

**UNIVERSITI TEKNOLOGI MARA**

**ON A NEW CLASS OF  $P$ -VALENT  
FUNCTIONS OF COMPLEX ORDER  
INVOLVING SALAGEAN  
DIFFERENTIAL OPERATOR**

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Thesis submitted in fulfilment  
of the requirements for the degree of  
**Master of Science**

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## AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of UniversitiTeknologi MARA. It is original and is the result of my own work, unless otherwise indicated or acknowledged as referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any other degree of qualification.

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## ABSTRACT

Let  $S$  be the class of univalent functions of the form  $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ , analytic in the open unit disk  $U = \{z : |z| < 1\}$  and normalized by the conditions  $f(0) = f'(0) - 1 = 0$ . Also let  $S_p$  denote the class of  $p$ -valent functions of the form of  $f(z) = z^p + \sum_{k=1}^{\infty} a_{p+k} z^{p+k}$ . In this thesis, a new class of  $p$ -valent function is defined

where functions in this class satisfy the condition  $1 + \frac{1}{b} \left( \frac{1}{p} \frac{z(D^\lambda f(z))'}{D^\lambda f(z)} - 1 \right) \prec \frac{1 + Az}{1 + Bz}$ .

$\prec$  denotes subordination,  $b$  is any non-zero complex number,  $A$  and  $B$  are the arbitrary fixed numbers,  $-1 \leq B < A \leq 1$ .  $D^\lambda f(z)$  is the operator introduced by Shenen et al. (2004) which is the extension of Salagean operator where

$D^\lambda f(z) = z^p + \sum_{k=1}^{\infty} \left( \frac{p+k}{p} \right)^\lambda a_{p+k} z^{p+k}$  and  $\lambda \in N_0 = \{0\} \cup N$ . The properties of the new

class such as the coefficient estimates, growth and distortion theorems and radius properties are investigated. The upper bounds of the Fekete Szego functional and the upper bound of the second Hankel dererminant are also found in this thesis.

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