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Piecewise Linear and Nonlinear Window Functions for Modelling of Nanostructured Memristor Device

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The present paper reports two new window functions viz. piecewise linear window function and nonlinear window function for modelling of the nanostructured memristor device. The piecewise linear window function can be used for modelling of symmetric pinched hysteresis loop in *I-V* plane (for digital memory applications) and the nonlinear window function can be used for modelling of nonlinear pinched hysteresis loop in *I-V* plane (for analog memory applications). Flexibility in the parameter selection is the main attractive feature of these window functions.

Keywords: Memristor, Simulation, Window function.

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1. INTRODUCTION

The memristor is a fourth fundamental circuit component and it has potential application in the area of Resistive Random Access Memory (RRAM), biomedical applications, new kind of electronic circuits and many more [1-5]. The pinched hysteresis loop in *I-V* plane is one of the fingerprint characteristics of memristor. This loop is analogues with the Low Resistance State (LRS) and High Resistance State (HRS) of RRAM based devices. The LRS and HRS loop can be symmetrical and asymmetrical (nonlinear) in the nature. The symmetrical loop can be used for digital memory application due to its instantaneous switching from LRS to HRS and vice versa. The asymmetrical loop can be used for analog memory application such as neuromorphic application where switching of memristor can be considered as smooth, highly nonlinear switching from LRS to HRS and vice versa. Recently Dongale et al. developed nanostructured TiO_2 thin film memristor with low symmetric voltage switching ($\pm$ 0.68 V) [3]. Similarly Kundozerova, et al. developed anodic oxides based memristor in which *I-V* curve is asymmetric [6].

The literature survey reveals that, most of the researchers applied the nonlinear window function or derived nonlinear dependence of the state variable for the simulation of nonlinear effects of memristor device [7-10]. The explicit relationship between memristance (*M*) and charge (*q*) cannot be obtained using above model and theories. If we would like to simulate the memistor with above window functions, then we require some numerical methods for solving the differential equations [11, 12]. To deal with this problem, we are proposing two window functions viz. Piecewise Linear Window Function and Nonlinear Window Function.

The rest of the paper portrayed as follows, the first section deals with the introduction and outline of the problem statement. The second section discusses background theory of memristor device and simulation of memristor device using linear drift model. The third section briefly describes the proposed window functions and at the end conclusion is reported.

2. BACKGROUND THEORY OF MEMRISTOR DEVICE

The memristor is popularly known as fourth fundamental and passive circuit component [1]. The memristor device was first of all predicted by Prof. L. Chua in his seminal research paper [1] in 1971. After forty years, in 2008, HP research group reported the first physical prototype of memristor device [2]. The pinched hysteresis Current-Voltage (*I-V*) loop and nonlinear curve between Charge (*q*)-Magnetic Flux (*φ*) are the fingerprint characteristics of nanostructured memristor device. The current controlled and voltage controlled are the two types of memristor device.

The HP memristor model considered the drift of oxygen vacancies as a state variable in the Pt / TiO_2 / Pt structure. The typical structure of HP memristor is depicted in the Fig. 1.

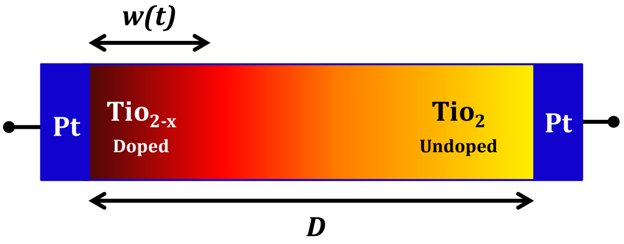

Fig. 1 – Structure of memristor reported by HP Lab [2]

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The Valency Change Mechanism (VCM) is the basis of memristor. The existing literature reveals that the most of the memristor is modeled around the ideal HP memristor model in which drift of oxygen vacancies considered as a state variable, which is similar to Chua's state variable '*w*' [1, 2]. The reported HP memristor consists of Pt / TiO_2 / Pt structure in which TiO_{2-x} oxygen rich doped conductive layer plays an important role to produce memristor like characteristics. The '*D*' is a thickness of active sandwich structure and '*w*' is the thickness of doped region. The HP memristor is based on linear drift model and considered the linear ionic drift. If average drift velocity of oxygen vacancies μ_v, then memristor current and voltage relation can be represented using following mathematical equations (1) [2]:

$$V(t)=\left[\left(\frac{R_{ON}w(t)}{D}\right)+R_{OFF}\left(1-\frac{w(t)}{D}\right)\right]i(t), \qquad (1)$$

Where state variable '*w*' can be represented as,

$$\frac{dw(t)}{dt}=\eta\frac{u_v R_{ON}}{D}i(t) \qquad (2)$$

Integrating equation (2) w.r. to '*t*' we get,

$$w=\eta\frac{u_v R_{ON}}{D}q(t) \qquad (3)$$

Inserting equation (3) into (1) we get memristance *M*(*q*),

If $R_{ON} << R_{OFF}$,

$$M(q)=R_{OFF}\left(1-\frac{\eta u_v R_{ON}q(t)}{D^2}\right) \qquad (4)$$

The simulation of linear drift model is represented in the Fig. 2. The experimental results suggested that the drifts of vacancies are highly nonlinear near the boundary interfaces and it is generally known as nonlinear dopent drift [13-16]. It is due to fact that, a small voltage across the nanoscale memristor produces large electric field across the device. This large electric field produces nonlinear drifting of vacancies near the boundary interfaces [13-16]. Another problem with linear drift model of memristor is that the state variable '*w*' never reaches to zero physical length which indicates oxygen vacancies are absent in the devices [17]. The boundary problem can be minimized using window function *f*(*x*). In general, the nonlinear dopent drift can be obtained by simply multiplying the state equation of memristor (equation 2) with the window function *f*(*x*) [1, 8].

$$\frac{dw(t)}{dt}=\left(\eta\frac{u_v R_{ON}}{D}i(t)\right)f(x) \qquad (5)$$

The function *f*(*x*) should have its highest value at the center of the device ($x = 0.5$) and zero at boundaries of memristor device [17].

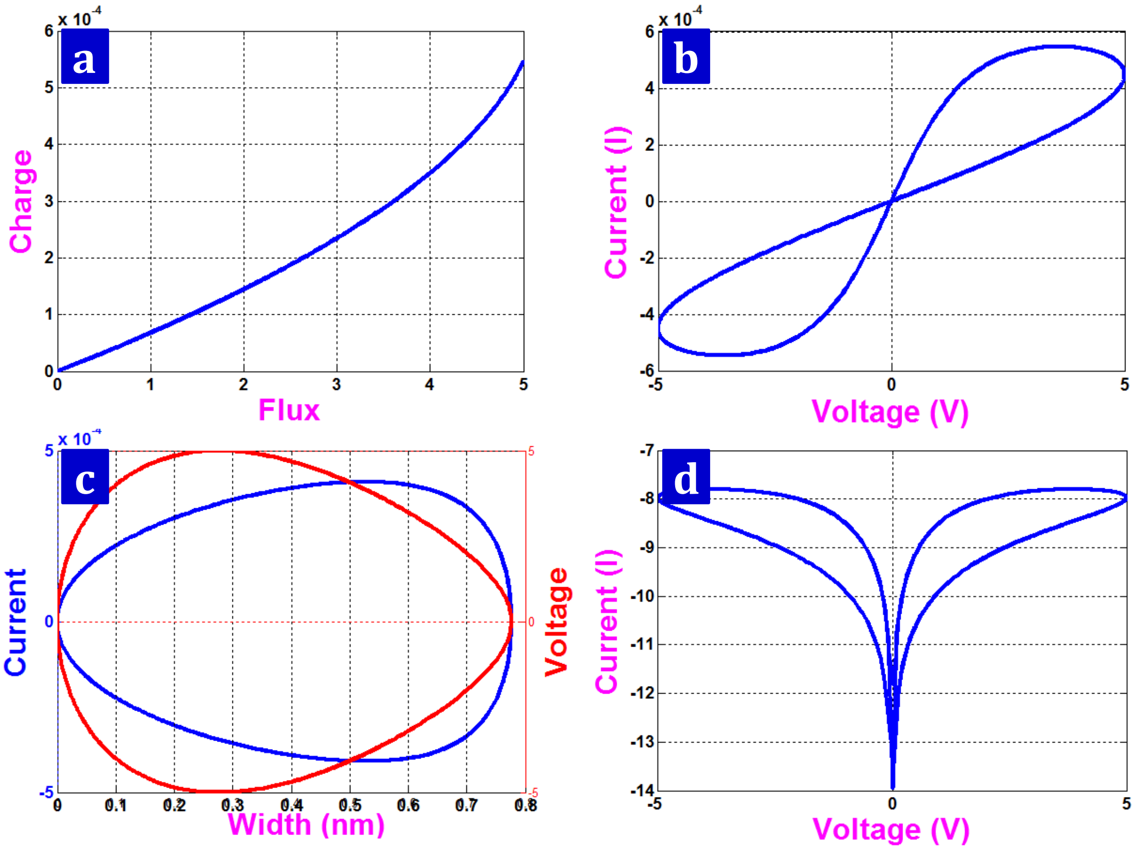

Fig. 2 – Simulation results of linear drift model of nanostructured memristor device (a) plot of flux vs charge. (b) plot of current vs voltage. (c) plot of relationship between current and voltage vs width (state variable). (d) plot of log (current (*I*)) vs log (voltage (*V*))

3. PROPOSED WINDOW FUNCTIONS

3.1 Piecewise Linear Window Function

The proposed window function is piecewise continuously differentiable at three regions viz. LHS bounds, middle region and RHS bounds and shows the nonlinear behaviour at lower values of control parameter 'p' and linear behaviour at higher values of control parameter 'p'. The equation (6) represents the generalized piecewise linear window function such as,

$$f(x) = \begin{cases} \dfrac{px}{mX_0} & \text{for } 0 \le x \le X_0 \\ \dfrac{p}{m} & \text{for } X_0 \le x \le Y_0 \\ \dfrac{p(1-x)}{m(1-Y_0)} & \text{for } Y_0 \le x \le 1 \end{cases} \tag{6}$$

Where, $0 < X_0 < Y_0 < 1$ and (p and $m \in R^+$). Consider a suitable case when LHS bounds lies between $0 \le x \le 1/3$ and RHS bounds lies between $2/3 \le x \le 1$ and constant for middle region such that,

$$f(x) = \begin{cases} \dfrac{3px}{20} & \text{for } 0 \le x \le 1/3 \\ \dfrac{p}{20} & \text{for } 1/3 \le x \le 2/3 \\ \dfrac{3p(1-x)}{20} & \text{for } 2/3 \le x \le 1 \end{cases} \tag{7}$$

Where, $x(t) = w/D$ is the normalized form of the state variable, and the control parameter 'p' is a positive integer and $m = 20$. Fig. 3 represents the proposed piecewise linear window function for various values of control parameter 'p'.

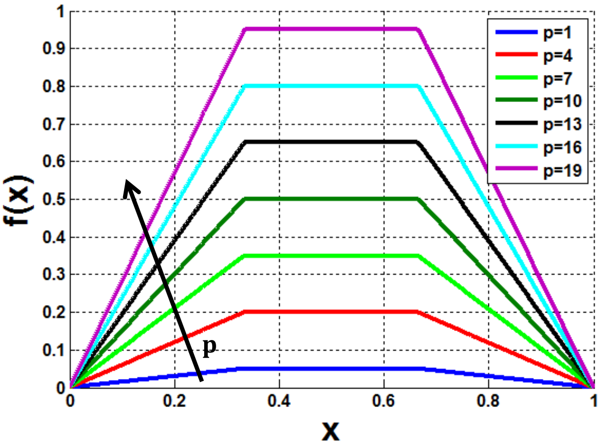

Fig. 3 – Plot of piecewise linear window function for modelling of nonlinearities of memristor device for various values of control parameter 'p'

The Fig. 3 clearly indicates that, as the value of control parameter 'p' increases from 1 to 20, LHS and RHS bounds of window function shows the quasi nonlinear behaviour of dopent kinetics at the boundaries and middle region represents the linear behaviour of memristor's dopent kinetics. This indicates proposed model accurately considered the linearity of dopent drift at the middle region of the device and quasi nonlinear behaviour at the boundaries. The proposed window function has more flexibility than other nonlinear window functions. The proposed window function simulates the nonlinearity at the boundaries only for the lower value of control parameter 'p'. This drawback is rectified by using nonlinear window function. One such approach is depicts in the next section. One can adjust the values of X_0, Y_0, m and control parameter 'p' for modelling of the memristor based digital memories.

3.2 Nonlinear Window Function

The proposed piecewise linear window function simulate the quasi nonlinear behaviour of dopent kinetics at the boundaries and linear behaviour in the middle region. A good window function must possess full nonlinear behaviour at the boundaries and linear behaviour in the middle region. It is found that the piecewise linear window function shows the nonlinear behaviour only at lower value of control parameter 'p'. This problem can be minimized by adopting the smooth nonlinear window function. The proposed nonlinear window function is similar to piecewise linear window function except, it has nonlinear characteristics at the boundaries. The equation (8) represents the proposed nonlinear window function such that,

$$f(x) = \begin{cases} x^{\frac{1}{p}} & \text{for } 0 \le x \le X_0 \\ x_0^{\frac{1}{p}} & \text{for } X_0 \le x \le Y_0 \\ \left|(x-1)^{\frac{1}{p}}\right| & \text{for } Y_0 \le x \le 1 \end{cases} \tag{8}$$

Where, $0 < X_0 < Y_0 < 1$, $Y_0 = (1 - X_0)$ and ($p \in R^+$). Consider a suitable case when LHS bounds lies between $0 \le x \le 1/5$ and RHS bounds lies between $4/5 \le x \le 1$ and constant for middle region such that,

$$f(x) = \begin{cases} x^{\frac{1}{p}} & \text{for } 0 \le x \le 1/5 \\ 5^{\frac{1}{p}} & \text{for } 1/5 \le x \le 4/5 \\ \left|(x-1)^{\frac{1}{p}}\right| & \text{for } 4/5 \le x \le 1 \end{cases} \tag{9}$$

Where, $x(t) = w/D$ is the normalized form of the state variable and the control parameter 'p' is a positive integer and $m = 20$. The nonlinear dopent drift can be obtained by simply multiplying the state equation of memristor with the window function $f(x)$. The Fig. 4 represents the proposed nonlinear window function for various values of control parameter 'p'.

Fig. 4 suggested that, as the value of control parameter 'p' increases from 1 to 20, the LHS and RHS bounds of window function show the nonlinear behaviour of dopent kinetics at the boundaries, and middle region represents the linear region of memristor's dopent kinetics. Moreover, it accurately fitted the nonlinearity for the higher values of control parameter 'p'.

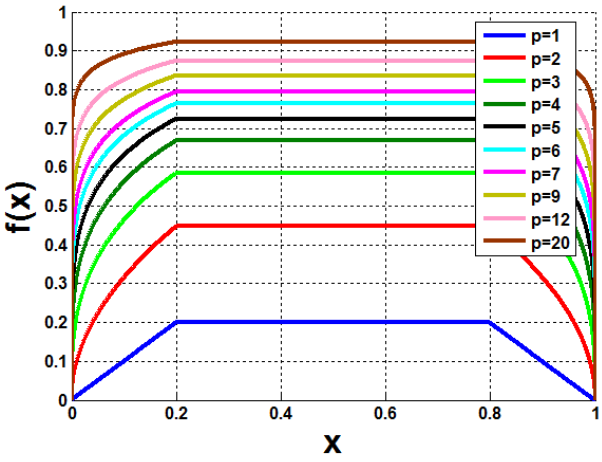

Fig. 4 – Plot of nonlinear window function for modelling of nonlinearities of memristor device for various values of control parameter '*p*'

This indicates that the model accurately considered the linearity of dopent drift at the middle region of the device and nonlinear behaviour at the boundaries. The proposed window function has more flexibility than other nonlinear window functions and proposed piecewise linear window function. The proposed nonlinear window function can be used for modelling of the memristor based analog memories.

4. CONCLUSION

The present paper discuses two new window functions for modelling of memristor based analog and digital memories. The piecewise linear window function accurately models the linearity of dopent drift at the middle region of the device and quasi nonlinear behaviour at the boundaries. The nonlinear window function models the linearity of dopent drift at the middle region of the device and nonlinear behaviour at the boundaries. The main feature of these models is the flexibility of its operation and one can adjust the parameters of window function for modelling of memristor based memory devices.

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