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Search for light tetraquark states in $\Upsilon(1S)$ and $\Upsilon(2S)$ decays

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We search for the $J^{PC}=0^{--}$ and 1^{+-} light tetraquark states with masses up to 2.46 GeV/c^2 in $\Upsilon(1S)$ and $\Upsilon(2S)$ decays with data samples of (102 ± 2) million and (158 ± 4) million events, respectively, collected with the Belle detector. No significant signals are observed in any of the studied production modes, and 90% credibility level (C.L.) upper limits on their branching fractions in $\Upsilon(1S)$ and $\Upsilon(2S)$ decays are obtained. The inclusive branching fractions of the $\Upsilon(1S)$ and $\Upsilon(2S)$ decays into final states with $f_1(1285)$ are measured to be $\mathcal{B}(\Upsilon(1S)\to f_1(1285)+\text{anything})=(46\pm 28(\text{stat})\pm 13(\text{syst}))\times 10^{-4}$ and $\mathcal{B}(\Upsilon(2S)\to f_1(1285)+\text{anything})=(22\pm 15(\text{stat})\pm 6.3(\text{syst}))\times 10^{-4}$. The measured $\chi_{b2}\to J/\psi+\text{anything}$ branching fraction is measured to be $(1.50\pm 0.34(\text{stat})\pm 0.22(\text{syst}))\times 10^{-3}$, and 90% C.L. upper limits for the $\chi_{b0,b1}\to J/\psi+\text{anything}$ branching fractions are found to be 2.3×10^{-3} and 1.1×10^{-3}, respectively. For $\mathcal{B}(\chi_{b1}\to\omega+\text{anything})$, the branching fraction is measured to be $(4.9\pm 1.3(\text{stat})\pm 0.6(\text{syst}))\times 10^{-2}$. All results reported here are the first measurements for these modes.

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I. INTRODUCTION

In the past decade, many experiments, both at lepton and hadron colliders, have reported evidence for a large number of particles having properties that cannot be readily explained within the framework of the expected heavy quarkonium states [1,2]. Among them, the $X(3872)$ [3], the $Z_c(3900)$ [4,5], the $X(3940)$ [6], the $Y(4260)$ [7,8], the $Z(4430)$ [9], the $Z_b(10610)$ and the $Z_b(10650)$ [10], are generally interpreted as possible tetraquark candidates with exotic properties.

In the low-mass region, the Dalitz analysis of the decay $D^0\to\pi^+\pi^-\pi^0$ [11] indicates the existence of a state decaying into a $\rho\pi$ final state with exotic quantum numbers $J^{PC}=0^{--}$ [12] at a mass of ≈ 1865 MeV/c^2, which cannot be composed of a quark-antiquark pair in the conventional quark model [13,14]. If such a resonance exists, it might be a hybrid or a tetraquark state [15].

The authors of Ref. [16] calculated the masses of such exotic four-quark states with $J^{PC}=0^{--}$ and 1^{+-} in Laplace sum rules (LSR) and finite-energy sum rules (FESR) using tetraquarklike currents. In the scalar channel, both LSR and FESR gave consistent mass predictions of a tetraquark state with a mass of (1.66 ± 0.14) GeV/c^2. This numerical result favors the tetraquark interpretation of the possible $\rho\pi$ dominance in the D^0 decays. In the vector channel, the authors also conservatively estimated the mass of a tetraquark state to be in the mass region $1.18-1.43$ GeV/c^2. Although the masses have been calculated, the width and couplings to any final states were not predicted.

Very recently, the Belle Collaboration reported the search for the $J^{PC}=0^{--}$ glueball ($G_{0^{--}}$) in the production modes $\Upsilon(1S,2S)\to\chi_{c1}+G_{0^{--}}$, $\Upsilon(1S,2S)\to f_1(1285)+G_{0^{--}}$, $\chi_{b1}\to J/\psi+G_{0^{--}}$, and $\chi_{b1}\to\omega+G_{0^{--}}$ with data samples of (102 ± 2) million $\Upsilon(1S)$ and (158 ± 4) million $\Upsilon(2S)$ events [17]. The masses of the putative glueballs were fixed at 2.800, 3.810, and 4.330 GeV/c^2, as predicted from quantum chromodynamics (QCD) sum rules [18] and distinct bottom-up holographic models of QCD [19]. Considering the kinematical constraints and the conservation of the quantum numbers J^{PC}, the production modes for glueball searches are also suitable for searches for the aforementioned light tetraquark states with $J^{PC}=0^{--}$ and 1^{+-}, denoted collectively as X_{tetra}.

In this paper, we utilize the low-mass recoil spectra of the χ_{c1}, $f_1(1285)$, J/ψ, and ω in bottomonium decays to search for X_{tetra} signals in the modes $\Upsilon(1S,2S)\to\chi_{c1}+X_{\text{tetra}}$, $\Upsilon(1S,2S)\to f_1(1285)+X_{\text{tetra}}$, $\chi_{b1}\to J/\psi+X_{\text{tetra}}$, and $\chi_{b1}\to\omega+X_{\text{tetra}}$ [17]. Since the X_{tetra} properties are unknown, we report our investigation for different assumed values for the X_{tetra} mass and width.

As byproducts of the X_{tetra} search, we measure the inclusive $f_1(1285)$ production in $\Upsilon(1S,2S)$, J/ψ production in $\chi_{bJ}(J=0,1,2)$, and ω production in χ_{b1} decays.

II. THE DATA SAMPLE AND BELLE DETECTOR

This analysis utilizes the Belle $\Upsilon(1S)$ and $\Upsilon(2S)$ data samples with a total luminosity of 5.74 and 24.91 fb^{-1}, respectively, corresponding to $(102\pm 2)\times 10^6$ $\Upsilon(1S)$ and $(158\pm 4)\times 10^6$ $\Upsilon(2S)$ events [20]. An 89.45 fb^{-1} data sample collected at $\sqrt{s}=10.52$ GeV is used to estimate the possible irreducible contributions from continuum ($e^+e^-\to q\bar{q}$, where $q\in\{u,d,s,c\}$). Here, $\sqrt{s}$ is the center-of-mass (C.M.) energy of the colliding e^+e^- system. The data were collected with the Belle detector [21,22] operated at the KEKB asymmetric-energy e^+e^- collider [23,24]. Large Monte Carlo (MC) samples of all of the investigated tetraquark modes are generated with EVTGEN [25] and simulated with a GEANT3-based [26] model for the detector response to determine the signal line shapes and efficiencies. The angular distribution for the decay $\Upsilon(2S)\to\gamma\chi_{bJ}$ is simulated assuming a pure E1 transition

($dN/d\cos\theta_\gamma \propto 1 + \alpha\cos^2\theta_\gamma$ with $\alpha = 1, -\frac{1}{3}, \frac{1}{13}$ for $J = 0$, 1, 2, respectively [27], where θ_γ is the polar angle of the $\Upsilon(2S)$ radiative photon in the e^+e^- C.M. frame); a phase space model in EVTGEN is used for the χ_{bJ} decays. We use the phase space model for other decays as well. Note that the X_{tetra} inclusive decays are modelled using PYTHIA [28]. Inclusive $\Upsilon(1S)$ and $\Upsilon(2S)$ MC samples, produced using PYTHIA with four times the total numbers of $\Upsilon(1S,2S)$ events of the data, are used to identify possible backgrounds showing peak distributions from $\Upsilon(1S)$ and $\Upsilon(2S)$ decays.

The Belle detector is a large solid-angle magnetic spectrometer that consists of a silicon vertex detector, a 50-layer central drift chamber, an array of aerogel threshold Cherenkov counters, a barrel-like arrangement of time-of-flight scintillation counters, and an electromagnetic calorimeter comprised of CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return yoke instrumented with resistive plate chambers located outside the coil is used to detect K_L^0 mesons and to identify muons. A detailed description of the Belle detector can be found in Refs. [21,22].

III. MEASUREMENTS OF $\Upsilon(1S,2S) \to f_1(1285) + \mathrm{anything}$

Candidate $f_1(1285)$ states are reconstructed via $\eta\pi^+\pi^-$, $\eta \to \gamma\gamma$. Considering the differences in the MC-determined reconstruction efficiencies for different $f_1(1285)$ momenta, we partition the data samples according to the scaled momentum $x = 2\sqrt{s} \times p^*_{f_1(1285)}/(s - m^2_{f_1(1285)})$, where $p^*_{f_1(1285)}$ is the momentum of the $f_1(1285)$ candidate in the C.M. system, and $m_{f_1(1285)}$ is the $f_1(1285)$ nominal mass [13]. The normalizing expression $(s - m^2_{f_1(1285)})/(2\sqrt{s})$ represents the maximum value of $p^*_{f_1(1285)}$ for the case where the $f_1(1285)$ candidate recoils against a massless particle. The use of x removes the beam-energy dependence in comparing the continuum data to those taken at the $\Upsilon(1S,2S)$ resonances. The event selections are identical to those used in Ref. [17]. Figure 1 shows the reconstruction efficiencies as a function of x for $f_1(1285)$ candidates from $\Upsilon(1S,2S)$ decays in each x interval. Here, the efficiencies are estimated using a MC signal sample generated on the basis of the relative weights of the differential branching fractions (discussed below) in the different x bins.

The invariant mass distributions for the $f_1(1285)$ candidates in $\Upsilon(1S,2S)$ data for the entire x region and for subranges in x are shown in Figs. 2 and 3. We observe clear $f_1(1285)$ signals in high-x bins and $\eta(1405)$ signals in the subregion $0.6 < x < 1.0$. In the figures, the cross-hatched histograms are from the normalized continuum contributions. See Ref. [17] for the definition of the normalization method of the continuum contribution. For $\Upsilon(2S) \to f_1(1285) + \mathrm{anything}$, a further background arises from the intermediate transition $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$ or $\pi^0\pi^0\Upsilon(1S)$ with $\Upsilon(1S)$ decaying to $f_1(1285)$. This contamination is removed by requiring the $\pi\pi$ recoil mass to be outside the [9.45, 9.47] GeV/c^2 range for all $\pi\pi$ combinations [17].

A binned extended simultaneous likelihood fit is applied to the x-dependent $\eta\pi^+\pi^-$ invariant mass spectra to extract the $f_1(1285)$ signal yields in the $\Upsilon(1S,2S)$ and continuum data samples. Due to the dependence on momentum, the $f_1(1285)$ and $\eta(1405)$ signal shapes in each x bin are described by Voigtian functions (a Breit-Wigner distribution convolved with a Gaussian function) that are obtained from the MC simulations directly; a third-order Chebyshev polynomial background shape is used for the $\Upsilon(1S,2S)$ decay backgrounds in addition to the normalized continuum contributions. The fit results are shown in Figs. 2 and 3 for the $\Upsilon(1S)$ and $\Upsilon(2S)$ decays, respectively. The fitted $f_1(1285)$ signal yields (N_{fit}) in each x bin from $\Upsilon(1S)$ and $\Upsilon(2S)$ decays are tabulated in Table I, together with the reconstruction efficiencies from MC signal simulations (ε), the total systematic uncertainties (σ_{syst}) discussed below (which are the sum of the common systematic errors, fit uncertainties and continuum-scale-factor uncertainties), and the corresponding branching fractions ($\mathcal{B}$). The total numbers of $f_1(1285)$ events, i.e., the sums of the signal yields in all of the x bins, the sums of

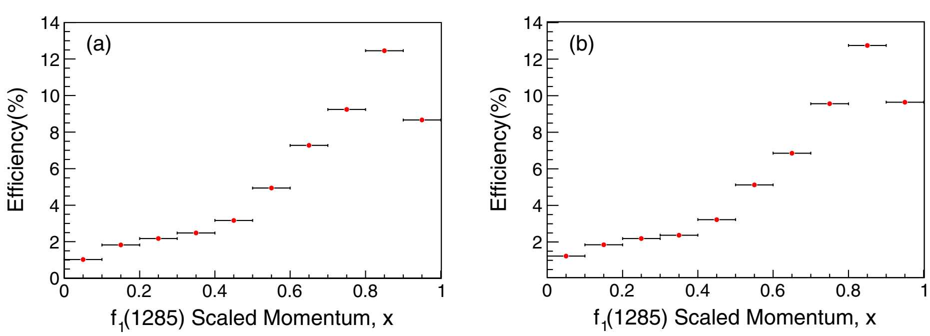

FIG. 1. MC efficiencies for reconstructed $f_1(1285)$ mesons in (a) $\Upsilon(1S)$ and (b) $\Upsilon(2S)$ decays as a function of the scaled momentum x.

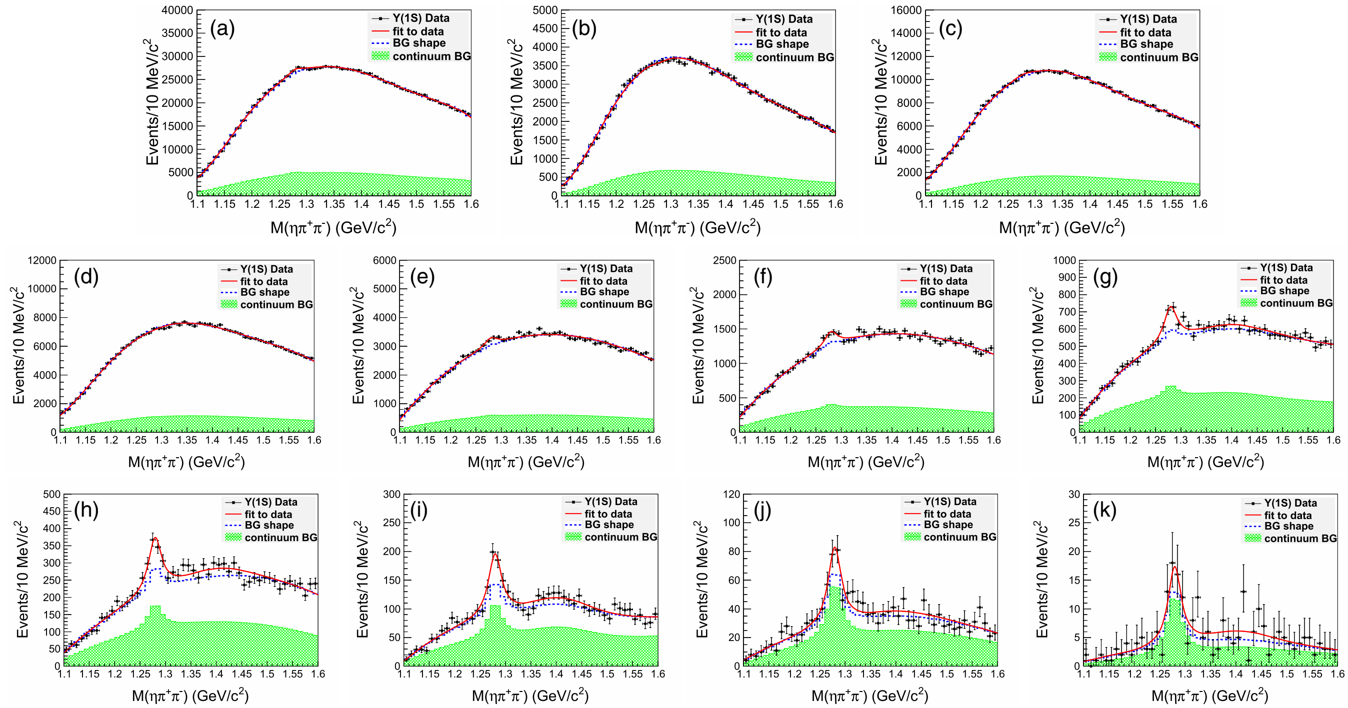

FIG. 2. Invariant mass distributions of the $f_1(1285)$ candidates in (a) the entire x region and (b–k) for x bins of size 0.1. The dots with error bars are the $\Upsilon(1S)$ data. The red solid lines are the best fits, and the blue dotted lines represent the total backgrounds. The cross-hatched green histograms are from the normalized continuum contributions.

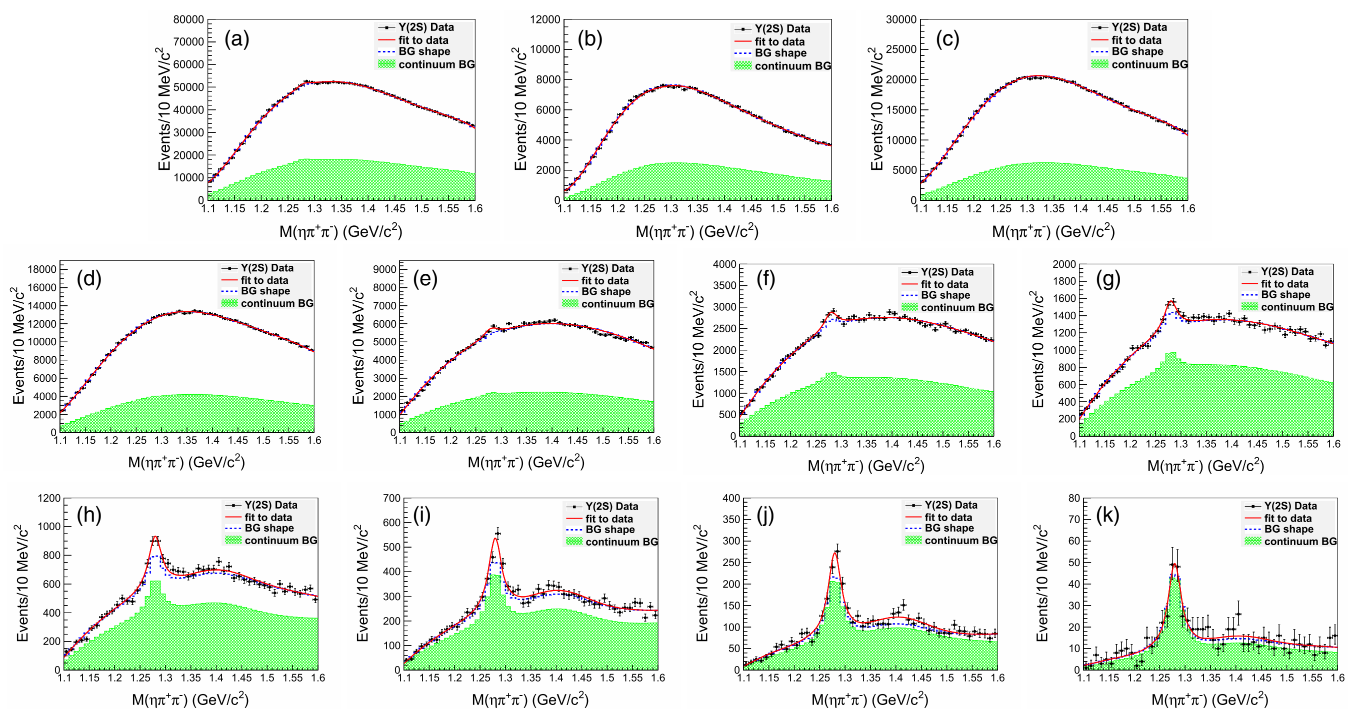

FIG. 3. Invariant mass distributions of the $f_1(1285)$ candidates in (a) the entire x region and (b–k) for x bins of size 0.1. The dots with error bars are the $\Upsilon(2S)$ data. The red solid lines are the best fits, and the blue dotted lines represent the total backgrounds. The green cross-hatched histograms are from the normalized continuum contributions.

TABLE I. Summary of the branching fraction measurements of $\Upsilon(1S, 2S)$ inclusive decays into $f_1(1285)$, where $N_{\rm fit}$ is the number of fitted signal events, ε is the reconstruction efficiency, $\sigma_{\rm syst}$ is the relative total systematic uncertainty, and $\mathcal{B}$ is the measured branching fraction.

	$\Upsilon(1S) \to f_1(1285)$ + anything				$\Upsilon(2S) \to f_1(1285)$ + anything			
x	$N_{\rm fit}$	$\varepsilon(\%)$	$\sigma_{\rm syst}(\%)$	$\mathcal{B}(10^{-4})$	$N_{\rm fit}$	$\varepsilon(\%)$	$\sigma_{\rm syst}(\%)$	$\mathcal{B}(10^{-4})$
(0.0, 0.1)	-480 ± 239	1.03	24.5	$-32 \pm 16 \pm 8.0$	-442 ± 253	1.23	29.8	$-16 \pm 9.2 \pm 4.8$
(0.1, 0.2)	727 ± 497	1.82	25.5	$28 \pm 19 \pm 7.1$	265 ± 192	1.85	26.9	$6.4 \pm 4.7 \pm 1.8$
(0.2, 0.3)	-432 ± 339	2.17	24.6	$-14 \pm 11 \pm 3.4$	-749 ± 333	2.19	26.0	$-15 \pm 6.8 \pm 4.0$
(0.3, 0.4)	1181 ± 240	2.48	28.9	$33 \pm 6.7 \pm 9.6$	1296 ± 348	2.37	25.3	$24 \pm 6.6 \pm 6.2$
(0.4, 0.5)	736 ± 165	3.16	24.2	$16 \pm 3.6 \pm 3.9$	801 ± 247	3.22	26.7	$11 \pm 3.5 \pm 3.0$
(0.5, 0.6)	645 ± 126	4.94	36.4	$9.0 \pm 1.8 \pm 3.3$	590 ± 189	5.12	34.9	$5.1 \pm 1.7 \pm 1.8$
(0.6, 0.7)	412 ± 88	7.27	31.3	$3.9 \pm 0.9 \pm 1.3$	563 ± 143	6.86	32.6	$3.7 \pm 1.0 \pm 1.2$
(0.7, 0.8)	229 ± 65	9.24	42.8	$1.7 \pm 0.5 \pm 0.8$	382 ± 70	9.56	35.6	$1.8 \pm 0.4 \pm 0.7$
(0.8, 0.9)	66 ± 38	12.46	48.0	$0.4 \pm 0.3 \pm 0.2$	205 ± 84	12.75	36.3	$0.7 \pm 0.3 \pm 0.3$
(0.9, 1.0)	16 ± 11	8.66	55.0	$0.1 \pm 0.1 \pm 0.1$	15 ± 11	9.65	48.9	$0.1 \pm 0.1 \pm 0.1$
All x	3100 ± 950	4.68	28.7	$46 \pm 28 \pm 13$	2926 ± 712	5.93	28.4	$22 \pm 15 \pm 6.3$

the x-dependent efficiencies weighted by the signal fraction in that x bin, and the measured branching fractions are listed in the bottom row of Table I. The branching fractions for $\Upsilon(1S, 2S) \to f_1(1285)$ + anything are measured to be

$$\mathcal{B}(\Upsilon(1S) \to f_1(1285) + \text{anything}) = (46 \pm 28(\text{stat}) \pm 13(\text{syst})) \times 10^{-4},$$

$$\mathcal{B}(\Upsilon(2S) \to f_1(1285) + \text{anything}) = (22 \pm 15(\text{stat}) \pm 6.3(\text{syst})) \times 10^{-4}.$$

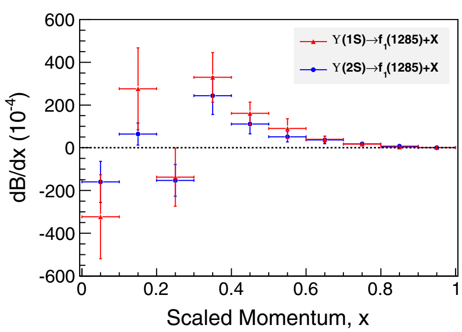

FIG. 4. Differential branching fractions for $\Upsilon(1S)$ and $\Upsilon(2S)$ inclusive decays into $f_1(1285)$ as a function of the scaled momentum x defined in the text. The error bar of each point is the sum of the statistical and systematic errors.

The differential branching fractions of $\Upsilon(1S, 2S)$ decays to $f_1(1285)$ are shown in Fig. 4.

IV. MEASUREMENTS OF $\chi_{bJ} \to J/\psi$ + anything

The χ_{bJ} is identified through the decay $\Upsilon(2S) \to \gamma\chi_{bJ}$. The same mass regions of the J/ψ signal and sidebands are used as in Ref. [17], i.e., we define the J/ψ signal region to be the window $|M_{\ell^+\ell^-} - m_{J/\psi}| < 0.03$ GeV/c^2 ($\sim 2.5\sigma$), where $m_{J/\psi}$ is the J/ψ nominal mass [13], while the J/ψ sideband is 2.97 GeV/$c^2 < M_{\ell^+\ell^-} < 3.03$ GeV/c^2 or 3.17 GeV/$c^2 < M_{\ell^+\ell^-} < 3.23$ GeV/c^2, which is twice as wide as the signal region. After requiring the lepton-pair mass to be within the J/ψ signal region, Figs. 5(a–c) show the distributions of the $\Upsilon(2S)$ radiative photon energy in the e^+e^- C.M. frame from MC simulated $\Upsilon(2S) \to \gamma\chi_{bJ}$, $\chi_{bJ} \to J/\psi$ + anything decays, where each χ_{bJ} signal shape is described by the convolution of a BW function with a Novosibirsk [29] function. Based on the fitted results, the efficiencies are $(23.87 \pm 0.42)\%$, $(32.21 \pm 0.53)\%$, and $(22.96 \pm 0.39)\%$ for χ_{b0}, χ_{b1} and χ_{b2}, respectively.

As shown in Fig. 6 of the spectrum of the $\Upsilon(2S)$ radiative photon energy in the C.M. frame, a clear χ_{b2} signal may be observed. After all selection requirements, no backgrounds showing peak distributions are found in the distribution estimated from J/ψ mass sideband data, nor in the

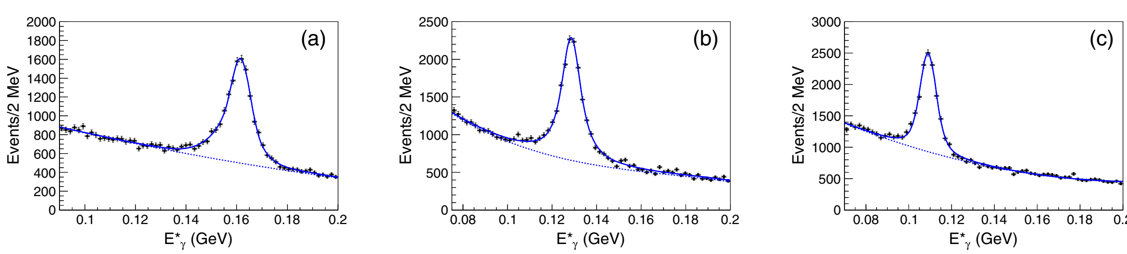

FIG. 5. The spectra of the $\Upsilon(2S)$ radiative photon energy in the e^+e^- C.M. frame from MC simulated $\Upsilon(2S) \to \gamma\chi_{bJ}$, $\chi_{bJ} \to J/\psi$ + anything signal samples for (a) χ_{b0}, (b) χ_{b1}, and (c) χ_{b2}, respectively.

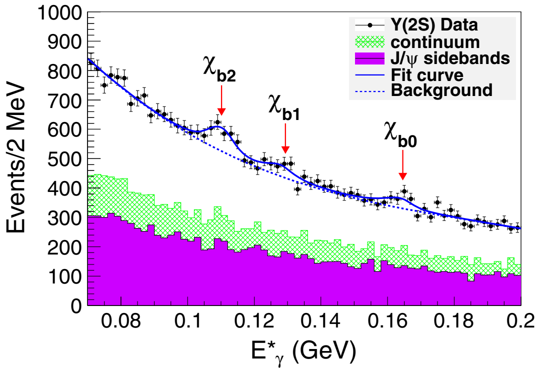

FIG. 6. The spectra of the $\Upsilon(2S)$ radiative photon energy in the e^+e^- C.M. frame in $\Upsilon(2S)$ data. The dots with error bars are the $\Upsilon(2S)$ data. The blue solid line is the best fit, and the blue dotted line represents the backgrounds. The magenta shaded histogram is from the normalized J/ψ sideband and the green cross-hatched histogram is from the normalized continuum contributions described in the text.

continuum production in the χ_{bJ} signal regions, in agreement with the expectation from the $\Upsilon(2S)$ generic MC samples. An unbinned extended maximum-likelihood fit to the spectrum is performed to extract the signal and background yields in the $\Upsilon(2S)$ data samples. In the fit, the probability density function (PDF) of each χ_{bJ} signal is a BW function convolved with a Novosibirsk function with all the parameters free; for the background PDF, a third-order Chebyshev polynomial function is adopted. The fit yields 243 ± 101, 269 ± 120, and 462 ± 105 events for the χ_{b0}, χ_{b1}, and χ_{b2} signals, respectively, in the $\Upsilon(2S)$ data sample. The statistical significances of the χ_{b0}, χ_{b1} and χ_{b2} signals are estimated to be 1.5σ, 1.1σ and 3.5σ, from the differences of the logarithmic likelihoods, $-2\ln(\mathcal{L}_0/\mathcal{L}_{\rm max})$, where $\mathcal{L}_0$ and $\mathcal{L}_{\rm max}$ are the likelihoods of the fits without and with a signal component, respectively (taking the number of degrees of freedom in each fit into account). For $\chi_{b2} \to J/\psi + \text{anything}$, the branching fraction is measured for the first time using

$$\mathcal{B}(\chi_{b2} \to J/\psi + \text{anything}) = \frac{N_{\chi_{b2}}}{N_{\Upsilon(2S)} \times \varepsilon_{\chi_{b2}} \times \mathcal{B}(\Upsilon(2S) \to \gamma\chi_{b2}) \times \mathcal{B}(J/\psi \to \ell^+\ell^-)},$$

where $N_{\chi_{b2}}$ is the number of fitted χ_{b2} signal events and $\varepsilon_{\chi_{b2}}$ is the signal detection efficiency given above. We measure a value of $(1.50 \pm 0.34(\text{stat}) \pm 0.22(\text{syst})) \times 10^{-3}$. The systematic uncertainties are discussed below. The $\chi_{b0,b1}$ branching fractions are computed in a similar way. Since the $\chi_{b0,b1}$ signal significances are less than 3σ, we compute 90% credibility level (C.L.) upper limits $x^{\rm UL}$ on the $\chi_{b0,b1}$ signal yields and the branching fractions. For this purpose, we solve the equation $\int_0^{x^{\rm UL}} \mathcal{L}(x)dx / \int_0^{+\infty} \mathcal{L}(x)dx = 0.9$, where x is the assumed signal yield or branching fraction, and $\mathcal{L}(x)$ is the corresponding likelihood of the data. To take into account the systematic uncertainties discussed below, the likelihood is convolved with a Gaussian function whose width equals the total systematic uncertainty. The upper limits for the yields of χ_{b0} and χ_{b1} are 380 and 432 respectively, and the corresponding upper limits on the branching fractions are $\mathcal{B}^{\rm UL}(\chi_{b0} \to J/\psi + \text{anything}) = 2.3 \times 10^{-3}$ and $\mathcal{B}^{\rm UL}(\chi_{b1} \to J/\psi + \text{anything}) = 1.1 \times 10^{-3}$ at 90% C.L.

V. MEASUREMENTS OF $\chi_{b1} \to \omega + \text{anything}$

Candidate ω mesons are reconstructed via $\pi^+\pi^-\pi^0$. We perform a mass-constrained kinematic fit to the selected π^0 candidate and require $\chi^2 < 10$. To remove the backgrounds with K_S^0, the $\pi^+\pi^-$ invariant mass is required to be outside the $[0.475, 0.515]$ GeV$/c^2$ range. After requiring the $\pi^+\pi^-\pi^0$ invariant mass to be within the ω signal region of $0.755\ \text{GeV}/c^2 < M(\pi^+\pi^-\pi^0) < 0.805\ \text{GeV}/c^2$, Fig. 7 shows the distributions of the energy of the $\Upsilon(2S)$ radiative photon in the C.M. frame, where the dots represent the $\Upsilon(2S)$ data and the cross-hatched histogram is from the normalized continuum contributions. We define the χ_{b1} signal region as $0.12\ \text{GeV} < E_\gamma^* < 0.14$ GeV and its sideband as $0.075\ \text{GeV} < E_\gamma^* < 0.095$ GeV or $0.18\ \text{GeV} < E_\gamma^* < 0.20$ GeV, which is twice as wide as the signal region. From the histogram, no χ_{b1} signal is present in the continuum contributions.

After the application of the above requirements, the $\pi^+\pi^-\pi^0$ invariant mass distribution from MC simulated $\chi_{b1} \to \omega + \text{anything}$ signal sample is shown in Fig. 8(a). In the fit to this distribution, a Voigtian function is used for the ω signal shape and a second-order Chebyshev polynomial function is used for the background shape. Based on the fitted result, the efficiency is $(10.9 \pm 0.1)\%$. Figure 8(b)

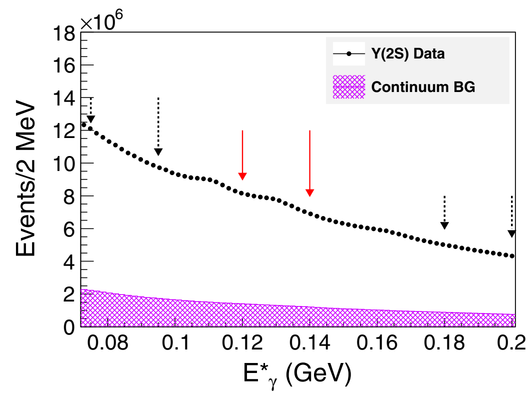

FIG. 7. The spectra of the $\Upsilon(2S)$ radiative photon energy in the e^+e^- C.M. frame, where the dots with imperceptible error bars are the $\Upsilon(2S)$ data and the magenta cross-hatched histogram is from the normalized continuum contributions. The red solid arrows indicate the selected χ_{b1} signal region, and the black dashed arrows show the two ranges of the χ_{b1} sideband.

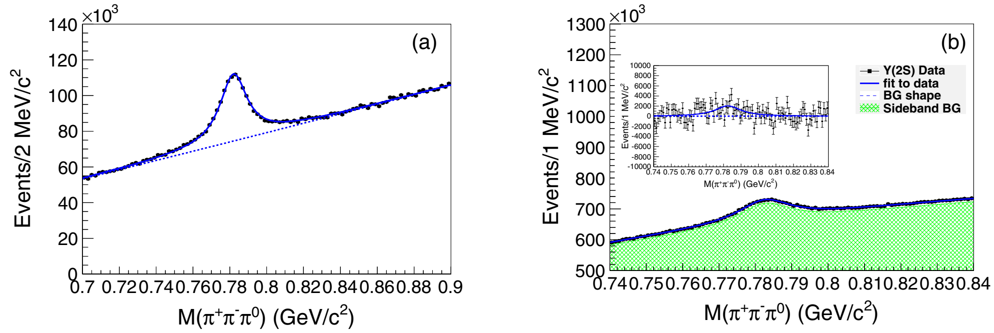

FIG. 8. The $\pi^+\pi^-\pi^0$ invariant mass spectra from (a) MC simulated $\chi_{b1} \to \omega +$ anything signal sample and (b) $\Upsilon(2S)$ data. The dots represent the data. The cross-hatched histogram in (b) represents the normalized χ_{b1} sideband; the inset shows the fitted background-subtracted distribution. The blue solid lines are the best fits, and the blue dotted lines represent the backgrounds.

shows the distributions of the $\pi^+\pi^-\pi^0$ invariant mass from the $\Upsilon(2S)$ data (the dots with error bars) and the normalized χ_{b1} sideband events (the cross-hatched histogram). From the plot, the observed ω signals in the normalized χ_{b1} sideband account for most of the events in the χ_{b1} signal region.

A simultaneous binned extended maximum likelihood fit is applied to the $\pi^+\pi^-\pi^0$ invariant mass spectra to extract the ω signal yields in the χ_{b1} signal region and its sideband. The ω signal shape is described by a Voigtian function with the values of the parameters fixed to those from the fit to MC-simulated signals; a second-order Chebyshev polynomial background shape is used for the χ_{b1} decay backgrounds in addition to the normalized χ_{b1} sideband. The fitted ω signal yield is 51054 ± 12943 and the estimated statistical significance is 4.1σ. Hence, the branching fraction for $\chi_{b1} \to \omega +$ anything is measured for the first time to be

$$\mathcal{B}(\chi_{b1} \to \omega + \text{anything}) = (4.9 \pm 1.3(\text{stat}) \pm 0.6(\text{syst})) \times 10^{-2}.$$

VI. SEARCH FOR X_{tetra} IN $\Upsilon(1S)$, $\Upsilon(2S)$, AND χ_{b1} DECAYS

We generate a large number of MC samples for $\Upsilon(1S, 2S) \to \chi_{c1} + X_{\text{tetra}}$, $\Upsilon(1S, 2S) \to f_1(1285) + X_{\text{tetra}}$, $\chi_{b1} \to J/\psi + X_{\text{tetra}}$, and $\chi_{b1} \to \omega + X_{\text{tetra}}$ with X_{tetra} masses varying from 1.16 to 2.46 GeV/c^2 in steps of 0.10 GeV/c^2 and widths varying from 0.0 to 0.3 GeV in steps of 0.1 GeV, using the same decay modes as in Ref. [17]. After applying all the event selections in Ref. [17], all relevant efficiencies are obtained; they are displayed graphically in Fig. 9. Since the event selection requirements are independent of the recoil part of the χ_{c1},

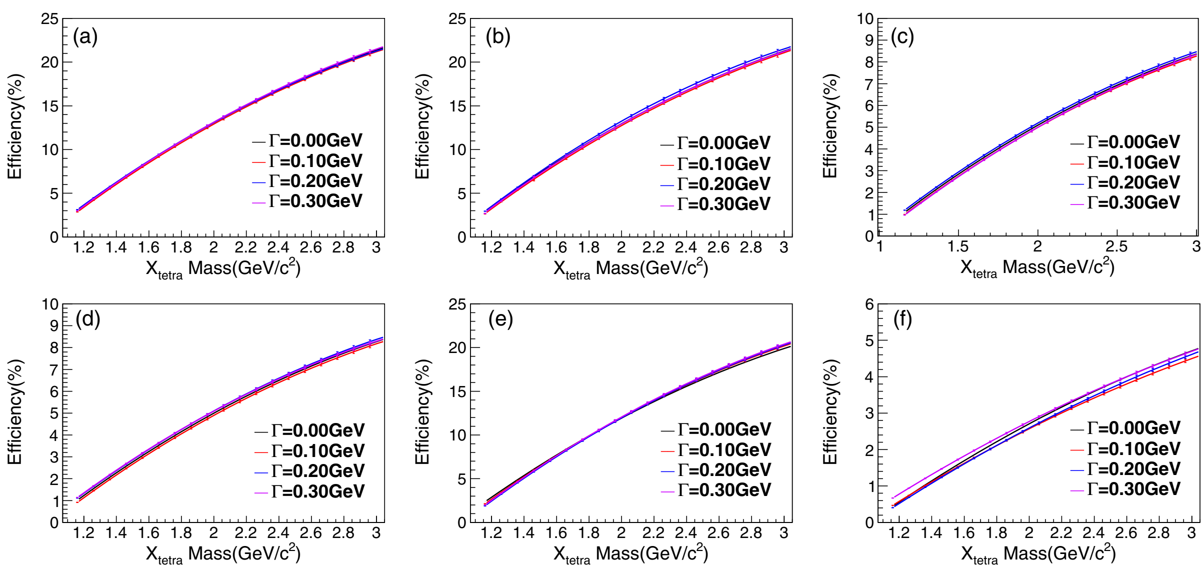

FIG. 9. Reconstruction efficiencies for (a) $\Upsilon(1S) \to \chi_{c1} + X_{\text{tetra}}$, (b) $\Upsilon(2S) \to \chi_{c1} + X_{\text{tetra}}$, (c) $\Upsilon(1S) \to f_1(1285) + X_{\text{tetra}}$, (d) $\Upsilon(2S) \to f_1(1285) + X_{\text{tetra}}$, (e) $\chi_{b1} \to J/\psi + X_{\text{tetra}}$ and (f) $\chi_{b1} \to \omega + X_{\text{tetra}}$ as a function of the assumed X_{tetra} masses, with X_{tetra} widths varying from 0.0 to 0.3 GeV in steps of 0.1 GeV. The four solid lines in each panel, one for each X_{tetra} width, are the fits of a second-order Chebyshev polynomial to these data.

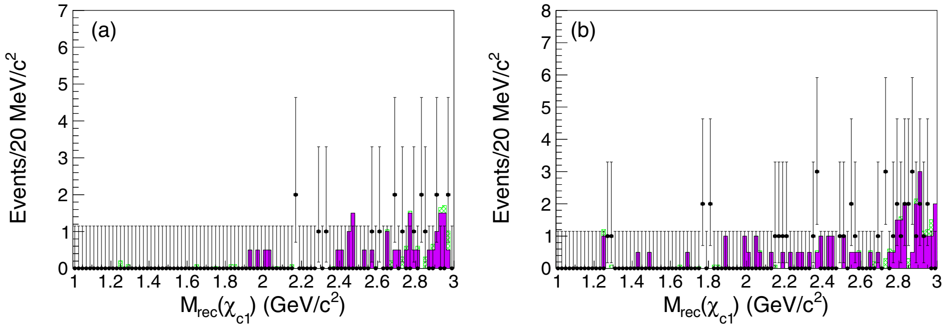

FIG. 10. The χ_{c1} recoil mass spectra in the (a) $\Upsilon(1S)$ and (b) $\Upsilon(2S)$ data samples. The shaded histograms are from the normalized χ_{c1} sideband and the cross-hatched histograms show the normalized continuum contributions [17].

$f_1(1285)$, J/ψ, and ω in the studied channels, the detection efficiencies are only related to the recoil masses. The efficiencies versus $X_{\rm tetra}$ mass in the entire region from 1.16 to 3.0 GeV/c^2 are displayed graphically in Fig. 9 for the studied production modes. The fitted curves show the second-order Chebyshev polynomials used to model these efficiencies.

In the channels analyzed below, $\Upsilon(1S,2S) \to \chi_{c1} + X_{\rm tetra}$, $\Upsilon(1S,2S) \to f_1(1285) + X_{\rm tetra}$, $\chi_{b1} \to J/\psi + X_{\rm tetra}$, and $\chi_{b1} \to \omega + X_{\rm tetra}$, we search for the $X_{\rm tetra}$ signals in the recoil mass spectra of the χ_{c1}, $f_1(1285)$, J/ψ, and ω, respectively, with $X_{\rm tetra}$ widths between 0.0 and 0.3 GeV in steps of 0.1 GeV. All recoil mass spectra are taken from Ref. [17] with a focused view of the low-mass region.

For $\Upsilon(1S,2S) \to \chi_{c1} + X_{\rm tetra}$, the χ_{c1} is reconstructed via its decay into $\gamma J/\psi$, $J/\psi \to \ell^+\ell^-$ ($\ell = e$ or μ). Figure 10 shows the recoil mass spectra of χ_{c1} candidates in the $\Upsilon(1S,2S)$ data, where the shaded histograms are from the normalized χ_{c1} sideband and the cross-hatched histograms show the normalized continuum contributions. See Ref. [17] for the definition of the χ_{c1} sideband and the normalization method of the continuum contribution. There are no evident signals for any of the $X_{\rm tetra}$ states at any of the masses. In the entire region of study, the most significant signal is observed at an $X_{\rm tetra}$ mass of 2.46 (2.26) GeV/c^2 and width of 0.3 (0.0) GeV with a statistical significance of 1.4σ (0.6σ) in $\Upsilon(1S)$ ($\Upsilon(2S)$) data. Since the number of selected signal candidate events is small, we obtain the 90% C.L. upper limit of the signal yield ($N^{\rm UL}$) at each $X_{\rm tetra}$ mass point by using the frequentist approach [30] implemented in the POLE (Poissonian limit estimator) program [31], where each mass region is selected to contain 95% of the signal according to MC simulations, the number of observed signal events is counted directly, and the number of expected background events is estimated from the sum of the normalized χ_{c1} sideband and continuum contributions. The systematic uncertainties discussed below are taken into account.

The calculated upper limits on the numbers of signal events ($N^{\rm UL}$) and branching fraction ($\mathcal{B}^{\rm UL}$) for each $X_{\rm tetra}$ state with $X_{\rm tetra}$ masses from 1.16 to 2.46 GeV/c^2 and widths from 0.0 to 0.3 GeV in $\Upsilon(1S,2S)$ data are listed in Table II, together with the reconstruction efficiencies (ε) and the systematic uncertainties ($\sigma_{\rm syst}$). The results are displayed graphically in Fig. 11.

For $\Upsilon(1S,2S) \to f_1(1285) + X_{\rm tetra}$, $f_1(1285)$ candidates are reconstructed via $\eta\pi^+\pi^-$, $\eta \to \gamma\gamma$. Figure 12 shows the recoil mass spectra of the $f_1(1285)$ in $\Upsilon(1S,2S)$ data, together with the backgrounds from the normalized $f_1(1285)$ sideband and the normalized continuum contributions. No evident $X_{\rm tetra}$ signals are seen. An unbinned extended maximum-likelihood fit repeated with $X_{\rm tetra}$ masses from 1.46 to 2.46 GeV/c^2 in steps of 0.10 GeV/c^2, and with $X_{\rm tetra}$ widths from 0.0 to 0.3 GeV in steps of 0.1 GeV, is applied to the recoil mass spectra. The signal shape of each $X_{\rm tetra}$ signal is described with a BW function convolved with a Novosibirsk function, where all parameter values are fixed to those from the fit to the MC-simulated signals. Since no backgrounds showing peak distributions are found, a second-order Chebyshev polynomial shape is used for the backgrounds. The fit result for the $X_{\rm tetra}$ signal with its mass fixed at 1.66 GeV/c^2 (a theoretically predicted mass for a scalar tetraquark state [16]) and width fixed at 0.10 GeV is shown in Fig. 12. The fit yields 1.7 ± 4.7 (-0.3 ± 9.8) events for the $X_{\rm tetra}$ signals in the $\Upsilon(1S)$ ($\Upsilon(2S)$) data sample. In the whole mass region of interest, the most significant signal is observed at an $X_{\rm tetra}$ mass of 2.26 (2.16) GeV/c^2 and width of 0.0 (0.3) GeV with a statistical significance of 1.1σ (1.8σ) in $\Upsilon(1S)$ ($\Upsilon(2S)$) data.

For $\chi_{b1} \to J/\psi + X_{\rm tetra}$, the χ_{b1} is identified through the decay $\Upsilon(2S) \to \gamma\chi_{b1}$. Figure 13 shows the recoil mass spectrum of $\gamma J/\psi$ in $\Upsilon(2S)$ data, together with the background estimated from the normalized J/ψ sideband and the normalized continuum contributions. No evident $X_{\rm tetra}$ signal is observed. An unbinned extended maximum-likelihood fit is applied to the $\gamma J/\psi$ recoil mass spectrum. The result of the fit with the $X_{\rm tetra}$ mass fixed at 1.66 GeV/c^2 and width fixed at 0.10 GeV is shown in Fig. 13. This fit yields 8.9 ± 5.8 $X_{\rm tetra}$ signal events. In the entire region of study, the most significant signal is observed at an $X_{\rm tetra}$

TABLE II. Summary of the upper limits for $\Upsilon(1S,2S) \to \chi_{c1} + X_{\rm tetra}$, $f_1(1285) + X_{\rm tetra}$, and $\chi_{b1} \to J/\psi + X_{\rm tetra}$, $\omega + X_{\rm tetra}$ under different assumptions of $X_{\rm tetra}$ mass (m in GeV/c^2) and width (Γ in GeV), where $N^{\rm UL}$ is the upper limit on the number of signal events taking into account systematic errors, ε is the reconstruction efficiency, $\sigma_{\rm syst}$ is the total relative systematic uncertainty on the branching fraction and $\mathcal{B}^{\rm UL}$ is the 90% C.L. upper limit on the branching fraction.

	$\Upsilon(1S) \to \chi_{c1} + X_{\rm tetra}$ (for $\Gamma = 0.0/0.1/0.2/0.3$ GeV)				$\Upsilon(2S) \to \chi_{c1} + X_{\rm tetra}$ (for $\Gamma = 0.0/0.1/0.2/0.3$ GeV)			
m	ε(%)	$N^{\rm UL}$	$\sigma_{\rm syst}$(%)	$\mathcal{B}^{\rm UL}(\times 10^{-6})$	ε(%)	$N^{\rm UL}$	$\sigma_{\rm syst}$(%)	$\mathcal{B}^{\rm UL}(\times 10^{-6})$
1.16	3.1/2.9/3.1/3.0	2.3	6.2	18.3/19.6/17.9/18.7	3.0/2.6/3.0/2.7	4.7/4.7/4.7/6.0	6.3	26.2/29.1/25.7/36.7
1.26	4.4/4.2/4.5/4.3	2.3	6.2	12.7/13.3/12.5/12.9	4.3/4.0/4.2/4.1	4.7/4.7/6.0/7.6	6.3	17.8/19.1/23.6/29.5
1.36	5.7/5.5/5.8/5.7	2.3	6.2	9.8/10.1/9.7/9.9	5.7/5.4/5.6/5.5	4.7/4.7/7.9/7.9	6.3	13.6/14.3/23.3/23.7
1.46	7.0/6.8/7.0/7.0	2.3/2.3/2.3/4.2	6.2	8.0/8.3/8.0/15.1	6.7/6.5/7.0/6.9	2.3/5.9/7.6/10.0	6.3	5.4/15.3/17.4/23.0
1.56	8.2/8.0/8.2/8.2	2.3/2.3/2.3/5.5	6.2	6.8/7.0/6.8/16.4	8.2/8.0/8.1/8.1	2.3/5.9/7.0/10.5	6.3	4.4/12.5/14.2/21.2
1.66	9.4/9.2/9.4/9.4	2.3/2.3/4.2/6.1	6.2	6.0/6.1/11.1/16.4	9.2/9.0/9.3/9.3	4.7/5.8/10.0/14.4	6.3	8.4/10.4/17.1/24.9
1.76	10.5/10.3/10.5/10.5	2.3/2.3/5.5/6.5	6.2	5.3/5.4/12.7/15.5	10.3/10.1/10.4/10.3	5.8/6.7/8.8/14.9	6.3	9.0/10.7/13.5/23.0
1.86	11.6/11.4/11.6/11.6	2.3/4.2/6.1/7.1	6.2	4.8/9.2/13.2/15.7	11.4/11.2/11.3/11.4	6.7/8.7/12.1/17.8	6.3	9.5/12.2/18.0/25.3
1.96	12.7/12.7/12.5/12.7	2.3/5.5/6.5/9.2	6.2	4.4/10.6/13.0/17.8	12.5/12.5/12.4/12.5	4.2/9.3/13.5/17.3	6.3	5.4/11.6/17.4/22.8
2.06	13.7/13.5/13.7/13.7	4.1/6.1/7.1/10.2	6.2	8.7/11.3/13.3/19.3	13.6/13.4/13.5/13.5	6.7/11.2/16.7/21.1	6.3	8.0/13.3/19.9/26.8
2.16	14.7/14.5/14.6/14.7	5.5/7.2/9.2/12.9	6.2	9.1/12.9/15.4/22.3	14.6/14.4/14.5/14.5	4.7/10.5/17.3/24.6	6.3	5.3/11.9/19.8/28.8
2.26	15.6/15.4/15.6/15.7	6.5/7.3/10.2/16.4	6.2	10.7/12.4/17.0/25.9	15.5/15.3/15.5/15.4	10.1/13.5/20.7/30.4	6.3	10.4/14.1/22.0/33.1
2.36	16.6/16.4/16.5/16.6	4.1/9.3/13.8/20.7	6.2	7.2/13.8/20.8/32.4	16.4/16.2/16.4/16.4	10.1/14.3/29.3/30.0	6.3	9.9/14.8/28.3/30.6
2.46	17.4/17.2/17.4/17.5	4.1/9.3/16.9/24.3	6.2	6.8/13.2/24.0/37.8	17.3/17.1/17.3/17.3	10.4/18.7/27.7/32.3	6.3	10.1/18.1/26.3/31.2
	$\Upsilon(1S) \to f_1(1285) + X_{\rm tetra}$ (for Γ =0.0/0.1/0.2/0.3 GeV)				$\Upsilon(2S) \to f_1(1285) + X_{\rm tetra}$ (for $\Gamma = 0.0/0.1/0.2/0.3$ GeV)			
m	ε(%)	$N^{\rm UL}$	$\sigma_{\rm syst}$(%)	$\mathcal{B}^{\rm UL}(\times 10^{-6})$	ε(%)	$N^{\rm UL}$	$\sigma_{\rm syst}$(%)	$\mathcal{B}^{\rm UL}(\times 10^{-6})$
1.16	1.2/1.1/1.0/1.0	4.4/4.4/7.8/9.1	20.2	24.5/26.8/51.7/60.9	1.0/0.9/1.0/1.0	5.1/6.0/10.8/15.1	20.2	21.6/28.6/46.1/64.7
1.26	1.6/1.5/1.7/1.5	2.3/4.4/12.7/14.1	20.2	9.4/19.6/49.7/62.4	1.5/1.4/1.6/1.5	6.4/8.4/13.5/16.6	20.2	18.3/25.7/36.2/47.6
1.36	2.1/2.0/2.2/2.0	4.4/7.8/12.7/14.8	20.2	14.0/25.9/38.4/48.8	2.1/2.0/2.2/2.0	7.2/9.8/15.3/22.7	20.2	14.7/20.9/29.8/48.4
1.46	2.6/2.7/2.5/2.5	7.6/10.7/11.0/11.6	20.9/21.6/22.2/24.4	19.1/26.2/29.1/31.2	2.6/2.5/2.5/2.4	15.0/19.7/23.3/27.3	20.4/24.9/27.8/28.6	24.5/33.6/39.9/48.6
1.56	3.2/3.2/3.1/3.0	9.7/12.1/12.9/13.4	20.2/20.2/21.5/22.2	20.0/25.2/27.7/29.9	3.0/3.0/3.1/3.0	12.1/17.9/24.2/28.9	20.1/20.4/22.3/24.4	16.6/25.4/33.4/41.4
1.66	3.6/3.5/3.7/3.5	5.2/10.3/13.4/13.9	20.5/21.7/22.0/24.5	9.4/19.4/24.1/26.3	3.5/3.4/3.6/3.5	9.3/14.3/18.1/22.5	22.0/23.2/23.5/28.3	11.4/17.9/21.4/27.7
1.76	4.1/4.0/4.2/4.0	4.5/6.8/9.8/13.3	20.5/21.6/24.2/24.3	7.4/11.3/15.2/21.9	4.0/3.9/4.1/4.0	12.5/14.9/18.2/21.4	20.8/23.6/24.5/29.1	13.4/16.3/18.8/23.1
1.86	4.5/4.4/4.6/4.4	5.5/6.6/7.1/9.9	20.3/20.5/21.0/22.3	8.2/9.2/10.2/14.9	4.4/4.3/4.5/4.4	12.7/15.4/17.4/21.7	20.5/21.4/21.5/23.0	12.8/15.3/16.7/21.2
1.96	4.9/4.8/5.0/4.8	5.1/5.8/6.2/8.3	20.3/20.3/20.6/21.0	6.8/7.7/8.2/11.4	4.8/4.7/4.9/4.7	10.6/12.9/15.9/20.8	20.2/20.6/27.0/23.7	9.4/11.8/13.8/18.9
2.06	5.3/5.2/5.4/5.2	3.7/5.0/6.2/8.1	20.0/20.2/20.2/20.3	4.6/6.3/7.6/10.4	5.2/5.1/5.3/5.1	9.9/12.0/14.3/18.8	24.8/26.0/27.0/27.5	8.1/10.1/11.6/15.7
2.16	5.7/5.6/5.8/5.6	5.3/6.9/8.2/10.8	20.4/21.7/23.7/24.2	6.1/8.2/9.4/12.7	5.6/5.5/5.7/5.5	10.2/12.6/14.8/19.0	26.7/27.4/30.8/33.6	7.8/9.7/11.1/14.7
2.26	6.1/6.0/6.2/6.0	12.6/14.7/17.0/19.0	21.4/24.0/24.2/30.9	13.6/16.2/18.2/21.0	6.0/5.9/6.1/5.9	12.1/15.3/17.8/22.0	21.5/24.8/28.8/35.2	8.6/11.1/12.4/15.9
2.36	6.5/6.4/6.6/6.4	14.2/22.8/25.7/30.7	24.5/27.7/28.8/32.9	14.6/23.9/25.8/32.0	6.4/6.3/6.5/6.4	19.4/23.0/25.3/28.6	20.6/25.6/26.5/28.0	12.9/15.7/16.6/19.4
2.46	6.8/6.7/6.9/6.7	15.9/24.8/32.4/40.1	20.6/21.1/21.2/22.3	15.4/24.6/31.7/40.8	6.7/6.6/6.8/6.6	29.7/36.8/40.1/42.7	20.9/22.2/23.5/29.9	19.1/23.9/25.4/28.7
	$\chi_{b1} \to J/\psi + X_{\rm tetra}$ (for $\Gamma = 0.0/0.1/0.2/0.3$ GeV)				$\chi_{b1} \to \omega + X_{\rm tetra}$ (for $\Gamma = 0.0/0.1/0.2/0.3$ GeV)			
m	ε(%)	$N^{\rm UL}$	$\sigma_{\rm syst}$(%)	$\mathcal{B}^{\rm UL}(\times 10^{-5})$	ε(%)	$N^{\rm UL}$	$\sigma_{\rm syst}$(%)	$\mathcal{B}^{\rm UL}(\times 10^{-5})$
1.16	2.4/2.5/2.3/2.3	1.9/2.3/3.3/5.6	7.8	6.2/7.2/11.8/19.1	0.4/0.5/0.4/0.6	5.7/7.2/15.3/24.0	9.3	13.5/16.2/40.0/44.4
1.26	3.7/3.8/3.6/3.6	2.1/3.1/4.6/8.0	7.8	4.4/6.4/10.2/17.4	0.7/0.7/0.7/0.7	8.6/11.6/21.1/25.6	9.3	12.2/16.4/32.2/26.3
1.36	4.9/5.0/4.8/4.9	3.1/3.7/5.6/9.0	7.8	4.9/5.7/9.1/14.5	1.0/1.0/1.0/1.1	12.6/16.5/24.6/34.4	9.3	12.8/17.2/26.8/33.9
1.46	6.1/6.2/6.0/6.1	3.3/6.0/8.4/12.5	7.9/8.7/10.1/13.9	4.2/7.6/10.9/16.2	1.3/1.3/1.2/1.4	9.4/14.5/22.3/27.1	19.0/20.3/21.4/23.5	7.5/12.0/18.9/20.8
1.56	7.3/7.4/7.2/7.3	5.2/9.4/15.3/20.2	7.9/8.1/8.3/9.1	5.5/9.9/16.7/21.7	1.6/1.5/1.5/1.6	6.6/10.1/16.4/19.3	11.4/14.0/18.6/19.8	4.3/7.0/11.4/12.6
1.66	8.4/8.5/8.3/8.4	9.4/14.6/19.8/24.4	7.9/8.0/11.4/12.4	8.7/13.4/18.6/22.7	1.9/1.8/1.8/1.9	8.8/13.2/18.0/21.2	13.1/13.9/16.4/16.9	5.0/7.9/10.8/12.0
1.76	9.5/9.6/9.4/9.5	18.6/21.3/25.4/27.1	8.1/9.0/9.9/12.0	15.2/17.2/21.0/22.3	2.1/2.0/2.0/2.1	13.9/19.0/23.8/27.1	9.5/10.9/13.4/13.7	6.9/9.9/12.4/13.5
1.86	10.6/10.7/10.5/10.6	10.2/18.2/24.8/28.7	7.9/8.6/8.9/10.4	7.5/13.2/18.4/21.4	2.4/2.2/2.2/2.4	15.0/21.3/27.3/30.2	10.0/11.0/11.1/11.3	6.6/9.9/12.7/13.5
1.96	11.6/11.7/11.5/11.6	3.4/6.8/11.2/19.5	8.0/8.1/8.9/11.1	2.3/4.6/7.7/13.1	2.6/2.5/2.5/2.6	13.2/17.5/24.4/27.5	9.3/9.4/9.5/10.7	5.3/7.4/10.3/11.2
2.06	12.6/12.7/12.5/12.6	3.8/5.2/6.8/10.4	7.9/8.0/8.1/8.3	2.3/3.2/4.3/6.4	2.8/2.7/2.7/2.8	9.0/13.7/21.2/25.7	9.7/9.9/10.1/10.4	3.3/5.3/8.1/9.6
2.16	13.6/13.7/13.5/13.6	3.8/5.0/6.2/7.8	7.8/8.1/8.2/8.4	2.2/2.8/3.6/4.5	3.1/3.0/3.0/3.0	11.7/21.3/29.8/36.1	10.4/11.4/11.6/12.0	4.0/7.7/10.5/12.6
2.26	14.5/14.6/14.4/14.5	3.3/4.6/5.7/7.2	7.9/8.1/8.6/10.8	1.8/2.4/3.1/3.9	3.3/3.1/3.2/3.2	39.1/52.9/64.9/76.7	10.0/10.3/10.7/12.8	12.5/17.9/21.6/25.4
2.36	15.4/15.5/15.3/15.4	3.8/5.2/6.1/7.5	7.9/9.2/9.5/12.9	1.9/2.6/3.1/3.8	3.5/3.3/3.4/3.4	30.2/54.9/84.4/96.7	13.0/14.3/15.2/16.2	9.0/17.3/26.2/30.2
2.46	16.2/16.3/16.1/16.2	5.7/6.4/7.3/8.3	8.2/8.5/8.6/8.7	2.7/3.1/3.5/4.0	3.7/3.5/3.6/3.6	32.4/55.7/86.8/100.9	15.0/15.3/17.3/18.0	9.1/16.5/25.5/29.6

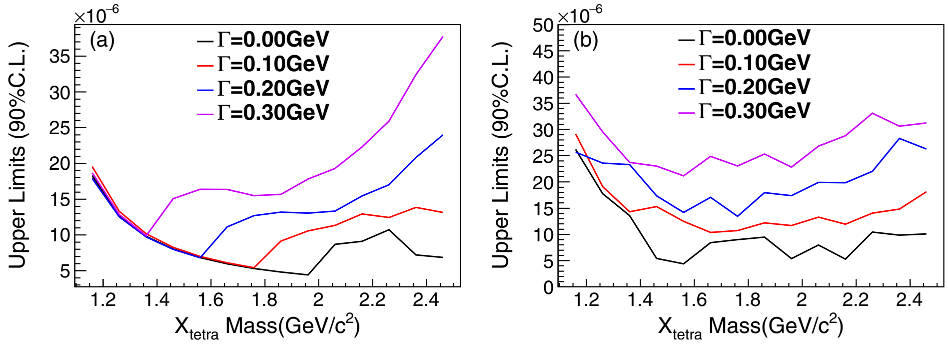

FIG. 11. The upper limits on the branching fractions for (a) $\Upsilon(1S) \to \chi_{c1} + X_{\rm tetra}$ and (b) $\Upsilon(2S) \to \chi_{c1} + X_{\rm tetra}$ as a function of the assumed $X_{\rm tetra}$ mass with widths fixed at 0.0, 0.1, 0.2, and 0.3 GeV.

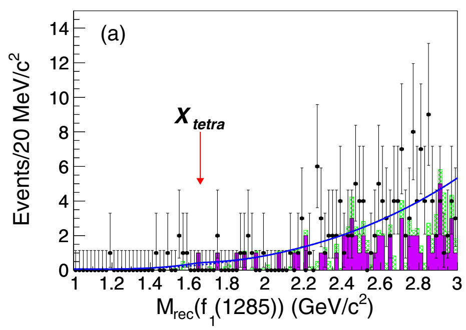

FIG. 12. The $f_1(1285)$ recoil mass spectra in the (a) $\Upsilon(1S)$ and (b) $\Upsilon(2S)$ data samples. The blue solid curves show the results of the fit described in the text, including the $X_{\rm tetra}$ states with widths fixed at 0.10 GeV and masses fixed at 1.66 GeV$/c^2$ indicated by the arrows. The nearly imperceptible blue dashed curves show the fitted background. The magenta shaded histograms are from the normalized $f_1(1285)$ sideband and the green cross-hatched histograms show the normalized continuum contributions.

mass of 1.76 GeV$/c^2$ and width of 0.1 GeV, with a statistical significance of 2.8σ.

For $\chi_{b1} \to \omega + X_{\rm tetra}$, ω candidates are reconstructed via $\pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$. Figure 14 shows the recoil mass spectrum of $\gamma\omega$ for events in the ω signal region, along with the backgrounds from the normalized ω sideband and the normalized continuum contributions. No evident $X_{\rm tetra}$ signal is observed. An unbinned extended maximum-likelihood fit

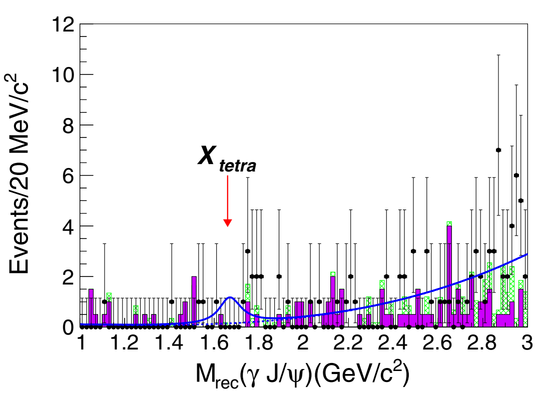

FIG. 13. The $\gamma J/\psi$ recoil mass spectrum for $\Upsilon(2S) \to \gamma\chi_{b1} \to \gamma J/\psi + \text{anything}$ in the $\Upsilon(2S)$ data sample. The blue solid curve shows the result of the fit described in the text, including the $X_{\rm tetra}$ state with a width fixed to 0.10 GeV and a mass fixed at 1.66 GeV$/c^2$ indicated by the arrow. The blue dashed curve shows the fitted background. The magenta shaded histogram is from the normalized J/ψ sideband and the green cross-hatched histogram shows the normalized continuum contributions.

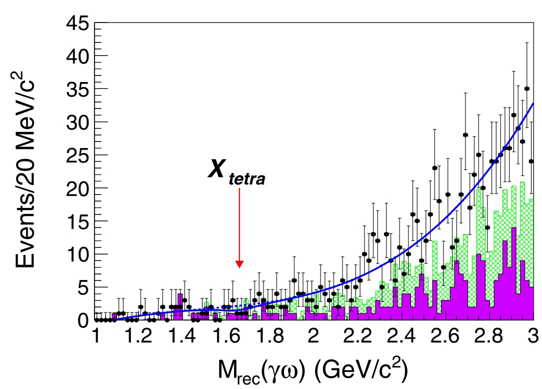

FIG. 14. The $\gamma\omega$ recoil mass spectrum for $\Upsilon(2S) \to \gamma\chi_{b1} \to \gamma\omega + \text{anything}$ in the $\Upsilon(2S)$ data sample. The blue solid curve shows the result of the fit described in the text, including the $X_{\rm tetra}$ state with a width fixed to 0.10 GeV and a mass fixed at 1.66 GeV$/c^2$ indicated by the arrow. The blue dashed curve shows the fitted background. The magenta shaded histogram is from the normalized ω sideband and the green cross-hatched histogram shows the normalized continuum contributions.

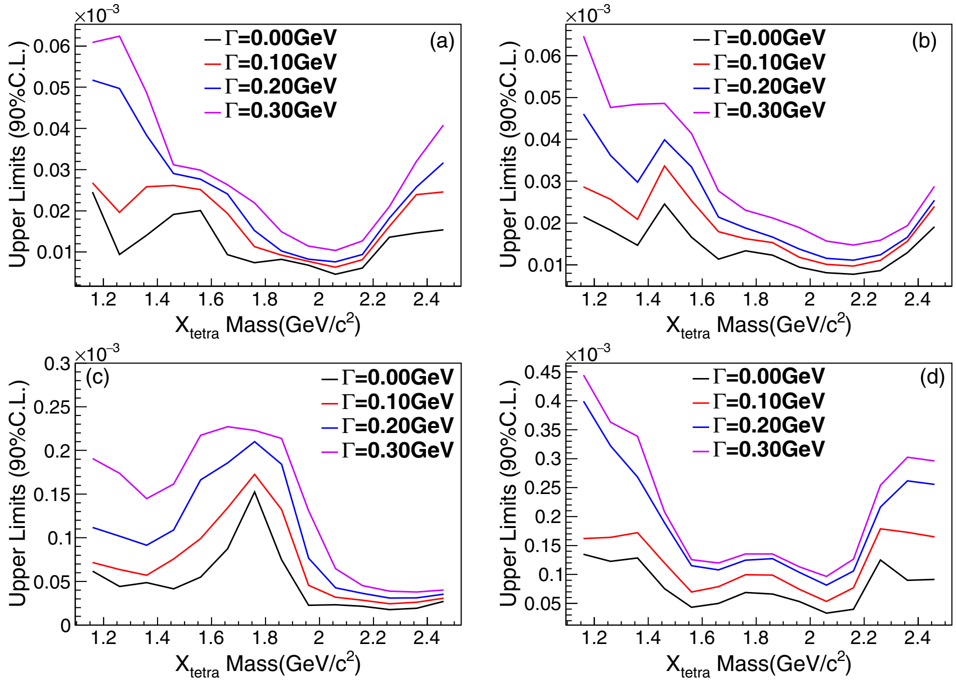

FIG. 15. The upper limits on the branching fractions for (a) $\Upsilon(1S) \to f_1(1285) + X_{\rm tetra}$, (b) $\Upsilon(2S) \to f_1(1285) + X_{\rm tetra}$, (c) $\chi_{b1} \to J/\psi + X_{\rm tetra}$, and (d) $\chi_{b1} \to \omega + X_{\rm tetra}$ as a function of the assumed $X_{\rm tetra}$ mass with widths fixed at 0.0, 0.1, 0.2, and 0.3 GeV, respectively.

is applied to the $\gamma\omega$ recoil mass spectrum. The result of the fit including the $X_{\rm tetra}$ signal with its mass fixed at 1.66 GeV$/c^2$ and width fixed at 0.10 GeV is shown in Fig. 14. This fit yields -7.8 ± 9.1 $X_{\rm tetra}$ signal events. In the entire region of study, the most significant signal is observed at an $X_{\rm tetra}$ mass of 2.26 GeV$/c^2$ and width of 0.1 GeV, with a statistical significance of 2.2σ.

Considering the yields for $\Upsilon(1S,2S) \to f_1(1285) + X_{\rm tetra}$, $\chi_{b1} \to J/\psi + X_{\rm tetra}$ and $\chi_{b1} \to \omega + X_{\rm tetra}$ are very small, we determine the 90% C.L. upper limits on the $X_{\rm tetra}$ signal yields ($N^{\rm UL}$) for $M(X_{\rm tetra}) < 1.46$ GeV$/c^2$ following the procedure in Ref. [31] as described above for $\Upsilon(1S,2S) \to \chi_{c1} + X_{\rm tetra}$, and for $M(X_{\rm tetra}) > 1.46$ GeV$/c^2$ using the same method as described for $\chi_{b0,b1} \to J/\psi +$ anything. Here, the systematic errors have been taken into account in the determination of $N^{\rm UL}$.

The calculated upper limits on the numbers of signal events ($N^{\rm UL}$) and branching fraction ($\mathcal{B}^{\rm UL}$) for $\Upsilon(1S,2S) \to f_1(1285) + X_{\rm tetra}$, $\chi_{b1} \to J/\psi + X_{\rm tetra}$ and $\chi_{b1} \to \omega + X_{\rm tetra}$ with $X_{\rm tetra}$ masses from 1.16 to 2.46 GeV$/c^2$ and widths from 0.0 to 0.3 GeV are listed in Table II, together with the reconstruction efficiencies (ε) and the systematic uncertainties ($\sigma_{\rm syst}$). The results are displayed graphically in Fig. 15.

VII. SYSTEMATIC UNCERTAINTIES

Most of the systematic errors in the branching fraction measurements are the same as in Ref. [17], including tracking reconstruction, photon reconstruction, particle identification, trigger efficiency, the branching fractions of the intermediate states, and the total numbers of $\Upsilon(1S)$ and $\Upsilon(2S)$ events; the notable exception is the dominant systematic error from the fit uncertainty. By changing the order of the background polynomial and the range of the fit, the model-dependent relative difference in the signal yields (or the upper limits for those modes with statistically insignificant branching fractions) is obtained; this is taken as the systematic error due to the uncertainty of the fit. The estimation of the continuum contributions in the $f_1(1285)$ inclusive production processes assumes a $1/s^2$ dependence. The analysis is repeated assuming a $1/s$ or $1/s^3$ dependence and the largest change in the fitted $f_1(1285)$ signal yield is taken as a systematic uncertainty. Assuming that all of these systematic-error sources are independent, the total systematic errors are summed in quadrature and listed in Table II for all the studied modes for each hypothesized $X_{\rm tetra}$ mass.

VIII. SUMMARY

In summary, utilizing the recoil mass spectra of the χ_{c1}, $f_1(1285)$, J/ψ, and ω in the channels $\Upsilon(1S,2S) \to \chi_{c1} + G_{0^{--}}$, $\Upsilon(1S,2S) \to f_1(1285) + G_{0^{--}}$, $\chi_{b1} \to J/\psi + G_{0^{--}}$, and $\chi_{b1} \to \omega + G_{0^{--}}$ [17], respectively, we report the first search for the light tetraquark states predicted with a mass of 1.66 ± 0.14 GeV$/c^2$ and $J^{PC} = 0^{--}$, and with a mass in the region 1.18–1.43 GeV$/c^2$ and $J^{PC} = 1^{+-}$ [16].

No evident signal is found below 3 GeV/c^2 in the above processes and 90% C.L. upper limits are set on the branching fractions. Figures 11 and 15 show the upper limits on the branching fractions as a function of the tetraquark masses. In addition, as byproducts of the search, we measure the inclusive $f_1(1285)$ production in $\Upsilon(1S,2S)$, J/ψ production in $\chi_{bJ}(J=0,1,2)$, and ω production in χ_{b1}. The corresponding branching fractions are measured for the first time to be $\mathcal{B}(\Upsilon(1S)\to f_1(1285)+\text{anything})=(46\pm28(\text{stat})\pm13(\text{syst}))\times10^{-4}$, $\mathcal{B}(\Upsilon(2S)\to f_1(1285)+\text{anything})=(22\pm15(\text{stat})\pm6.3(\text{syst}))\times10^{-4}$, $\mathcal{B}(\chi_{b2}\to J/\psi+\text{anything})=(1.50\pm0.34(\text{stat})\pm0.22(\text{syst}))\times10^{-3}$, and $\mathcal{B}(\chi_{b1}\to\omega+\text{anything})=(4.9\pm1.3(\text{stat})\pm0.6(\text{syst}))\times10^{-2}$, and the 90% C.L. upper limits on the branching fractions $\mathcal{B}(\chi_{b0}\to J/\psi+\text{anything})<2.3\times10^{-3}$ and $\mathcal{B}(\chi_{b1}\to J/\psi+\text{anything})<1.1\times10^{-3}$ are determined for the first time.

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