Integrity analysis of the RTCA tropospheric delay model

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***Abstract*—Electromagnetic signals broadcast by GNSS satellites suffer considerable delays while travelling through the atmosphere. Apart from the ionosphere, the troposphere also has a significant effect on the propagation. The delay caused can be separated into two different parts: the effect of gases in hydrostatic equilibrium and the effect of water vapour and condensed water present in the troposphere.**

In navigation applications of GNSS not only the accuracy of the positioning needs to be known, but the integrity of the positioning service should be evaluated, too. The integrity information includes the maximum positioning error at an extremely rare probability level (10^{-7}), called the protection level. The RTCA (Radio Technical Commission for Aeronautics) specifies the minimum operational performance standard for GNSS systems used in the aeronautics. This standard recommends 0.12 m as the maximum tropospheric error in terms of standard deviation in the zenith direction, but it neglects both the geographical and seasonal variation of this error.

Our study focuses on the derivation of a new integrity model for the troposphere, which takes into consideration both the seasonal and geographical behaviour of the model performance using the extreme value theory.

The results show that the original RTCA recommendation is too conservative. Our study shows that the standard deviation is in the order of only 5 cm with a seasonal amplitude of 2-3 cm at the mid-latitudes. The application of the derived – more realistic – integrity model helps to improve the availability of GNSS positioning service in aviation.

I. Introduction

The global navigation satellite systems (GNSS) use range observations between the satellites and the receivers to derive the position of the user. These ranges are measured by measuring the duration of signal propagation and the results is multiplied by the velocity of light in vacuum to obtain the distance between the satellite and the receiver.

It is well known that radio waves propagate slower in the lower neutral part of the atmosphere, therefore this atmospheric layer (i.e. the troposphere) causes a significant signal delay. This delay is called tropospheric delay and it is modelled with models derived from various meteorological observations.

To assess the integrity of the satellite signal, the performance of these tropospheric delay models must be evaluated on an extremely rare probability level to ensure that the safety-of-life users (e.g. aviation,) can absolutely rely on the coordinates provided by the GNSS receivers.

Error models used in current 'standard' for safety-of-life GNSS [1] applications are considered very conservative when it comes to residual error modelling. In recent times, there has been much interest in revisiting these models with the aim of making them less conservative in order to assess the availability of satellite positioning more reliably.

The current tropospheric delay model from the RTCA Satellite-Based Augmentation System (SBAS) Minimal Operations Standards (RTCA MOPS) [1] possesses an associated residual error that is equal to 0.12 meters in the vertical sense. The value is derived from the results reported in [2]. While this approach gives a resulting standard deviation that is much higher than the estimated standard variance that best fits the data (0.05 m), it can surely be considered conservative for most applications. [2] also states however, that characterizing the delay errors beyond the $\pm 4\sigma$ level using a normal distribution is not recommended as it drastically underestimates the true distribution. The probability level denoted by $\pm 4\sigma$ corresponds to 99.994% which is obviously high, however safety critical systems may demand even higher levels. These considerations leave room for doubt whether the current model is safe to use under all circumstances. Additionally, the current residual error model has also been inspected in [3], where it is concluded that the model seems to be too conservative. Furthermore, it also lacks the ability to take into account the latitude dependency of the tropospheric delay estimations.

In near future, more demanding applications are expected to arise and as most of these will be based on multi-frequency and multi-constellation use of GNSS, they suffer from ionospheric delays less than today. This creates a demand for more accurate tropospheric error modelling and ensures its importance in approximating integrity while maintaining sufficient system availability. Recent investigations have already been done on the performance of the European Geostationary Navigation Overlay Service (EGNOS) in aiding localizer performance and vertical guidance (LPV) approaches of airplanes [4]. The calculation and validation of the protection levels established using such an overlay service has also been of interest recently, using open-source software for the computation [5].

The approach proposed in this paper can be summarized as analyzing tropospheric delay data using state-of-the-art knowledge on tropospheric modelling, in order to characterize the performance of the RTCA MOPS model by simple overbounding models that safety-of-life users can employ to derive error bounds on their positioning performance (e.g. in the form of protection levels). To this end we employed a dedicated processing methodology using reference dataset generated by a raytracing algorithm on numerical weather models and a combination of statistical concepts and techniques to rigorously prove the correctness of error bounds to an associated confidence level. To establish the overbounding relation between the model and the reference data and deal with the tails of the distribution, the extreme value theory was employed.

II. THE RTCA-MOPS TROPOSPHERIC DELAY MODEL

The tropospheric delay model described in [1] calculates the total slant delay for satellite i as:

$$TC_i = \left(d_{\text{hyd}} + d_{\text{wet}}\right) \cdot m(El_i)\,, \tag{1}$$

where TC_i denotes the total tropospheric delay [m], d_{hyd} and d_{wet} correspond to the hydrostatic and wet part of the delay in the zenith direction [m], while $m(El_i)$ is the value of the mapping function [-] at a given El elevation angle that is used to scale the zenith delay to the actual elevation angle.

The hydrostatic and wet parts of the delay are computed from the receiver's height and the estimation of five meteorological parameters: air pressure, temperature, water vapour pressure, temperature lapse rate and water vapour lapse rate. Each parameter (ξ) is estimated for the receiver's latitude (ϕ) and day-of-year (DOY) from the mean value (ξ_0) and its seasonal variation ($\Delta\xi$):

$$\xi(\phi, D) = \xi_0(\phi) + \\ + \Delta\xi(\phi) \cdot \cos\left(\frac{2\pi(DOY - DOY_{\text{min}})}{365.25}\right). \tag{2}$$

The value of DOY_{min} is different for the northern and southern hemisphere. The model works with a predefined value set for each meteorological parameter given for latitudes 15° (or less), 30°, 45°, 60° and 75° (or greater) and linearly interpolates for intermediate latitudes using the two closest values. The equation of the mapping function used to scale the zenith delays to slant range is the same as equation (5) for the integrity calculation.

III. INTEGRITY MODELLING IN RTCA–MOPS

According to [1], the following formula is used in the RTCA-MOPS to calculate the residual error for GPS pseudorange measurements for satellites used for the positioning:

$$\sigma_i^2 = \sigma_{i,\,\text{flt}}^2 + \sigma_{i,\,\text{UIRE}}^2 + \sigma_{i,\,\text{air}}^2 + \sigma_{i,\,\text{tropo}}^2\,, \tag{3}$$

where:

- σ_i is the standard deviation of satellite i pseudorange measurement [m],
- $\sigma_{i,flt}^2$ is the model variance of the residual errors for fast and long-term corrections [m],
- $\sigma_{i,\,UIRE}^2$ is the model variance of the slant range ionospheric delay estimation error [m],
- $\sigma_{i,\,air}^2$ is variance of the airborne receiver errors [m],
- $\sigma_{i,\,tropo}^2$ is the variance of tropospheric delay estimation error [m].

The standard deviation of the residual tropospheric error is modeled as a random integer with the standard deviation of $\sigma_{i,\,tropo}$, which is calculated as:

$$\sigma_{i,\,\text{tropo}} = \left(\sigma_{\text{TVE}} \cdot m(\theta_i)\right), \tag{4}$$

$$m(\theta_i) = \frac{1.001}{\sqrt{0.002001 + \sin^2(\theta_i)}}, \tag{5}$$

where σ_{TVE} denotes the vertical residual error of the tropospheric delay estimation and is equal to 0.12 meters and θ_i is the satellite elevation angle. Note that the vertical residual error of the tropospheric delay estimation is a constant value which globally overbounds the standard deviation of the residuals, but as it neglects the effect of latitude on the accuracy of the tropospheric delay estimation, leads to an overly conservative model in many regions.

Combining these terms, one ends up with the variance of the total residual error which enables the system to calculate the horizontal and vertical protection levels (HPL and VPL) for a given position as follows:

$$HPL = K_H \cdot d_{\text{major}}\,, \tag{6}$$

$$VPL = K_V \cdot d_{\text{major}}\,, \tag{7}$$

where K_H and K_V are constants depending on the different approach type and d_{major} [m] corresponds to the uncertainty along the semimajor axis of the error ellipse:

$$d_{\text{major}} \equiv \\ \equiv \sqrt{\frac{d_{\text{east}}^2 + d_{\text{north}}^2}{2} + \sqrt{\left(\frac{d_{\text{east}}^2 - d_{\text{north}}^2}{2}\right)^2 + d_{EN}^2}}\,. \tag{8}$$

The terms in the equation stand for the following:

- d_{east}^2 is the variance of model distribution that overbounds the true error distribution in the east axis [m^2],
- d_{north}^2 is the variance of model distribution that overbounds the true error distribution in the north axis [m^2],
- d_{EN}^2 is the covariance of the model distribution in the east and the north axes [m^2],
- d_U^2 is the variance of model distribution that overbounds the true error distribution in the vertical axis [m^2].

All the model variances are calculated using the partial derivatives of the position error in the respective direction with respect to the pseudorange error on each satellite.

Using the HPL and the VPL values, the instrument can decide whether current accuracy of the position is suitable for navigational purposes during the different approach types.

IV. REFERENCE DATA

A. Meteorological data

In order to assess model performance, a reference data set of tropospheric delays was needed. Four European Centre for Medium-Range Weather Forecasts (ECMWF) ERA-Interim solutions per day were used to calculate this data set with ray-tracing the various atmospheric layers. Relative humidity, temperature and geopotential values estimated on 37 pressure levels (from 1000 hPa to 1 hPa) with a resolution of 1° x 1° were collected for the years 2000-2016 for this study. Besides ECMWF ERA-Interim solutions International Standard Atmosphere (ISA) [6] values were used to expand the atmospheric profiles up to the height of 86 km.

B. Computation of reference tropospheric delays

The ray-tracing method supposes specific layers of the atmosphere, where the path of a beam is traced. The beam starting at a certain elevation angle continuously refracts at different layers and changes direction [7]. The tropospheric delay can be calculated by multiplying the length of the refracted beam with the refractivity in the given layer.

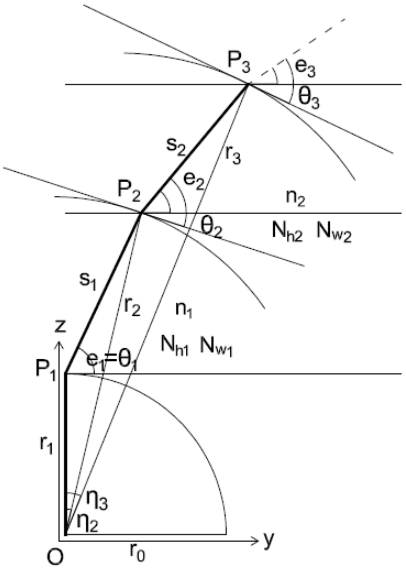

Fig. 1. The principle of the ray tracing showed with a beam starting at the surface of a sphere (modelling the Earth) and refracting at each layer of the atmosphere with different refractivity

To obtain optimal results, the resolution of the meteorological data needs to be increased. The interpolation is done linearly for the temperature and exponentially for the air pressure and water vapour pressure. Then the hydrostatic and the wet refractivity can be calculated for each layer as well as the distance travelled in the layer. The hydrostatic and wet delays are defined:

$$ds_{h,w} = \sum_{i=1}^{k-1} s_i \cdot N_{h,w,i}\,, \tag{9}$$

where s_i is the length of the refracted beam [m] and $N_{h,w,i}$ is the hydrostatic and wet refractions [-] in the i-th layer.

V. METHODOLOGY

A. Principles

The general integrity requirements of radio navigational aids used in civil aviation is formulated in [8]. According to this document, the integrity of GNSS positioning service must be evaluated at the extremely rare probability level of 2×10^{-7} in any approach. Assuming the duration of an average approach of 150 seconds and no concurrent approaches in the same time, the recurrence interval of an integrity event would be 25 years.

Since only a limited number of observation samples are available to assess the performance of the tropospheric delay models, one must use a probabilistic approach for such a study. It would be straightforward to fit a normal distribution to the residuals of the estimated tropospheric delays, and extrapolate it to the tails of the distribution. However, the probability plot of the residuals (Fig. 2) clearly indicates that the tails of the residuals significantly deviate from the normal distribution. Thus, the extreme value theory must be applied for this problem.

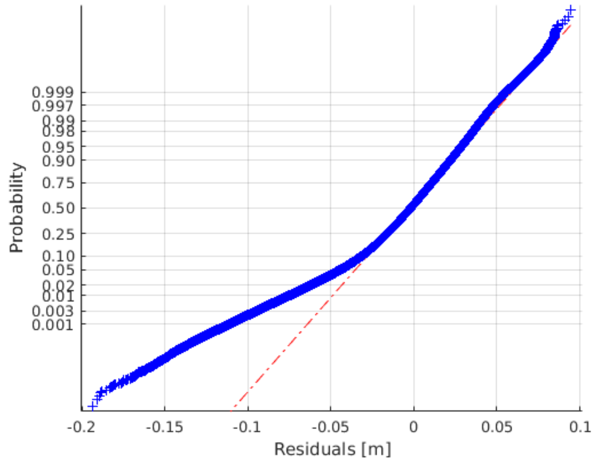

Fig. 2. Normal probability plot of the hydrostatic tropospheric delay model residuals for the latitude band N40°-N50°

B. Principles of Extreme value theory

The Fisher-Tippett theorem states that the maximum of a sample of independent and identically distributed probability variables after proper renormalization can converge to one of the three possible distributions, the Gumbel, the Fréchet or the Weibull distribution.

The three distribution functions are the following:

$$H(x) = \begin{cases} 0 & \text{if } x \le 0 \\ \exp\{-x^{-\alpha}\} & \text{if } x > 0 \end{cases}, \tag{10}$$

for the Fréchet,

$$H(x) = \begin{cases} \exp\{-(-x)^{-\alpha}\} & \text{if } x < 0 \\ 1 & \text{if } x \ge 0 \end{cases}, \tag{11}$$

for the Weibull, and

$$H(x) = \exp\{-\exp\{-x\}\} \quad x \in R\,, \tag{12}$$

for the Gumbel distribution.

The general extreme value (GEV) theory [9] combines the previous three distributions to the general extreme value distribution. The distribution function is:

$$H(x) = \begin{cases} \exp\{-\{1 - k(x - \xi)/\alpha\}^{1/k}\} & \text{if } k \neq 0 \\ \exp\{-\exp\{-(x - \xi/\alpha)\}\} & \text{if } k = 0 \end{cases} \quad (13)$$

with x bounded by $\xi + \alpha/k$ from above if $k > 0$ and from below if $k < 0$. Here ξ and α are the location and scale parameters, while k is the shape parameter. The shape parameter determines which original extreme value is represented by the GEV distribution:

- for $k > 0$ the Fréchet distribution (heavy tailed)
- for $k = 0$ the Gumbel distribution (light tailed)
- for $k < 0$ the short tailed negative Weibull distribution

is described by the GEV distribution.

C. *Estimation of extreme tropospheric error using GEV theory*

To study the performance of tropospheric delay models under extreme conditions, firstly, the tropospheric model error must be calculated. To achieve this, the hydrostatic and wet tropospheric delays were computed using the RTCA-MOPS troposphere model based on surface meteorological parameters obtained from the numerical weather models. Since numerical weather model data are given in constant pressure levels instead of elevation levels, therefore an interpolation or extrapolation of the air pressure, water vapour pressure and the ambient temperature was needed to calculate the parameters on the ground.

Afterwards these tropospheric delays were subtracted from the reference values calculated with ray-tracing the entire atmosphere. These residuals were calculated in 18, equally sized latitude bands for the whole globe. Fig. 3. shows the time series of the hydrostatic delay residuals for all the grid points in the latitude band between N41 to N50 latitudes. The figure shows, that both the spread of the daily residuals have a significant seasonal variation. To derive an appropriate model for the integrity assessment, this seasonal variation must be removed from the residuals and later restored in the derived model to be able to represent the seasonal behavior

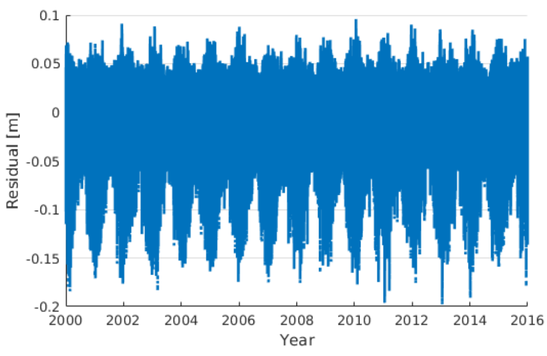

Fig. 3. Time series of the residuals of the hydrostatic delays with w.r.t. the raytraced reference values

of the tropospheric model performance. Basically, this is equivalent with the normalization of the time series of the residuals. Thus, the daily standard deviation of the residuals was calculated and a periodic function was fit to these mean and standard deviation values considering both the annual and the semi-annual components of the seasonal variations (Fig. 4).

The model function for the daily standard deviation values:

$$\sigma(DOY) = \bar{\sigma} + A_1 \cos\left(\frac{DOY - DOY_0}{365.25} 2\pi\right) + +A_2 \sin\left(\frac{DOY - DOY_0}{365.25} 2\pi\right) + +A_3 \sin\left(\frac{DOY - DOY_0}{365.25} 4\pi\right) + +A_4 \sin\left(\frac{DOY - DOY_0}{365.25} 4\pi\right), \quad (14)$$

where the unknown parameters are: $\bar{\sigma}$ is the mean value of the daily mean residuals for the total time series, DOY_0 is the day of the annual minimum of the standard deviation of the daily residuals (the phase), while A_1 and A_2 are the amplitudes of annual, and A_3 and A_4 of the semi-annual terms of the seasonal variation.

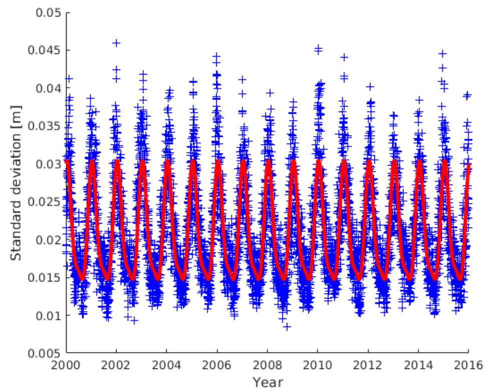

Fig. 4. The seasonal variation of the daily standard deviations of the residuals and the fitted model

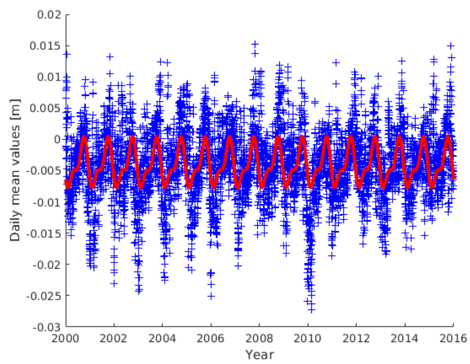

Fig. 5. The seasonal variation of the daily mean values of the residuals and the fitted model

Afterwards, the residuals (δ) were normalized using a zero-mean assumption with the following equation:

$$\delta_n = \frac{\delta}{\sigma(DOY)}. \quad (15)$$

In the next step, the normalized residuals were used for extreme value analysis. Since the samples covered 17 years of data, 17 annual extremes (maximum and minimum values) were identified and selected for the extreme value analysis. The GEV distribution was fit to these extremes using the MATLAB software [8], and finally the extreme value representing the recurrence time of 25 years was estimated using the fitted distribution for both the maximal (positive) and minimal (negative) extremes. From these two values, the one with the larger absolute value was chosen as the maximal expected error of the normalized residuals ($\Delta_{n,\max}$).

Since the RTCA-MOPS proposes a calculation of the protection levels based on the standard deviation of parameters defined as normally distributed probabilistic variables, the previously estimated extreme values had to be converted to the standard deviation of normally distributed probabilistic variables. Thus:

$$\sigma_{n,\max} = \frac{\Delta_{n,\max}}{K} \tag{16}$$

where K is the value of the probability density function of the standard normal distribution at the probability level (meaning the probability of non-exceedance) of $1\text{-}10^{-7}$.

To estimate the seasonal variations of the troposphere model errors the following overbounding model is formulated for each latitude band:

$$\sigma_{\max}(DOY, band) = \frac{\Delta_0}{K} + \sigma(DOY) \cdot \sigma_{n,\max}\,, \tag{17}$$

where Δ_0 is an offset parameter, that is necessary for achieving the overbounding of model error. This offset parameter is calculated by fitting another extreme value distribution function to the annual extremes of the daily mean values (Fig. 5).

VI. RESULTS

The overbounding models of the troposphere model error were calculated for all the 18 latitude bands of the global grid for both the hydrostatic and the wet component of the tropospheric delay models (TABLE I. and TABLE II.). The results can be seen on Fig. 6 for the northern hemisphere for both components

The figures indicate that the $\sigma_{\max}$ values of the hydrostatic and wet components are in all scenarios better than ±6 and ±10 centimeters, respectively. Since the total delay can be computed as the sum of the two components, the maximum standard deviation of the total tropospheric delay error can be computed using the law of error propagation:

$$\sigma_{TD,\max} = \sqrt{\sigma_{HD,\max}^2 + \sigma_{WD,\max}^2} \cong 0.12 \text{ m} \tag{18}$$

where $\sigma_{HD,\max}$ and $\sigma_{WD,\max}$ are the maximum standard deviation for the hydrostatic and wet delay in the zenith direction, respectively. This value perfectly agrees with the recommendations of the RTCA-MOPS. However, Fig. 6. shows that this value is too conservative for large regions of the world.

The maximum tropospheric error, i.e. the integrity model for the tropospheric delays, can be estimated by reformatting Eq. (17):

$$\Delta_{\max}(DOY, band) = \Delta_0 + \sigma(DOY) \cdot K \cdot \sigma_{n,\max}\,. \tag{19}$$

TABLE I.
INTEGRITY MODEL PARAMETERS FOR THE HYDROSTATIC DELAY

Model parameters								
Band	Δ_0 **[mm]**	$\bar{\sigma}$ **[mm]**	A_1 **[mm]**	A_2 **[mm]**	A_3 **[mm]**	A_4 **[mm]**	DOY_0 **[day]**	$\sigma_{n,\max}$
Northern hemisphere								
90 – 81	87.8	14.1	2.8	0.4	-0.2	0.2	2	2.0
80 – 71	51.0	21.6	6.0	1.6	-0.1	0.4	0	1.3
70 – 61	43.2	22.9	8.4	1.5	0.1	0.0	0	1.3
60 – 51	29.7	24.3	10.0	1.8	0.5	0.1	1	1.5
50 – 41	26.6	20.9	7.0	2.5	2.0	0.7	0	1.7
40 – 31	20.7	15.6	1.3	1.8	2.3	1.1	0	2.1
30 – 21	15.2	11.6	-3.6	0.4	1.5	1.0	3	2.7
20 – 11	16.0	7.1	-2.1	0.1	0.6	0.4	8	3.9
10 – 0	17.5	4.6	-0.2	-0.1	0.4	0.2	1	3.3
Southern hemisphere								
1 – 10	17.3	5.0	-0.2	-0.5	0.4	0.2	3	2.6
11 – 20	15.3	6.7	0.8	-0.3	0.5	0.4	2	3.6
21 – 30	10.6	10.2	0.3	-0.9	0.7	0.5	2	2.3
31 – 40	21.1	16.4	-2.8	-1.6	0.5	0.1	0	2.0
41 – 50	41.8	25.1	-3.4	-1.5	0.0	0.0	0	1.4
51 – 60	73.9	31.3	-3.4	-1.3	-0.9	0.4	2	1.3
61 – 70	101.1	26.6	-5.2	-2.1	-1.0	0.5	0	1.8
71 – 80	97.1	23.0	-8.6	-5.4	-0.3	-0.4	1	2.8
81 – 90	92.4	13.2	-5.4	-3.3	-0.3	0.0	1	4.0

TABLE II.
INTEGRITY MODEL PARAMETERS FOR THE WET DELAY

Model parameters								
Band	Δ_0 **[mm]**	$\bar{\sigma}$ **[mm]**	A_1 **[mm]**	A_2 **[mm]**	A_3 **[mm]**	A_4 **[mm]**	DOY_0 **[day]**	$\sigma_{n,\max}$
Northern hemisphere								
90 – 81	70.4	8.5	-3.8	-2.7	0.8	1.5	6	2.9
80 – 71	54.6	15.5	-5.6	-3.5	1.1	1.5	1	1.9
70 – 61	55.7	22.3	-6.7	-3.9	1.8	1.5	2	1.6
60 – 51	59.8	29.0	-6.0	-4.5	1.8	1.4	3	1.2
50 – 41	60.2	37.3	-6.1	-5.8	0.8	1.2	1	1.1
40 – 31	72.5	47.7	-10.7	-6.7	2.1	1.1	2	1.0
30 – 21	89.9	59.7	-13.6	-5.1	2.8	0.0	0	0.8
20 – 11	117.6	57.0	-1.2	-1.4	1.3	-5.4	0	1.0
10 – 0	58.6	46.8	6.7	1.6	1.1	2.9	1	0.9
Southern hemisphere								
1 – 10	74.6	55.3	2.4	-6.5	3.4	-2.0	2	0.7
11 – 20	120.1	61.0	9.0	2.2	2.0	-1.3	1	0.9
21 – 30	100.8	53.6	9.5	3.9	1.3	1.0	0	0.8
31 – 40	111.3	42.6	7.0	5.1	0.1	1.1	2	0.9
41 – 50	97.1	34.1	4.6	4.5	-0.2	0.7	0	1.1
51 – 60	94.6	25.1	2.3	3.0	-0.5	0.5	1	1.1
61 – 70	86.4	17.2	1.0	1.5	-0.4	0.2	2	1.3
71 – 80	60.8	13.9	6.6	4.4	-0.8	-0.2	1	2.5
81 – 90	48.2	9.2	5.9	3.8	-0.7	-0.5	3	5.1

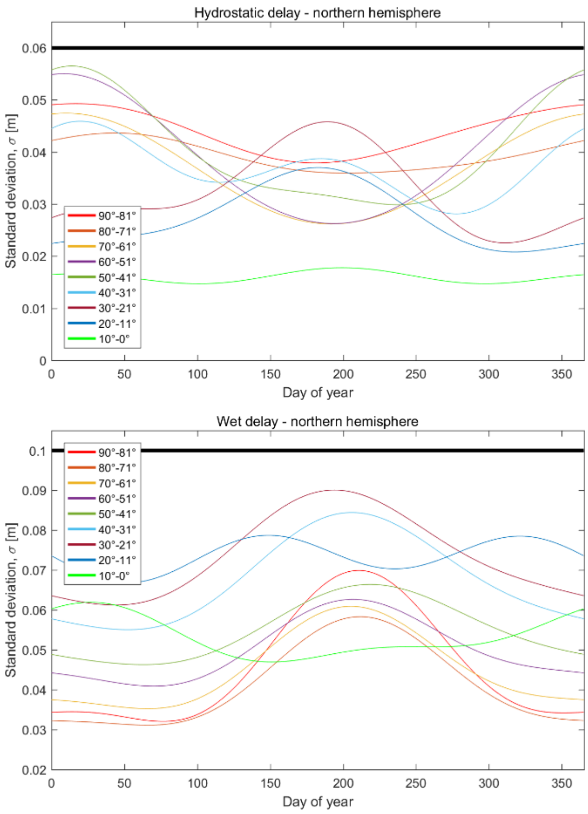

Fig. 6. The seasonal variation of the σ_{max} in the latitude bands on the northern hemisphere

Fig. 7 depicts the unnormalized hydrostatic and wet residuals and the derived integrity model for the latitude band between N40° and N50° latitudes. It can be clearly seen that the derived model truly overbounds the tropospheric delay error and it is significantly less conservative than the original RTCA model. Moreover, the derived model takes into consideration the seasonal variations of the tropospheric delays caused by the climate.

VII. CONCLUSION

The results of our study confirmed that the RTCA MOPS recommendation of 0.12m for modelling the maximal tropospheric delay error in the zenith direction is appropriate, but it can also be stated that it is too conservative for a large part of the globe.

In this paper, a less conservative, nevertheless reliable model was derived for the globe, which provides the users a more realistic limit for the maximal error of the tropospheric models. This leads to smaller level of the expected error, thus a smaller protection level for the assessment of the integrity of the system, which increases the availability of the satellite positioning services for safety-of-life users.

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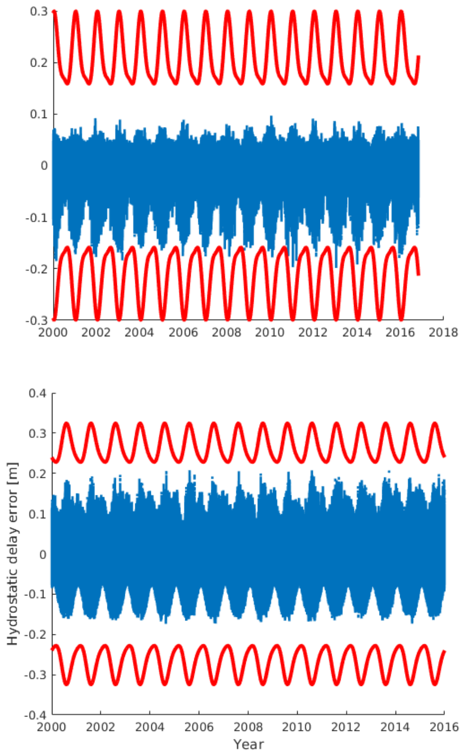

Fig. 7. The integrity model of the hydrostatic (top) and the wet (bottom) tropospheric delays for the latitude band N40°-N50°.

REFERENCES

[1] *Minimum Operational Performance Standards for Global Positioning System/Satellite-Based Augmentation System Airborne Equipment*, RTCA DO-229, 2006.

[2] J. P. Collins, R. B. Langley, "The residual tropospheric propagation delay: How bad can it get?", *11th International Technical Meeting of the Institute of Navigation*, Nashville, Tennessee, 1998.

[3] S. Storm van Leeuwen, H. van der Marel, M. Toussaint, A. Martelluci, "Validation of SBAS MOPS troposphere model over the EGNOS service area", *European Navigation Conference (GNSS-2004)*, Rotterdam, The Netherlands, 2004.

[4] R. Markovits-Somogyi, B. Takács, A. de la Fuente, P. Lubrani, "Introducing E-GNSS navigation in the Hungarian Airspace – The BEYOND experience and the relevance of GNSS monitoring and vulnerabilities", in *Selected papers of 3rd International Conference on Research, Technology and Education of Space*, Budapest, Hungary, 2017.

[5] B. Takács, Z. Siki, R. Markovits-Somogyi, "Extension of RTKLIB for the calculation and validation of protection levels", in *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, vol. XLII-4/W2, 2017.

[6] *Standard Atmosphere*, ISO 2533:1975, 1975.

[7] J. Boehm, H. Schuh, "Vienna Mapping Functions", in *16th Working Meeting on European VLBI for Geodesy and Astrometry*, pp. 131-143, 2003.

[8] International Civil Aviation Organization (ICAO), Aeronautical Telecommunication, vol. 1., Radio Navigation Aids, in *Annex 10 to the Convention on International Civil Aviation*, p.578, 2006

[9] Jenkinson, A.F, The frequency distribution of the annual maximum (or minimum) of meteorological elements, *Quarterly Journal of the Royal Meteorological Society*, Vol 81., pp 158-171., 1955

[10] MATLAB Statistics and Machine Learning Toolbox Release 2016a, The MathWorks Inc., Natick, Massachusetts, United States.