

Development of group method of data handling based on genetic algorithm to predict incipient motion in rigid rectangular storm water channel

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Abstract. Sediment transport is a prevalent vital process in uial and coastal environments, and "incipient motion" is an issue inseparably bound to this topic. This study utilizes a novel hybrid method based on Group Method of Data Handling (GMDH) and Genetic Algorithm (GA) to design GMDH structural (GMDH-GA). Also, Singular Value Decomposition (SVD) was utilized to compute the linear coecient vectors. In order to predict the densimetric Froude number (Fr), the ratio of median diameter of particle size to hydraulic radius ($d=R$) and the ratio of sediment deposit thickness to hydraulic radius ($t_s=R$) are utilized as eective parameters. Using three dierent sources of experimental data and GMDH-GA model, a new equation is proposed to predict incipient motion. The performance of development equation is compared using GMDH-GA and traditional equations. The results indicate that the presented equation is more accurate (RMSE = 0:18 and MAP E = 6:48%) than traditional methods. Also, a sensitivity analysis is presented to study the performance of each input combination in predicting incipient motion.

KEYWORDS

Genetic Algorithm (GA);
Group Method of Data Handling (GMDH);
Rigid rectangular channel;
Incipient motion;

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1. Introduction

Urban drainage systems, commonly used to transport storm water runos, are capable of easily transporting storm water runos, but sedimentation can occur after a specic period of time. Sedimentation in the urban drainage system leads to reduction in the hydraulic

capacity of the drain, which in return can cause ash ooding [1]. One of the essential issues in sediment transport is determining the minimum velocity needed in order for the sediment to start to transport from a stagnant state (incipient motion). Therefore, it is to fully understand the denition of "initial velocity" in designing urban drainage systems. It can be stated that generally the initial velocity occurs when the ow around the particle is more powerful than the resistance force of the particle weight [2]. So, "incipient motion" occurs when the calculated shear stress is greater than the critical shear stress of the bed substances in the channel.

Evaluating the ow conditions that produce incip-

ient motion of coarse river-bed material dates back to several decades [2-5]n the subject area of determining a constant velocity in order to consider self-cleansing within a channel, many researchers have tried to present a shear stress or velocity threshold to prevent sedimentation. The studies conducted in Germany indicate no sedimentation for shear stresses greater than 4 N/m^2 , while considerable amount of sedimentation occurs when the shear stress is less than 1.8 N/m^2 [6]. American Society of Civil Engineering [7] considers the minimum velocity to be equal to 0.9 m/s , while the British Standard [8] suggests 0.75 m/s for storm sewer and 1 m/s for combined sewer [9]. Using constant minimum velocity may lead to success in some cases, but using these values may not always be sufficient, since they do not consider the sediment specifications and the hydraulic conditions of the channel [10,11]. Therefore, utilizing an equation, which considers the sediment specifications and hydraulics of the channel when designing an urban drainage system may result in better designs.

Due to the availability and the capacity of soft computing in solving complex problems, its methods have been widely applied by different disciplines such as hydrology, hydraulic, and sediment transport [12-21]. Group Method of Data Handling network is one of the self-organized methods amongst soft computing methods based on artificial intelligence, capable of solving different problems in extremely complex nonlinear systems [22-25]. The GMDH method has been used to recognize behavior of nonlinear systems in different subjects in hydraulic such as friction factor in pipeline [26], scour depth [27-29], discharge coefficient [30,31], flow discharge in straight compound channels [32,33], basin sediment yield [34], and longitudinal dispersion in water networks [35].

In this study, in order to predict the densimetric Froude number (Fr) for incipient motion, the GMDH-type neural network was coded using Genetic Algorithm (GA) (GMDH-GA) and Singular Value Decomposition (SVD). Three sets of experimental data, namely those of Shvidchenko and Pender [36], Bong et al. [37], and Salem [5], were used. The existing equations were first examined, and then a novel equation was presented by considering the parameters in unifying incipient motion.

2. Review of the existing equations for incipient motion

Novak and Nalluri [38] conducted a series of experiments on rectangular and circular channels with smooth and rough beds in order to develop an equation for determining the incipient motion of sediment particles. The median diameter of particles ranged from 3.6 to 37.2 mm in their experimental tests. They balanced

the densimetric Froude number (Fr) and relative flow depth ($d=R$) parameters, presenting their equation as follows:

$$Fr = \frac{V}{\sqrt{gd(s-1)}} = 0.5 \frac{d}{R}^{0.4}; \quad (1)$$

where Fr is the densimetric Froude number, V is the critical velocity, g is the acceleration of gravity, $s (= \frac{\rho_s}{\rho})$ is the specific gravity, d is the median diameter of particles, and R is the hydraulic radius.

EI-Zaemey [39] conducted different experiments, in which the range of the median diameter of particles used was 2.9 to 8.4 mm , in order to present an equation to predict the critical velocity. Like Novak and Nalluri [38], EI-Zaemey carried out his experiments on rough and smooth beds, but the cross sections were only circular. EI-Zaemey [39] presented his equation as follows:

$$Fr = \frac{V}{\sqrt{gd(s-1)}} = 0.75 \frac{d}{R}^{0.34}; \quad (2)$$

Salem [5] conducted a series of experiments on a rectangular channel with median diameter range of particles of 0.55 to 4.78 millimeters and considered Fr to be dependent on ($d=R$) like the previous equations. The author presented different equations for different bed thicknesses with their general form being like the equations presented by Novak and Nalluri [38] and EI-Zaemey [39]. Salem [5] eventually presented the equation as Eq. (3) below, with a higher accuracy in comparison with the other equations. He was also inspired by the equations presented by Ackers and White [40] and Garde and Ranga Raju [41], thus presenting Eq. (4) using the logarithmic function:

$$Fr = \frac{V}{\sqrt{gd(s-1)}} = 0.937 \frac{d}{R}^{0.255}; \quad (3)$$

$$Fr = \frac{V}{\sqrt{gd(s-1)}} = 0.523 \ln \frac{d}{R} + 0.476; \quad (4)$$

Bong et al. [37] conducted a number of experiments on a rectangular channel in order to consider the sediment deposit thickness in presenting the critical velocity essential to moving the sediments. The median diameter of the particles used ranged from 0.81 to 4.78 mm . They considered Fr to be proportionate to the dimensionless parameters of ($d=R$) and (t_s/d) when presenting their equation through using dimensional analysis. The equation presented by Bong et al. [37] is as follows:

$$Fr = \frac{V}{\sqrt{gd(s-1)}} = 1.17 \frac{d}{R}^{0.167} \frac{t_s}{d}^{0.0378}; \quad (5)$$

where t_s is the sediment deposit thickness.

The incipient motion in all equations is represented by Fr parameter as a function of $(d=R)$; Bong et al. [37] used $(t_s=d)$ dimensionless parameter, with considering the effect of the bed thickness and $(d=R)$, to predict Fr.

3. Data presentation

The experimental studies, conducted on the minimum velocity essential for the incipient motion of the sediment, have considered velocity as a parameter dependent on hydraulic flow and the flow in the cross section [5,37-39,42]. To define a two-phase flow phenomenon, the components include noncohesive granular particles and fluid and flow [37]. The fluid characteristic was considered as its density (ρ); the noncohesive granular particles were represented by the median diameter of particles (d) and its density (ρ_s); the flow specifications were considered by hydraulic radius (R) and gravity acceleration (g). Also, sediment deposit thickness as an additional parameter was added to consider the effect of sediment deposit thickness. Therefore, the parameters affecting the estimation of the minimum velocity essential for the incipient motion of the sediment can be presented by the following equation:

$$V = f(\rho_s; d; R; g; \delta) \quad (6)$$

By selecting d and ρ_s as the repeating parameters, using Buckingham theorem and considering the form of existing equations (Eqs. (3)-(5)), the following functional relationship can be considered:

$$Fr = \rho \frac{V}{gd(s-1)} = f\left(\frac{d}{R}; \frac{t_s}{R}\right) \quad (7)$$

where s is the specific gravity of sediment ($s = \rho_s / \rho$).

Thus, Eq. (7) was applied in this study to present a novel equation using GMDH-type neural network for forecasting the incipient motion in an alluvial channel. For the purpose of presenting an equation capable of predicting this incipient motion, three different sets of data were used, consisting of those of Shvidchenko and Pender [36], Bong et al. [37] and Salem [5].

Shvidchenko and Pender [36] conducted their studies in an 8 m \times 3 m \times 3 m (L , W , and D) rectangular channel. The non-cohesive sediments used had particles of 1.5, 2.4, 3.4, and 4.5 mm in median diameter (d) and with the specific gravity ranging from 2.6 to 2.65. The slopes used for conducting the experiments were 0.0019 to 0.02870.

Bong et al. [37] conducted their experiments on a 6 m \times 6 m \times 4 m (L , W , and D) rectangular channel. The sediments had median diameters of 0.81, 1.53, and 4.78 mm in median diameter (d), and the specific gravities of the non-cohesive sediments were equal to

2.54, 2.55, and 2.57. The experiments were carried out on four different slopes, equaling 1.200, 1.350, 1.500, and 1.1000, respectively.

Salem [5] conducted his experiments on a rectangular channel with dimensions of 10 \times 3 \times 0.45 m (L ; W , and D). In order to change the channel slope, he used a mechanical jack near the flume exit and a tail gate at the end of the flume to keep the flow depth constant. The author also experimented with different bed thicknesses ($t_b = d$, 5 mm, 10 mm, and 24 mm) to examine the effect of the bed thickness on incipient motion. The median diameter of the particles used in this experiment ranged between 0.5 < d (mm) < 4.78. The details on how these experiments were carried out are presented in Shvidchenko and Pender [36], Bong et al. [37], and Salem [5].

4. Group Method of Data Handling (GMDH)

The GMDH neural network is a self-organizing system and a method for categorizing a group of data, presenting a proper output based on the observed data [43]. As a basic tool of the inductive modeling theory, GMDH belongs to the most modern methods of Computational Intelligence and Soft Computing. This method is the original and effective tool for solving wide spectrum of problems and discovery of relationships and excellent parallelization on multiple processors (very fast processing). Thus, it appears that to the obvious advantage of GMDH, an automatic forming of the network structure, simplicity, speed of learning the parameters, and also the possibility of "reducing" the adjusted network can be directly attributed to an explicit mathematical expression.

Also, here are some of the other benefits of using GMDH approach:

1. The optimal complexity of model structure is found adequate for level of noise in data sample. For real problems solution with noised or short data, simplified forecasting models are more accurate;
2. The number of layers and neurons in hidden layers, model structure, and other optimal NN parameters is determined automatically;
3. It guarantees that the most accurate or unbiased models will be found; the method does not miss the best solution during the sorting process of all variants (in a given class of functions);
4. As input variables are used by any non-linear functions or features, they can influence the output variable;
5. It automatically finds interpretable relationships in data and selects effective input variables;
6. GMDH sorting algorithms are rather simple for programming.

In contrast to the majority of neural networks with weak mathematical support, these kinds of neural networks use a nonlinear function known as the Volterra series function, as shown by the following mathematical expression:

$$\hat{y} = a_0 + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n a_{ijk} x_i x_j x_k + \dots \quad (8)$$

The above-mentioned series can be broken down into a second-degree polynomial as follows:

$$G(x_i x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i^2 + a_4 x_j^2 + a_5 x_i x_j \quad (9)$$

Regression methods are used for each pair of x_i and x_j input variables in order to find unknown coefficient a_i [43]. G is presented according to the function of least squared error principle [44], as is shown below:

$$E = \frac{1}{M} \sum_{i=1}^M (y_i - G_i O)^2 \quad (10)$$

$$y_i = f(x_{i1}; x_{i2}; x_{i3}; \dots; x_{im}) \quad i = 1; 2; 3; \dots; m \quad (11)$$

Considering Eq. (9), the system of equations, including N equations and 6 unknown values, must be solved; therefore, we use the SVD method to solve it.

4.1. Singular Value Decomposition (SVD)

The Singular Value Decomposition (SVD) has been used to compute the linear coefficient vectors. SVD is a method for solving least squared error problems in which singularity is likely to occur. Like $A \in \mathbb{R}^{M \times 6}$, SVD matrix is made up of three matrices: column orthogonal matrix, nonnegative diagonal matrix (singular values), $W \in \mathbb{R}^{6 \times 6}$, and orthogonal matrix, $V \in \mathbb{R}^{M \times 6}$, which could be written as follows [45]:

$$A = U W V^T \quad (12)$$

In order to obtain the efficient coefficients for Eq. (12), we calculated the corrected inverse matrix of W diagonal matrix and selected the zero values and near-zero values as zero in this calculation:

$$a = V \text{diag} \left(\frac{1}{W_j} \right) U^T Y \quad (13)$$

4.2. Development of GMDH using GA

GA belongs to the group of evolutionary algorithms that selects the GMDH network for the best structure, a method where the inputs are coded as numbers. Afterwards, different types of neural networks are generated through the connection formed between the

inputs and the neurons of the hidden layers. One of the simplest types of these neural networks is the CS-GMDH (Conventional Structure) which uses only the neurons of the adjacent layer to construct neurons in the new layer, but another type is known as GS-GMDH (Generalized Structure) which is utilized in this study and is not limited to using the nonadjacent neurons [30,44]. The length of the neurons is calculated as $2HL + 1$ in GS-GMDH, 'HL', indicating the hidden layers. In GS-GMDH network, neurons of different lengths are combined with each other; in other words, shorter neurons can jump through a number of the hidden layers and be combined with a longer neuron, such that regulations of neuron connection in these networks are not limited merely to the adjacent layer [44]. The flowchart of the proposed method is presented in Figure 1.

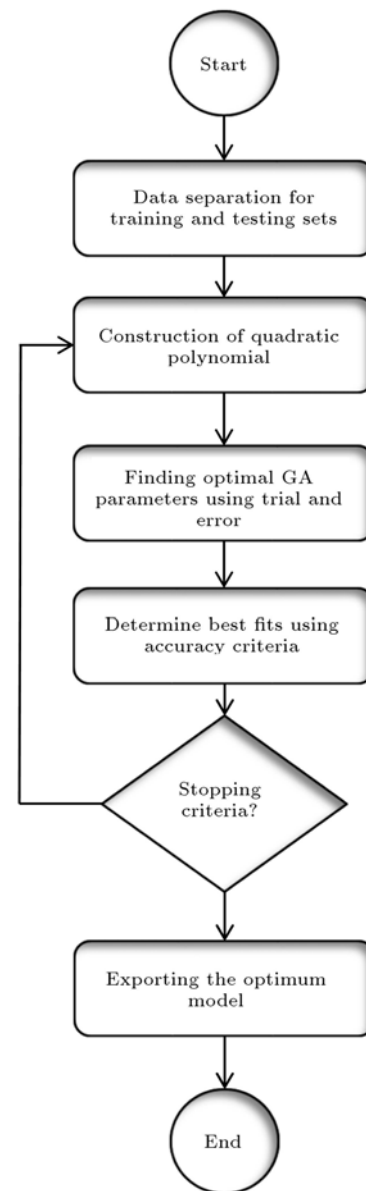


Figure 1. The flowchart for the proposed method.

5. Results and discussion

The results of the comparison between the existing equations and those proposed in this study are presented here with respect to the criteria for Root Mean Square of Error (RMSE) and the Mean Absolute Percentage Error (MAPE), as dened below:

$$RMSE = \sqrt{\frac{1}{n} \sum (Fr_{observed} - Fr_{predicted})^2}; \quad (14)$$

$$MAPE = \frac{100}{n} \sum \frac{|Fr_{observed} - Fr_{predicted}|}{Fr_{observed}}; \quad (15)$$

The Root Mean Squared Error (RMSE) is a criterion of mean error, which has no upper limit and has the lowest possible value of zero, representing the best estimation by the model. The Mean Absolute Percentage Error (MAPE) expresses the estimated value in relation to the observed value. MAPE and RMSE are non-negative indices, which have no higher limit. The considered model has the best possible performance when the value of this index is zero.

The GMDH-GA is used in this study in order to resolve the shortcomings of the existing equations. As explained in Section 3, the factors in uencing incipient motion are presented as in Eq. (7). Here, 220 items of varied data presented by Shvidchenko and Pender [36], Salem [5], and Bong et al. [37] are used for the purpose of calculating Fr for a novel equation. To do so, by random sampling without replacement, 55 items (25%) of the data were selected to verify the chosen model, then Eq. (20) was presented using 75% (165 items) of the remaining data and applying the GMDH-GA method. The evolved structure of the GMDH neural network generalized for modeling and prediction of densimetric Froude number is provided in Figure 2:

$$Y_1 = 2:586 \frac{d}{R} + 16:31 \frac{t_s}{R} + 1:989 \frac{t_s}{R} + 105:4 \frac{d}{R} + 2:375 \frac{t_s^2}{R} + 12:8 \frac{d}{R} \frac{t_s}{R}; \quad (16)$$

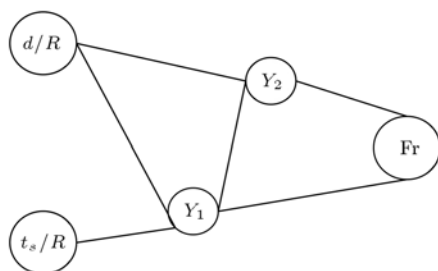


Figure 2. Evolved structure of the generalized GMDH neural network for modeling and predicting densimetric Froude number.

$$Y_2 = 16:82 + 89:13 \frac{d}{R} + 12:718 Y_1 + 56:96 \frac{d}{R} + 2:749 Y_1^2 + 39:725 \frac{d}{R} Y_1; \quad (17)$$

$$Fr = 4:649 + 15:254 Y_2 + 20:48 Y_1 + 8:91 Y_2^2 + 1:25 Y_1^2 + 11:12 Y_2 Y_1; \quad (18)$$

Table 1 presents the results of examining the existing equations and GMDH model in predicting Fr for MAPE and RMSE indices. The results of the different equations are presented in this table for three different data series [5,37-38]. Each of the experiments corresponds to each different median diameter of the particles. The equations, presented by Novak and Nalluri [38] and El-Zaemey [39], do not present good results most of the time, so much so that they have a MAPE greater than 20% in some cases, which makes them unreliable. El-Zaemey's [39] equation gives better results for all the data in comparison with that of Novak and Nalluri. Salem [5] presented two different equations in the form of Eqs. (3) and (4), which give different results under different conditions. But, Eq. (3) is relatively more accurate with regard to all the data. Equation of Bong et al. [37] gives different results under different conditions just as the other equations do. The best results by the equations presented in Table 1 are related to Eq. (3) [5], which were presented in $t_s = d$ and in Salem's [5] experiments. In $t_s = 5$ mm, there is a MAPE of 14%, which is more than twice the value of this index in $t_s = d$. Therefore, it can be seen that this equation does not always give satisfactory results, not even when using the set of data used to predict the model.

Figure 3 shows the performance of different methods in prediction of incipient motion using two statistical indices. It can be seen that both statistical indices presented in this figure have lesser values for the

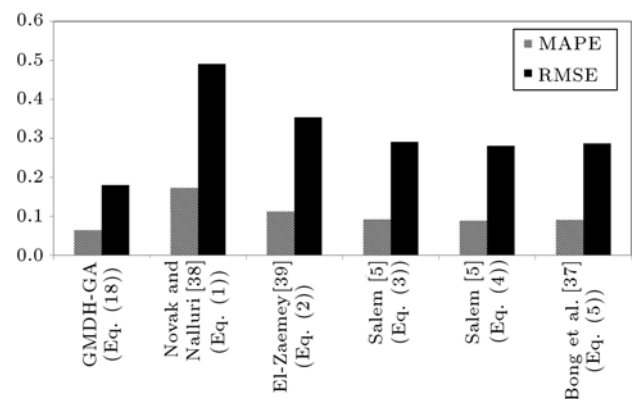


Figure 3. Bar plot of different incipient motions using statistical indices.

Table 1. Examination of the accuracy of different existing equations and GMDH-GA model under different experimental conditions through using statistical indices.

		GMDH-GA (Eq. (18))	Novak and Nalluri [38]	El-Zaemey [39]	Salem (Eq. (3)) [5]	Salem (Eq. (4)) [5]	Bong et al. (Eq. (5)) [37]	
Shvidchenko and Pender [36]	d = 1:5 mm	MAPE	7.346	11.604	12.226	8.763	9.123	12.156
		RMSE	0.221	0.370	0.404	0.289	0.301	0.368
	d = 2:4 mm	MAPE	5.822	13.832	11.008	9.037	9.083	9.714
		RMSE	0.169	0.394	0.318	0.264	0.263	0.286
	d = 3:4 mm	MAPE	5.139	21.519	7.396	7.530	7.040	7.724
		RMSE	0.135	0.492	0.188	0.198	0.185	0.202
d = 4:5 mm	MAPE	7.739	32.297	13.945	14.058	13.298	12.622	
	RMSE	0.194	0.729	0.354	0.360	0.343	0.335	
Bong et al. [37]	d = 0:81 mm	MAPE	6.389	17.889	8.626	9.739	9.556	10.152
		RMSE	0.189	0.501	0.250	0.311	0.307	0.304
	d = 1:53 mm	MAPE	6.428	16.803	14.235	11.809	12.118	11.089
		RMSE	0.134	0.375	0.339	0.279	0.286	0.245
Salem [5]	t _s = d	MAPE	7.754	16.876	8.564	6.575	6.560	7.481
		RMSE	0.186	0.407	0.232	0.173	0.176	0.221
	t _s = 5 mm	MAPE	7.819	20.963	23.080	14.033	11.713	9.184
		RMSE	0.197	0.600	0.726	0.418	0.348	0.286
	t _s = 10 mm	MAPE	6.102	21.112	10.484	10.209	9.770	9.971
		RMSE	0.182	0.593	0.296	0.319	0.311	0.327
	t _s = 24 mm	MAPE	5.755	17.097	14.423	9.717	9.640	9.959
		RMSE	0.164	0.454	0.435	0.295	0.287	0.289
All	MAPE	6.480	17.272	11.272	9.224	8.9	9.096	
	RMSE	0.180	0.491	0.354	0.291	0.281	0.287	

GMDH-GA model (MAPE = 6:5% and RMSE = 0:18) in comparison with the existing equations. Therefore, it can be stated that this model presents Fr, essential for incipient motion, with relatively higher accuracy when compared with the existing equations.

Figure 4 indicates the error distribution of the existing incipient motion equations for the three sets of data used in this study all at the same time. It can be seen that Eq. (1) only predicts approximately 40% of the data with a relative error less than 20%, while Eqs. (2) to (4) present similar results in almost all states, meaning that nearly 50% of the predictions by these equations have less than 10% relative error, and approximately 90% of the data have a relative error less than 20%. The point to be gathered from the figure is that all the equations have a relative error greater than 20% for at least 10% of the predicted values, and in some cases, the accuracy of the predictions made reaches 30% and more. Therefore, considering the provided explanations, the existing equations (Eq. (1)-(5)) cannot be reliably used to predict incipient motion.

Figure 5 indicates the error distribution of the presented GMDH-GA model for both states of test and

train. The maximum error presented for the two states is approximately 16%, while according to Figure 4, it is at least 30% for the existing equations. GMDH-GA predicts almost 90% of the values with a relative error

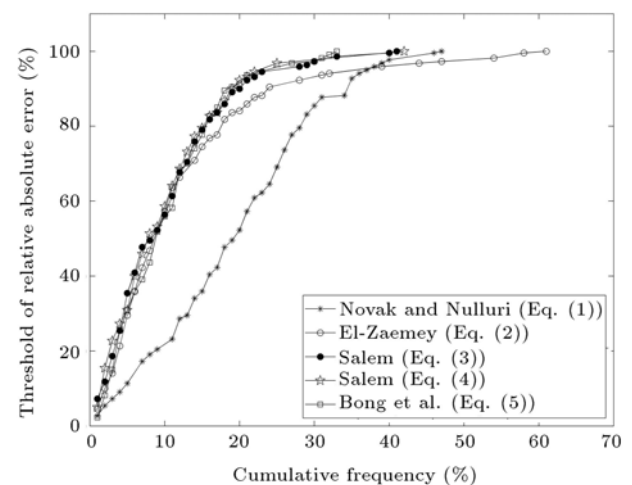


Figure 4. Error distribution of the existing incipient motion equations for all three data sets.

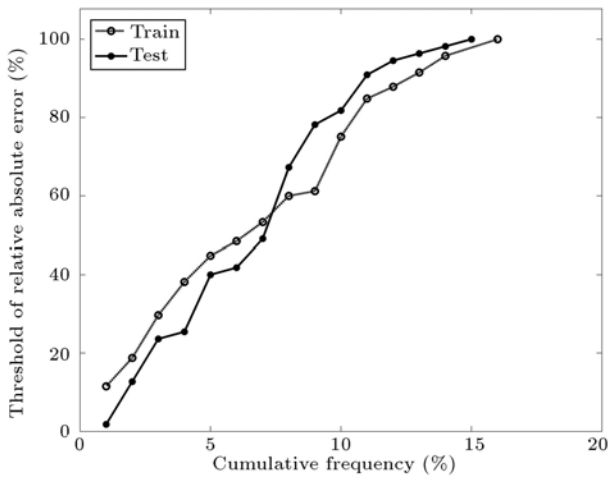


Figure 5. Error distribution of GMDH-GA model for test and train.

less than 12%, while the existing equations predict less than 65% of the actual values with a relative error less than 12%.

It is clear from Figures 3 and 4 that examining the accuracy of the presented model using the data, which had no role in predicting the model, presented similar results to those of the training state of the model, which indicates that this model is reliable. The efficiency of the GMDH-GA results for test and train states is presented in the Figure 6. The proposed GMDH-GA

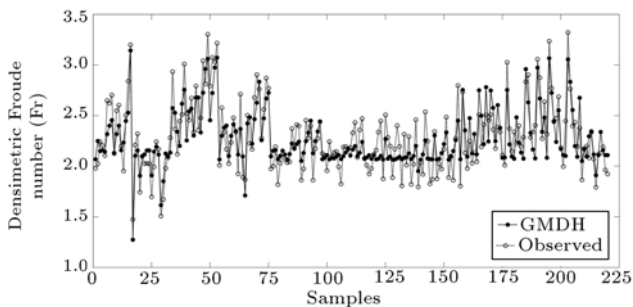


Figure 6. Performances of results for training and testing stages of the GMDH-GA model.

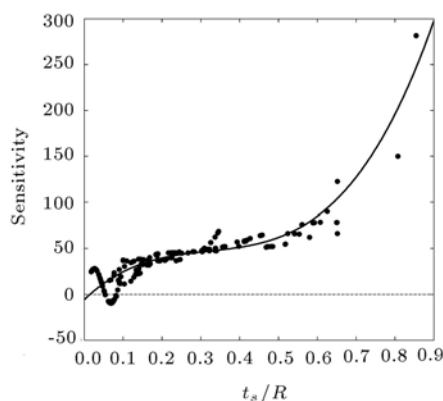
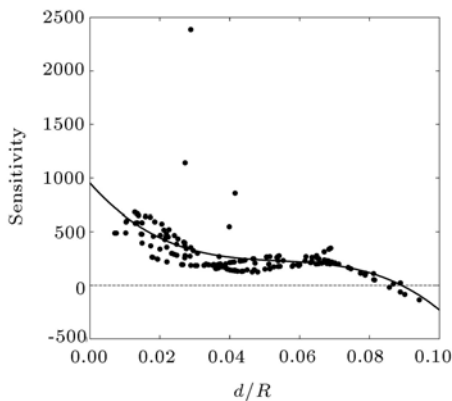


Figure 7. Results of sensitivity analysis for different input variables ($d=R$ and $t_s=R$).

model predicts Fr fairly accurately; in addition, it is more accurate in comparison with the already existing equations.

To survey the variation trend regarding Fr based on input parameters ($d=R$ and $t_s=R$), the partial derivative sensitivity analysis approach is employed [46-47]. In this method, the sensitivity of goal variable (Fr) on different input variables ($d=R$ and $t_s=R$) is investigated by partial derivative ($\partial(Fr)/\partial x_i$; x_i is the input variable). The value of partial derivative related to a certain variable has a direct relationship with the sensitivity of target variable to that variable. In fact, the positive value of partial derivative indicates that the increase of input variable leads to an increase in the value of target variable; for negative values of partial derivative, the relationship of input and target variable is in contrast to each other (i.e., the increase of input variable leads to the decrease of target variable, and vice versa). Figure 7 shows the results of sensitivity analysis of GMDH-GA (Eq. (18)) for both input variables. The sensitivity value for $d=R > 0.085$ is negative, and for other values, the results of $\partial(Fr)/\partial(d=R)$ are positive. Actually, within range of $d=R < 0.085$, the increase of this value leads to the increase of Fr . Like for $d=R$ as an input variable, most of sensitivity for $t_s=R$ are positive. The results of sensitivity for $t_s=R$ variable are negative only in range of $0.054 < t_s=R < 0.07$. Therefore, both of effective variables in prediction of Fr have a direct relationship with Fr .

6. Conclusions

In this study, the existing equations for incipient motion were examined using three different sets of data, namely those of Shvidchenko and Pender [36], Bong et al. [37], and Salem [5]. The results of the examination indicated that the results of the existing equations differ under different conditions. The worst result among the existing equations is related to Novak

and Nalluri [38] (MAPE = 17:27 & RMSE = 0:491). The error distribution indicated that only 20% of all predicted Fr by this equation have a relative error less than 10%. SALEM (Eq. (4)) [5] used $\ln(d=R)$ instead of $(d=R)$ for predicting Fr , which had the best performance (RMSE = 0:281 & MAPE = 8:9%). However, this equation presented Fr with a relative error of approximately 30% in some cases. Therefore, considering that the existing equations are not sufficiently reliable, a new equation was presented using GMDH-GA coding (Eq. (18)). This equation leads to reliable results (RMSE = 0:18 and MAPE = 6:5%), in a way, as the prediction error distribution indicates, that in 90% of the predictions, this equation presents Fr with a relative error less than 12%. The worst performance of this equation is in $\xi = 5$ mm (RMSE = 0:197 and MAPE = 7:2%). However, this equation has better results related to the best existing equations (Eq. (4)). The partial derivative sensitivity analysis method represents the direct relation of inputs ($d=R$ and $t_s=R$) with Fr . Actually, the increase of inputs results in increasing Fr value calculated by Eq. (18) in most cases.

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Nomenclature

a_i	Unknown coefficients (Eq. (9))
d	Median diameter of particles
Fr	Densimetric Froude number
g	Acceleration of gravity
R	The hydraulic radius
s	Specific gravity
t_s	Sediment deposit thickness
V	Critical velocity
x_i	Input variables (Eq. (9))
x_j	Input variables (Eq. (9))

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