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SV-WAVE WITH EXTERNAL FORCE IN SATURATED MEDIUM

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Graphical abstract

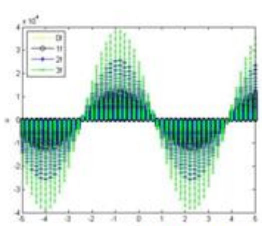

Figure 1.1 SV-wave's propagation with various forces in low density medium.

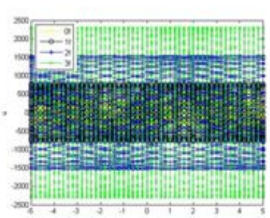

Figure 1.2 SV-wave's propagation with various forces in high density medium.

Abstract

There are past researches done for SV-wave with the absence of external force. This work aims to investigate the influence of external force in the propagation of SV-wave in the soluble and insoluble mediums. Isotropic wave model is derived for SV-wave with various forces in accordance to Duhamel Principle and their analytical solutions are used to compare with each other's' amplitudes. With the existence of high density fluid, diffusive waves with larger external force will induce higher amplitude. However, SV-waves are non-diffusive in low density medium and hence, this work concludes that wave diffusion by external force is subjected to the properties of the targeted medium.

Keywords: Diffusion, external force, fluid saturated medium, Rayleigh wave, SV-wave

Abstrak

Terdapat kajian-kajian lepas yang dilakukan untuk gelombang SV tanpa mempertimbangkan daya luaran. Kajian ini bertujuan untuk mengkaji kesan daya luaran dalam perambatan gelombang SV dalam medium yang larut dan tidak larut. Model isotropi gelombang diperolehi untuk gelombang SV dengan pelbagai daya berdasarkan Prinsip Duhamel dan penyelesaian analisis dibandingkan amplitude masing-masing. Dengan kewujudan medium cecair ketumpatan tinggi, gelombang bersifat penyebar dengan daya luaran yang lebih besar mendorong amplitud yang lebih tinggi. Walau bagaimanapun, gelombang SV tidak bersifat penyebar di dalam medium berketumpatan rendah. Oleh itu, kajian ini menyimpulkan bahawa penyebaran gelombang dengan daya luaran adalah tertakluk kepada sifat-sifat medium yang disasarkan.

Kata kunci: Sifat penyebaran, daya luaran, medium cecair tepu, gelombang Rayleigh, gelombang SV

1.0 INTRODUCTION

There are many researches done for SV-wave. One of the earliest theories is proposed by Biot, which relates the propagating stress wave to miscible porous solid, which is later named as Biot's theory [1]. Subsequently, exact theory of attenuation and dispersion of seismic wave in porous medium is obtained to show water saturation has significance effect on propagating wave [2]. Since then, investigation on topographic effect incidence at critical SV-wave has revealed amplification of SV-wave is independent on source types and spectrum [3]. Next, a method to calculate P, SV and Rayleigh wave by irregular topographic characteristic in half-space is given. The outcome shows topography has significance effect on amplification and reduction of waves at irregular feature or nearby region [4].

Later on, an analytical solution is derived for SV wave dispersion by enclosing layer of a circular-arc canyon. The result emphasizes stiffness and thicknesses of enclosing layer have major influence on incident SV wave [5]. Afterward, an analytical solution for motion of incident plane SV-wave of a semi-cylindrical surface is developed. The result shows an incident plane SV-wave is greatly intensified by a hill [6]. At the same time, investigation on scattering of SV-wave due to a crack on interface between an elastic substrate and piezoelectric layer has been done. They have found that the incident angle of SV-wave and the ratio of crack's length to the layer's width have responded to the increment in the crack of the interface [7]. Consequently, researchers found that in the half-space, interference field in the principal estimation away from the source does not rely on relation between phase velocity and the longitudinal and transversal velocities [8].

Later a solution of displacement with torsional force of a miscible porous medium is found. The result reveals two phase medium decays to a single phase medium [9]. A new method is introduced to solve wave with external force. By the new method, the elastic wave produced in half-space by external force situated on or under body surface can be classified. The effects of inhomogeneities have linked to wave propagation [10]. Besides, a solitary wave moves through external force and when small amplitude change is done, the result shows solitary wave either partially or completely been caught in location of external force [11].

A lot of studies have been done on SV-wave and external force separately. However to the best of our knowledge, not many research focus on SV-wave with external force. In this work, investigation on the influence of external force on propagating SV wave in the soluble and insoluble medium has been done. The isotropic wave model is derived for various forces, and the comparison of amplitude and diffusion has been done for wave with various forces. This paper aims to obtain diffusion profiles of SV-wave under the effects of external forces.

2.0 METHODOLOGY

In this work, solution of SV-wave with sinusoidal force is given and the steps of derivation are shown in Appendix A. The equation below is the solution of SV-wave with sinusoidal force which moves along the characteristic line:

$$\mathbf{u}_{SV} = B(-\eta_\beta a_x + a_z)\left(\frac{1}{4k^2c^2}\begin{pmatrix} e^{-ik(x+\eta_\beta z - ct)} \\ -e^{-ik(x+\eta_\beta z + ct)} \end{pmatrix} + \frac{1}{2ikc} t e^{-ik(x+\eta_\beta z - ct)} \right) \quad (1)$$

$x + ct =$ constant as $t \to \infty$, where $\eta_\beta = \sqrt{\frac{c^2}{\beta^2} - 1}$

Given that dispersion relation $\omega = ck$, where ω is the angular velocity or frequency, c is the phase velocity, and *k* is the wave number.

According to Snell's law

$$\frac{V_{app}}{\sin f} = \frac{c}{\sin e},$$

where V_{app} is the apparent velocity which is measured along the boundary of similar density medium. f is the refraction angle made by the P-wave and S-wave, c is wave velocity, and e is the incident angle.

For the case of fluid saturated medium, there exists difference in density medium and the slowness induced by refraction, i.e. V_{app} the apparent velocity of the wave at the boundary is expected to be slower than the phase velocity in the medium [15].

$$V_{app} < c$$

In the above equation, B is the displacement of SV-wave, η_β is the velocity of refracted SV-wave, k is the wave number and c is the wave velocity. The unit vectors a_x, a_z show that polarization of SV-wave is in $x - z$ direction. β is the displacement velocity of SV-wave at $z = 0$. When SV-wave at $z = 0$, meaning the SV-wave is propagated at the surface. When angular velocity, $\omega = \eta_\beta k$ is in complex form, plane waves are diffusive at depth z with particular case where the SV-wave velocity in the medium is greater than the velocity of envelope, $c > \beta$ [12]. In this case, β is assumed analogous to the apparent velocity along the boundary, V_{app}, which yields $\beta = V_{app}$.

In this work, sinusoidal force $\mathbf{f} = e^{-ik(x-ct)}$ is presumed to act constantly along direction of propagating SV-wave. The elastic wave equation with $0\mathbf{f}, 1\mathbf{f}, 2\mathbf{f}, 3\mathbf{f}$ are solved to show the effects of external forces on SV-wave. (Figure 1.1) and (Figure 1.2) show the amplitude $\mathbf{u}$ is increasing along the characteristic line as $t \to \infty$, and it is polarized at $x-z$ directions. Polarization of the wave gives frequency equation where its amplitude decreases with depth z as below:

$$\mathbf{u}_{SV} = B\tan(-\eta_\beta)\begin{pmatrix} \dfrac{1}{4k^2c^2}\begin{pmatrix} e^{-ik(x-ct)} \\ -e^{-ik(x+ct)} \end{pmatrix} \\ +\dfrac{1}{2ikc}te^{-ik(x-ct)} \end{pmatrix}\cdot e^{-ik\eta_\beta z} \quad (2)$$

$$\omega = n_\beta k$$

Refraction is changing of wave direction due to variation in the optical medium. The result shows that wave in deeper water is moving faster as compared with the wave in shallow water [13]. Subsequently, many non-homogeneous linear evolution equations are solved by using a Duhamel's theorem; such are applicable to the vibrating plate equation, wave equation, and the heat equation [14]. In this research, Duhamel's theorem is used to verify refraction of an SV-wave in miscible medium. Furthermore, refraction of water waves in shallow water which is caused by landslides is shown. Analytical expression is obtained by Duhamel's theorem and given in the integral form for both water particle and water elevation, and it is applicable for rigid sliding body of any shape or velocity [15].

In the following, SV-wave is considered to pass through three different mediums with various forces.

Case 1: $c > \beta, \beta = V_{app}$

Research shows that slowness has initiated refraction in immiscible medium [16]. Equations (3) till (6) below represent the analytical solutions of SV-wave with various forces in low density medium. In the case where the velocity is higher than apparent velocity, $c > \beta$, it is believed that the low density fluid saturated in immiscible medium. In this study, displacement $\mathbf{u}$ is assumed as the amplitude of wave, and hence the (Figure 1) has been plotted. (Figure 1) portrays the amplitudes of SV-wave are higher in the low density medium than amplitudes of SV-wave in high density medium. Besides, SV-wave with larger force also induced higher amplitude in the same medium.

$$0\mathbf{f}: \mathbf{u}_{SV} = B\tan(-\eta_\beta)\left(e^{-ik(x-ct)}\right)\cdot e^{-ik\eta_\beta z} \quad (3)$$

$$1\mathbf{f}: \mathbf{u}_{SV} = B\tan(-\eta_\beta)\begin{pmatrix} \dfrac{1}{4k^2c^2}\begin{pmatrix} e^{-ik(x-ct)} \\ -e^{-ik(x+ct)} \end{pmatrix} \\ +\dfrac{1}{2ikc}te^{-ik(x-ct)} \end{pmatrix}\cdot e^{-ik\eta_\beta z} \quad (4)$$

$$2\mathbf{f}: \mathbf{u}_{SV} = B\tan(-\eta_\beta)\begin{pmatrix} \dfrac{1}{2k^2c^2}\begin{pmatrix} e^{-ik(x-ct)} \\ -e^{-ik(x+ct)} \end{pmatrix} \\ +\dfrac{1}{ikc}te^{-ik(x-ct)} \end{pmatrix}\cdot e^{-ik\eta_\beta z} \quad (5)$$

$$3\mathbf{f}: \mathbf{u}_{SV} = B\tan(-\eta_\beta)\begin{pmatrix} \dfrac{3}{4k^2c^2}\begin{pmatrix} e^{-ik(x-ct)} \\ -e^{-ik(x+ct)} \end{pmatrix} \\ +\dfrac{3}{2ikc}te^{-ik(x-ct)} \end{pmatrix}\cdot e^{-ik\eta_\beta z} \quad (6)$$

where $\eta_\beta = \sqrt{\dfrac{c^2}{\beta^2} - 1}$

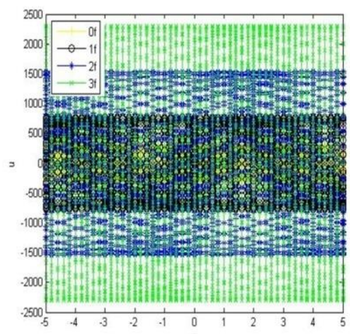

Figure 1 SV-wave's propagation with various forces in low density medium.

Case 2: $c = \beta, \beta = V_{app}$

$c = \beta$ means the fluid is completely mixed with the medium, the phase velocity is equivalent to apparent velocity, $c = \beta$, and hence $\eta_\beta = \sqrt{c^2/\beta^2 - 1}$ becomes zero. Subsequently, $\eta_\beta = 0$ gives $\tan(-\eta_\beta) = 0$. Therefore, SV-wave with $0\mathbf{f}, 1\mathbf{f}, 2\mathbf{f}, 3\mathbf{f}$ give zeros amplitudes. As a result, the SV-wave with various forces is invalid for $c = \beta$. This result indicates that the amplitude of an SV-wave does not exist in miscible medium and it implies SV-wave cannot propagate in liquid.

Case 3: $c < \beta, \beta = V_{app}$

With the concept of low density medium induced slowness of SV-wave [16], we can predict when an SV-wave passes through high density medium, and the velocity of the wave will get amplified. Equations (7) till (10) are analytical solutions for SV-wave with various forces in high density medium. As the apparent velocity is higher than phase velocity, $c < \beta$, it is assumed that high density fluid saturated in immiscible medium. By assuming amplitude $\mathbf{u}$ as the wave amplitude, (Figure 2) has been plotted. (Figure 2) portrays all SV-waves in high density medium give the highest amplitudes among the three mediums. In addition, SV-wave with larger force also gives higher amplitude in the same medium.

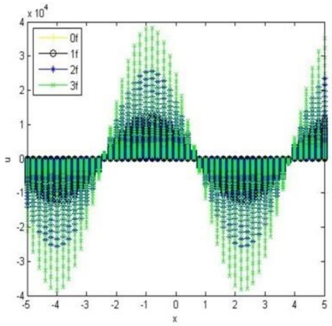

Figure 2 SV-wave's propagation with various forces in high density medium.

$$0\mathbf{f}:\mathbf{u}_{SV}=B\tan(-\eta_\beta)\left(e^{-ik(x-ct)}\right)\cdot e^{-ik\eta_\beta z} \quad (7)$$

$$1\mathbf{f}:\mathbf{u}_{SV}=B\tan(-\eta_\beta)\left(\begin{matrix}\frac{1}{4k^2c^2}\left(\begin{matrix}e^{-ik(x-ct)}\\-e^{-ik(x+ct)}\end{matrix}\right)\\+\frac{1}{2ikc}te^{-ik(x-ct)}\end{matrix}\right)\cdot e^{-ik\eta_\beta z} \quad (8)$$

$$2\mathbf{f}:\mathbf{u}_{SV}=B\tan(-\eta_\beta)\left(\begin{matrix}\frac{1}{2k^2c^2}\left(\begin{matrix}e^{-ik(x-ct)}\\-e^{-ik(x+ct)}\end{matrix}\right)\\+\frac{1}{ikc}te^{-ik(x-ct)}\end{matrix}\right)\cdot e^{-ik\eta_\beta z} \quad (9)$$

$$3\mathbf{f}:\mathbf{u}_{SV}=B\tan(-\eta_\beta)\left(\begin{matrix}\frac{3}{4k^2c^2}\left(\begin{matrix}e^{-ik(x-ct)}\\-e^{-ik(x+ct)}\end{matrix}\right)\\+\frac{3}{2ikc}te^{-ik(x-ct)}\end{matrix}\right)\cdot e^{-ik\eta_\beta z} \quad (10)$$

where $\eta_\beta=\sqrt{\frac{c^2}{\beta^2}-1}$

Figure 1.1 and Figure 1.2 have shown that the amplitudes of SV-wave become higher with larger forces. It is shown that sinusoidal force does cause significant changes in wave motion.

3.0 RESULTS AND DISCUSSION

(Figure 1.1) and (Figure 1.2) are plotted for propagating SV-wave with various forces through a low density medium $c>\beta$, and a high density medium $c<\beta$. These figures have shown that SV-wave in high density medium give higher amplitude. Meanwhile, in the same medium, SV-wave with larger force will give higher amplitude. It is proven that the sinusoidal force does affect the propagating SV-wave in medium of different densities. Besides, mathematical modeling of SV-wave also showed that in a similar dense medium $c=\beta$, it gives zero amplitude. This implies that SV-wave does not exist in soluble medium, and thus it cannot propagate in liquid.

4.0 CONCLUSIONS

This study has investigated the effect of external force to propagating SV-wave. The results show that the SV-wave with larger force induced higher amplitude in both low and high density media, which implies that a sinusoidal force has significant effect on the propagating SV-wave. In addition, the mathematical model showed that SV-wave cannot propagate in soluble medium. In this work, we have shown the effect of external force on propagating SV-wave, and thus we can conclude that external force should be involved in the seismic wave calculation.

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Appendix A

Problem formulation: The governing equation for the amplitude of the particle of an anisotropic elastic solid is given by:

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\nabla(\nabla\cdot\mathbf{u})+\rho\mathbf{f}=\rho\frac{\partial^2\mathbf{u}}{\partial t^2} \quad (11)$$

with boundary conditions and stresses are shown as below:

$$\sigma_z=(\lambda+\mu)\frac{\partial u_z}{\partial z}+\lambda(\frac{\partial u_z}{\partial x}+\frac{\partial u_y}{\partial y})=0$$

$$\tau_{xz}=\mu(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x})=0 \quad (12)$$

$$\tau_{yz}=\mu(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y})=0$$

and initial condition for $t=0$:

$$u(x,y,z)=0 \quad (13)$$

$$\frac{\partial u_x}{\partial t}=\frac{\partial u_y}{\partial t}=\frac{\partial u_z}{\partial t}=0 \quad (14)$$

For isotropic medium, λ and μ are the Lame parameters for the stress tensors $\sigma_z,\tau_{xz},\tau_{yz}$ and the displacements u_x,u_y,u_z are continuous everywhere. Simultaneously, ρ is the density of fluid and $\mathbf{f}$ is the external force exerts in the x,y,z directions such as gravity, electric force, magnetic force, inertia force, and more.

In this work, a sinusoidal force $\mathbf{f}=e^{-ik(x-ct)}$ is applied in the direction of propagating SV-wave, which is constant along the characteristic line $x+ct=\text{constant as } t\to\infty$. The elastic wave equations with $0\mathbf{f}$, $1\mathbf{f},2\mathbf{f},3\mathbf{f}$ [17] are solved and the results are plotted with MATLAB to show the effect of external force on propagating SV-wave.

By using the Laplacian of a vector $\nabla^2\mathbf{u}=\nabla(\nabla\cdot\mathbf{u})-\nabla\times(\nabla\times\mathbf{u})$, together with irrotational of P-wave and solenoidal of S-wave, $\mathbf{u}=\mathbf{u}_p+\mathbf{u}_s$ and $\nabla\times\mathbf{u}_p=0;\nabla\cdot\mathbf{u}_s=0$ [18]. Additionally, the force is assuming acting against both P wave and S wave, e.g. $\mathbf{f}=\mathbf{f}_P+\mathbf{f}_s$. All equations are substituted into equation (11), and the results are given as below:

$$\alpha^2(\nabla^2\mathbf{u}_p)+\beta^2(\nabla^2\mathbf{u}_s)+(\mathbf{f}_p+\mathbf{f}_s)=\frac{\partial^2}{\partial t^2}(\mathbf{u}_p+\mathbf{u}_s)$$

Rearrange to give,

$$\left(\frac{\partial^2\mathbf{u}_p}{\partial t^2}-\alpha^2\nabla^2\mathbf{u}_p-\mathbf{f}_p\right)+\left(\frac{\partial^2\mathbf{u}_s}{\partial t^2}-\beta^2\nabla^2\mathbf{u}_s-\mathbf{f}_s\right)=0 \quad (15)$$

In the following, equation (16) is the inhomogeneous equation for P and S waves [10]:

$$\frac{\partial^2\mathbf{u}_p}{\partial t^2}=\alpha^2\nabla^2\mathbf{u}_p+\mathbf{f}_p;\qquad \frac{\partial^2\mathbf{u}_s}{\partial t^2}=\beta^2\nabla^2\mathbf{u}_s+\mathbf{f}_s \quad (16)$$

Here, the inhomogeneous equations, i.e. elastic wave equation with $1\mathbf{f}$, are solved by using Duhamel's principle [18].

$$\mathbf{u}=\frac{1}{4k^2c^2}\left(e^{-ik(x-ct)}-e^{-ik(x+ct)}\right)+\frac{1}{2ick}te^{-ik(x-ct)} \quad (17)$$

The elastic wave equation can further be transformed into three dimensional space:

$$\mathbf{u}=\frac{1}{4k^2c^2}\left(e^{-ikc(\frac{lx+nz}{c}-t)}-e^{-ikc(\frac{lx+nz}{c}+t)}\right)+\frac{1}{2ick}te^{-ik(\frac{lx+nz}{c}-t)} \quad (18)$$

where $c=\alpha,\beta$

Furthermore, by Hansen's vector[16], then the amplitudes of P-wave, SV-wave and SH-wave are given by

$$\mathbf{u}_P=A(la_x+na_z)\left(-\frac{1}{4k^2c^2}\left(e^{-ikc(\frac{lx+nz}{\alpha}-t)}-e^{-ikc(\frac{lx+nz}{\alpha}+t)}\right)+\frac{1}{2ick}te^{-ikc(\frac{lx+nz}{\alpha}-t)}\right) \quad (19)$$

$$\mathbf{u}_{SV} = B(-na_x + la_z)\begin{pmatrix} -\dfrac{1}{4k^2c^2}\begin{pmatrix} e^{-ikc(\frac{lx+nz}{\beta}-t)} \\ -e^{-ikc(\frac{lx+nz}{\beta}+t)} \end{pmatrix} \\ +\dfrac{1}{2ikc}te^{-ikc(\frac{lx+nz}{\beta}-t)} \end{pmatrix} \quad (20)$$

$$\mathbf{u}_{SH} = Ca_y\begin{pmatrix} -\dfrac{1}{4k^2c^2}\begin{pmatrix} e^{-ikc(\frac{lx+nz}{\beta}-t)} \\ -e^{-ikc(\frac{lx+nz}{\beta}+t)} \end{pmatrix} \\ +\dfrac{1}{2ikc}te^{-ikc(\frac{lx+nz}{\beta}-t)} \end{pmatrix} \quad (21)$$

where a_x, a_y, a_z are unit vectors, l, n are the vector components, $\mathbf{u}_p$ is amplitude of P wave, $\mathbf{u}_{SV}$ is amplitude of vertical S wave, and $\mathbf{u}_{SH}$ is amplitude of horizontal S wave. In this paper, only equation (20) is considered, it is modifying to give SV-wave equation on amplitude reduces with depth as follow:

$$\mathbf{u}_{SV} = B(-\eta_\beta a_x + a_z)\begin{pmatrix} \dfrac{1}{4k^2c^2}\begin{pmatrix} e^{-ik(x+\eta_\beta z-ct)} \\ -e^{-ik(x+\eta_\beta z+t)} \end{pmatrix} \\ +\dfrac{1}{2ikc}te^{-ik(x+\eta_\beta z-t)} \end{pmatrix} \quad (22)$$

where $\eta_\beta = \sqrt{\dfrac{c^2}{\beta^2} - 1}$. η_β is always positive, c is the velocity of SV-wave in the medium, β is velocity of SV-wave measured at $z = 0$, which is assumed analogous to apparent velocity, V_{app} .