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About properties of linear stochastic optimization problems on arrangements

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Abstract – The article deals with the properties of linear combinatorial optimization problems on a set of arrangements under probabilistic uncertainty. The statement of the problem, considering the possibility of stochastic uncertainty of the initial data, is considered. The properties of the formulated stochastic problems are explored.

Keywords – optimization problem; optimization problem on arrangements; stochastic optimization.

INTRODUCTION

Actual trend of the modern theory of optimization is to study the problems of combinatorial nature ([1–14] and others). Important results have been obtained as a result of immersion of combinatorial sets in Euclidean space and study the properties of such problems ([6–14] and others). This paper is a continuation and development of a research within the Euclidean combinatorial optimization, it considers such an important class of Euclidean combinatorial optimization problems as arrangement problems.

We also should note that the problems with uncertainty, including probabilistic, attract the attention of researchers recently ([11–20] and others). Such problems arise and in combinatorial optimization. Stochastic combinatorial models can be used to describe and solve many other important practical problems. Earlier the authors [14] proposed an approach for stochastic optimization problems formalization through the introduction of the order relation on the set of random variables. Some properties of the problems on arrangements in this type of statement are discussed in this article.

I. Statement of the Problem

Let some of the initial data in the optimization problem be discrete random variables. The last will be denoted by Latin capital letters ($A,B,\dots$), their possible values – by small ($a^i,b^i,\dots$). In this paper, we consider only those discrete random variables among the possible values of which there is the least. We also believe that the possible values of the random variable are in ascending order, and the least value has an index 1.

Let $P(\cdot)$ denote the probability of a random event, $M(A)$ and $D(A)$ - respectively the mathematical expectation and dispersion A. Let characteristic vector of random variable A be $H(A)=(M(A);-D(a))$, $<_l$ – symbol of lexicographic order. Let us suppose also that the order on the set of discrete random variables is introduced by the following definition.

Definition 1. We will call two discrete random variables A,B organized in ascending (A preceded B) order $\prec$ (and denote this fact $A\prec B$), if one of these conditions:

1. $H(A)<_l H(B)$;

2. $H(A)=H(B)$ and there is such t that $a^i=b^i$, $P(A=a^i)=P(B=b^i)$ for all $1\le i<t$, and:

   2.1. or $a^t<b^t$,

   2.2. or $a^t=b^t$ and $P(A=a^i)>P(B=b^i)$

is true.

We will call two discrete random variables A,B ordered by the lack of growth ($A\preceq B$) if $A\prec B$ or $A=B$.

Using introduced linear order, let us order the elements of a given finite subset Ω of the set of discrete random variables: $X_1\preceq X_2\preceq\dots\preceq X_s$. X_s is the maximum value and X_1 is the minimum value. The definition of the minimum and maximum allows setting the optimization problem for finding the extreme elements in the given conditions. Let $X=(X_1,X_2,\dots,X_k)$ — random vector. Consider a linear function $L(X)=\sum_{j=1}^{k}c_jX_j$, where $c_j\in R^1$, $X_j\in\Omega$ $\forall j\in J_k$ (here and after J_k defines set of k first natural numbers), the values of the function also belong to the set Ω under all $X_j\in\Omega$ $\forall j\in J_k$. Then the linear optimization problem on a sphere Q can be formulated as follows: find a pair $\langle L(X^*),X^*\rangle$ such that

$$L(X^*)=\min_{X\in Q}L(X),\ X^*=\arg\min_{X\in Q}L(X).\quad (1)$$

In particular, sphere Q can be Euclidian combinatorial set. Let $\Gamma=\left\{G_1,G_2,...,G_\eta\right\}$ — stochastic multiset, whose elements are independent discrete random variables, $E_\eta^k\left(\Gamma\right)$ — a common set of k -arrangements from the elements of the stochastic multiset Γ [6]. Let us consider the problem of the representation (1) when $Q=E_\eta^k\left(G\right)$: find a pair $\left\langle L\left(X^*\right),X^*\right\rangle$ such as

$$L\left(X^*\right)=\min_{X\in E_\eta^k(\Gamma)}L(X),\ X^*=\arg\min_{X\in E_\eta^k(\Gamma)}L(X)\ .\quad(2)$$

We assume that the elements of multiset satisfy

$$G_1\preceq G_2\preceq...\preceq G_\eta.\quad(3)$$

Also we assume that the coefficients of objective function satisfy

$$c_1\geq...\geq c_\alpha>0=c_{\alpha+1}=...=c_{\beta-1}>c_\beta\geq...\geq c_k.\quad(4)$$

and

$$\text{if } c_i\neq c_j \text{ then } c_i^2\neq c_j^2 \text{ for all } i,j\in J_k.\quad(5)$$

II. Properties of Solving Stochastic Optimization Problem on Arrangements

Suppose characteristic vector satisfy the condition

$$H(G_1)\leq_l H(G_2)\leq_l ...\leq_l H(G_\eta).\quad(6)$$

Let also $\Gamma^M=\left\{M\left(G_1\right),M\left(G_2\right),...,M\left(G_\eta\right)\right\}$, multiset Γ^M have the base $\left(\bar{M}_1,...,\bar{M}_s\right)$ and primary specification $\left(\bar{q}_1,...,\bar{q}_s\right)$. Let's denote

$$q_1=1,\ q_{i+1}=q_i+\bar{q}_i=1+\sum_{j=1}^{i}\bar{q}_j \text{ for } i\in J_s;$$

$$\Gamma_i^M=\left\{G_{q_i},...,G_{q_{i+1}-1}\right\} \text{ for all } i\in J_s;$$

$$r=\min\{j\,|\,q_{j+1}>\alpha\},$$

$$t=\max\{j\,|\,q_j\leq\eta-k+\beta\},$$

$$u_i=\begin{cases}q_i,\ \text{if } i\leq r,\\ k-\eta+q_i, \text{if } i>t,\\ \beta, \text{if } i=t>r;\end{cases}$$

$$v_i=\begin{cases}q_i+\bar{q}_i-1,\ \text{if } i<r,\\ k-\eta+q_i+\bar{q}_i-1, \text{if } i\geq t,\\ \alpha, \text{if } i=r<t,\end{cases}$$

$$k_i=v_i-u_i+1.$$

Then for all $i\in J_r\cup J_s^t$ $\left(X_{u_i},...,X_{v_i}\right)\in E_{\bar{q}_i}^{k_i}(\Gamma_i^M)$ and there is minimal in solution of the problem (2) such that $i\in J_r\cup J_s^t$ $\left(X_{u_i},...,X_{v_i}\right)\in E_{\bar{q}_i}^{k_i}(\Gamma_i^M)$.

Let's denote $T=J_r\cup J_s^t$. If $r=t$ then $T=J_s$ and $\sum_{i\in J_r\cup J_s^t}\sum_{u_i}^{v_i}c_jX_j=\sum_{j=1}^{k}c_jX_j$. If $r<t$ then $v_r=\alpha$, $u_t=\beta$, $c_{\alpha+1}=...=c_{\beta-1}=0$. Thus

$$\sum_{i\in T}\sum_{u_i}^{v_i}c_jX_j=\sum_{i=1}^{r}\sum_{u_i}^{v_i}c_jX_j+\sum_{i=t}^{k}\sum_{u_i}^{v_i}c_jX_j=$$

$$\sum_{j=1}^{\alpha}c_jX_j+\sum_{j=\beta}^{k}c_jX_j=L(X).$$

Let's denote

$$\tilde{X}=\left(X_{u_i},...,X_{v_i}\right),\ L_i(\tilde{X})=\sum_{j=u_i}^{v_i}c_jX_j.$$

We prove the follow theorem.

Theorem 1. If for all $i\in J_r\cup J_s^t$ $\tilde{X}^*=\left(X_{u_i}^*,...,X_{v_i}^*\right)$ is the minimal of function $L_i(\tilde{X})$ on the set of arrangements $E_{\bar{q}_i}^{k_i}(\Gamma_i^M)$ then $X^*=\left(X_1^*,...,X_k^*\right)\in E_\eta^k(\Gamma)$ is the minimal in solution of the problem (2).

From the properties of minimal of (deterministic) linear unconstrained optimization problem [6] on arrangements it follows that

$$M\left(X_j\right)=M\left(G_j\right)\ \forall j\in J_{q_r},$$

$$M\left(X_j\right)=M\left(G_{\eta-k+j}\right)\ \forall j\in J_\eta^{\eta-k+q_t}.$$

Let us consider solving of problem of optimization of function $L_i(\tilde{X})$ on the stochastic set $E_{\bar{q}_i}^{k_i}(\Gamma_i^M)$. Since mathematical expectation of all random variables from stochastic multiset Γ_i^M are equal and (6) is true, then $-D(G_{u_i})\leq_l ...\leq_l -D(G_{v_i})$. If $i<r$ then $c_j>0$ $\forall j\in J_{v_i}^{u_i}$. Then it implies from (4) that $c_{u_i}^2\geq c_{u_i+1}^2\geq...\geq c_{v_i}^2$. Hence there is minimal of the function $L_i(\tilde{X})$ such as $M\left(X_j\right)=M\left(G_j\right)$ $\forall j\in J_{v_i}^{u_i}$. Thus $\forall i\in J_{r-1}$

$$H\left(X_j\right)=H\left(G_j\right)\ \forall j\in J_{v_i}^{u_i}.\quad(7)$$

If $i > t$ then $c^2_{v_i} \geq c^2_{u_i+1} \geq ... \geq c^2_{v_i}$. Hence there is minimal of function $L_i(\tilde{X})$ such as $D\left(X_{v_{i-j}}\right) = D\left(G_{u_{i+j}}\right) \quad \forall j \in J^0_{k_i-1}$. Taking into consideration that $M\left(G_{u_{i+j}}\right) = M\left(G_{u_i}\right) \quad \forall j \in J^0_{k_i-1}$, we obtain that $\forall i \in J^{t+1}_k$

$$H\left(X_{v_i-j}\right) = H\left(G_{u_i+j}\right) \;\; \forall j \in J^0_{k_i-1}. \qquad (8)$$

If $r < t$ then $v_r = \alpha$ and (7) is true for $i = r$. Thus (7) is true for all $i \in J_{\bar{r}}$ where $\bar{q} = \min\{r, t-1\}$. Similar from $u_t = \beta$ obtain that (8) is true for $i = t$. Hence (8) is true for all $i \in J^{\bar{t}}_k$, where $\bar{t} = \max\{r+1, t\}$.

Suppose now that $r = t$ and coefficients of the objective function satisfy

$$c^2_{u_r+p_1} \geq c^2_{u_r+p_2} \geq ... \geq c^2_{u_r+p_k}, \qquad (9)$$

where $k = v_r - u_r + 1$, $p_j \in J^0_{k-1} \quad \forall j \in J_k$. Then $\forall j \in J_k$ the correlation $H\left(X_{u_i+p_j}\right) = H\left(G_{u_i+j-1}\right)$ is true. In the case when all the coefficients of the objective function are positive there is minimal in the solution of the problem (2) such as (7) is true for all $i \in J_s$. Hence $H\left(X_j\right) = H\left(G_j\right) \;\; \forall j \in J_k$.

And if the condition

$$H\left(G_i\right) \neq H\left(G_j\right) \text{ if } G_i \neq G_j \;\; i, j \in J_\eta \qquad (10)$$

is true then the arrangement

$$X^*_j = G_j \;\; \forall j \in J_k \qquad (11)$$

is the minimal in the solution of the problem (2).

Note that if elements of the multiset H are satisfy (3) but (10) is not true then the arrangement (11) may not be a minimal in solution of the problem (2).

III. Conclusions

Optimization problems on arrangements, in which the elements of the multiset are independent discrete random variables, are considered. Minimum in the problem refers in accordance with the linear order introduced to the set of random variables. Some properties of the problems on arrangements in this type of statement are discussed. Subsequent studies suggest further study of the properties of the considered problems that will allow developing methods and algorithms to solve them.

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