Comparing Voting by Committees According to Their Manipulability[†]

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We consider the class of voting by committees to be used by a society to collectively choose a subset from a given set of objects. We offer a simple criterion to compare two voting by committees without dummy agents according to their manipulability. This criterion is based on the set-inclusion relationships between the two corresponding pairs of sets of objects, those at which each agent is decisive and those at which each agent is vetoer. We show that the binary relation "to be as manipulable as" endows the set of equivalence classes of anonymous voting by committees (i.e., voting by quotas) with a complete upper semilattice structure, whose supremum is the equivalence class containing all voting by quotas with the property that the quota of each object is strictly larger than one and strictly lower than the number of agents. Finally, we extend the comparability criterion to the full class of all voting by committees. (*JEL* D71, D72)

In this paper, we compare voting by committees according to their manipulability when they operate on the full domain of preference profiles, and, hence, they are manipulable. To do so, we apply a criterion introduced in Arribillaga and Massó (2016). Specifically, we consider a set of agents who have to collectively choose a subset from a given set of objects K. There are many social choice problems where the set of social alternatives is the family of all subsets of a given set. For instance, when the set of agents is the tenured members of a department and the set of objects is the set of junior candidates under consideration to become new assistant professors or a scientific society whose current fellows have to elect new fellows from a given list of candidates. Voting by committees (a subclass of social choice functions mapping preferences profiles into subsets of K) have been proposed to solve this

class of problems. Following Barberà, Sonnenschein, and Zhou (1991) a voting by committees is defined by specifying for each object $x \in K$ a monotonic family of winning coalitions $\mathcal{W}_x$ (a committee). Then the choice of the subset of objects made by a voting by committees at a preference profile is done object-by-object as follows. Fix a voting by committees $\mathcal{W} = (\mathcal{W}_x)_{x \in K}$ and a preference profile, and consider object x. Then, x belongs to the chosen set (the one selected by $\mathcal{W}$ at the preference profile) if and only if the set of agents whose best subset of objects contains x belongs to the committee $\mathcal{W}_x$. Hence, voting by committees can be seen as a family of extended majority voting (one for each object x), where the two alternatives at stake are whether or not x belongs to the collectively chosen subset of objects.

A social choice function is strategy-proof if it is always in the best interest of agents to reveal their preferences truthfully and it respects voter sovereignty if all subsets of objects can be chosen, for some revealed profile of agents' preferences. An agent's preferences are *separable* on the family of all subsets of objects 2^K if they are guided by the partition separating the set of objects into the set of *good* objects (as singleton sets, objects that are better than the empty set) and *bad* objects (as singleton sets, objects that are worse than the empty set). Adding a good object to any set leads to a better set, while adding a bad object leads to a worse set. Note that all additively representable preferences are separable. One of the main reasons why voting by committees are attractive is that they constitute the class of all strategy-proof social choice functions respecting voter sovereignty on the domain of separable preference profiles. So, voting by committees are attractive because they induce good strategic incentives to agents whenever they have separable preferences. But in addition voting by committees are appealing because they are simple.[1]

Agent i is *dummy* at object x in the committee $\mathcal{W}_x$ if i does not belong to any minimal winning coalition of $\mathcal{W}_x$; that is, i's opinion about object x is not used at all in the decision of whether or not x belongs to the chosen subset. An especially interesting subclass of voting by committees are those without dummy agents. And among the class of voting by committees without dummy agents the subclass of *voting by quotas* is particularly appealing. A voting by committees is a voting by quotas if the set of winning coalitions for each object x are the sets of agents with equal or larger cardinality than a given strictly positive integer q_x, the quota of x. Hence, in any voting by quotas all agents play a symmetric role when determining whether or not objects belong to the chosen subset. Using the main characterization result in Barberà, Sonnenschein, and Zhou (1991), it is easy to see that the class of all anonymous and strategy-proof social choice functions satisfying voter sovereignty on the domain of separable preference profiles coincides with all voting by quotas.

Voting by committees are simple for two reasons. First, they are tops-only because they only depend on the profile of top subsets of objects, one for each agent. Second, they are object-by-object decomposable, and this is precisely the reason why they are strategy-proof on the domain of separable preference profiles: agent i, when

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[1] See Barberà, Sonnenschein, and Zhou (1991) for a description of this class of problems and this axiomatic characterization of voting by committees.

considering whether or not to give support to object x, does not need to know the other set of elected objects because i wants to support x if and only if x is a good object as a singleton set (i.e., x belongs to i's best subset of objects), since its addition improves, by separability, any subset of objects.

However, in many applications, the set of conceivable preferences of agents may be larger than the set of separable preferences. When adding an object to a set, some considerations regarding complementarities or substitutabilities among the added object and the objects already in the set may become relevant, yet they are not admissible if preferences are separable. For instance, although a voter for new assistant professors in the department considers that candidates x and y are the best and the second best candidates, both working in a similar research area, the voter may consider that, for the sake of diversity, the subset $\{x, z\}$ is better than the subset $\{x, y, z\}$, where z is a third good candidate who works in a very different area from the one that x and y work, contravening separability. For this and similar cases we know that nontrivial voting by committees become manipulable, once nonseparable preferences are admitted in the domain where they operate. But since separable preferences may be conceivable too, voting by committees still have to be used to ensure that the social choice procedure remains strategy-proof on the subdomain of separable preference profiles. But the large mechanism design literature characterizing strategy-proof social choice functions on restricted domains of preferences has mainly neglected the potential interest to compare two social choice functions (operating on the full domain of preferences) according to their manipulability. And this is specially relevant if the designer has some doubts on whether agents' preferences are indeed restricted.

In Arribillaga and Massó (2016), we have already argued that the manipulability of a social choice function does not indicate the degree of its lack of strategy-proofness. There may be only one instance at which the social choice function is manipulable or there may be many such instances. The mechanism design literature contains alternative measures (or lower and upper bounds) of the degree of manipulability to be applied to a given social choice function—see, for instance, Peleg (1979); Nitzan (1985); Kelly (1993); Smith (1999); Ehlers, Peters, and Storcken (2004); Maus, Peters, and Storcken (2007a, 2007b, 2007c, 2007d); Campbell and Kelly (2009); and Carroll (2013)—or direct criteria (in the form of partial orders) to compare pairs of social choice functions according to their manipulability—see, for instance, Pathak and Sönmez (2013); Chen et al. (2016); and Arribillaga and Massó (2016).

As in Arribillaga and Massó (2016) our criterion to compare two social choice functions takes the point of view of individual agents. We say that an agent is able to manipulate a social choice function at a preference (the true one) if there exist preference relations, one for each of the other agents, and another preference for the agent (the strategic one), such that if submitted, the social choice function selects a strictly better alternative according to the agent's true preference. Consider two voting by committees, $\mathcal{W}$ and $\mathcal{V}$, operating on the universal domain of preference profiles. Assume that for each agent the set of preferences under which the agent is able to manipulate $\mathcal{W}$ is contained in the set of preferences under which the agent is able to manipulate $\mathcal{V}$. Then, from the point of view of all agents, $\mathcal{V}$ is more manipulable than $\mathcal{W}$. Hence, we think that $\mathcal{W}$ is unambiguously a better voting by committees

than $\mathcal{V}$ according to the strategic incentives induced to agents.[2] Often, it may be reasonable to think that agents' preferences are separable, but if the designer foresees that agents also may have non-separable preferences, then $\mathcal{W}$ may be a better choice than $\mathcal{V}$ if strategic incentives are relevant and important to the designer.

Before presenting our general result in Theorem 2, we focus on voting by committees without dummy agents. In Theorem 1, we provide a simple necessary and sufficient condition for the comparability of two voting by committees without dummy agents in terms of their manipulability. This condition reflects the power of agents to influence the choice of the objects in the two voting by committees. Two notions are relevant to describe this power. Fix a voting by committees $\mathcal{W} = (\mathcal{W}_x)_{x \in K}$, an agent i, and an object x. We say that agent i is *decisive* at x if the singleton set $\{i\}$ belongs to $\mathcal{W}_x$; that is, $\{i\}$ is a winning coalition of x and, hence, i can impose object x in the final chosen subset by voting for x (i.e., by declaring that x belongs to i's top subset of objects). We say that agent i is *vetoer* at x if i belongs to all winning coalitions of $\mathcal{W}_x$; that is, i can make sure that x is not in the final chosen subset by not voting for x (i.e., by declaring that x does not belong to i's top subset of objects). Then, the voting by committees without dummy agents $\mathcal{V}$ is more manipulable than the voting by committees without dummy agents $\mathcal{W}$ if and only if, for each agent, the decisive and vetoer sets of objects at $\mathcal{V}$ are contained, respectively, in the decisive and vetoer sets of objects at $\mathcal{W}$. Given the binary relation "to be as manipulable as" defined on the family of voting by committees we construct the set of equivalence classes of voting by committees, relative to this binary relation, by grouping together the voting by committees that are equally manipulable to each other, and extend this relation to the quotient set of equivalence classes in the natural way. Then we show that, when the number of agents n is greater than or equal to three, the binary relation "to be at least as manipulable as" defined on the family of equivalence classes of voting by quotas (i.e., anonymous voting by committees) is a complete upper semilattice, whose maximal element is the equivalence class containing all voting by quotas where all quotas are strictly larger than 1 and strictly smaller than n. On the other hand, the equivalence classes of voting by quotas that are not more manipulable than any other equivalence class of voting by quotas are those where all objects have either quota 1 or quota n. We also identify, in Proposition 2, among all voting by committees without dummies (a larger set than the set of voting by quotas) those that are less manipulable. They can be characterized by two properties. First, the set of objects at which each agent is decisive contains the set of objects at which all agents are not vetoers. Second, the set of objects at which each agent is not a vetoer is contained in the set of objects at which some agent is decisive. In Theorem 2, we give the necessary and sufficient condition for

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[2]Pathak and Sönmez (2013) proposed two comparability criteria based on the inclusion of sets of *preference profiles* and use them to compare two different matching mechanisms (in school choice problems) according to their manipulability. In contrast, our notion is based on the inclusion of sets of *preference relations* at which an agent is able to manipulate. In applications, preference profiles are not common knowledge while each agent knows his preference relation. A more manipulable social choice function requires that each agent has to worry about his potential capacity to manipulate in a larger set. In a remark at the end of the paper we present the two Pathak and Sönmez (2013) criteria and relate them to our comparability criterion. Chen et al. (2016) proposed an agent-by-agent comparability criterion to compare stable matching mechanisms based on the inclusion of sets of *alternatives* that represent an improvement that an agent, at a preference profile, can obtain by misreporting.

the comparability according to their manipulability of two voting by committees, potentially with dummy agents. The condition is more involved than the one used to compare voting by committees without dummy agents, and it incorporates, in addition to the inclusion of sets of objects at which each agent is decisive and vetoer, the inclusion of sets of objects at which each agent is a dummy. However, the inclusions of the three sets of objects are not necessary, but we show that they may not hold only in two very special circumstances, that we fully identify.

The paper is organized as follows. Section I contains preliminary notation and definitions. Separable preferences and voting by committees are defined in Section II. Section III presents preliminary results. Section IV compares voting by committees without dummy agents and Theorem 2 in Section V provides the complete criterion to compare any pair of voting by committees. Section VI relates our comparability criterion with the two criteria proposed by Pathak and Sönmez (2013).

I. Preliminaries

Agents are the elements of a finite set $N = \{1, \ldots, n\}$. The set of *objects* is a finite set K. Generic agents will be denoted by i and j and generic objects by x and y. Subsets of agents will be represented by S and T and subsets of objects by A and B. The set of agents N has to choose a subset of K. Hence, the set of *alternatives* is the family 2^K of all subsets of objects. Given $S \subseteq N$ and $A \in 2^K$, we denote by $|S|$ and $|A|$ their cardinalities and by $\overline{S}$ and $\overline{A}$ their complementary sets. We assume that $|N| = n \geq 2$ and $|K| \geq 2$.

The (strict) *preference* of each agent $i \in N$ is a linear order P_i on the set of alternatives; namely, P_i is a complete, antisymmetric, and transitive binary relation on 2^K. As usual, let R_i denote the weak preference relation induced by P_i; namely, for all $A, B \in 2^K$, AR_iB if and only if either AP_iB or $A = B$. The top alternative according to P_i is the most preferred alternative and we denote it by $t(P_i)$; i.e., $t(P_i) P_i A$ for all $A \in 2^K \backslash \{t(P_i)\}$. Let $\mathcal{P}$ be the set of all preferences. A *preference profile* $P = (P_1, \ldots, P_n) \in \mathcal{P}^n$ is an n-tuple of preferences, one for each agent. To emphasize the role of agent i a preference profile P will be represented by (P_i, P_{-i}).

A Cartesian product subset $\hat{\mathcal{P}}^n \subseteq \mathcal{P}^n$ of preference profiles (or the set $\hat{\mathcal{P}}$ itself) will be called a *domain*. A *social choice function* is a function $f : \hat{\mathcal{P}}^n \to 2^K$ selecting, for each preference profile P in the domain $\hat{\mathcal{P}}^n$, a subset of objects $f(P) \in 2^K$.

A social choice function $f : \hat{\mathcal{P}}^n \to 2^K$ satisfies *voter sovereignty* if for all $A \in 2^K$ there exists a profile $P \in \hat{\mathcal{P}}^n$ such that $f(P) = A$; namely, f is onto.

A social choice function $f : \hat{\mathcal{P}}^n \to 2^K$ is *tops-only* if for all $P, P' \in \hat{\mathcal{P}}^n$ such that $t(P_i) = t(P'_i)$ for all $i \in N$, $f(P) = f(P')$.

A social choice function requires that each agent reports a preference on a domain $\hat{\mathcal{P}}$. A social choice function is *strategy-proof* on $\hat{\mathcal{P}}^n$ if it is always in the best interest of agents to reveal their preferences truthfully. Formally, a social choice function $f : \hat{\mathcal{P}}^n \to 2^K$ is *strategy-proof* if for all $P \in \hat{\mathcal{P}}^n$, all $i \in N$, and all $P'_i \in \hat{\mathcal{P}}$,

$$(1) \qquad\qquad f(P_i, P_{-i}) R_i f(P'_i, P_{-i}).$$

That is, a social choice function $f : \hat{\mathcal{P}}^n \to 2^K$ is strategy-proof (on the domain $\hat{\mathcal{P}}^n$) if, for each preference profile $P \in \hat{\mathcal{P}}^n$ and each agent, truth-telling is a weakly dominant strategy in the normal form game induced by f at P. We will say that a social choice function $f : \hat{\mathcal{P}}^n \to 2^K$ is *not manipulable by* $i \in N$ *at* $P_i \in \hat{\mathcal{P}}$ if (1) holds for all $(P_i', P_{-i}) \in \hat{\mathcal{P}}^n$. To compare social choice functions according to their manipulability, our reference set of preferences will be the full set $\mathcal{P}$.

The set of *manipulable preferences of* $i \in N$ *at* $f : \mathcal{P}^n \to 2^K$ is given by

$$\mathcal{M}_i^f = \{P_i \in \mathcal{P} \mid f(P_i', P_{-i})\, P_i f(P_i, P_{-i}) \text{ for some } (P_i', P_{-i}) \in \mathcal{P}^n\}.$$

Obviously, a social choice function $f : \mathcal{P}^n \to 2^K$ is strategy-proof if and only if $\mathcal{M}_i^f = \{\emptyset\}$ for all $i \in N$. We say that $f : \mathcal{P}^n \to 2^K$ is *more manipulable than* $g : \mathcal{P}^n \to 2^K$ *for* $i \in N$ if $\mathcal{M}_i^g \subsetneq \mathcal{M}_i^f$.

Now, we introduce our criteria to compare social choice functions according to their manipulability.

DEFINITION 1: *A social choice function* $f : \mathcal{P}^n \to 2^K$ *is at least as manipulable as social choice function* $g : \mathcal{P}^n \to 2^K$ *if* $\mathcal{M}_i^g \subseteq \mathcal{M}_i^f$ *for all* $i \in N$.

DEFINITION 2: *A social choice function* $f : \mathcal{P}^n \to 2^K$ *is equally manipulable as social choice function* $g : \mathcal{P}^n \to 2^K$ *if* f *is at least as manipulable as* g *and vice versa; i.e.,* $\mathcal{M}_i^g = \mathcal{M}_i^f$ *for all* $i \in N$.

DEFINITION 3: *A social choice function* $f : \mathcal{P}^n \to 2^K$ *is more manipulable than social choice function* $g : \mathcal{P}^n \to 2^K$ *if* f *is at least as but not equally manipulable as social choice function* g; *i.e.,* $\mathcal{M}_i^g \subseteq \mathcal{M}_i^f$ *for all* $i \in N$ *and there exists* $j \in N$ *such that* $\mathcal{M}_j^g \subsetneq \mathcal{M}_j^f$.

Given two social choice functions $f : \mathcal{P}^n \to 2^K$ and $g : \mathcal{P}^n \to 2^K$, we write (i) $f \succsim g$ to denote that f is at least as manipulable as g, (ii) $f \approx g$ to denote that f is equally manipulable as g, and (iii) $f \succ g$ to denote that f is more manipulable than g. Obviously, there are many pairs of social choice functions that cannot be compared according to their manipulability.

II. Separable Preferences and Voting by Committees

Barberà, Sonnenschein, and Zhou (1991) characterizes, on the restricted domain of separable preferences, the family of all strategy-proof social choice functions satisfying voter sovereignty as the class of voting by committees. A preference P_i is separable if the division between good objects (x is a good object if $\{x\} P_i \{\emptyset\}$) and bad objects (x is a bad object if $\{\emptyset\} P_i \{x\}$) guides the ordering of (some) subsets in the sense that adding a good object to any set leads to a better set, while adding a bad object to any leads to a worse set. Formally,

DEFINITION 4: *A preference* $P_i \in \mathcal{P}$ *on* 2^K *is separable if for all* $A \in 2^K$ *and* $x \notin A$,

$$A \cup \{x\} P_i A \quad \textit{if and only if} \quad \{x\} P_i \{\emptyset\}.$$

Let $\mathcal{S}$ be the set of all separable preferences on 2^K. Observe that for any separable preference its top is the subset of good objects. That is, for any separable preference $P_i \in \mathcal{S}$,

$$t(P_i) = \{x \in K \mid \{x\} P_i \{\emptyset\}\}.$$

The following remark characterizes separable preferences. It follows from transitivity of the preference and it says that if we modify any given set of objects A by removing good objects and adding bad objects, the new set is less preferred.

REMARK 1: *A preference P_i is separable if and only if for all $A \in 2^K$, $T_1 \subset t(P_i) \cap A$, and $T_2 \subset \overline{t(P_i)} \cap \overline{A}$,*

$$A R_i (A \backslash T_1) \cup T_2.$$

We now define the class of social choice functions known as voting by committees. Let $x \in K$ be an object. A committee $\mathcal{W}_x$ for x is a nonempty set of nonempty coalitions (subsets) of N, which satisfies the following monotonicity condition:

$$\text{if } M \in \mathcal{W}_x \text{ and } M \subset M', \text{ then } M' \in \mathcal{W}_x.$$

A social choice function $f : \mathcal{P}^n \to 2^K$ is a *voting by committees* if for each $x \in K$ there exists a committee $\mathcal{W}_x$, such that for all $P \in \mathcal{P}^n$,

(2) $x \in f(P) \quad \text{if and only if} \quad \{i \in N \mid x \in t(P_i)\} \in \mathcal{W}_x.$

Observe that voting by committees are very simple. They are tops-only and the selected subset of objects at each preference profile is obtained in a decomposable way, object-by-object. Barberà, Sonnenschein, and Zhou (1991) characterizes this class when they operate on the restricted domain of separable preferences as follows.

PROPOSITION 1 (Barberà, Sonnenschein, and Zhou 1991): *A social choice function $f : \mathcal{S}^n \to 2^K$ is strategy-proof and satisfies voter sovereignty if and only if it is voting by committees.*

III. Preliminary Results

Let $\mathcal{W}_x$ be a committee for object $x \in K$. The subset of agents $M \in \mathcal{W}_x$ is a *minimal winning coalition* on $\mathcal{W}_x$ if there is no $M' \in \mathcal{W}_x$, such that $M' \subsetneq M$. Given a committee $\mathcal{W}_x$, we denote by $\mathcal{W}_x^m$, the set of its minimal winning coalitions.

Assume $f : \mathcal{P}^n \to 2^K$ is a voting by committees and let $\mathcal{W} = (\mathcal{W}_x)_{x \in K}$ be its associated family of committees, one for each object. Abusing notation we will often write f directly as $\mathcal{W} : \mathcal{P}^n \to 2^K$; hence, for $P \in \mathcal{P}^n$, $\mathcal{W}(P)$ will denote the subset of objects chosen by the voting by committees $\mathcal{W}$ at P.

Let $\mathcal{W}$ be a voting by committees. We define three different notions of power that agents may have at $\mathcal{W}$ with respect to their role on the choice of the subset of objects. These notions will be relevant to compare voting by committees according to their manipulability.

First, agent i is *dummy* at x if i does not belong to any minimal winning coalition on $\mathcal{W}_x$; hence, i does not play any role on the choice of whether or not x belongs to the chosen set of objects according to $\mathcal{W}$. The set of objects at which agent i is dummy, given $\mathcal{W}$, is defined as

$$Du_i^{\mathcal{W}} = \{x \in K \mid \text{for all } S \in \mathcal{W}_x^m, i \notin S\}.$$

We say that a voting by committees $\mathcal{W} : \mathcal{P}^n \to 2^K$ has no dummies if, for all $i \in N, Du_i^{\mathcal{W}} = \{\emptyset\}$. Without loss of generality we assume that no agent is dummy at all objects. That is, for all $i \in N$,

$$(3) \qquad\qquad\qquad Du_i^{\mathcal{W}} \neq K;$$

otherwise if $Du_i^{\mathcal{W}} = K$, then $\mathcal{M}_i^{\mathcal{W}} = \{\emptyset\}$ and therefore we may proceed by setting $N := N\backslash\{i\}$.

Second, agent i is *decisive* at x if i, as a singleton set, belongs to $\mathcal{W}_x$; hence, i can impose object x in the chosen subset by declaring it as an element in the top subset of objects. The set of objects at which agent i is decisive, given $\mathcal{W}$, is defined as

$$De_i^{\mathcal{W}} = \{x \in K \mid \{i\} \in \mathcal{W}_x\}.$$

Third, agent i is a *vetoer* at x if i belongs to all coalitions on $\mathcal{W}_x$; hence, i can veto object x by not declaring it as an element in the top subset of objects. The set of objects at which agent i is vetoer, given $\mathcal{W}$, is defined as

$$Ve_i^{\mathcal{W}} = \{x \in K \mid i \in \underset{S \in \mathcal{W}_x}{\cap} S\}.$$

Example 1 illustrates how voting by committees work and the three notions of power.

Example 1: Let $N = \{1, 2, 3, 4\}$ be the set of agents and $K = \{x, y, z, w\}$ be the set of objects. Consider the voting by committees $\mathcal{W}$ defined by the following (minimal) committees:

$$\mathcal{W}_x^m = \{S \subset N \mid |S| = 2\},$$

$$\mathcal{W}_y^m = \{\{1, 2\}, \{2, 3\}\},$$

$$\mathcal{W}_z^m = \{\{1, 2, 3, 4\}\}, \text{ and}$$

$$\mathcal{W}_w^m = \{\{1\}, \{4\}\}.$$

Take any pair of preference profiles $P, P' \in \mathcal{P}^4$ with the properties that

$$t(P_1) = \{x, y\}, \ t(P_2) = \{y, w\}, \ t(P_3) = \{x, y\}, \ \text{and} \ t(P_4) = \{y, z, w\} \ \text{and}$$

$$t(P_1') = \{y\}, \ t(P_2') = \{z, w\}, \ t(P_3') = \{y, z\}, \ \text{and} \ t(P_4') = \{w\}.$$

Then, $\mathcal{W}(P) = \{x, y, w\}$ and $\mathcal{W}(P') = \{w\}$. Observe that the sets related with the power of the agents at $\mathcal{W}$ are

$$Du_1^{\mathcal{W}} = \{\emptyset\}, \ Du_2^{\mathcal{W}} = \{w\}, \ Du_3^{\mathcal{W}} = \{w\}, \ \text{and} \ Du_4^{\mathcal{W}} = \{y\},$$

$$De_1^{\mathcal{W}} = \{w\}, \ De_2^{\mathcal{W}} = \{\emptyset\}, \ De_3^{\mathcal{W}} = \{\emptyset\}, \ \text{and} \ De_4^{\mathcal{W}} = \{w\}, \ \text{and}$$

$$Ve_1^{\mathcal{W}} = \{z\}, \ Ve_2^{\mathcal{W}} = \{y, z\}, \ Ve_3^{\mathcal{W}} = \{z\}, \ \text{and} \ Ve_4^{\mathcal{W}} = \{z\}.$$

Our first preliminary result states that agent i cannot affect the choice of the objects at which i is a dummy agent.

LEMMA 1: *Let* $\mathcal{W} : \mathcal{P}^n \to 2^K$ *be a voting by committees. Then, for all* $(P_i, P_{-i}) \in \mathcal{P}^n$ *and* $P_i' \in \mathcal{P}$,

$$\mathcal{W}(P_i, P_{-i}) \cap Du_i^{\mathcal{W}} = \mathcal{W}(P_i', P_{-i}) \cap Du_i^{\mathcal{W}}.$$

PROOF:
Fix $P_{-i} \in \mathcal{P}^{n-1}$. It will be sufficient to show that, for any pair $P_i, P_i' \in \mathcal{P}$, $\mathcal{W}(P_i, P_{-i}) \cap Du_i^{\mathcal{W}} \subset \mathcal{W}(P_i', P_{-i})$. Assume $x \in \mathcal{W}(P_i, P_{-i}) \cap Du_i^{\mathcal{W}}$. Then, there exists $S \in \mathcal{W}_x^m$ such that $x \in t(P_j)$ for all $j \in S$. Since $x \in Du_i^{\mathcal{W}}$, $i \notin S$. Thus, $x \in \mathcal{W}(P_i', P_{-i})$. ∎

The comparability between two voting by committees in terms of their manipulability will relay strongly on the inclusion relationship between the two induced sets of alternatives that may be selected by them, once the preference of a fixed agent i is given.

DEFINITION 5: *Let* $f : \mathcal{P}^n \to 2^K$ *be a social choice function and let* $P_i \in \mathcal{P}$. *The set of options left open by* $P_i \in \mathcal{P}$ *at* f *is defined as follows:*

$$o^f(P_i) = \{A \in 2^K \mid f(P_i, P_{-i}) = A \ \text{for some} \ P_{-i} \in \mathcal{P}^{n-1}\}.$$

Given two subsets of objects $A, B \in 2^K$, such that $A \subset B$, let $[A, B]$ be the family of all subsets of objects that can be obtained from A by adding to it objects in $B \backslash A$. Namely, for any pair of alternatives $A \subset B \subset K$,
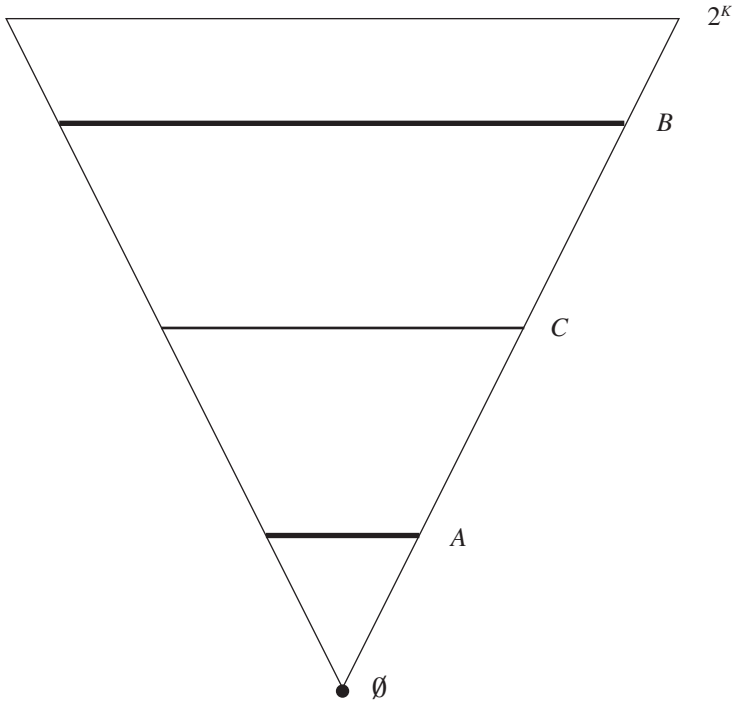
$$[A, B] = \{C \subset K \mid A \subset C \subset B\}.$$

Although imperfectly, to obtain an intuitive geometric idea, the set $[A, B]$ can be seen as the elements in the cone lying between A and B, where $[\{\emptyset\}, 2^K]$ would be the cone containing all subsets of K (see Figure 1).

Lemma 2 characterizes the set of options left open by P_i at $\mathcal{W}$ in terms of $t(P_i)$ and the sets of objects at which i is decisive and no vetoer. This result plays a crucial role in the sequel and it will be intensively used. The intuition why it holds is as follows. Fix a preference $P_i \in \mathcal{P}$. First, any subset of objects belonging to the set of options left open by P_i has to contain the objects in $t(P_i)$ for which i is simultaneously decisive at them; this is so because agent i has voted for them and i has the power to include them. Second, any subset of objects in the set of options left open by P_i has to be contained in the set made by the union of the set of objects in $t(P_i)$ and the subset of objects at which i is not a vetoer; this is so because any object for which agent i has not voted for and simultaneously i is a vetoer at will never belong to the chosen subset of objects. Moreover, any subset of objects that does satisfy the two conditions above will belong to the set of option left open by P_i at $\mathcal{W}$ because, whenever all remaining agents declare this set as their top subset of objects, it will be selected by $\mathcal{W}$ since the vote of i is not required (because i is not a vetoer) against the unanimous vote of the remaining set of agents. Figure 2 illustrates the set of options left open by P_i at $\mathcal{W}$.

LEMMA 2: *Let* $\mathcal{W} : \mathcal{P}^n \to 2^K$ *be a voting by committees. Then, for all* $P_i \in \mathcal{P}$,

$$o^{\mathcal{W}}(P_i) = \left[t(P_i) \cap De_i^{\mathcal{W}}, t(P_i) \cup \overline{Ve_i^{\mathcal{W}}} \right].$$
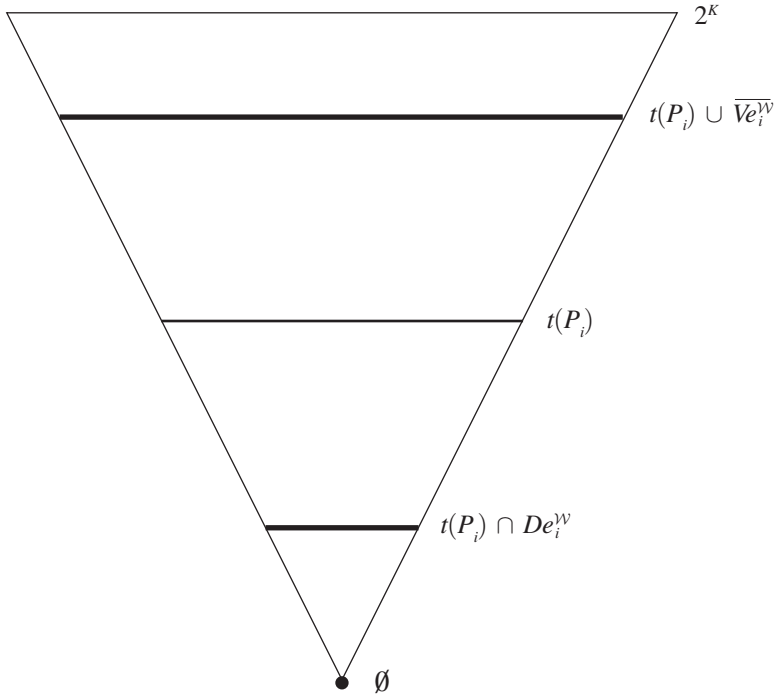
2^K

$t(P_i) \cup \overline{Ve_i^{\mathcal{W}}}$

$t(P_i)$

$t(P_i) \cap De_i^{\mathcal{W}}$

$\emptyset$

FIGURE 2

PROOF:

Assume $A \in o^{\mathcal{W}}(P_i)$. Then, there exists $P_{-i} \in \mathcal{P}^{n-1}$, such that $A = \mathcal{W}(P_i, P_{-i})$. Let $x \in t(P_i) \cap De_i^{\mathcal{W}}$. Then, $x \in t(P_i)$ and $\{i\} \in \mathcal{W}_x$. Hence, $x \in \mathcal{W}(P_i, P_{-i}) = A$. Thus, $t(P_i) \cap De_i^{\mathcal{W}} \subset A$. Let $x \in A$ and assume that $x \notin t(P_i)$. We will prove that $x \in \overline{Ve_i^{\mathcal{W}}}$. Since $x \in \mathcal{W}(P_i, P_{-i})$ and $x \notin t(P_i)$, there exists $S \in \mathcal{W}_x^m$ such that $S \subset \{j \in N \mid x \in t(R_j)\}$ and $i \notin S$. Hence, $x \in \overline{Ve_i^{\mathcal{W}}}$. Thus, $A \subset t(P_i) \cup \overline{Ve_i^{\mathcal{W}}}$.

Now, let $A \in \left[t(P_i) \cap De_i^{\mathcal{W}}, t(P_i) \cup \overline{Ve_i^{\mathcal{W}}} \right]$. For each $j \in N \backslash \{i\}$, let P_j be any preference such that $t(P_j) = A$. We will prove that $\mathcal{W}(P_i, P_{-i}) = A$. To prove one of the two inclusions, assume $x \in A$. If $x \in t(P_i)$, then $\{j \in N \mid x \in t(P_j)\} = N$. Hence, $x \in \mathcal{W}(P_i, P_{-i})$. If $x \notin t(P_i)$, then $x \in \overline{Ve_i^{\mathcal{W}}}$ (since $A \subset t(P_i) \cup \overline{Ve_i^{\mathcal{W}}}$). Hence, $\{j \in N \mid x \in t(P_j)\} = N \backslash \{i\} \in \mathcal{W}_x$ and $x \in \mathcal{W}(P_i, P_{-i})$. To prove the other inclusion, assume $x \in \mathcal{W}(P_i, P_{-i})$. By the definition of $t(P_j)$, either $x \in A$ or else $x \in t(P_i) \backslash A$ and $\{i\} \in \mathcal{W}_x$. Hence, either $x \in A$ or else $x \in t(P_i) \cap De_i^{\mathcal{W}}$. Thus, since by assumption $t(P_i) \cap De_i^{\mathcal{W}} \subset A$, $x \in A$. ∎

In light of Lemma 2, it is easy to see that as the decisive and vetoer sets of objects become larger the option set left open by a preference becomes smaller. Figure 3 illustrates this statement and Lemma 3 states it formally.

LEMMA 3: *Let $\mathcal{W} : \mathcal{P}^n \to 2^K$ and $\mathcal{V} : \mathcal{P}^n \to 2^K$ be two voting by committees with the property that $De_i^{\mathcal{V}} \subset De_i^{\mathcal{W}}$ and $Ve_i^{\mathcal{V}} \subset Ve_i^{\mathcal{W}}$. Then, $o^{\mathcal{W}}(P_i) \subset o^{\mathcal{V}}(P_i)$ for all $P_i \in \mathcal{P}$.*
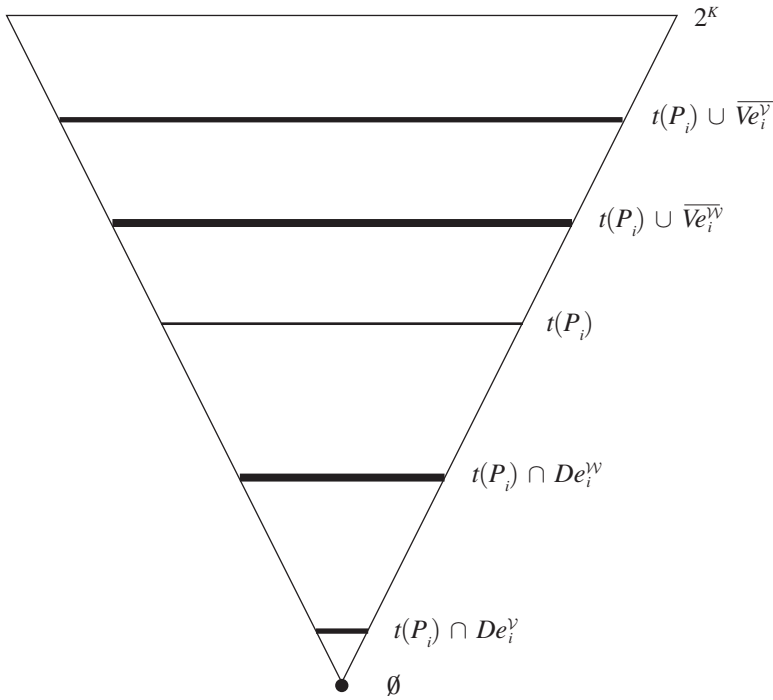
<div align="center">FIGURE 3</div>

PROOF:

It follows immediately from Lemma 2. ∎

In the last preliminary result of this section, we identify the necessary and sufficient conditions under which a voting by committees $\mathcal{W}$ is not manipulable by agent i at a particular preference P_i. These conditions can be seen as a weakening of the separability conditions because they require to compare in a specific way fewer pairs of subsets of objects. These pairs are composed of two kinds of sets. First, any subset A that is selected by $\mathcal{W}$ when i votes for $t(P_i)$ (i.e., $A \in o^{\mathcal{W}}(P_i)$). Second, any subset that can be obtained from A by taking out objects in $t(P_i)$ at which i is not a dummy and by simultaneously adding objects not in A that are in $\overline{t(P_i)}$ and at which i is not a dummy. Lemma 4 can be seen as providing a general maximal domain result for all voting by committees, which depends on the sets of decisive, vetoers and dummy objects of agent i at $\mathcal{W}$.[3] Figure 4 illustrates a particular pair of distinct subsets (A and $(A\backslash T_1) \cup T_2$) in the cone $[\{\emptyset\}, 2^K]$ that have to be comparable by P_i (i.e., $AP_i(A\backslash T_1) \cup T_2$) to ensure that $\mathcal{W}$ is not manipulable by i at P_i.

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[3] See Barberà, Massó, and Neme (1999); Barberà, Sonnesnchein, and Zhou (1991); and Serizawa (1995) for related results identifying maximal domains of preferences under which voting by committees remain strategy-proof. The results on the second and third papers are less general since they apply only to voting by committees without vetoer and dummy agents or without dummy agents, respectively. The results in the first paper are presented in the different setting of multidimensional generalized median voters schemes and hence, they are stated in terms of left and right coalition systems instead of committees.
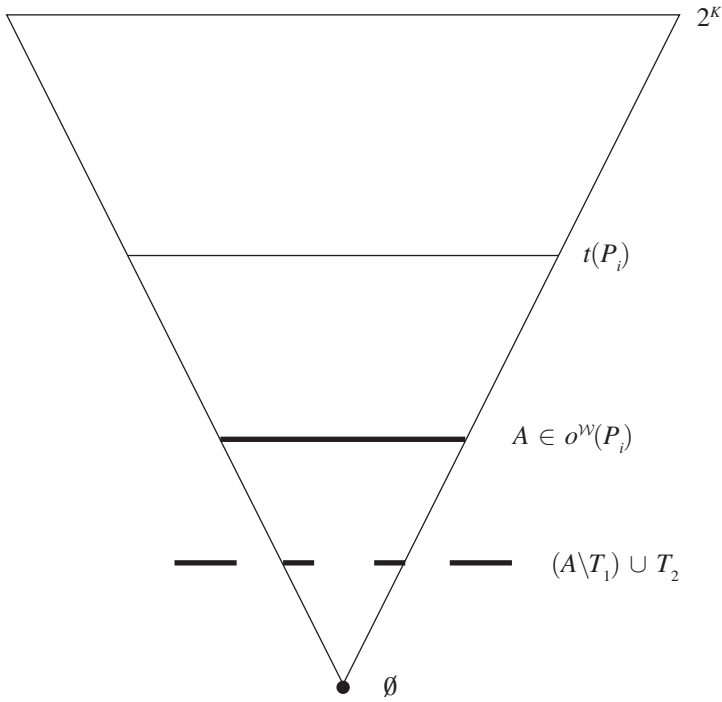
FIGURE 4

LEMMA 4: *Let $\mathcal{W} : \mathcal{P}^n \to 2^K$ be a voting by committees. Then, $\mathcal{W}$ is not manipulable by $i \in N$ at $P_i \in \mathcal{P}$ if and only if for all $A \in o^{\mathcal{W}}(P_i)$, $T_1 \subset t(R_i) \cap A \cap \overline{Du_i}^{\mathcal{W}}$, and $T_2 \subset \overline{t(R_i)} \cap \overline{A} \cap \overline{Du_i}^{\mathcal{W}}$,*

$$AR_i (A \backslash T_1) \cup T_2.$$

PROOF:

$\Rightarrow$) Consider any $P_i \in \mathcal{P}$ at which $\mathcal{W}$ is not manipulable by $i \in N$ and let $A \in o^{\mathcal{W}}(P_i)$, $T_1 \subset t(P_i) \cap A \cap \overline{Du_i}^{\mathcal{W}}$, and $T_2 \subset \overline{t(P_i)} \cap \overline{A} \cap \overline{Du_i}^{\mathcal{W}}$ be arbitrary. Then, there exists $P_{-i} \in \mathcal{P}^{n-1}$ such that

$$\mathcal{W}(P_i, P_{-i}) = A.$$

Since $T_1 \cup T_2 \subset \overline{Du_i}^{\mathcal{W}}$, for each $x \in T_1 \cup T_2$ there is $S_x^i \in \mathcal{W}_x^m$ such that $i \in S_x^i$. For each $j \in N\backslash\{i\}$, consider any $P_j' \in \mathcal{P}$ such that

(4) $t(P_j') = (t(P_j)\backslash\{x \in T_1 \cup T_2 \mid j \notin S_x^i\}) \cup \{x \in T_1 \cup T_2 \mid j \in S_x^i\},$

and let $P_i' \in \mathcal{P}$ be any preference with the property that

$$t(P_i') = (A\backslash T_1) \cup T_2.$$

Claim 1: $\mathcal{W}(P_i, P'_{-i}) = A$ and $\mathcal{W}(P'_i, P'_{-i}) = (A \backslash T_1) \cup T_2$.

PROOF OF CLAIM 1:

We first show that $\mathcal{W}(P_i, P'_{-i}) = A$. Assume $x \in T_2$. By (4), $\{j \in N \backslash \{i\} \mid x \in t(P'_j)\} = S^i_x \backslash \{i\}$. Hence, and since $T_2 \subset \overline{t(P_i)}$, $x \notin t(P_i)$. Observe that $i \in S^i_x \in \mathcal{W}^m_x$ implies $S^i_x \backslash \{i\} \notin \mathcal{W}_x$. Thus, $x \notin \mathcal{W}(P_i, P'_{-i})$. Therefore,

$$\mathcal{W}(P_i, P'_{-i}) \cap T_2 = \{\emptyset\}. \tag{5}$$

Assume $x \in T_1$. By (4), $\{j \in N \backslash \{i\} \mid x \in t(P'_j)\} = S^i_x \backslash \{i\}$. Furthermore, as $T_1 \subset t(P_i)$, $x \in t(R_i)$. Observe that $S^i_x \in \mathcal{W}^m_x$. Thus, $x \in \mathcal{W}(P_i, P'_{-i})$. Therefore,

$$\mathcal{W}(P_i, P'_{-i}) \cap T_1 = T_1. \tag{6}$$

Assume $x \notin T_1 \cup T_2$. By (4), $\{j \in N \backslash \{i\} \mid x \in t(P'_j)\} = \{j \in N \backslash \{i\} \mid x \in t(P_j)\}$. Hence,

$$(\overline{T_1 \cup T_2}) \cap \mathcal{W}(P_i, P'_{-i}) = (\overline{T_1 \cup T_2}) \cap \mathcal{W}(P_i, P_{-i}). \tag{7}$$

By (5), (6), and (7),

$$\mathcal{W}(P_i, P'_{-i}) = [(\overline{T_1 \cup T_2}) \cap \mathcal{W}(P_i, P'_{-i})] \cup T_1$$

$$= [(\overline{T_1 \cup T_2}) \cap \mathcal{W}(P_i, P_{-i})] \cup T_1$$

$$= [(\overline{T_1 \cup T_2}) \cap A] \cup T_1$$

$$= A,$$

where the last equality follows from the facts that $T_1 \subset A$ and $T_2 \subset \overline{A}$.

We now show that $\mathcal{W}(P'_i, P'_{-i}) = (A \backslash T_1) \cup T_2$. Assume $x \in A \backslash T_1$. If $x \in t(P_i)$, and since $\mathcal{W}(P_i, P'_{-i}) = A$, $\{j \in N \backslash \{i\} \mid x \in t(P'_j)\} \cup \{i\} \in \mathcal{W}_x$ holds. If $x \notin t(P_i)$, and since $\mathcal{W}(P_i, P'_{-i}) = A$, $\{j \in N \backslash \{i\} \mid x \in t(P'_j)\} \in \mathcal{W}_x$ holds. Hence, in both cases,

$$\{j \in N \backslash \{i\} \mid x \in t(P'_j)\} \cup \{i\} \in \mathcal{W}_x. \tag{8}$$

Since $x \in t(P'_i)$, $\{j \in N \backslash \{i\} \mid x \in t(P'_j)\} \cup \{i\} = \{j \in N \mid x \in t(P'_j)\}$. Hence, by (8), $\{j \in N \mid x \in t(P'_j)\} \in \mathcal{W}_x$. Thus, $x \in \mathcal{W}(P'_i, P'_{-i})$. Assume now $x \in T_2$. By (4), $\{j \in N \backslash \{i\} \mid x \in t(P'_j)\} = S^i_x \backslash \{i\}$. Hence, and since $x \in t(P'_i)$, $x \in \mathcal{W}(P'_i, P'_{-i})$. Therefore, we have showed that $(A \backslash T_1) \cup T_2 \subset \mathcal{W}(P'_i, P'_{-i})$. To show that the other inclusion holds, assume $x \in \mathcal{W}(P'_i, P'_{-i})$. If $x \in t(P'_i)$, by the definition of $t(P'_i)$, $x \in (A \backslash T_1) \cup T_2$. If, on the other hand, $x \notin t(P'_i)$,

$$(9) \qquad \{\,j \in N \mid x \in t(P'_j)\,\} = \{\,j \in N \setminus\{i\} \mid x \in t(P'_j)\,\}.$$

Hence, and since $x \in \mathcal{W}(P'_i, P'_{-i})$, by (9), $x \in \mathcal{W}(P_i, P'_{-i}) = A$. Now, to obtain a contradiction assume $x \in T_1$. Hence, $\{\,j \in N \setminus\{i\} \mid x \in t(P'_j)\,\} = S^i_x \setminus\{i\}$ and $x \notin t(P'_i)$. Therefore, $x \notin \mathcal{W}(P'_i, P'_{-i})$ which is a contradiction. Thus, $x \in A \setminus T_1$. ∎

Therefore, by Claim 1 and the fact that $\mathcal{W}$ is not manipulable by i at P_i,

$$A R_i (A \setminus T_1) \cup T_2.$$

$\Longleftarrow$) Let $P_i \in \mathcal{P}$ be arbitrary and assume that for all $A \in o^{\mathcal{W}}(P_i)$, $T_1 \subset t(R_i) \cap A \cap \overline{Du_i^{\mathcal{W}}}$, and $T_2 \subset \overline{t(R_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{W}}}$, $AR_i(A \setminus T_1) \cup T_2$ holds. We will show that, for all $(P'_i, P_{-i}) \in \mathcal{P}^n$,

$$\mathcal{W}(P_i, P_{-i}) \, R_i \, \mathcal{W}(P'_i, P_{-i}).$$

Assume $(P'_i, P_{-i}) \in \mathcal{P}^n$ and let $\mathcal{W}(P_i, P_{-i}) = A \in o^{\mathcal{W}}(P_i)$, $T_1 = (A \setminus \mathcal{W}(P'_i, P_{-i})) \cap \overline{Du_i^{\mathcal{W}}}$, and $T_2 = (\mathcal{W}(P'_i, P_{-i}) \setminus A) \cap \overline{Du_i^{\mathcal{W}}}$. Then,

$$
\begin{aligned}
(A \setminus T_1) \cup T_2 &= [(A \cap Du_i^{\mathcal{W}} \cup A \cap \overline{Du_i^{\mathcal{W}}}) \setminus T_1] \cup T_2 \\[4pt]
&= (A \cap Du_i^{\mathcal{W}}) \cup [(A \cap \overline{Du_i^{\mathcal{W}}}) \setminus T_1] \cup T_2 && \text{since } T_1 \subset \overline{Du_i^{\mathcal{W}}} \\[4pt]
&= (A \cap Du_i^{\mathcal{W}}) \cup [(A \cap \overline{Du_i^{\mathcal{W}}}) \cap \overline{T_1}] \cup T_2 \\[4pt]
&= (A \cap Du_i^{\mathcal{W}}) \cup [(A \cap \overline{Du_i^{\mathcal{W}}}) \cap (\overline{A} \cup \mathcal{W}(P'_i, P_{-i}) \cup Du_i^{\mathcal{W}})] \cup T_2 \\[4pt]
&= (A \cap Du_i^{\mathcal{W}}) \cup [(A \cap \overline{Du_i^{\mathcal{W}}}) \cap \mathcal{W}(P'_i, P_{-i})] \cup T_2 \\[4pt]
&= (A \cap Du_i^{\mathcal{W}}) \cup [(A \cap \overline{Du_i^{\mathcal{W}}}) \cap \mathcal{W}(P'_i, P_{-i})] \\
&\qquad\qquad \cup (\mathcal{W}(P'_i, P_{-i}) \setminus A) \cap \overline{Du_i^{\mathcal{W}}}) \\[4pt]
&= (A \cap Du_i^{\mathcal{W}}) \cup [(A \cap \mathcal{W}(P'_i, P_{-i}) \cup (\mathcal{W}(P'_i, P_{-i}) \setminus A)] \cap \overline{Du_i^{\mathcal{W}}} \\[4pt]
&= (A \cap Du_i^{\mathcal{W}}) \cup (\mathcal{W}(P'_i, P_{-i}) \cap \overline{Du_i^{\mathcal{W}}}) \\[4pt]
&= (\mathcal{W}(P'_i, P_{-i}) \cap Du_i^{\mathcal{W}}) \cup (\mathcal{W}(P'_i, P_{-i}) \cap \overline{Du_i^{\mathcal{W}}}) && \text{by Lemma 1} \\[4pt]
&= \mathcal{W}(P'_i, P_{-i}).
\end{aligned}
$$

Furthermore, $T_1 \subset t(R_i) \cap A \cap \overline{Du_i^{\mathcal{W}}}$ and $T_2 \subset \overline{t(R_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{W}}}$ since $A \setminus \mathcal{W}(P'_i, P_{-i}) \subset t(R_i)$ and $\mathcal{W}(P'_i, P_{-i}) \setminus A \subset \overline{t(P_i)}$. Thus, by hypothesis,

$$\mathcal{W}(P_i, P_{-i}) = A R_i (A \setminus T_1) \cup T_2 = \mathcal{W}(P'_i, P_{-i}). \; \blacksquare$$

IV. Comparing Voting by Committees without Dummies

A. *Main Result without Dummies*

Theorem 1 gives an easy and operative way to compare voting by committees without dummies according to their manipulability. A voting by committees $\mathcal{V}$ is at least as manipulable as voting by committees $\mathcal{W}$ if and only if, for each agent $i \in N$, the sets of objects at which agent i is decisive and vetoer in $\mathcal{V}$ is each contained in the corresponding sets in $\mathcal{W}$. The results in the preceding lemmata are key to understand this characterization. Larger decisive and vetoer sets of objects make the option sets left open smaller and this leaves more freedom on the comparability between subsets of objects, reducing, hence, the set of preferences under which the agent is able to manipulate.

THEOREM 1: *Let $\mathcal{V} : \mathcal{P}^n \to 2^K$ and $\mathcal{W} : \mathcal{P}^n \to 2^K$ be two voting by committees without dummies. Then, $\mathcal{V} \succsim \mathcal{W}$ if and only if, for all $i \in N$, $De_i^{\mathcal{V}} \subset De_i^{\mathcal{W}}$ and $Ve_i^{\mathcal{V}} \subset Ve_i^{\mathcal{W}}$.*

PROOF:
$\Leftarrow$) To prove sufficiency, assume that for all $i \in N$, $De_i^{\mathcal{V}} \subset De_i^{\mathcal{W}}$ and $Ve_i^{\mathcal{V}} \subset Ve_i^{\mathcal{W}}$. We prove that $\mathcal{V} \succsim \mathcal{W}$ by showing that, for all $i \in N$, $\mathcal{M}_i^{\mathcal{W}} \subset \mathcal{M}_i^{\mathcal{V}}$. Suppose $P_i \in \mathcal{M}_i^{\mathcal{W}}$. Observe that since $\mathcal{V}$ and $\mathcal{W}$ have no dummies, $\overline{Du_i^{\mathcal{W}}} = \overline{Du_i^{\mathcal{V}}}$ $= K$ holds. Then, by Lemma 4, there exist $A \in o^{\mathcal{W}}(P_i)$, $T_1 \subset t(P_i) \cap A$, and $T_2 \subset \overline{t(P_i)} \cap \overline{A}$ such that

$$(A \backslash T_1) \cup T_2 \, P_i A.$$

By Lemma 3, $o^{\mathcal{W}}(P_i) \subset o^{\mathcal{V}}(P_i)$. Hence, $A \in o^{\mathcal{V}}(P_i)$. Thus, by Lemma 4, $P_i \in \mathcal{M}_i^{\mathcal{V}}$. Therefore, $\mathcal{V}$ is at least as manipulable as $\mathcal{W}$.
$\Rightarrow$) To prove necessity assume that $\mathcal{V}$ is at least as manipulable as $\mathcal{W}$. Hence, for all $i \in N$,

(10) $$\mathcal{M}_i^{\mathcal{W}} \subset \mathcal{M}_i^{\mathcal{V}}.$$

Assume, to obtain a contradiction, that there exists $x \in K$ such that $x \in Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}}$ or $x \in De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}}$. We distinguish between these two cases.

Case 1: There exists $x \in Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}}$. Consider any $P_i' \in \mathcal{P}$ such that $t(P_i') = \{\emptyset\}$ satisfying in addition the following properties:

(i) $BP_i'CP_i'\{x\}$ if $x \notin B$ and $x \in C$,

(ii) $AR_i'B$ if $A \subset B$ and $x \notin B$.

Three comments on the preference P_i' are appropriate. First, such preference exists and, because we assumed that $|K| \geq 2$, there exists $C \subset K$ such that $x \in C$ and $\{x\} \neq C$. Second, any pair of subsets of objects that are unrestricted by conditions (i) and (ii) can be ordered arbitrarily by P_i'. Third, P_i' is not separable since $KP_i'\{x\}$ while all objects are bad.

Since $x \in Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}}$, $\{x\} \in o^{\mathcal{W}}(P_i')$. Set $A = \{x\}$, $T_1 = \{\emptyset\}$, and $T_2 = C \backslash \{x\}$ where C is such that $x \in C$ and $\{x\} \neq C$ (which exists because $|K| \geq 2$). Observe first that $A \in o^{\mathcal{W}}(P_i')$ and, by (i) in the definition of P_i', $(A \backslash T_1) \cup T_2 = CP_i'\{x\} = A$. Hence, by Lemma 4, $P_i' \in \mathcal{M}_i^{\mathcal{W}}$. Second, take any $B \in o^{\mathcal{V}}(P_i')$. Since $x \in Ve_i^{\mathcal{V}}$ and $x \notin t(P_i')$, $x \notin B$. Let $T_2 \subset \overline{A}$ be arbitrary. By (i) and (ii) in the definition of P_i', $BR_i'B \cup T_2$. Hence, by Lemma 4, $P_i' \notin \mathcal{M}_i^{\mathcal{V}}$. Thus, $P_i' \in \mathcal{M}_i^{\mathcal{W}} \backslash \mathcal{M}_i^{\mathcal{V}}$ holds, which contradicts (10).

Case 2: There exists $x \in De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}}$. Consider any $P_i'' \in \mathcal{M}_i^{\mathcal{W}}$ such that $t(P_i'') = K$ satisfying in addition the following properties:

(i) $BP_i''CP_i''K \backslash \{x\}$ if $x \in B$ and $x \notin C$,

(ii) $AR_i''B$ if $B \subset A$ and $x \in B$.

Three comments on the preference P_i'' are also appropriate. First, such preference exists and, because we assumed that $|K| \geq 2$, there exists $C \subset K$, such that $x \notin C$ and $C \neq \{\emptyset\}$. Second, any pair of subsets of objects that are unrestricted by conditions (i) and (ii) can be ordered arbitrarily by P_i''. Third, P_i'' is not separable since $\{\emptyset\}P_i''K \backslash \{x\}$ while all objects are good.

Since $x \in De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}}$, $K \backslash \{x\} \in o^{\mathcal{W}}(P_i'')$. Set first $A = K \backslash \{x\}$, $T_2 = \{\emptyset\}$, and $T_1 = C$, where C is such that $x \notin C$ and $C \neq \{\emptyset\}$. Observe that $A \in o^{\mathcal{W}}(P_i'')$, $x \notin (A \backslash T_1) \cup T_2$, and $(A \backslash T_1) \cup T_2 \neq \{\emptyset\}$. Then, by (i) in the definition of P_i'', $(A \backslash T_1) \cup T_2 P_i''K \backslash \{x\} = A$. Hence, by Lemma 4, $P_i'' \in \mathcal{M}_i^{\mathcal{W}}$. Second, take any $B \in o^{\mathcal{V}}(P_i'')$. Since $x \in De_i^{\mathcal{V}}$ and $x \in t(P_i'')$, $x \in B$. Let $T_1 \subset B$ be arbitrary. By (i) and (ii) in the definition of P_i'', $BR_i''B \backslash T_1$. Hence, by Lemma 4, $P_i'' \notin \mathcal{M}_i^{\mathcal{V}}$. Thus, $P_i'' \in \mathcal{M}_i^{\mathcal{W}} \backslash \mathcal{M}_i^{\mathcal{V}}$ holds, which contradicts (10). ∎

Next proposition identifies those voting by committees without dummies that are less manipulable. They have the properties that (i) the set of objects at which all agents are not vetoers is a subset of the set of decisive objects of each agent, and (ii) the set of objects at which each agent is not a vetoer is contained in the set of objects at which some agent is decisive. But before stating Proposition 2 we present a simple remark that will be used in its proof.

REMARK 2: *Assume $\mathcal{W} : \mathcal{P}^n \to 2^K$ is a voting by committees without dummies. Then, for all $i \in N$, $De_i^{\mathcal{W}} \subset \overline{Ve_i^{\mathcal{W}}}$.*

To see why Remark 2 holds, assume there exists $x \in De_i^{\mathcal{W}}$ such that $x \in Ve_i^{\mathcal{W}}$. Then, $\mathcal{W}_x^m = \{\{i\}\}$. Hence, each $j \neq i$ is a dummy at x, which is a contradiction.

PROPOSITION 2: *Let $\mathcal{W} : \mathcal{P}^n \to 2^K$ be a voting by committees without dummies. Then, there does not exist a voting by committee without dummies $\mathcal{V} : \mathcal{P}^n \to 2^K$ such that $\mathcal{W} \succ \mathcal{V}$ if and only if*

$$\text{(11)} \qquad \cap_{i \in N} \overline{Ve_i^{\mathcal{W}}} \subset De_j^{\mathcal{W}} \text{ for all } j \in N,$$

$$\text{(12)} \qquad \overline{Ve_j^{\mathcal{W}}} \subset \cup_{i \in N} De_i^{\mathcal{W}} \text{ for all } j \in N.$$

PROOF:

$\Rightarrow$) We distinguish between two cases.

Case 1: There exist $j \in N$ and $x \in K$ such that $x \in \cap_{i \in N} \overline{Ve_i^{\mathcal{W}}}$ and $x \notin De_j^{\mathcal{W}}$. Define $\mathcal{V}$ as follows:

$$\mathcal{V}_y^m = \begin{cases} \mathcal{W}_y^m & \text{if } y \neq x \\ \{S \in \mathcal{W}_x^m \mid j \notin S\} & \\ \quad \cup \{S \backslash \{j\} \subset N \mid S \in \mathcal{W}_x^m \text{ such that } j \in S\} \cup \{\{j\}\} & \text{if } y = x. \end{cases}$$

Since $x \notin De_j^{\mathcal{W}}$ and $\mathcal{W}$ does not have dummies, $\{\emptyset\} \notin \{S \backslash \{j\} \subset N \mid S \in \mathcal{W}_x^m$ such that $j \in S\}$ and $\mathcal{V}$ is a well-defined voting by committees. Now, we prove that $\mathcal{V}$ is a voting by committees without dummies. For each object $y \neq x$, it is immediate to see that $\mathcal{V}_y$ has no dummies since $\mathcal{W}_y$ does not have any. We prove that no agent is a dummy at x at $\mathcal{V}$. Let $i \in N$. If $i = j$, it is immediate to see that i is not dummy at x since $\{i\} \in \mathcal{V}_x^m$ by definition. If $i \neq j$, there exists $S \in \mathcal{W}_x^m$ such that $i \in S$ (since $\mathcal{W}$ is a voting by committees without dummies). Hence, $S \backslash \{j\} \in \mathcal{V}_x^m$ and i is not dummy at x at $\mathcal{V}$. Now, we prove that $\mathcal{W} \succ \mathcal{V}$. By Theorem 1, it is sufficient to prove that $De_i^{\mathcal{W}} \subset De_i^{\mathcal{V}}$ and $Ve_i^{\mathcal{W}} \subset Ve_i^{\mathcal{V}}$ for all $i \in N$ and that $De_j^{\mathcal{V}} \neq De_j^{\mathcal{W}}$. The last inequality follows from the fact that $x \in De_j^{\mathcal{V}}$ and $x \notin De_j^{\mathcal{W}}$. Then, we only have to show that $De_i^{\mathcal{W}} \subset De_i^{\mathcal{V}}$ and $Ve_i^{\mathcal{W}} \subset Ve_i^{\mathcal{V}}$ for all $i \in N$.

Assume $y \in De_i^{\mathcal{W}}$. Then, $\{i\} \in \mathcal{W}_y^m$, hence, by definition of $\mathcal{V}_y^m$, $\{i\} \in \mathcal{V}_y^m$. Hence, $y \in De_i^{\mathcal{V}}$.

Assume $y \in Ve_i^{\mathcal{W}}$. Then, $i \in \cap_{S \in \mathcal{W}_y^m} S$. Since $y \in Ve_i^{\mathcal{W}}$, and $x \in \cap_{i \in N} \overline{Ve_i^{\mathcal{W}}}$ holds, $y \neq x$. Hence, by definition of $\mathcal{V}_y^m$, $j \in \cap_{S \in \mathcal{V}_y^m} S$ and $y \in Ve_i^{\mathcal{V}}$.

Case 2: Assume that there exist $j \in N$ and $x \in K$ such that $x \in \overline{Ve_j^{\mathcal{W}}}$ and $x \notin \cup_{i \in N} De_i^{\mathcal{W}}$. To define $\mathcal{V}$ set $S' = \cap_{S \in \mathcal{W}_x^m} S$ and consider any $i \neq j$. Define the collection of subsets of agents $\mathcal{T}_i$ associated to i as follows:

$$T \in \mathcal{T}_i \text{ if and only if } i \notin S' \text{ and } T = \{i, j\} \cup S'.$$

Define now $\mathcal{V}$ by setting

$$\mathcal{V}_y^m = \begin{cases} \mathcal{W}_y^m & \text{if } y \neq x \\ \{\mathcal{T}_i\}_{i \neq j} & \text{if } y = x. \end{cases}$$

Since $x \in \overline{Ve_j^{\mathcal{W}}}$, $j \notin S'$. Therefore, $\mathcal{V}_x^m$ is minimal. Furthermore, as $x \in \overline{Ve_j^{\mathcal{W}}}$ and $\mathcal{W}$ is a voting by committees without dummies, there exists $S \subsetneqq N$ such that $j \in S \in \mathcal{W}_x^m$. Therefore, $S' \subsetneqq N \setminus \{j\}$. Hence, $\mathcal{V}_x^m \neq \{\emptyset\}$ and $\mathcal{V}$ is a well-defined voting by committees. Moreover, $\mathcal{V}_y$ is a committee without dummies because $\mathcal{W}_y$ is a committee without dummies at any $y \neq x$. Furthermore, by its definition, $\mathcal{V}_x$ is a committee without dummies. Then, $\mathcal{V}$ is a voting by committees without dummies. Now, we prove that $\mathcal{W} \succ \mathcal{V}$. By Theorem 1, it is sufficient to show that $De_i^{\mathcal{W}} \subset De_i^{\mathcal{V}}$ and $Ve_i^{\mathcal{W}} \subset Ve_i^{\mathcal{V}}$ for all $i \in N$ and that $Ve_j^{\mathcal{W}} \neq Ve_j^{\mathcal{V}}$. The last inequality follows from the fact that $x \notin Ve_j^{\mathcal{W}}$ and $x \in Ve_j^{\mathcal{V}}$. Hence, we only have to show that $De_i^{\mathcal{W}} \subset De_i^{\mathcal{V}}$ and $Ve_i^{\mathcal{W}} \subset Ve_i^{\mathcal{V}}$ for all $i \in N$.

Assume $y \in De_i^{\mathcal{W}}$. Then, $\{i\} \in \mathcal{W}_y^m$ and, since $x \notin \cup_{i \in N} De_i^{\mathcal{W}}$, $y \neq x$. Hence, by the definition of $\mathcal{V}_y^m$, $\{i\} \in \mathcal{V}_y^m$. Thus, $y \in De_i^{\mathcal{V}}$.

Assume $y \in Ve_i^{\mathcal{W}}$ and $y \neq x$. Hence, by the definition of $\mathcal{V}_y^m$, $y \in Ve_i^{\mathcal{V}}$.

Now, assume $x \in Ve_i^{\mathcal{W}}$. Then, $i \in \cap_{S \in \mathcal{W}_x^m} S = S'$. Then, by definition of $\mathcal{V}_x^m$, $x \in Ve_i^{\mathcal{V}}$.

$\Leftarrow$) Assume (11) and (12) hold and let $\mathcal{V} : \mathcal{P}^n \to 2^K$ be a voting by committees without dummies such that $\mathcal{W} \succeq \mathcal{V}$. By Theorem 1, $De_i^{\mathcal{W}} \subset De_i^{\mathcal{V}}$ and $Ve_i^{\mathcal{W}} \subset Ve_i^{\mathcal{V}}$ for all $i \in N$. We prove that $De_i^{\mathcal{W}} = De_i^{\mathcal{V}}$ and $Ve^{\mathcal{W}} = Ve_i^{\mathcal{V}}$ for all $i \in N$ and hence, $\mathcal{W} \not\succ \mathcal{V}$.

Assume there exist $j \in N$ and $x \in K$ such that $x \in De_j^{\mathcal{V}} \setminus De_j^{\mathcal{W}}$. By Remark 2, $x \in \overline{Ve_j^{\mathcal{V}}} \setminus De_j^{\mathcal{W}}$. Since $Ve_j^{\mathcal{W}} \subset Ve_j^{\mathcal{V}}$, $x \in \overline{Ve_j^{\mathcal{W}}} \setminus De_j^{\mathcal{W}}$. By (11), there exists $i \neq j$ such that $x \in Ve_i^{\mathcal{W}}$. Thus, $x \in Ve_i^{\mathcal{V}}$ which contradicts that $x \in De_j^{\mathcal{V}}$ and $i \neq j$ since $i \notin \{j\} \in \mathcal{V}_x^m$.

Now, assume there exist $j \in N$ and $x \in K$ such that $x \in Ve_j^{\mathcal{V}} \setminus Ve_j^{\mathcal{W}}$. Then, $x \in \overline{Ve_j^{\mathcal{W}}} \setminus \overline{Ve_j^{\mathcal{V}}}$. By Remark 2, $x \in \overline{Ve_j^{\mathcal{W}}} \setminus De_j^{\mathcal{V}}$. Since $De_i^{\mathcal{W}} \subset De_i^{\mathcal{V}}$, $x \in \overline{Ve_j^{\mathcal{W}}} \setminus De_j^{\mathcal{W}}$. By (12), there exists $i \neq j$ such that $x \in De_i^{\mathcal{W}}$. Thus, $x \in De_i^{\mathcal{V}}$ which contradicts that $x \in Ve_j^{\mathcal{V}}$ and $i \neq j$ since $j \notin \{i\} \in \mathcal{V}_x^m$. ∎

B. Anonymity

In this subsection, we restrict further the class of voting by committees without dummies by requiring that they are indeed anonymous (and, hence, no agent is dummy at any object). We show that the binary relation arising from our comparability criterion endows the (quotient) set of anonymous voting by committees with a complete upper semilattice structure that we fully identify.

A social choice function $f : \hat{\mathcal{P}}^n \to 2^K$ is *anonymous* if it is invariant with respect to the agents' names; namely, for all one-to-one $\sigma : N \to N$ and all $P \in \hat{\mathcal{P}}^n, f(P_1, \ldots, P_n) = f(P_{\sigma(1)}, \ldots, P_{\sigma(n)})$.

REMARK 3: *Let* $\mathcal{W} : \mathcal{P}^n \rightarrow 2^K$ *be an anonymous voting by committees. Then, for all* $x \in K$, $\mathcal{W}_x$ *does not have dummy agents.*[4]

A voting by committees $\mathcal{W} : \mathcal{P}^n \rightarrow 2^K$ is a *voting by quotas* if for each object $x \in K$ there exists an integer $q_x^{\mathcal{W}}$ between 1 and n such that

$$\mathcal{W}_x = \{S \subset N \mid |S| \geq q_x^{\mathcal{W}}\}.$$

Observe that Barberà, Sonnenschein, and Zhou (1991) call voting by quota to a voting by quotas that is in addition neutral in the sense that $q_x^{\mathcal{W}} = q_y^{\mathcal{W}}$ for all $x, y \in K$. The following remark states that the subclass of anonymous voting by committees coincides with all voting by quotas (not necessarily neutral).

REMARK 4: *A voting by committees is anonymous if and only if it is voting by quotas.*

Let $\mathcal{W} : \mathcal{P}^n \rightarrow 2^K$ be a voting by quotas. Denote by $A_1^{\mathcal{W}}$ and $A_n^{\mathcal{W}}$ the set of objects in K that have quota 1 and quota n, respectively; namely, $A_1^{\mathcal{W}} = \{x \in K \mid q_x^{\mathcal{W}} = 1\}$ and $A_n^{\mathcal{W}} = \{x \in K \mid q_x^{\mathcal{W}} = n\}$. Since for all voting by quotas $\mathcal{W} : \mathcal{P}^n \rightarrow 2^K$ it holds that $De_i^{\mathcal{W}} = A_1^{\mathcal{W}}$ and $Ve_i^{\mathcal{W}} = A_n^{\mathcal{W}}$ for all $i \in N$, we obtain as a consequence of Theorem 1 the following Corollary.

COROLLARY 1: *Let* $\mathcal{W} : \mathcal{P}^n \rightarrow 2^K$ *and* $\mathcal{V} : \mathcal{P}^n \rightarrow 2^K$ *be two voting by quotas. Then,* $\mathcal{V} \succsim \mathcal{W}$ *if and only if* $A_1^{\mathcal{V}} \subset A_1^{\mathcal{W}}$ *and* $A_n^{\mathcal{V}} \subset A_n^{\mathcal{W}}$.

The next corollary identifies the class of anonymous voting by committees that do not admit a less manipulable anonymous voting by committees. This class is the family of all voting by quotas such that the quota of each object is either 1 or n.

COROLLARY 2: *Let* $\mathcal{W} : \mathcal{P}^n \rightarrow 2^K$ *be a voting by quotas. Then, there does not exist a voting by quotas (i.e., an anonymous voting by committees)* $\mathcal{V} : \mathcal{P}^n \rightarrow 2^K$ *such that* $\mathcal{W} \succ \mathcal{V}$ *if and only if* $A_n^{\mathcal{W}} = K \backslash A_1^{\mathcal{W}}$.

Let *VbQ* be the family of all voting by quotas relative to a given set of agents N. Using Theorem 1 we can partition the set of all voting by quotas *VbQ* into equivalence classes in such a way that each equivalence class contains voting by quotas that are all equally manipulable. Denote the (quotient) set of those equivalence classes by $VbQ/\approx$. Furthermore, we can extend $\succsim$ on *VbQ* to the set of equivalence classes $VbQ/\approx$ in a natural way. Denote this extension by $[\succsim]$. In this subsection we will show that the pair $(VbQ/\approx, [\succsim])$ is a complete upper semilattice; namely, any nonempty subset of equivalence classes in $VbQ/\approx$ has a supremum according

---

[4] To see that, assume i is dummy at x. Then, by anonymity, j must be dummy at x for all $j \in N$. Hence, $\mathcal{W}_x = \{\emptyset\}$ which is a contradiction.

to $[\succsim]$. Formally, given $\mathcal{W} \in VbQ$, denote by $[\mathcal{W}]$ the equivalence class of $\mathcal{W}$ with respect to $\approx$; i.e.,

$$[\mathcal{W}] = \{\mathcal{V} \in VbQ \mid \mathcal{W} \approx \mathcal{V}\}.$$

By Corollary 1, $[\mathcal{W}]$ can be identified with the pair $(A_1^{\mathcal{W}}, A_n^{\mathcal{W}})$. Denote by $VbQ/\approx$ the set of all equivalence classes induced by $\approx$ on VbQ and consider the binary relation $[\succsim]$ on $VbQ/\approx$ defined as follows. For any pair $[\mathcal{W}],[\mathcal{V}] \in VbQ/\approx$, set $[\mathcal{V}][\succsim][\mathcal{W}]$ if and only if $\mathcal{V} \succsim \mathcal{W}$. Since $\succsim$ is a preorder on VbQ, it follows that $[\succsim]$ is a partial order on $VbQ/\approx$. Furthermore, by Corollary 1,

$$[\mathcal{V}][\succsim][\mathcal{W}] \text{ if and only if } A_1^{\mathcal{V}} \subset A_1^{\mathcal{W}} \text{ and } A_n^{\mathcal{V}} \subset A_n^{\mathcal{W}}.$$

We can now state and prove the main result of this subsection.

PROPOSITION 3: *Assume $n \geq 3$. Then, the pair $(VbQ/\approx,[\succsim])$ is a complete upper semilattice.*

PROOF:
Let $\{\emptyset\} \neq Z \subseteq VbQ/\approx$. Consider any voting by quotas $\mathcal{V} : \mathcal{P}^n \rightarrow 2^K$ such that

$$A_1^{\mathcal{V}} = \cap_{[\mathcal{W}]\in Z} A_1^{\mathcal{W}}, \quad A_n^{\mathcal{V}} = \cap_{[\mathcal{W}]\in Z} A_n^{\mathcal{W}},$$

and for all $x \notin A_1^{\mathcal{V}} \cup A_n^{\mathcal{V}}$ (if any) set $q_x^{\mathcal{V}}$ equal to any integer other than 1 and n (which exists because $n \geq 3$). Since $A_1^{\mathcal{W}} \cap A_n^{\mathcal{W}} = \{\emptyset\}$ for all $[\mathcal{W}] \in Z$, $\mathcal{V}$ is well defined as a voting by quotas. By Corollary 1 and the definition of $[\mathcal{V}]$, $\sup_{[\succsim]} Z = [\mathcal{V}]$. Thus, $(VbQ/\approx,[\succsim])$ is a complete upper semilattice. ∎

Example 2 and Figure 5 show, for any $n \geq 3$, the complete upper semilattice structure of $(VbQ/\approx,[\succsim])$ when $|K| = 3$.

Example 2: Assume $n \geq 3$ and let $K = \{x, y, z\}$ be the set of objects. Given $[\mathcal{W}] \in VbQ/\approx$, we identify the equivalence class $[\mathcal{W}]$ by the pair $(A_1^{\mathcal{W}}, A_n^{\mathcal{W}})$ and furthermore, we denote $\{x\}$ by x, $\{x, y\}$ by xy, and $\{x, y, z\}$ by xyz, and similarly for $\{y\},\{z\},\{x, z\}$, and $\{y, z\}$ (and $\{\emptyset\}$ by $\emptyset$). By Proposition 3, the set of equivalence classes of voting by quotas is a complete upper semilattice. Figure 5 represents this partial order where if two pairs $(A_1^{\mathcal{W}}, A_n^{\mathcal{W}})$ and $(A_1^{\mathcal{V}}, A_n^{\mathcal{V}})$ are directly connected, the one above is more manipulable than the one below and moreover, all connections that can be obtained by transitivity are omitted.

V. Comparing All Voting by Committees

In this section, we state and prove the main result of the paper which identifies a set of conditions under which two voting by committees (with or without dummy
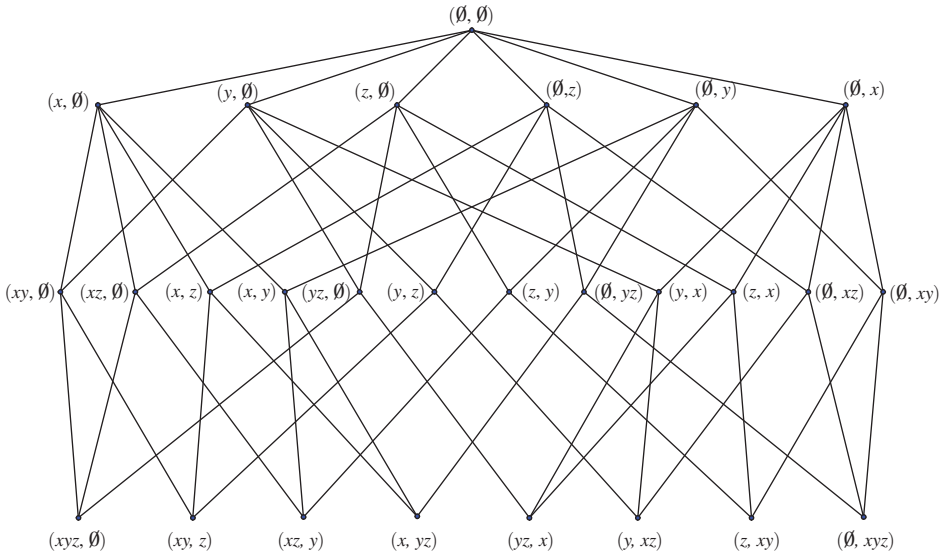
$(\emptyset, \emptyset)$

$(x, \emptyset)$ $(y, \emptyset)$ $(z, \emptyset)$ $(\emptyset, z)$ $(\emptyset, y)$ $(\emptyset, x)$

$(xy, \emptyset)$ $(xz, \emptyset)$ (x, z) (x, y) $(yz, \emptyset)$ (y, z) (z, y) $(\emptyset, yz)$ (y, x) (z, x) $(\emptyset, xz)$ $(\emptyset, xy)$

$(xyz, \emptyset)$ (xy, z) (xz, y) (x, yz) (yz, x) (y, xz) (z, xy) $(\emptyset, xyz)$

FIGURE 5

agents) are comparable according to their manipulability. But before doing so we state a remark and give a definition.

REMARK 5: *Let* $\mathcal{W} : \mathcal{P}^n \to 2^K$ *be a voting by committees and let* $x \in K$ *and* $i \in N$ *be such that* $x \in Du_i^{\mathcal{W}}$. *Then,* $x \in \overline{Ve_i^{\mathcal{W}}}$ *and* $x \in \overline{De_i^{\mathcal{W}}}$.

We say that a voting by committees $\mathcal{W} : \mathcal{P}^n \to 2^K$ is *dictatorial* if there exists $i \in N$ such that for all $x \in K$, $\mathcal{W}_x^m = \{\{i\}\}$. Obviously, all dictatorial voting by committees are strategy-proof in any domain and hence, they are less manipulable than any other voting by committees. Thus, it is not necessary to include them in the main result of the paper that we state below as Theorem 2.

THEOREM 2: *Let* $\mathcal{V} : \mathcal{P}^n \to 2^K$ *and* $\mathcal{W} : \mathcal{P}^n \to 2^K$ *be two non-dictatorial voting by committees. Then,* $\mathcal{V} \succsim \mathcal{W}$ *if and only if for all* $i \in N$,

(A) $$De_i^{\mathcal{V}} \subset De_i^{\mathcal{W}} \text{ and } Ve_i^{\mathcal{V}} \subset Ve_i^{\mathcal{W}} \text{ and } Du_i^{\mathcal{V}} \subset Du_i^{\mathcal{W}} \text{ or}$$

(B) $$(De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}}) \cup (Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}}) = \{x\} = \overline{Du_i^{\mathcal{W}}} \text{ or}$$

(C) $$\overline{De_i^{\mathcal{W}} \cap Ve_i^{\mathcal{W}}} = \{x\} = Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}}.$$

Before proving Theorem 2 we exhibit in Example 3 two voting by committees that illustrate conditions (A), (B), and (C) above.

Example 3: Let $N = \{1, 2, 3\}$ be the set of agents and $K = \{x, y, z\}$ the set of objects. Consider two voting by committees $\mathcal{V}$ and $\mathcal{W}$ defined by the following (minimal) committees:

$$\mathcal{V}_x^m = \{\{1, 2\}, \{3\}\}, \qquad \mathcal{W}_x^m = \{\{3\}\},$$

$$\mathcal{V}_y^m = \{\{1\}, \{2\}\}, \qquad \mathcal{W}_y^m = \{\{1\}, \{2, 3\}\},$$

$$\mathcal{V}_z^m = \{\{3\}\}, \qquad \mathcal{W}_z^m = \{\{3\}\}.$$

We consider the decisive, vetoer, and dummy sets of objects for each of the three agents separately.

For agent 1, $De_1^{\mathcal{V}} = De_1^{\mathcal{W}} = \{y\}$, $Ve_1^{\mathcal{V}} = Ve_1^{\mathcal{W}} = \{\emptyset\}$, $Du_1^{\mathcal{V}} = \{z\}$, and $Du_1^{\mathcal{W}} = \{x, z\}$. Thus,

$$De_1^{\mathcal{V}} = De_1^{\mathcal{W}} \text{ and } Ve_1^{\mathcal{V}} = Ve_1^{\mathcal{W}} \text{ and } Du_1^{\mathcal{V}} \subset Du_1^{\mathcal{W}},$$

which means that condition (A) holds for agent 1. Observe that conditions (B) and (C) do not hold for agent 1 since $\overline{Du_1^{\mathcal{W}}} = \{y\}$ and $(De_1^{\mathcal{V}} \backslash De_1^{\mathcal{W}}) \cup (Ve_1^{\mathcal{V}} \backslash Ve_1^{\mathcal{W}}) = \{\emptyset\}$, and $\overline{De_1^{\mathcal{W}} \cap Ve_1^{\mathcal{W}}} = K$, respectively.

For agent 2, $De_2^{\mathcal{V}} = \{y\}$, $De_2^{\mathcal{W}} = \{\emptyset\}$, $Ve_2^{\mathcal{V}} = Ve_2^{\mathcal{W}} = \{\emptyset\}$, $Du_2^{\mathcal{V}} = \{z\}$, and $Du_2^{\mathcal{W}} = \{x, z\}$. Hence, $De_2^{\mathcal{V}} \backslash De_2^{\mathcal{W}} = \{y\}$, $Ve_2^{\mathcal{V}} \backslash Ve_2^{\mathcal{W}} = \{\emptyset\}$, and $\overline{Du_2^{\mathcal{W}}} = \{y\}$. Thus,

$$(De_2^{\mathcal{V}} \backslash De_2^{\mathcal{W}}) \cup (Ve_2^{\mathcal{V}} \backslash Ve_2^{\mathcal{W}}) = \{y\} = \overline{Du_2^{\mathcal{W}}},$$

which means that condition (B) holds for agent 2. Observe that conditions (A) and (C) do not hold for agent 2 since $De_2^{\mathcal{V}} \not\subseteq De_2^{\mathcal{W}}$, and $\overline{De_2^{\mathcal{W}} \cap Ve_2^{\mathcal{W}}} = K$, respectively.

For agent 3, $De_3^{\mathcal{V}} = De_3^{\mathcal{W}} = \{x, z\}$, $Ve_3^{\mathcal{V}} = \{z\}$, $Ve_3^{\mathcal{W}} = \{x, z\}$, $Du_3^{\mathcal{V}} = \{y\}$ and $Du_3^{\mathcal{W}} = \{\emptyset\}$. Hence, $De_3^{\mathcal{W}} \cap Ve_3^{\mathcal{W}} = \{x, z\}$ and $Du_3^{\mathcal{V}} \backslash Du_3^{\mathcal{W}} = \{y\}$. Thus,

$$\overline{De_3^{\mathcal{W}} \cap Ve_3^{\mathcal{W}}} = \{y\} = Du_3^{\mathcal{V}} \backslash Du_3^{\mathcal{W}},$$

which means that condition (C) holds for agent 3. Observe that conditions (A) and (B) do not hold for agent 3 since $Du_3^{\mathcal{V}} \not\subseteq Du_3^{\mathcal{W}}$, and $\overline{Du_3^{\mathcal{W}}} = K$, respectively.

Thus, by Theorem 2, $\mathcal{V} \succsim \mathcal{W}$. Moreover, $Du_1^{\mathcal{W}} \backslash Du_1^{\mathcal{V}} = \{x\}$, $\overline{Du_1^{\mathcal{V}}} = \{x, y\}$, and $\overline{De_1^{\mathcal{V}} \cap Ve_1^{\mathcal{V}}} = \{x, y, z\}$. Hence, by Theorem 2, $\mathcal{W} \succsim \mathcal{V}$ does not hold. Thus, $\mathcal{V} \succ \mathcal{W}$.

We next exhibit a preference P_1 such that $P_1 \in \mathcal{M}_1^{\mathcal{V}}$ but $P_1 \notin \mathcal{M}_1^{\mathcal{W}}$. Let P_1 be the following preference:

$$\{y\} P_1 \{x, y, z\} P_1 \{y, z\} P_1 \{y, x\} P_1 \{\emptyset\} P_1 \{x\} P_1 \{z\} P_1 \{x, z\}.$$

Let $(P_1', P_2', P_3') \in \mathcal{P}^3$ be any preference profile with the property that $t(P_1') = \{x, y\}$, $t(P_2') = \{x\}$, and $t(P_3') = \{z\}$. Then,

$$\mathcal{V}(P_1', P_2', P_3') = \{x, y, z\} P_1 \{y, z\} = \mathcal{V}(P_1, P_2', P_3')$$

and, hence, $P_1 \in \mathcal{M}_1^{\mathcal{V}}$. On the other hand, consider any arbitrary preference profile $(P_1^*, P_2, P_3) \in \mathcal{P}^3$. Then, for some (potentially empty subset) $A \subset K \backslash \{y\}$,

$$\mathcal{W}(P_1, P_2, P_3) = \{y\} \cup A \text{ and } A \subset \mathcal{W}(P_1^*, P_2, P_3) \subset \{y\} \cup A.$$

Hence, by the definition of P_1,

$$\mathcal{W}(P_1, P_2, P_3) R_1 \mathcal{W}(P_1^*, P_2, P_3).$$

Thus, $P_1 \notin \mathcal{M}_1^{\mathcal{W}}$.

PROOF OF THEOREM 2:

$\Leftarrow$) To prove sufficiency, fix $i \in N$ and assume that $P_i \in \mathcal{M}_i^{\mathcal{W}}$. We want to show that $P_i \in \mathcal{M}_i^{\mathcal{V}}$. By Lemma 4, there exist $A \in o^{\mathcal{W}}(P_i)$, $T_1 \subset t(P_i) \cap A \cap \overline{Du_i^{\mathcal{W}}}$, and $T_2 \subset \overline{t(P_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{W}}}$ such that

$$(14) \qquad\qquad (A \backslash T_1) \cup T_2 P_i A.$$

We distinguish among three cases, depending on whether (A), (B), or (C) holds.

Case 1: Suppose that (A) holds; namely, $De_i^{\mathcal{V}} \subset De_i^{\mathcal{W}}$, $Ve_i^{\mathcal{V}} \subset Ve_i^{\mathcal{W}}$, and $Du_i^{\mathcal{V}} \subset Du_i^{\mathcal{W}}$. Hence,

$$(15) \qquad\quad T_1 \subset t(P_i) \cap A \cap \overline{Du_i^{\mathcal{V}}} \text{ and } T_2 \subset \overline{t(P_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{V}}}.$$

By Lemma 2, $A \in o^{\mathcal{V}}(P_i)$. By (14), (15), and Lemma 4, $P_i \in \mathcal{M}_i^{\mathcal{V}}$.

Case 2: Suppose that (B) holds; namely, $(De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}}) \cup (Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}}) = \{x\}$ $= \overline{Du_i^{\mathcal{W}}}$. Since $x \in (De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}}) \cup (Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}})$, either $x \in De_i^{\mathcal{V}}$ or $x \in Ve_i^{\mathcal{V}}$. By Remark 5, $x \in \overline{Du_i^{\mathcal{V}}}$. Hence,

$$(16) \qquad\quad T_1 \subset t(P_i) \cap A \cap \overline{Du_i^{\mathcal{V}}} \text{ and } T_2 \subset \overline{t(P_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{V}}}.$$

By (14), (16), and Lemma 4, it is sufficient to show that $A \in o^{\mathcal{V}}(P_i)$; or equivalently, by Lemma 2, that $t(P_i) \cap De_i^{\mathcal{V}} \subset A$ and $A \subset t(P_i) \cup \overline{Ve_i^{\mathcal{V}}}$. We distinguish between two subcases.

Case 2.1: Suppose $x \notin t(P_i)$. Then, since $De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}} \subset \{x\}$, $t(P_i) \cap De_i^{\mathcal{V}}$ $\subset t(P_i) \cap De_i^{\mathcal{W}}$ holds. By assumption, $A \in o^{\mathcal{W}}(P_i)$. Hence, $t(P_i) \cap De_i^{\mathcal{W}} \subset A$. Thus,

$$
(17) \qquad\qquad t(P_i) \cap De_i^{\mathcal{V}} \subset A.
$$

Since $x \notin t(P_i)$ and $\overline{Du_i^{\mathcal{W}}} = \{x\}$ hold, we have that $t(P_i) \cap \overline{Du_i^{\mathcal{W}}} = \{\emptyset\}$ and $\overline{t(P_i)} \cap \overline{Du_i^{\mathcal{W}}} = \{x\}$. Moreover, since $T_1 \subset t(P_i) \cap A \cap \overline{Du_i^{\mathcal{W}}}$, $T_2 \subset \overline{t(P_i)} \cap \overline{A}$ $\cap \overline{Du_i^{\mathcal{W}}}$, and (14) hold, we have that $T_1 = \{\emptyset\}$ and $T_2 = \{x\}$ hold as well. Since $T_2 \subset \overline{A}$, $x \notin A$. Hence, since $A \in o^{\mathcal{W}}(P_i)$, $A \subset t(P_i) \cup (\overline{Ve_i^{\mathcal{W}}} \backslash \{x\})$. But by the hypothesis that $Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}} \subset \{x\}$, we have

$$
(18) \qquad\qquad A \subset t(P_i) \cup \overline{Ve_i^{\mathcal{V}}}.
$$

Case 2.2: Suppose $x \in t(P_i)$. Then, since $\overline{Du_i^{\mathcal{W}}} = \{x\}$, $t(P_i) \cap \overline{Du_i^{\mathcal{W}}} = \{x\}$ and $\overline{t(P_i)} \cap \overline{Du_i^{\mathcal{W}}} = \{\emptyset\}$. Since $T_1 \subset t(P_i) \cap A \cap \overline{Du_i^{\mathcal{W}}}$, $T_2 \subset \overline{t(P_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{W}}}$, and (14) hold, we have that $T_1 = \{x\}$ and $T_2 = \{\emptyset\}$ hold as well. Since $T_1 \subset A$, $x \in A$. Hence, since $A \in o^{\mathcal{W}}(P_i)$, $(t(P_i) \cap De_i^{\mathcal{W}}) \cup \{x\} \subset A$. But by the hypothesis that $De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}} \subset \{x\}$, we have

$$
(19) \qquad\qquad t(P_i) \cap De_i^{\mathcal{V}} \subset A.
$$

Since $Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}} \subset \{x\}$ and $x \in t(P_i)$, $t(P_i) \cup \overline{Ve_i^{\mathcal{W}}} \subset t(P_i) \cup \overline{Ve_i^{\mathcal{V}}}$. Furthermore, since $A \in o^{\mathcal{W}}(P_i)$, $A \subset t(P_i) \cup \overline{Ve_i^{\mathcal{W}}}$. Thus,

$$
(20) \qquad\qquad A \subset t(P_i) \cup \overline{Ve_i^{\mathcal{V}}}.
$$

Case 3: Suppose that (C) holds; namely, $\overline{De_i^{\mathcal{W}}} \cap Ve_i^{\mathcal{W}} = \{x\} = Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}}$. Since $x \in Du_i^{\mathcal{V}}$, by Remark 5, $x \in \overline{Ve_i^{\mathcal{V}}}$ and $x \in \overline{De_i^{\mathcal{V}}}$. Hence,

$$
(21) \qquad De_i^{\mathcal{V}} \subset K \backslash \{x\} = \overline{\overline{De_i^{\mathcal{W}} \cap Ve_i^{\mathcal{W}}}} = \overline{De_i^{\mathcal{W}}} \cap \overline{Ve_i^{\mathcal{W}}} \subset \overline{De_i^{\mathcal{W}}} \text{ and}
$$

$$
(22) \qquad \overline{Ve_i^{\mathcal{W}}} \subset \overline{De_i^{\mathcal{W}}} \cup \overline{Ve_i^{\mathcal{W}}} = \overline{De_i^{\mathcal{W}} \cap Ve_i^{\mathcal{W}}} = \{x\} \subset \overline{Ve_i^{\mathcal{V}}}.
$$

By Lemma 3, (21), and (22), $o^{\mathcal{W}}(P_i) \subset o^{\mathcal{V}}(P_i)$. Hence, and since $A \in o^{\mathcal{W}}(P_i)$, $A \in o^{\mathcal{V}}(P_i)$. We want to prove that $P_i \in \mathcal{M}_i^{\mathcal{V}}$. By Lemma 4 and (14), it is sufficient to show that

$$
(23) \qquad T_1 \subset t(P_i) \cap A \cap \overline{Du_i^{\mathcal{V}}} \text{ and } T_2 \subset \overline{t(P_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{V}}}
$$

hold. We distinguish between two subcases.

Case 3.1: Suppose $x \in t(P_i)$. As (C) holds, $\overline{Ve_i^{\mathcal{W}}} \subset \{x\}$ and $K \backslash \{x\} \subset De_i^{\mathcal{W}}$. Hence, by Lemma 2, $o^{\mathcal{W}}(P_i) \subset \{t(P_i) \backslash \{x\}, t(P_i)\}$. By (14), $A = t(P_i) \backslash \{x\}$. Then, since $x \notin A$ and $x \in t(P_i)$, $x \notin T_1$ and $x \notin T_2$ hold, respectively. Hence, and since $T_1 \subset t(P_i) \cap A \cap \overline{Du_i^{\mathcal{W}}}$, $T_2 \subset \overline{t(P_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{W}}}$, and $Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}} = \{x\}$ hold, we have that $T_1 \subset t(P_i) \cap A \cap \overline{Du_i^{\mathcal{V}}}$ and $T_2 \subset \overline{t(P_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{V}}}$ hold as well.

Case 3.2: Suppose $x \notin t(P_i)$. As (C) holds, $\overline{Ve_i^{\mathcal{W}}} \subset \{x\}$ and $K \backslash \{x\} \subset De_i^{\mathcal{W}}$. Hence, by Lemma 2, $o^{\mathcal{W}}(P_i) \subset \{t(P_i), t(P_i) \cup \{x\}\}$. By (14), $A = t(P_i) \cup \{x\}$. Then, since $x \notin t(P_i)$ and $x \in A$, $x \notin T_1$ and $x \notin T_2$ hold, respectively. Hence, and since $T_1 \subset t(P_i) \cap A \cap \overline{Du_i^{\mathcal{W}}}$, $T_2 \subset \overline{t(P_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{W}}}$, and $Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}} = \{x\}$ hold, we have that $T_1 \subset t(P_i) \cap A \cap \overline{Du_i^{\mathcal{V}}}$ and $T_2 \subset \overline{t(P_i)} \cap \overline{A} \cap \overline{Du_i^{\mathcal{V}}}$ hold as well.

$\Rightarrow$) To prove necessity, assume $\mathcal{V}$ and $\mathcal{W}$ are non-dictatorial voting by committees and $\mathcal{V} \succsim \mathcal{W}$; i.e.,

$$(24) \qquad\qquad \mathcal{M}_i^{\mathcal{W}} \subset \mathcal{M}_i^{\mathcal{V}} \text{ for all } i \in N.$$

Fix $i \in N$ and assume that (A) and (B) do not hold. We will show that (C) holds; i.e.,

$$\overline{De_i^{\mathcal{W}}} \cap \overline{Ve_i^{\mathcal{W}}} = \{x\} = Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}}.$$

Claim 1: If $\left| Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}} \right| \geq 2$ and $\mathcal{V} \succsim \mathcal{W}$, then i is a dictator in $\mathcal{W}$.

PROOF OF CLAIM 1:

Assume $x \in Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}}$ and suppose that there exists $y \in K$ such that $y \notin De_i^{\mathcal{W}}$. Since $\left| Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}} \right| \geq 2$, we can assume without loss of generality that $y \neq x$. Let $P_i' \in \mathcal{P}$ be any preference such that $t(P_i') = \{x, y\}$ and satisfying in addition the following properties:

(i) $\{x, y\} P_i' \{y\} P_i' \{\emptyset\} P_i' \{x\} P_i' A$,

　　　for all $A \notin \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$,

(ii) $A R_i' B$ if $B \cap t(P_i') \subset A \cap t(P_i')$ and $A \cap \overline{t(P_i')} \subset B \cap \overline{t(P_i')}$,

　　　for all $A, B \notin \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$.

The preference P_i' can be seen as having two separate blocks. The first one orders the subsets $\{x, y\}, \{y\}, \{\emptyset\}, \{x\}$ in a nonseparable way. All other subsets are dispreferred to each of these four subsets but, among those that are different to these four, any set A that can be obtained from B by adding objects in $t(P_i')$ and deleting objects in $\overline{t(P_i')}$ is preferred to B. Moreover, any pair of subsets of objects that are unrestricted by conditions (i) and (ii) can be ordered arbitrarily by P_i'.

Since $y \notin De_i^{\mathcal{W}}$, $\{x\} \in o^{\mathcal{W}}(P_i')$. Set $A = \{x\}$, $T_2 = \{\emptyset\}$, and $T_1 = \{x\}$. Observe that $A \in o^{\mathcal{W}}(P_i')$ and, by (i) in the definition of P_i', $(A \backslash T_1) \cup T_2 = \{\emptyset\} P_i' \{x\} = A$. Hence, by Lemma 4, $P_i' \in \mathcal{M}_i^{\mathcal{W}}$. Take now any $A \in o^{\mathcal{V}}(P_i')$. Let $T_1 \subset t(P_i') \cap A \cap \overline{Du_i^{\mathcal{Y}}}$ and $T_2 \subset \overline{t(P_i')} \cap \overline{A} \cap \overline{Du_i^{\mathcal{Y}}}$ be arbitrary. We can assume that either $T_1 \neq \{\emptyset\}$ or $T_2 \neq \{\emptyset\}$; otherwise, $A = (A \backslash T_1) \cup T_2$ and $AI_i'(A \backslash T_1) \cup T_2$ trivially. Since $x \in \overline{Du_i^{\mathcal{Y}}}$ and $t(P_i') = \{x, y\}$, $T_1 \subset \{y\}$ and $x, y \notin T_2$. We distinguish between two cases.

Case 1.a: If $(A \backslash T_1) \cup T_2 \notin \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$, then, by (i) and (ii) in the definition of P_i', $AR_i'(A \backslash T_1) \cup T_2$.

Case 2.a: If $(A \backslash T_1) \cup T_2 \in \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$, then $T_2 = \{\emptyset\}$ and $T_1 = \{y\}$. Hence, $y \in A$ and $(A \backslash T_1) \cup T_2 = A \backslash \{y\}$. Since $(A \backslash T_1) \cup T_2 \in \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$ and $(A \backslash T_1) \cup T_2 = A \backslash \{y\}$, either $A = \{y\}$ and $(A \backslash T_1) \cup T_2 = \{\emptyset\}$ or else $A = \{x, y\}$ and $(A \backslash T_1) \cup T_2 = \{x\}$. But then, and according to (i) in the definition of P_i', $AP_i'(A \backslash T_1) \cup T_2$ must hold.

Hence, by Lemma 4, $P_i' \notin \mathcal{M}_i^{\mathcal{V}}$ in both cases. Thus, $P_i' \in \mathcal{M}_i^{\mathcal{W}} \backslash \mathcal{M}_i^{\mathcal{V}}$, which contradicts that $\mathcal{V} \succsim \mathcal{W}$. Therefore, we cannot suppose that there exists $y \in K$ such that $y \notin De_i^{\mathcal{W}}$. Thus,

$$(25) \qquad\qquad\qquad De_i^{\mathcal{W}} = K.$$

Now, suppose that there exists $y \in K$ such that $y \in \overline{Ve_i^{\mathcal{W}}}$. Since $x \in Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}}$ and $\left| Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}} \right| \geq 2$, we can assume without loss of generality that $y \neq x$. Let $P_i'' \in \mathcal{P}$ be any preference such that $t(P_i'') = \{x\}$ and satisfying in addition the following properties:

(i) $\{x\} P_i'' \{\emptyset\} P_i'' \{y\} P_i'' \{x, y\} P_i'' A$,

        for all $A \notin \{\{x\}, \{\emptyset\}, \{y\}, \{x, y\}\}$,

(ii) $AR_i'' B$ if $B \cap t(P_i'') \subset A \cap t(P_i'')$ and $A \cap \overline{t(P_i'')} \subset B \cap \overline{t(P_i'')}$,

        for all $A, B \notin \{\{x\}, \{\emptyset\}, \{y\}, \{x, y\}\}$.

The preference P_i'' can also be seen as having two separate blocks. The first one orders the subsets $\{x\}, \{\emptyset\}, \{y\}, \{x, y\}$ in a nonseparable way. All other subsets are dispreferred to each of these four subsets but, among those that are different to these four, any set A that can be obtained from B by adding objects in $t(P_i'')$ and deleting objects in $\overline{t(P_i'')}$ is preferred to B. Moreover, any pair of subsets of objects that are unrestricted by conditions (i) and (ii) can be ordered arbitrarily by P_i''.

Since $y \in \overline{Ve_i^{\mathcal{W}}}$, $\{x, y\} \in o^{\mathcal{W}}(P_i'')$. Set $A = \{x, y\}$, $T_1 = \{x\}$, and $T_2 = \{\emptyset\}$. Observe that $A \in o^{\mathcal{W}}(P_i'')$ and, by (i) in the definition of P_i'', $(A \backslash T_1) \cup T_2$

$= \{y\} P_i''\{x, y\} = A$. Hence, by Lemma 4, $P_i'' \in \mathcal{M}_i^{\mathcal{W}}$. Take now any $A \in o^{\mathcal{V}}(P_i'')$. Let $T_1 \subset t(P_i'') \cap A \cap \overline{Du_i}^{\mathcal{V}}$ and $T_2 \subset \overline{t(P_i'')} \cap \overline{A} \cap \overline{Du_i}^{\mathcal{V}}$ be arbitrary. We can assume that either $T_1 \neq \{\emptyset\}$ or $T_2 \neq \{\emptyset\}$; otherwise, $A = (A \backslash T_1) \cup T_2$ and $AI_i''(A \backslash T_1) \cup T_2$ trivially. Since $x \in \overline{Du_i}^{\mathcal{V}}$ and $t(P_i'') = \{x\}$, $T_1 = \{\emptyset\}$ and $x \notin T_2$. We distinguish between two cases.

Case 1.b: If $(A \backslash T_1) \cup T_2 \notin \{\{x\}, \{\emptyset\}, \{y\}, \{x, y\}\}$, then, by (i) and (ii) in the definition of P_i'', $AR_i''(A \backslash T_1) \cup T_2$.

Case 2.b: If $(A \backslash T_1) \cup T_2 \in \{\{x\}, \{\emptyset\}, \{y\}, \{x, y\}\}$, then $T_2 \subset \{x, y\}$. Since $T_1 = \{\emptyset\}$, $x \notin T_2$, and $T_2 \neq \{\emptyset\}$, $T_2 = \{y\}$, and $y \notin A$. Then, either $A = \{\emptyset\}$ and $(A \backslash T_1) \cup T_2 = \{y\}$ or else $A = \{x\}$ and $(A \backslash T_1) \cup T_2 = \{x, y\}$. But then, and according to (i) in the definition of P_i'', $AP_i''(A \backslash T_1) \cup T_2$ must hold.

Hence, by Lemma 4, $P_i'' \notin \mathcal{M}_i^{\mathcal{V}}$ in both cases. Thus, $P_i'' \in \mathcal{M}_i^{\mathcal{W}} \backslash \mathcal{M}_i^{\mathcal{V}}$, which contradicts that $\mathcal{V} \succsim \mathcal{W}$. Therefore, we can not suppose that there exists $y \in K$ such that $y \in \overline{Ve_i}^{\mathcal{W}}$. Thus,

$$(26) \qquad\qquad \overline{Ve_i}^{\mathcal{W}} = \{\emptyset\}.$$

By (25) and (26), i is a dictator in $\mathcal{W}$. This finishes the proof of Claim 1. ∎

By hypothesis, $\mathcal{W}$ is non dictatorial. By Claim 1,

$$\left| Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}} \right| \leq 1.$$

We distinguish between two cases.

Case I: Suppose that $Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}} = \{x\}$. We want to show that $\overline{De_i}^{\mathcal{W}} \cap \overline{Ve_i}^{\mathcal{W}} = \{x\}$. As $x \in Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}}$, applying the same argument than the one used in the proof of Claim 1, we can obtain that $\overline{De_i}^{\mathcal{W}} \subset \{x\}$ and $\overline{Ve_i}^{\mathcal{W}} \subset \{x\}$. Therefore,

$$(27) \qquad\qquad \overline{De_i}^{\mathcal{W}} \cap \overline{Ve_i}^{\mathcal{W}} = \overline{De_i}^{\mathcal{W}} \cup \overline{Ve_i}^{\mathcal{W}} \subset \{x\}.$$

Furthermore, since i is not a dictator in $\mathcal{W}$, $\overline{De_i}^{\mathcal{W}} \neq \{\emptyset\}$ or $\overline{Ve_i}^{\mathcal{W}} \neq \{\emptyset\}$. Hence, by (27),

$$\overline{De_i}^{\mathcal{W}} \cap \overline{Ve_i}^{\mathcal{W}} = \overline{De_i}^{\mathcal{W}} \cup \overline{Ve_i}^{\mathcal{W}} = \{x\} = Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}}.$$

Case II: Suppose that $Du_i^{\mathcal{V}} \backslash Du_i^{\mathcal{W}} = \{\emptyset\}$. We will obtain a contradiction. Since (A) does not hold there exists $x \in K$ such that $x \in (De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}}) \cup (Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}})$.

Claim 2: $\overline{Du_i}^{\mathcal{W}} = \{x\}$.

PROOF OF CLAIM 2:

We distinguish between two cases.

Case 1: Assume $x \in De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}}$ and there exists $y \neq x$ such that $y \in \overline{Du_i^{\mathcal{W}}}$. Let $P_i''' \in \mathcal{P}$ be any preference such that $t(P_i''') = \{x, y\}$ and satisfying in addition the following properties:

(i) $\{x, y\} P_i''' \{x\} P_i''' \{\emptyset\} P_i''' \{y\} P_i''' A$

for all $A \notin \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$,

(ii) $A R_i''' B$ if $B \cap t(P_i''') \subset A \cap t(P_i''')$ and $A \cap \overline{t(P_i''')} \subset B \cap \overline{t(P_i''')}$,

for all $A, B \notin \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$.

As previously the preference P_i''' can also be seen as having two separate blocks (we omit the details).

Since $x \in De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}}$, $\{y\} \in o^{\mathcal{W}}(P_i''')$. Set $A = \{y\}$, $T_2 = \{\emptyset\}$, and $T_1 = \{y\}$. Observe that $A \in o^{\mathcal{W}}(P_i''')$ and, by (i) in the definition of P_i''', $(A \backslash T_1) \cup T_2 = \{\emptyset\} P_i''' \{y\} = A$. Hence, by Lemma 4, $P_i''' \in \mathcal{M}_i^{\mathcal{W}}$. Take now any $A \in o^{\mathcal{V}}(P_i''')$. Since $x \in De_i^{\mathcal{V}} \backslash De_i^{\mathcal{W}}$ and $x \in t(P_i''')$, $x \in A$. Let $T_1 \subset t(P_i''') \cap A \cap \overline{Du_i^{\mathcal{V}}}$ and $T_2 \subset \overline{t(P_i''')} \cap \overline{A} \cap \overline{Du_i^{\mathcal{V}}}$ be arbitrary. We can assume that either $T_1 \neq \{\emptyset\}$ or $T_2 \neq \{\emptyset\}$; otherwise, $A = (A \backslash T_1) \cup T_2$ and $A I_i''' (A \backslash T_1) \cup T_2$ trivially. Since $t(P_i''') = \{x, y\}$, $T_1 \subset \{x, y\}$ and $x, y \notin T_2$. Now we will consider two subcases.

Case 1.1: If $(A \backslash T_1) \cup T_2 \notin \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$, then by (i) and (ii) in the definition of P_i''', $A R_i''' (A \backslash T_1) \cup T_2$.

Case 1.2: If $(A \backslash T_1) \cup T_2 \in \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$, then $T_2 = \{\emptyset\}$; otherwise, if there exists $z \in T_2 \backslash \{x, y\}$, then $z \in (A \backslash T_1) \cup T_2$ and therefore $(A \backslash T_1) \cup T_2 \notin \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$. Hence, $T_1 \neq \{\emptyset\}$ and $(A \backslash T_1) \cup T_2 = A \backslash T_1$. We distinguish among three different subcases.

Case 1.2.1: $T_1 = \{x\}$. Then, $x \in A$ and $(A \backslash T_1) \cup T_2 = A \backslash \{x\}$. Since $(A \backslash T_1) \cup T_2 \in \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$ and $(A \backslash T_1) \cup T_2 = A \backslash \{x\}$, either $(A \backslash T_1) \cup T_2 = \{y\}$ and $A = \{x, y\}$ or else $(A \backslash T_1) \cup T_2 = \{\emptyset\}$ and $A = \{x\}$ (since $x \in A$). But in both cases, by (i) in the definition of P_i''', $A P_i''' (A \backslash T_1) \cup T_2$.

Case 1.2.2: $T_1 = \{y\}$. Then, $y \in A$ and $(A \backslash T_1) \cup T_2 = A \backslash \{y\}$. Since $(A \backslash T_1) \cup T_2 \in \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$ and $(A \backslash T_1) \cup T_2 = A \backslash \{y\}$, $(A \backslash T_1) \cup T_2 = \{x\}$ and $A = \{x, y\}$ (since $x \in A$). But then, by (i) in the definition of P_i''', $A P_i''' (A \backslash T_1) \cup T_2$.

Case 1.2.3: $T_1 = \{x, y\}$. Then, $x, y \in A$ and $(A \backslash T_1) \cup T_2 = A \backslash \{y, x\}$. Since $(A \backslash T_1) \cup T_2 \in \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$ and $(A \backslash T_1) \cup T_2 = A \backslash \{y, x\}$, $(A \backslash T_1) \cup$

$T_2 = \{\emptyset\}$ and $A = \{x, y\}$ (since $x, y \in A$). But then, by (i) in the definition of P_i''', $A P_i'''(A \backslash T_1) \cup T_2$.

Hence, by Lemma 4, $P_i''' \notin \mathcal{M}_i^{\mathcal{V}}$. Thus, $P_i''' \in \mathcal{M}_i^{\mathcal{W}} \backslash \mathcal{M}_i^{\mathcal{V}}$ holds, which contradicts that $\mathcal{V} \succsim \mathcal{W}$.

Case 2: Assume $x \in Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}}$ and there exists $y \neq x$ such that $y \in \overline{Du_i^{\mathcal{W}}}$. Let $P_i'''' \in P$ be any preference such that $t(P_i'''') = \{y\}$ and satisfying in addition the following properties:

(i) $\{y\} P_i''''\{\emptyset\} P_i''''\{x\} P_i''''\{x, y\} P_i''''A$

for all $A \notin \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$,

(ii) $A R_i'''' B$ if $B \cap t(P_i'''') \subset A \cap t(P_i'''')$ and $A \cap \overline{t(P_i'''')} \subset B \cap \overline{t(P_i'''')}$,

for all $A, B \notin \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$.

As previously the non-separable preference P_i'''' can also be seen as having two separate blocks (we omit the details).

Since $x \in Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}}$, $\{x, y\} \in o^{\mathcal{W}}(P_i'''')$. Set $A = \{x, y\}$, $T_1 = \{y\}$, and $T_2 = \{\emptyset\}$. Observe that $A \in o^{\mathcal{W}}(P_i'''')$ and, by (i) in the definition of P_i'''', $(A \backslash T_1) \cup T_2 = \{x\} P_i''''\{x, y\} = A$. Hence, by Lemma 4, $P_i'''' \in \mathcal{M}_i^{\mathcal{W}}$. Take now any $A \in o^{\mathcal{V}}(P_i'''')$. Since $x \in Ve_i^{\mathcal{V}} \backslash Ve_i^{\mathcal{W}}$ and $x \notin t(P_i'''')$, $x \notin A$. Let $T_1 \subset t(P_i'''') \cap A \cap \overline{Du_i^{\mathcal{V}}}$ and $T_2 \subset \overline{t(P_i'''')} \cap \overline{A} \cap Du_i^{\mathcal{V}}$ be arbitrary. We can assume that either $T_1 \neq \{\emptyset\}$ or $T_2 \neq \{\emptyset\}$; otherwise, $A = (A \backslash T_1) \cup T_2$ and $A I_i''''(A \backslash T_1) \cup T_2$ trivially. Since $t(P_i'''') = \{y\}$, $T_1 \subset \{y\}$ and $y \notin T_2$. We distinguish between two subcases.

Case 2.1: If $(A \backslash T_1) \cup T_2 \notin \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$, then by (i) and (ii) in the definition of P_i'''', $A R_i''''(A \backslash T_1) \cup T_2$.

Case 2.2: If $(A \backslash T_1) \cup T_2 \in \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$, then $T_2 \subset \{x\}$. We distinguish among three different subcases.

Case 2.2.1: $T_1 = \{\emptyset\}$ and $T_2 = \{x\}$. Then, $(A \backslash T_1) \cup T_2 = A \cup \{x\}$. Since $(A \backslash T_1) \cup T_2 \in \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$ and $(A \backslash T_1) \cup T_2 = A \cup \{x\}$, either $(A \backslash T_1) \cup T_2 = \{x, y\}$ and $A = \{y\}$ or else $(A \backslash T_1) \cup T_2 = \{x\}$ and $A = \{\emptyset\}$ (since $x \notin A$). But then, by (i) in the definition of P_i'''', $A P_i''''(A \backslash T_1) \cup T_2$.

Case 2.2.2: $T_1 = \{y\}$ and $T_2 = \{\emptyset\}$. Then, $y \in A$ and $(A \backslash T_1) \cup T_2 = A \backslash \{y\}$. Since $(A \backslash T_1) \cup T_2 \in \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$ and $(A \backslash T_1) \cup T_2 = A \backslash \{y\}$, either $(A \backslash T_1) \cup T_2 = \{\emptyset\}$ and $A = \{y\}$ or else $(A \backslash T_1) \cup T_2 = \{x\}$ and $A = \{y\}$ (since $x \notin A$). But then, by (i) in the definition of P_i'''', $A P_i''''(A \backslash T_1) \cup T_2$.

Case 2.2.3: $T_1 = \{y\}$ and $T_2 = \{x\}$. Then, $y \in A$ and $(A\backslash T_1) \cup T_2 = A\backslash\{y\} \cup \{x\}$. Since $(A\backslash T_1) \cup T_2 \in \{\{x, y\}, \{y\}, \{\emptyset\}, \{x\}\}$ and $(A\backslash T_1) \cup T_2 = A\backslash\{y\} \cup \{x\}$, $(A\backslash T_1) \cup T_2 = \{x\}$ and $A = \{y\}$ (since $x \notin A$). But then, by (i) in the definition of P_i'''', AP_i'''' $(A\backslash T_1) \cup T_2$.

Hence, by Lemma 4, $P_i'''' \notin \mathcal{M}_i^{\mathcal{V}}$. Thus, $P_i'''' \in \mathcal{M}_i^{\mathcal{W}} \backslash \mathcal{M}_i^{\mathcal{V}}$ holds, which contradicts that $\mathcal{V} \succsim \mathcal{W}$.

Therefore, $\overline{Du_i^{\mathcal{W}}} \subset \{x\}$. Furthermore, and since we have assumed without loss of generality that $\overline{Du_i^{\mathcal{W}}} \neq \{\emptyset\}$ (see (3)),

$$(28) \qquad\qquad \overline{Du_i^{\mathcal{W}}} = \{x\}.$$

This finishes the proof of Claim 2. ∎

Finally, assume there exists $y \neq x$ such that $y \in (De_i^{\mathcal{V}}\backslash De_i^{\mathcal{W}}) \cup (Ve_i^{\mathcal{V}}\backslash Ve_i^{\mathcal{W}})$. By Claim 2, $\overline{Du_i^{\mathcal{W}}} = \{y\}$ which contradicts (28). Therefore,

$$(De_i^{\mathcal{V}}\backslash De_i^{\mathcal{W}}) \cup (Ve_i^{\mathcal{V}}\backslash Ve_i^{\mathcal{W}}) = \{x\} = \overline{Du_i^{\mathcal{W}}},$$

which contradicts that (B) does not hold. Therefore, Case II is not possible. ∎

VI. Final Remark

Before finishing the paper we want to relate our comparability criterion to two alternative criteria proposed by Pathak and Sönmez (2013). To present them in a more general setting let X be a set of alternatives and now let $\mathcal{P}$ be the set of all strict preferences on X. Following Pathak and Sönmez (2013) we say that the profile $P \in \mathcal{P}^n$ is *vulnerable* under the social choice function $f : \mathcal{P}^n \to X$ if f is manipulable by some agent at P; i.e., there exist $i \in N$ and $P_i' \in \mathcal{P}$ such that $f(P_i', P_{-i})P_i f(P_i, P_{-i})$.

First, and following their definitions in Section I, a social choice function $f : \mathcal{P}^n \to X$ is at least as manipulable as social choice function $g : \mathcal{P}^n \to X$ according to Pathak and Sönmez (referred to as *at least as PS-manipulable as*, and written as $f \succeq^{PS} g$) if any profile that is vulnerable under g is also vulnerable under f; i.e.,

- if there exist $i \in N$ and $P_i' \in \mathcal{P}$ such that $g(P_i', P_{-i})P_i g(P_i, P_{-i})$, then there exist $j \in N$ and $P_j'' \in \mathcal{P}$ such that $f(P_j'', P_{-j})P_j f(P_j, P_{-j})$.

Second, and following their definitions in Section II, a social choice function $f : \mathcal{P}^n \to X$ is at least as strongly manipulable as social choice function $g : \mathcal{P}^n \to X$ according to Pathak and Sönmez (referred to as *at least as strongly PS-manipulable as*, and written as $f \succeq^{sPS} g$) if any profile that is vulnerable under g it is also

vulnerable under f but the *same* agent manipulates both social choice functions at the profile; i.e.,[5]

- if there exist $i \in N$ and $P_i' \in \mathcal{P}$ such that $g(P_i', P_{-i})P_i\, g(P_i, P_{-i})$, then there exists $P_i^* \in \mathcal{P}$ such that $f(P_i^*, P_{-i})P_i\, f(P_i, P_{-i})$.

Proposition 4 relates the three comparability notions for tops-only social choice functions that are strategy-proof on a restricted and rich domain of preferences. We say that a domain $\hat{\mathcal{P}}$ of preferences on X is *rich* if for for every $x \in X$ there exists $P_i \in \hat{\mathcal{P}}$ such that $t(P_i) = x$.

PROPOSITION 4: *Let $f : \mathcal{P}^n \to X$ and $g : \mathcal{P}^n \to X$ be two tops-only social choice functions. Assume f and g are strategy-proof on $\hat{\mathcal{P}} \subsetneq \mathcal{P}$, where $\hat{\mathcal{P}}$ is rich. Then,*

$$f \succeq^{sPS} g \Rightarrow f \succeq^{PS} g \Rightarrow f \succeq g.$$

PROOF:

The implication $f \succeq^{sPS} g \Rightarrow f \succeq^{PS} g$ follows immediately from the two definitions. To show that the second implication holds as well, assume $f \succeq^{PS} g$. Fix $i \in N$ and let $P_i \in \mathcal{M}_i^g$. This means that there exists $(P_i', P_{-i}) \in \mathcal{P}^n$ such that $g(P_i', P_{-i})P_i\, g(P_i, P_{-i})$. Since g is tops-only and $\hat{\mathcal{P}}$ is rich, we may assume that $P_{-i} \in \hat{\mathcal{P}}^{n-1}$. By assumption, there exist $j \in N$ and $P_j'' \in \mathcal{P}$ such that

$$(29) \qquad\qquad\qquad f(P_j'', P_{-j})P_j\, f(P_j, P_{-j}).$$

Assume $j \neq i$ and consider any $\hat{P}_i \in \hat{\mathcal{P}}$ such that $t(\hat{P}_i) = t(P_i)$; since $\hat{\mathcal{P}}$ is rich, at least one such preference exists. Set $\hat{P} = (\hat{P}_i, P_{-i}) \in \hat{\mathcal{P}}^n$. Since f is tops-only, $f(\hat{P}) = f(P_j, P_{-j})$. By (29), j can manipulate f at a profile $\hat{P} \in \hat{\mathcal{P}}^n$, a contradiction with strategy-proofness of f on $\hat{\mathcal{P}} \subsetneq \mathcal{P}$. Hence, $j = i$. But then, by (29), $P_i \in \mathcal{M}_i^f$. Thus, $P_i \in \mathcal{M}_i^f$ whenever $P_i \in \mathcal{M}_i^g$ which implies that f is at least as manipulable as g. ∎

Example 4 shows that the reverse of the second implication does not hold in our setting where $X = 2^K$, and the social choice functions under consideration are voting by committees, strategy-proof on the restricted and rich domain of separable preferences. Namely, there exist two voting by committees $\mathcal{W}$ and $\mathcal{V}$ such that $\mathcal{W}$ is at least as manipulable as $\mathcal{V}$ but $\mathcal{W}$ is not at least as PS-manipulable as $\mathcal{V}$ (and, by Proposition 4 above, $\mathcal{W}$ is not at least as strongly PS-manipulable as $\mathcal{V}$). In particular, Example 4 together with Proposition 4 show that our notion of being *at least as manipulable as* is different and strictly stronger than the two notions proposed by Pathak and Sönmez (2013).

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[5]Observe that the notions of *at least as PS-manipulable as* and *at least as strongly PS-manipulable as* are relative to the inclusion of the sets of vulnerable *profiles* of preferences, while our notion of *at least as manipulable as* is relative to the inclusion of the sets of manipulable *individual* preferences.

Example 4: Let $N = \{1, 2, 3\}$ be the set of agents and $K = \{x, y, z\}$ the set of objects. Consider the two voting by quota $\mathcal{V}$ and $\mathcal{W}$ defined by the following committees:

$$\mathcal{V}_x^m = \mathcal{V}_y^m = \mathcal{V}_z^m = \{S \subset N : |S| = 2\},$$
$$\mathcal{W}_x^m = \mathcal{W}_y^m = \mathcal{W}_z^m = \{S \subset N : |S| = 1\}.$$

By Corollary 1, $\mathcal{V} \succ \mathcal{W}$. On the one hand, consider any profile $P = (P_1, P_2, P_3) \in \mathcal{P}^3$ and any preference $P_3' \in \mathcal{P}$ such that (i) $t(P_i) = \{z\}$ for $i = 1, 2$, (ii) $t(P_3) = \{x\}$, and $\{x, y, z\} P_3 \{x, z\}$, and (iii) $t(P_3') = \{x, y\}$. Therefore, $\mathcal{W}(P_1, P_2, P_3') = \{x, y, z\} P_3 \{x, z\} = \mathcal{W}(P)$ and hence, P is vulnerable under $\mathcal{W}$. Moreover, $\mathcal{V}(P) = \{z\}$ and P is not vulnerable under $\mathcal{V}$. Thus, $\mathcal{V}$ is not at least as PS-manipulable as $\mathcal{W}$ and hence, by Proposition 4, $\mathcal{V}$ is not at least as strongly PS-manipulable as $\mathcal{W}$. On the other hand, consider any profile $\hat{P} = (\hat{P}_1, \hat{P}_2, \hat{P}_3) \in \mathcal{P}^3$ and any preference $\hat{P}_3' \in \mathcal{P}$ such that (i) $t(\hat{P}_1) = \{x, z\}$, $\{x, y, z\} \hat{P}_1 \{y, z\}$, (ii) $t(\hat{P}_2) = \{z\}$, (iii) $t(\hat{P}_3) = \{y\}$ and $\{x, y, z\} \hat{P}_3 \{x, z\} \hat{P}_3 \{z\}$, and (iv) $t(\hat{P}_3') = \{x, z\}$. Therefore, $\mathcal{W}(\hat{P}) = \{x, y, z\}$ and $\hat{P}$ is not vulnerable under $\mathcal{W}$. Moreover, $\mathcal{V}(\hat{P}_1, \hat{P}_2, \hat{P}_3') = \{x, z\} \hat{P}_3 \{z\} = \mathcal{V}(\hat{P})$ and hence, $\hat{P}$ is vulnerable under $\mathcal{V}$. Thus, $\mathcal{W}$ is not at least as PS-manipulable as $\mathcal{V}$; hence, by Proposition 4, $\mathcal{W}$ is not at least as strongly PS-manipulable as $\mathcal{V}$. Therefore, $\mathcal{W}$ and $\mathcal{V}$ are not comparable according to the two notions proposed by Pathak and Sönmez (2013). ∎

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