, 2) : 1 \leq k \leq m\}$$

For each variable x_j we specify four demands, two for each of the literals:

$$\begin{aligned} D = & \quad \{d_j^1 = (l_j^1, t_3, 0, 1) : 1 \leq j \leq m\} \cup \{d_j^2 = (l_j^2, t_3, 0, 1) : 1 \leq j \leq m\} \\ \cup & \quad \{\bar{d}_j^1 = (\bar{l}_j^1, t_3, 0, 1) : 1 \leq j \leq m\} \cup \{\bar{d}_j^2 = (\bar{l}_j^2, t_3, 0, 1) : 1 \leq j \leq m\}. \end{aligned}$$

The substitution rules are defined as $\mathcal{S} = \{(2t_1, t_3), (t_2, t_3)\}$. Note that each supply $s_i \in S$ is allowed for every demand $d_j \in D$. The timetable $\mathcal{T}$ is defined as follows:

$$\begin{aligned} \mathcal{T} &= \{(l_i, 0, l_j^1, \infty, 0) : c_i \text{ is the first clause containing } x_j\} \\ &\cup \{(l_i, 0, l_j^2, \infty, 0) : c_i \text{ is the second clause containing } x_j\} \\ &\cup \{(l_i, 0, \bar{l}_j^1, \infty, 0) : c_i \text{ is the first clause containing } \neg x_j\} \\ &\cup \{(l_i, 0, \bar{l}_j^2, \infty, 0) : c_i \text{ is the second clause containing } \neg x_j\} \\ &\cup \{(l_k^1, 0, l_j^1, \infty, 0), (l_k^1, 0, l_j^2, \infty, 0) : k = j\} \\ &\cup \{(l_k^2, 0, l_j^2, \infty, 0), (l_k^2, 0, \bar{l}_j^1, \infty, 0) : k = j\} \\ &\cup \{(l_k^3, 0, \bar{l}_j^1, \infty, 0), (l_k^3, 0, \bar{l}_j^2, \infty, 0) : k = j\} \\ &\cup \{(l_a, 0, l_b, \infty, M) : \text{for each other combination of locations}\} \end{aligned}$$

Note that all supplies s_i and and demands d_j are in time with respect to $\mathcal{T}$, while only some corresponding connections θ_{ij} have associated cost zero. Nevertheless all supplies match all demands.

We can visualize matching supplies and demands corresponding to a single variable x_j in α as shown in Figure 1: solid vertices and lines correspond to supplies in S_v (which always exist for variable x_j) matching demands with associated costs zero. The numbers at the vertices represent the number of available and demanded cars respectively. The clauses in α containing either x_j or $\neg x_j$ are indicated by the dotted vertices in correspondence to associated supplies. The dotted lines also represent connections with associated cost zero. Figure 1 shows four dotted vertices, as S contains up to two supplies associated with a clause containing x_j (upper part of the figure) and up to two supplies associated with a clause containing $\neg x_j$ (lower part of the figure). Still the total number of such supplies in S_c is three.

Lemma 1. *The total supply of S_v can only be dispatched at cost zero by either of the following dispositions for each j with $1 \leq j \leq m$:*

(i) dispose two cars of s_j^1 to d_j^1, one car of s_j^2 to d_j^2 and the two cars of s_j^3 either both to $\bar{d}_j^1$ or both to $\bar{d}_j^2$ or one to each of them.

(ii) dispose two cars of s_j^3 to $\bar{d}_j^2$, one car of s_j^2 to $\bar{d}_j^1$ and the two cars of s_j^1 either both to d_j^1 or both to d_j^2 or one to each of them.

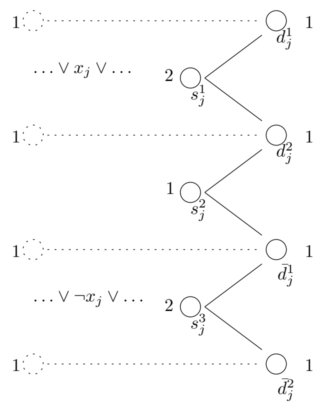

Figure 1: Matching supplies and demand associated with a variable x_j (solid) and its occurrence in clauses of α (dotted).

Proof: The one car of s_j^2 can either be disposed to d_j^2 or to $\bar{d}_j^1$ with zero cost and satisfies either of their demands completely. Otherwise it cannot be disposed with zero cost. Consequently, either s_j^1 is completely disposed to d_j^1 or s_j^3 is completely disposed to $\bar{d}_j^2$ respectively, as otherwise the supply cannot be disposed with zero cost. Possibly remaining supply may be distributed to the other demands with zero cost.

□

In either case, the total demand of four cars associated with a variable x_j is not satisfied. We associate the only two possible dispositions with zero cost per variable with its two possible truth values. We set x_j to false if the dispatching satisfies both demands d_j, and we set x_j to true if the dispatching satisfies both demands $\bar{d}_j$. Observe that the first variant allows no disposition of a supply $s_i \in S_c$ to a demand d_j with zero cost. The second variant allows both of the possibly two matching supplies from S_c to be disposed with zero cost, each to either of the demands. The latter disposition corresponds to the truth assignment of x_j satisfying the clause c_i. A similar argument holds for clauses containing $\neg x_j$. Given that any supply corresponding to a clause c_i can only be disposed with zero cost to one of the demands corresponding to a literal which occurs in c_i, a dispatching which disposes the total supply with zero cost corresponds to a satisfying truth assignment.

Theorem 2. *Let $I_\alpha = \{S, D, \mathcal{S}, \mathcal{T}\}$ be defined as above. The formula α is satisfiable if and only if all supply can be disposed with zero cost.*

Proof: Let α be satisfiable and x, a satisfying assignment. Dispose the supply $e_v = 5m$ of S_v by the variant of Lemma 1 which corresponds to the satisfying truth assignment of x_j for each j. Then e_v is completely disposed at zero costs, and only demands corresponding to literals set to true are not yet completely satisfied. Since x satisfies every clause, each corresponding supply c_i is at least a match with a connection of zero associated cost for one not yet satisfies demand. Dispose c_i to the latter demand, which corresponds to a true literal. Thus the additional unit supply for each clause $e_c = n$ is also disposable at zero cost.

Conversely, suppose α is not satisfiable. Each dispatching of e_v with zero cost according to the variants in Lemma 1 corresponds to a consistent truth assignment and therefore blocks at least one clause from all matching demands with associated connections of zero cost. Hence at most $5m + n - 1$ supply can be disposed at zero cost. If on the other hand, we allow any other dispatching of e_v, by Lemma 1, not all supply e_v can be disposed at zero costs and any complete dispatching of the total supply $5m + n$ needs at least cost of $M > c_I$.

□

As we can check in polynomial time for a given dispatching if it disposes the total supply at zero costs and does not violate the constraints, we have:

Corollary 3. *(DDP) is NP-complete for total* 2*-heterogeneous instances.*

□

By omitting connections with cost M in the timetable, the previous construction also proves the existence of a feasible dispatching if and only if the formula α is satisfiable. Hence (DDP) is strongly NP-complete. We chose the variant with costs in order to keep the proof closely related to the optimization problem. Also remember that in our practical applications the instances are 2-heterogeneous so that we have to find a compromise between running times and solution quality in practice.

3. Generalized Network Model

Let a generalized network $N = (V, A)$ be a directed graph with the following four functions on arcs and nodes, respectively: a *capacity function* $u : A \to \mathbb{R}$, a *multiplier function* $m : A \to \mathbb{R}$, a *cost function* $c : A \to \mathbb{R}$ and a *balance function* $b : V \to \mathbb{R}$.

As usual, we call a node $s \in V$ a source if $b(s) > 0$ and a node $t \in V$ a sink if $b(t) < 0$. A feasible generalized flow $f : A \to \mathbb{R}$ in N is a function which assigns a flow value $f(a)$ to each arc $a = (u, v) \in A$ such that the following capacity and node balancing constraints are satisfied:

$$0 \leq f(a) \leq u(a) \qquad \text{for all } a \in A \tag{3}$$

$$\sum_{a=(u,v)\in A} f(a) - \sum_{a=(w,u)\in A} m(a) f(a) = b(u) \qquad \text{for all } u \in V \tag{4}$$

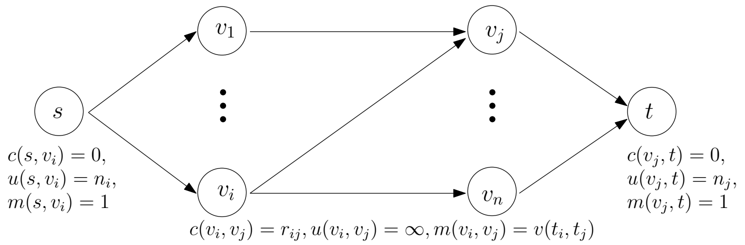

Figure 2: Generalized Network $N(I)$ associated with a DP instance I.

The cost $c(f)$ of f is the sum of the costs of all arcs weighted with the flow on this arc: $c(f) = \sum_{a \in A} f(a)c(a)$. A generalized minimum cost flow f^* in N is a feasible generalized flow with minimal cost [1, 2, 5].

Similar to the reduction of bipartite matchings to network flows, we construct a generalized network $N(I) = (V, A)$ for a given instance $I = (S, D, \mathcal{S}, \mathcal{T})$ of (DP) (cf. Figure 2). The node set V consists of three sets $V_S \cup V_D \cup \{s, t\}$. For each supply $s_i \in S$ we have a node $v_i \in V_S$, for each $d_j \in D$ we have a node $v_j \in V_D$ and an additional source s and sink t. The set A is the union of the arc sets $A_S = \{(s, v_i) | v_i \in V_S\}$, $A_D = \{(v_j, t) | v_j \in V_D\}$ and arcs $(v_i, v_j) \in A_T$ for which the corresponding supply s_i and demand d_j match with respect to I. We call A_T the set of *transit arcs*.

The capacity $u(a)$ of arcs $a = (s, v_i)$ $(a = (v_j, t))$ is set to n_i (n_j) of the associated supply (demand) and $u(a) = \infty$ for transit arcs. The cost of transit arcs $a = (v_i, v_j)$ are set to the actual transport costs of the associated connection $\theta_{ij} \in \mathcal{T}$: $c(a) = r_{ij}$. All other arcs have cost zero. The multiplier $m(a)$ for a transit arc $a = (v_i, v_j)$ is the relative valency $v(t_i, t_j)$ and one for all other arcs. Node balances $b(v)$ are zero for all nodes $v \in V_S \cup V_D$ and $b(s) = \sum_{a \in A_S} u(a) = \sum_{s_i \in S} n_i$, $-b(t) = \sum_{a \in A_D} u(a) = \sum_{d_j \in D} n_j$. Note that $b(s) \leq -b(t)$ due to our assumption on I. Thus there may be no feasible generalized flow in N at all. We apply a maximum flow computation and adapt $b(t)$ accordingly before computing such a flow or assume f to be a feasible pseudoflow in N, such that only the balance of the sink is violated.

Theorem 4. *Every LP-feasible dispatching $\mathcal{D}(I)$ is equivalent to a feasible flow or pseudo flow $f(N)$.*

Proof: Let $\mathcal{D}(I)$ be an LP-feasible dispatching. On transit arcs, let $f^{\mathcal{D}}(v_i, v_j) = n_{ij}$ if $\mathcal{D}$ contains a disposition $\delta_{ij} = (s_i, d_j, n_{ij})$ and $f^{\mathcal{D}}(v_i, v_j) = 0$ otherwise. For arcs in $A_S \cup A_D$ we set:

$$f^{\mathcal{D}}(s, v_i) = \sum\nolimits_{\delta_{ij} \in \mathcal{D}} n_{ij} \tag{5}$$

$$f^{\mathcal{D}}(v_j, t) = \sum\nolimits_{\delta_{ij} \in \mathcal{D}} v(t_i, t_j) n_{ij} \tag{6}$$

Then $f^{\mathcal{D}}(N)$ fulfills the capacity constraint (3). The feasibility of $\mathcal{D}(I)$ implies that the sum of disposed cars is equal to the number of available cars for each supply node corresponding to $f(s, v_i) = u(s, v_i)$ and the sum of cars disposed to a demand (weighted with the relative valency of the supply car type) does not exceed the number of demanded cars. Flow $f^{\mathcal{D}}(N)$ further satisfies the node balancing constraints (4) in all nodes, eventually except in node t.

Conversely, let $f(N)$ be a feasible (pseudo-)flow. We define $\mathcal{D}^f(I)$ as follows:

$$\mathcal{D}^f(I) = \{\delta_{ij} = (s_i, d_j, n_{ij} = f(v_i, v_j)) | f(v_i, v_j) > 0\}.$$

By the construction of the network ($s_i \in S$ matches $d_j \in D$) and since f satisfies the constraints (4) for all nodes (possibly except t), each disposition $\delta_{ij} \in \mathcal{D}^f(I)$ is LP-feasible, i.e.

$$n_{ij} \leq \sum_{\delta_{ij} \in \mathcal{D}} n_{ij} \leq f(s, v_i) \leq u(s, v_i) = n_i$$

and

$$n_{ij} \leq \sum_{\delta_{ij} \in \mathcal{D}} v(t_i, t_j) n_{ij} \leq f(v_j, t) \leq u(v_j, t) = n_j.$$

Hence the complete dispatching is LP-feasible.

□

From this equivalence the following corollary is obvious:

Corollary 5. *A generalized minimal cost (pseudo) flow $f^*(N)$ provides an optimal dispatching $\mathcal{D}^*(I)$ if and only if $f^*(N)$ is integral.*

For homogeneous instances of (DP), $N(I)$ is a classical flow network. So if input values are integral (after scaling, if necessary), an integral minimum cost flow can be computed in polynomial time. Corollaries 3 and 5 imply that such a result is very unlikely for the 2-heterogeneous instances which occur in our application. On the other hand, suppose we drop the integrality constraint and obtain a generalized minimal cost flow solution f^* in polynomial time - how infeasible (or in other words how fractional) can $\mathcal{D}^*$ be?

We call a function *β-fractional* for some $\beta \in \mathbb{N}$ if all its values can be expressed as integer multiples of $\frac{1}{\beta}$. In the following, we investigate the fractionality of minimal cost flows in generalized networks which correspond to total instances.

Let $I = (S, D, \mathcal{S}, \mathcal{T})$ be a total instance of (DP) such that p is the least common multiple of the nominators and q is the least common multiple of the denominators of all total valencies $v(t_i), s_i \in S$. We define an instance $I_t = (S_t, D, \mathcal{S}_t, \mathcal{T})$ as:

$$S_t = \{(l_i, t_i, c_i, v(t_i) \cdot n_i) : s_i \in S\},\ \mathcal{S}_t = \{(t_s, t_d) : \sigma_{sd} \in \mathcal{S}\}.$$

We slightly change the definition of the associated network $N(I_t)$ by setting the arc costs $c_t(v_i, v_j) = v(t_i) r_{ij}$. By definition of $\mathcal{S}_t$, the relative valencies $v_t(t_s, t_d)$ are now one, i.e. the network $N(I_t)$ is a classical flow network. We further modify $N(I_t)$ to $N(I_t, q)$ such that the capacity and balance function are scaled by q to $u_q(a) = qu(a)$ and $b_q(v) = qb(v)$. Thus $N(I_t, q)$ is a classical flow network with integral input data such that an integral minimal cost flow f_q^* in $N(I_t, q)$ can be computed in polynomial time. By the linearity of the capacity and balance constraints, a flow $f_t = \frac{1}{q} f_q$ is feasible in $N(I_t)$ if and only if f_q is feasible in $N(I_t, q)$. Similarly, $f_t^* = \frac{1}{q} f_q^*$ is a minimal cost flow in $N(I_t)$ if f_q^* is a minimal cost flow in $N(I_t, q)$ and f_t^* is obviously q-fractional.

Lemma 6. *Let $\mathcal{D}_t = \{\delta_{ij}^t = (s_i, d_j, n_{ij} = \frac{1}{v(t_i)} \cdot f_t^*(v_i, v_j)) | f_t^*(v_i, v_j)) > 0\}$. Then $\mathcal{D}_t$ is a pq-fractional LP-optimal dispatching to I.*

Proof: By definition of I_t and $N(I_t)$, for each $\delta_{ij}^t \in \mathcal{D}_t$ supply $s_i \in S$ matches demand $d_j \in D$ with respect to I. The feasibility of f_t^* gives $f_t^*(v_i, v_j) \leq u(s, v_i) = v(t_i)n_i$ and $f_t^*(v_i, v_j) \leq u(v_j, t) = n_j$. Hence:

$$n_{ij} = \frac{1}{v(t_i)} \cdot f_t^*(v_i, v_j) \leq \frac{1}{v(t_i)} \cdot u(s, v_i) = \frac{1}{v(t_i)} \cdot v(t_i)n_i = n_i \tag{7}$$

$$v(t_i)n_{ij} = v(t_i)\frac{1}{v(t_i)} \cdot f_t^*(v_i, v_j) = f_t^*(v_i, v_j) \leq u(s, v_i) = n_j \tag{8}$$

Thus each $\delta_{ij}^t \in \mathcal{D}_t$ is pq-fractional LP-feasible, as f_q^* is integral and by definition of p, q, f_t^*. By construction of the network, the inequalities (7) and (8) and the node balance of the flow at s, $\mathcal{D}_t$ is a pq-fractional LP-feasible dispatching for I. Furthermore:

$$c(f_t^*) = \sum_{a \in A_T} f_t^*(a)c(a) = \sum_{\delta_{ij}^t \in \mathcal{D}_t} n_{ij} r_{ij} = c(\mathcal{D}_t)$$

Thus, by minimality of $c(f_t^*)$, $\mathcal{D}_t$ is a pq-fractional LP-optimal dispatching for I.

□

Lemma 6 answers the question of the fractionality of a LP-optimal dispatching. It assumes, however, that the instances are total. We can extend this result slightly as follows. Let $I = (S, D, \mathcal{S}, \mathcal{T})$ be a (non-total) instance of (DP) and $T_D = \{t = t_d | \sigma_{sd} \in \mathcal{S}\}$. We call I *totalizable* if there is a function $w : T_D \to \mathbb{Q}$, such that the instance $I' = (S, D', \mathcal{S}', \mathcal{T})$ with

$$D' = \{(l_j, t_j, c_j, w(t_j)n_j) | d_j \in D\} \text{ and } \mathcal{S}' = \{(n_s t_s, w(t_d) \cdot n_d t_d) | \sigma_{sd} \in \mathcal{S}\}$$

is total.

Lemma 7. *Let $\mathcal{D}'$ be a pq-fractional LP-optimal dispatching to I'. Then $\mathcal{D}'$ is also a pq-fractional LP-optimal dispatching for I.*

Proof: By definition of I and I', supply $s_i \in S$ matches demand $d_j \in D$ for each $\delta'_{ij} \in \mathcal{D}'$ with respect to I. Furthermore, $n_{ij} \leq n_i$ and $v'(t_i, t_j)n_{ij} =$

$w(t_j)v(t_i,t_j)n_{ij} \leq w(t_j)n_j$ implies $v(t_i,t_j)n_{ij} \leq n_j$. Hence each $\delta'_{ij} \in \mathcal{D}'$ is pq-fractional LP-feasible for I. Moreover:

$$\forall s_i \in S : \sum_{\delta'_{ij} \in \mathcal{D}'} n_{ij} = n_i$$

and

$$\forall d_j \in D : \sum_{\delta'_{ij} \in \mathcal{D}'} v'(t_i,t_j)n_{ij} \leq w(t_j)n_j \Leftrightarrow \sum_{\delta'_{ij} \in \mathcal{D}'} v(t_i,t_j)n_{ij} \leq n_j$$

such that $\mathcal{D}'$ is a pq-fractional LP-optimal dispatching for I.

□

By Lemma 7, we can obtain β-fractional LP-optimal dispatchings for totalizable instances of (DP) in polynomial time.

4. Approximate Dispatchings for Application Instances

As usual, we call a feasible dispatching a ρ-*approximation* if its cost are at most ρ times the optimal cost. In the sequel we will also consider a different type of approximation. For this, we call a 1-to-1-exchange of a car of type t_a with a car of type t_b an *upgrade* with respect to a demand d_j of type t_c, if $\sigma_{ac}, \sigma_{bc} \in \mathcal{S}$ and $v(t_a,t_c) < v(t_b,t_c)$ and a *downgrade* if $\sigma_{ac}, \sigma_{bc} \in \mathcal{S}$ and $v(t_a,t_c) > v(t_b,t_c)$. A ν-*upgraded dispatching* $\mathcal{D}$, is a dispatching containing only feasible dispositions, obeying condition 1 and $\sum_{d_{ij} \in \mathcal{D}} v(t_i,t_j) \cdot n_{ij} \leq n_j + \nu, d_j \in D$ and furthermore after downgrading a single car for all $d_j \in D$ with $\sum_{d_{ij} \in \mathcal{D}} v(t_i,t_j) \cdot n_{ij} > n_j$, we obtain a feasible dispatching. Observe that downgrading has no effect on the cost of the dispatching.

Since our DB-instances of the dispatching problem contain substitution rules of the form $(2t_x, t_z)$, they are heterogeneous. We can totalize these instances by setting $w(t_x) = 1, w(t_y) = w(t_z) = 2$ (and continue $w : T_D \to \{1,2\}$ for all other car types). Since we assume integrality of the input data, the previous results give a half-integral LP-optimal dispatching in polynomial time.

In the following, let $I = (S, D, \mathcal{S}|xyz, \mathcal{T})$ be such an instance with $\mathcal{S}|xyz = \{(t_x,t_x),(t_y,t_y),(2t_x,t_z),(t_y,t_z)\}$ and $\mathcal{D}$ a half-integral LP-optimal dispatching for I. Note that with w as above, the total valencies of I' are $v'(t_x) = 1, v'(t_y) = \frac{1}{2}, v'(t_z) = \frac{1}{2}$. Since t_z is an artificial type introduced to allow customers to demand either type t_x or t_y, t_z only occurs as demand. Hence half-integral dispositions only occur from supplies with type t_y to demands of type t_y or t_z. To obtain an integral dispatching from $\mathcal{D}$, we round the solution trying to keep (LP-)feasibility (as much as we can). Feasibility of a rounded integral dispatching can be guaranteed if for each supply s_i (demand d_j) the number of dispositions δ_{iy} (δ_{xj}) which are rounded up equals the number of those dispositions which are rounded down.

Consider the following undirected bipartite graph $G(\mathcal{D}) = (V = V_S \cup V_D, E)$ representing the fractional dispositions with

$$\begin{aligned} V_S &= \{v_i : \delta_{ij} = (s_i, d_j, n_{ij}) \in \mathcal{D}, n_{ij} \notin \mathbb{N}\} \\ V_D &= \{v_j : \delta_{ij} = (s_i, d_j, n_{ij}) \in \mathcal{D}, n_{ij} \notin \mathbb{N}\} \\ E &= \{(v_i, v_j) | \delta_{ij} \in \mathcal{D}\}. \end{aligned}$$

A feasible solution based on rounding half-integral dispositions can be guaranteed if the node degree $deg(v)$ is even for each $v \in V$ (i.e. $G(\mathcal{D})$ is Eulerian). While in general this may not be the case, at least $deg(v_i), v_i \in V_S$ has to be even: the supply is completely disposed by $\mathcal{D}$, i.e. $\sum_{\delta_{ij} \in \mathcal{D}} n_{ij} = n_i \in \mathbb{N}$. Hence the number of dispositions $\delta_{ij} = (s_i, d_j, n_{ij} \notin \mathbb{N}) \in \mathcal{D}$ is even.

Let $U \subseteq V_D$ be the set of nodes v_j with odd degree. We use the greedy Algorithm 1 to cover E by a set $P \subseteq E$ of paths and a set $C \subseteq E$ of cycles. P consists of paths p_{uv} between nodes $u, v \in U$ and $|P| = \frac{1}{2}|U|$. Then each edge $e \in E$ occurs exactly once in $P \cup C$. Since the parity of the nodes $v \in V_S$, $w \in V_D$ and $u \in U$ are invariant throughout the algorithm, the appropriate choice of edge e in lines 6 and 8 is always possible. Once U is empty, all remaining nodes have even degree with respect to the remaining edge set E', which justifies line 16.

Algorithm 1 Cover Construction

1: $Cover = \emptyset$
2: **while** $U \neq \emptyset$ **do**
3: Chose $x = u \in U$.
4: $p = \emptyset$
5: **repeat**
6: Chose $e = (x, v) \in E$
7: $p = p \cup e, E \setminus \{e\}$
8: Chose $e = (v, w) \in E$
9: $p = p \cup e, E \setminus \{e\}, x = w$
10: **until** $w \in U$
11: $Cover = Cover \cup p$
12: **if** $u \neq w$ **then**
13: $U = U \setminus \{u, w\}$
14: **end if**
15: **end while**
16: Cover the remaining arcs $e \in E$ by cycles c.
17: $Cover = Cover \cup c$.

Let $Cover$ be the set of paths and cycles in $G(\mathcal{D})$ provided by Algorithm 1. Observe that both paths and cycles have even length. We obtain an integral dispatching $\mathcal{D}_r$ by rounding the half-integral dispositions corresponding to edges e alternately up and down along the paths and cycles $p, c \in Cover$. Let $p =$

$v_1, v_2, \ldots, v_r \in Cover$ be a path between $u = v_1, v = v_r \in U$. Then either the dispositions corresponding to the odd edges $e_{2k+1} = (v_{2k+1}, v_{2k+2}) \in p, k \in \mathbb{N}$ are rounded up and the dispositions corresponding to the even edges $e_{2k} = (v_{2k}, v_{2k+1}) \in p, k \in \mathbb{N}$ are rounded down or vice versa. The same possibilities or *rounding directions* occur for cycles $c \in Cover$. In the following we will speak of rounding edges when referring to rounding the corresponding dispositions. We call a demand d_j with $\sum_{\delta_{ij} \in \mathcal{D}_r} v(t_i, t_j) n_{ij} = n_j + \frac{1}{2}$ *over-satisfied*.

So far, we did not specify the rounding direction for paths and cycles in $Cover$. Let $c(p) = \sum_{e=(v_i,v_j)\in p} \frac{1}{2} r_{ij},\ p \in Cover$ account for the cost of the strictly half-integral portion of the associated dispositions $\delta_{ij} \in \mathcal{D}$. (The cost of a cycle is defined analogously.) Then we define $c_{odd}(p) = \sum_{e_{2k+1}=(v_i,v_j)\in p, k\in\mathbb{N}} r_{ij}$ as the cost of p with respect to $\mathcal{D}_r$, if we chose to round up the odd edges in p and $c_{even}(p) = \sum_{e_{2k}=(v_i,v_j)\in p, k\in\mathbb{N}} r_{ij}$ in the opposite case. We chose the rounding direction for $\mathcal{D}_r$, such that $c_r(p) = \min\{c_{odd}(p), c_{even}(p)\}$. We first observe that rounding $\mathcal{D}$ to $\mathcal{D}_r$ in this way does not affect the cost:

Lemma 8. *The cost of $\mathcal{D}_r$ with appropriate choice of rounding directions does not exceed $c(\mathcal{D})$.*

Proof: The dispatchings $\mathcal{D}_r$ and $\mathcal{D}$ only differ in originally half-integral dispositions corresponding to edges $e \in p\ (c) \subseteq Cover$. Let $c|2(\mathcal{D})$ denote the sum of the costs of the strictly half-integral portion of dispositions $\delta_{ij} \in \mathcal{D}$ and $c|2(\mathcal{D}_r)$ denote the sum of the costs of the same dispositions after rounding, such that $c_r(p) = \min\{c_{odd}(p), c_{even}(p)\}$. Then:

$$
\begin{aligned}
& c|2(\mathcal{D}_r) \\
= & \sum_{p\in Cover} c_r(p) + \sum_{c\in Cover} c_r(c) \\
= & \sum_{p\in Cover} \min\{c_{odd}(p), c_{even}(p)\} + \sum_{c\in Cover} \min\{c_{odd}(c), c_{even}(c)\} \\
= & 2 \sum_{p\in Cover} \frac{1}{2} \min\{c_{odd}(p), c_{even}(p)\} + 2 \sum_{c\in Cover} \frac{1}{2} \min\{c_{odd}(c), c_{even}(c)\} \\
\leq & \sum_{p\in Cover} \frac{1}{2} c_{odd}(p) + \frac{1}{2} c_{even}(p) + \sum_{c\in Cover} \frac{1}{2} c_{odd}(c) + \frac{1}{2} c_{even}(c) \\
= & c|2(\mathcal{D})
\end{aligned}
$$

Thus $c(\mathcal{D}_r) \leq c(\mathcal{D})$.

□

Theorem 9. *The dispatching $\mathcal{D}_r$ is a 0.5-upgraded optimal dispatching for I with at most $\frac{1}{2}|U|$ over-satisfied demands.*

Proof: Each path $p \in Cover$ and cycle $c \in Cover$ enters and leaves $v \in V \setminus U$ the same number of times. Hence the number of incident edges rounded up and down is the same and $\mathcal{D}_r$ is feasible for these nodes. For each $v \in U$ there is exactly one path which only enters or leaves v via some edge e. If e is rounded down $\sum_{\delta_{ij} \in \mathcal{D}} v(t_i, t_j) n_{ij} \leq n_j$ holds. Otherwise, if e is rounded up and $\sum_{\delta_{ij} \in \mathcal{D}} v(t_i, t_j) n_{ij} = n_j$ the sum of disposed cars exceeds n_j exactly by half the freight car which was rounded up on e. This occurs at most $\frac{1}{2}|U|$ times at demands $d_j \in V_D$. Knowing that the number of half-integral dispositions $\delta_{ij} \in \mathcal{D}$ to d_j is odd (otherwise $v_j \notin U$), there must be an odd number of dispositions $\delta_{ij} \in \mathcal{D}$ with $t_i = t_x$, otherwise the dispositions $\delta_{ij} \in \mathcal{D}$ cannot sum up to $n_j \in \mathbb{N}$. Hence there is at least one feasible disposition of type t_x, thus $t_j = t_z$, and we can downgrade one car of type t_y to type t_x for each over-satisfied demand d_j. Given the relative valencies $v(t_x, t_x) = v(t_y, t_y) = v(t_y, t_z) = 1$, $v(t_x, t_z) = \frac{1}{2}$, the downgrade makes $\mathcal{D}_r$ feasible and we obtain a 0.5-upgraded dispatching. Optimality then follows directly by Lemma 8.

□

Obviously, an over-satisfied demand receives too many cars. Knowing that each over-satisfied demand is of type t_z and receives at least one car of type t_x, this is either half a car of type t_y or one car of type t_x. We can now choose the rounding directions of paths, such that the number of over-satisfied demands is minimized. If we do so, for each over-satisfied demand d_j, there is another demand d'_j, such that $p_{jj'}$ between v_j, v'_j is in *Cover*. The demand d'_j is of type t_z as a change of rounding direction over-satisfies d'_j, otherwise, we would choose this rounding direction and obtain a smaller number of over-satisfied demands. Further, d'_j lacks at least half a freight car of type t_y due to rounding down the incident edge $e \in p$ and we can view this also as a lack of one car of type t_x. Let s_i be the supply from which the spare freight car of type t_x is disposed to d_j. If s_i is also in time for d'_j, s_i matches d'_j and we can redispose a car of type t_x to d'_j. We assume that $\mathcal{T}$ allows all such redispositions and the cost function satisfies the triangle inequality, which is reasonable as we have transport costs per kilometre. Let $\mathcal{D}'_r$ be the appropriately rounded half-integral dispatching with redispositions for I.

Theorem 10. *The rounded dispatching $\mathcal{D}'_r$ is a 4-approximation to the optimal dispatching for I.*

Proof: Similar to $\mathcal{D}_r$, $\mathcal{D}'_r$ is feasible except for up to $\frac{1}{2}|U|$ demands d_j, which are over-satisfied by one car of type t_x. The latter are feasibly redisposed to matching demands d'_j as argued above. Further, $\mathcal{D}'_r$ and $\mathcal{D}$ differ in originally half-integral dispositions corresponding to edges $e \in p\ (c)\ \subseteq$ *Cover* and the redispositions $\mathcal{R} \subset \mathcal{D}'_r$. Note that each redisposed car corresponds to a path $p \in$ *Cover* and its cost can be charged to twice the cost of the path (as on each edge of the path only half a car was disposed). Let $c|2(\mathcal{D})$ and $c|2(\mathcal{D}'_r)$ denote the cost of both dispatchings with respect to these dispositions. Then:

$$\begin{aligned} c|2(\mathcal{D}'_r) &= \sum_{p\in Cover} c_r(p) + \sum_{c\in Cover} c_r(c) + \sum_{\delta_{ij'}\in\mathcal{R}} r_{ij'} \\ &\leq 2\sum_{p\in Cover} c(p) + 2\sum_{c\in Cover} c(c) + 2\sum_{p\in Cover} c(p) \\ &\leq 4c|2(\mathcal{D}) \end{aligned}$$

Thus $c(\mathcal{D}'_r) \leq 4c(\mathcal{D})$ and $\mathcal{D}'_r$ is a 4-approximation to the optimal dispatching for I.

□

Remember that we bounded the cost of an optimal dispatching $c(\mathcal{D}^*)$ by the cost of an LP-optimal dispatching $c(\mathcal{D})$, which suffices for the approximation guarantee as $c(\mathcal{D}) \leq c(\mathcal{D}^*)$. On the other hand, the deviation of $c(\mathcal{D}^*)$ from $c(\mathcal{D})$ can be quite large, such that the approximations are likely to perform better in practice.

5. Computational Results

Without further prerequisites (such as a 'friendly' freight train schedule allowing all necessary redispositions), applying the 4-approximation in practice does not always yield a fully feasible dispatching. We therefore combine the 0.5-upgraded dispatching with the idea of redispositions used in the 4-approximation. We now allow spare cars to be redisposed to any matching demand. For this, we modify the original (DP) instance I by reducing all supplies and demands appropriate to the dispositions $\delta_{ij} \in \mathcal{D}_r$, except for the dispositions of spare freight cars of type t_x. Let I' be the reduced instance. Then I' does not contain any supplies of type t_y any longer, as they are fully disposed by $\mathcal{D}_r$ and I' is (empirically) a homogeneous instance. Consequently $N(I')$ is a classical network and we obtain an integral minimal cost flow and thus an optimal solution $\mathcal{D}'$ in polynomial time. Unfortunately, we cannot bound the additional cost $c(\mathcal{D}')$, as the following example (see Figure 3) shows:

Let $I = (S, D, \mathcal{S}|xyz, \mathcal{T})$ with

$$S = \{s_1 = (l_1, c_1, t_x, 1), s_2 = (l_2, c_2, t_x, 1), s_3 = (l_3, c_3, t_y, 1)\},$$

$$D = \{d_4 = (l_4, c_4, t_x, 1), d_5 = (l_5, c_5, t_z, 1), d_6 = (l_6, c_6, t_z, 1), d_7 = (l_7, c_7, t_x, 1)\}$$

and $\mathcal{T}$ such that s_1 is in time for d_4 and d_5, s_2 for d_5 and d_6 and s_3 for d_6 and d_7. The transport cost are $r_{ij} = r$ for all supply-demand-pairs except for $r_{14} = r_{37} = R >> r$. Then the half-integral LP-optimal dispatching $\mathcal{D}(I)$ is as displayed by the dispositions in Figure 3. Let $\mathcal{D}(I)$ be w.l.o.g. rounded to $\mathcal{D}_r(I) = \{(1, 5, 1), (2, 6, 1)\}$, such that s_3 provides the spare car of type t_x as otherwise d_6 would be over-satisfied. Then Figure 4 shows the reduced instance

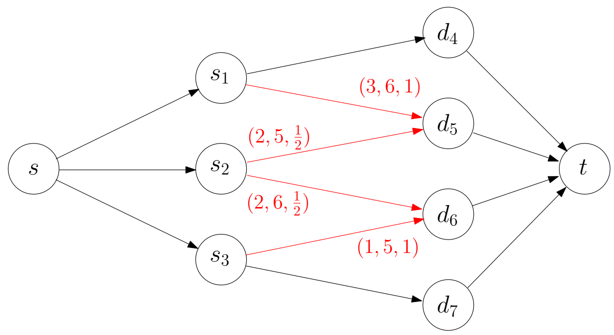

Figure 3: DP instance with half-integral dispatching.

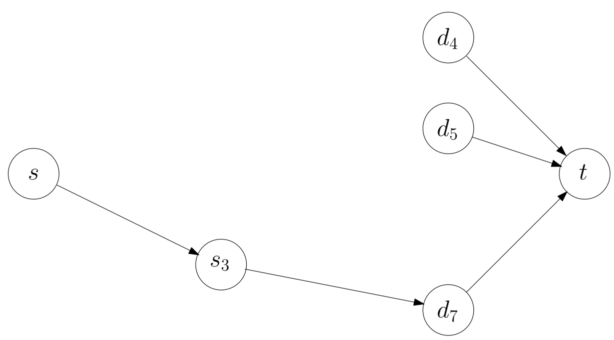

Figure 4: Reduced DP instance after rounding the half-integral dispatching: Solution $\delta_{37} = (3, 7, 1)$ with cost R is forced.

Table 1: Instances.

$\|S\|$	$\sum n_i$	(het.)	(%)	$\|D\|$	$\sum n_j$	(het.)	(%)	het.%
$2 \cdot 10^3$	17789	(327)	(1.84)	$2 \cdot 10^3$	19900	(916)	(4.6)	3.3
$3 \cdot 10^3$	26243	(670)	(2.55)	$3 \cdot 10^3$	30116	(1313)	(4.36)	3.52
$4 \cdot 10^3$	31439	(670)	(2.13)	$4 \cdot 10^3$	40095	(1815)	(4.53)	3.47
$5 \cdot 10^3$	40913	(935)	(2.29)	$5 \cdot 10^3$	49657	(2105)	(4.24)	3.36
$6 \cdot 10^3$	49427	(1285)	(2.6)	5715	62466	(2777)	(4.45)	3.63
$7 \cdot 10^3$	54926	(1285)	(2.34)	5715	62466	(2777)	(4.45)	3.46
$8 \cdot 10^3$	63655	(1476)	(2.32)	5715	62466	(2777)	(4.45)	3.37
$9 \cdot 10^3$	72130	(1835)	(2.54)	5715	62466	(2777)	(4.45)	3.43
10^4	78308	(1835)	(2.34)	5715	62466	(2777)	(4.45)	3.28

for which any dispatching ist forced to contain the single possible disposition $(3,7,1)$ at cost $r_{37} = R$. The cost ratio $\frac{c(\mathcal{D}_r)+c(\mathcal{D}')}{c(\mathcal{D}(I))} = \frac{r}{R}$ is then unbounded.

The following examples show the empirical approximation ratios achieved by the above described redispatching heuristic on instances generated from application data. DB Schenker Rail Deutschland AG provided supply and demand data of one calendar week comprising of more than 10^4 supplies and about 6000 demands, each for a single up to hundreds of cars. Usually, around $2000 - 3000$ supplies (the number of actual available cars scales roughly by a factor of 10) are available per day. About the same number of demands are considered, scattered over a time horizon of about two and a half days. The daily dispatching thus assigns about $3 \cdot 10^4$ cars. To keep typical supply-demand-structures in the considered instances, we did not draw supplies and demands from the data set randomly. Instead, from lists of supplies and demands sorted by availability and demand time respectively, we subsequently enlarged the supply and demand sets for our instances in steps of 10^3. This way we dispatched cars with a time horizon between one up to five dispatching days. Additionally, we incorprated operational storage as a kind of low priority demand. (Otherwise, cars may simply be undisposable in the redispatching process, but in practice each such car has to be stored somewhere nevertheless.)

Table 1 provides an overview over the structure of considered instances. Columns '$|S|$' and '$|D|$' contain the number of supplies and demands respectively, clustered by location, time and type (among more attributes, which are relevant to further side constraints in the application). Column 3 contains the total number of available cars, followed by the total number of cars involved in heterogeneous substitution and the percentage of such cars with respect to the total number of cars (in brackets). Column 5 contains analogous data for demands. The last column displays the percentage of cars involved in heterogeneous substitution with respect to both supply and demand. The latter shows that instances in the application are almost homogeneous. Yet, for the practical performance the integrated view is important, for example with respect to mixed type storage capacities.

Table 2: Running times in minutes for half-integral LP-optimal dispatching and heuristic dispatching.

$\|S\|$	$t(\mathcal{D})$	$t(\mathcal{D}_r + \mathcal{D}')$	t
$2 \cdot 10^3$	0.32	0.08	0.40
$3 \cdot 10^3$	1.02	0.12	1.14
$4 \cdot 10^3$	1.97	0.13	2.10
$5 \cdot 10^3$	3.76	0.17	3.93
$6 \cdot 10^3$	5.48	0.35	5.83
$7 \cdot 10^3$	6.90	0.72	7.62
$8 \cdot 10^3$	9.05	1.45	10.50
$9 \cdot 10^3$	11.92	3.12	15.04
10^4	13.51	6.87	20.38

Table 3: Costs for half-integral LP-optimal dispatching and heuristic dispatching.

$\|S\|$	$c(\mathcal{D})$	$c(\mathcal{D}_r + \mathcal{D}')$	$\frac{c(\mathcal{D}_r+\mathcal{D}')}{c(\mathcal{D})}$	$c(\mathcal{D}_r + \mathcal{D}') - c(\mathcal{D})$ (%)
$2 \cdot 10^3$	$1.09 \cdot 10^7$	$1.11 \cdot 10^7$	1.01	$1.16 \cdot 10^5$ (1.06)
$3 \cdot 10^3$	$1.32 \cdot 10^7$	$1.35 \cdot 10^7$	1.02	$2.34 \cdot 10^5$ (1.77)
$4 \cdot 10^3$	$1.43 \cdot 10^7$	$1.46 \cdot 10^7$	1.02	$2.93 \cdot 10^5$ (2.05)
$5 \cdot 10^3$	$1.84 \cdot 10^7$	$1.87 \cdot 10^7$	1.02	$3.34 \cdot 10^5$ (1.81)
$6 \cdot 10^3$	$2.05 \cdot 10^7$	$2.09 \cdot 10^7$	1.02	$3.46 \cdot 10^5$ (1.69)
$7 \cdot 10^3$	$2.40 \cdot 10^7$	$2.43 \cdot 10^7$	1.01	$3.49 \cdot 10^5$ (1.45)
$8 \cdot 10^3$	$3.04 \cdot 10^7$	$3.07 \cdot 10^7$	1.01	$3.54 \cdot 10^5$ (1.17)
$9 \cdot 10^3$	$3.47 \cdot 10^7$	$3.60 \cdot 10^7$	1.04	$13.24 \cdot 10^5$ (3.81)
10^4	$3.50 \cdot 10^7$	$4.01 \cdot 10^7$	1.14	$50.51 \cdot 10^5$ (14.42)

Table 2 shows running times for the computation of the half-integral dispatching in column '$t(\mathcal{D})$', the rounding and redisposition to obtain a feasible dispatching (column '$t(\mathcal{D}_r + \mathcal{D}')$') and their sum (column 't') in minutes with respect to the number of (clustered) supplies (column '$|S|$'). Running times are measured without input time (less than three seconds in each case) and network construction time (around two minutes maximum). All tests were carried out sequentially on one cluster node (Intel Xeon CPU E5410, 2.33 GHz, 6144 KB RAM).

The empirical approximation ratios drawn from the costs of the computed half-integral LP-optimal dispatching and the cost of the feasible dispatching found by the heuristic are presented in Table 3. Columns 2 and 3 contain the absolute cost $c(\mathcal{D})$ and $c(\mathcal{D}_r + \mathcal{D}')$ of the half-integral and the heuristical dispatching respectively. Column 4 displays the 'empirical approximation factor' $\frac{c(\mathcal{D}_r)+c(\mathcal{D}')}{c(\mathcal{D})}$. We also show the difference of both costs and its percentage with respect to $c(\mathcal{D})$ in columns 5 and 6. As remarked above, the latter cannot be seen as pure cost increase on $c(\mathcal{D})$, as the cost of an optimal solution can exceed the cost $c(\mathcal{D}(I))$ of a half-integral LP-optimal dispatching by far.

6. Acknowledgements

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