Does it matter whether God plays dice?

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Abstract

I argue that results from algorithmic number theory imply intrinsic randomness in models of quantum theory is much less important than has been assumed. I also point out some of the confusion in terminology of models of quantum theory, in particular the meaning of realism in relation to Bell's theorem.

1 Introduction

Bell's theorem says that models of quantum theory can't have both locality and realism. To look into this first we need to know what a model of quantum theory is. Suppose there are two experimenters, Alice and Bob each with a measuring device which allows two settings. Two entangled particles are emitted from a central source, and measured by the two devices. Alice gets result a_1 with setting 1 and a_2 with setting 2, and likewise Bob gets result b_1 or b_2, where each of a_1, a_2, b_1, b_2 is either zero or one. A model needs to explain the results in terms of the settings, possible properties of the particles and so on. This implies that a model is something like a computer program. For instance, [1] is a challenge to write a computer program which disproves Bell's theorem. I think that it is reasonable to allow a more general idea of a model, going beyond what is possible with a computer. For instance one would think of numbers as having infinite precision, and one might allow infinite structures. One could also allow a model to have intrinsically random elements. However, one shouldn't allow what is meant by a model to become too general, otherwise it may lose its predictive aspect.

I would argue that this more general idea of a model doesn't offer anything which is not available from a computer program when considering Bell's theorem. Infinite precision and infinite structures can be approximated by finite precision or structures, and such approximations are common in many areas of science, and so are well understood. If a such a model could be found which violates Bell's inequality, then it is almost certain that this violation could be reproduced in a finite model.

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2 Randomness

What about intrinsic randomness, which is often thought to be a central part of quantum theory? In a computer model a random process would be represented as the output of a pseudo-random number generator. Can an intrinsically random element of a model offer anything extra. I claim that it can't. Algorithmic number theory (e.g. [2]) tells us that we can never be sure that what we observe is random. However random a finite sequence of results may seem, there might be a pattern which we haven't been able to detect yet. Now if we are unable to detect a pattern then the hypothesis that there is no pattern seems reasonable. We might say that something is random 'for all practical purposes' so this may seem like an unimportant quibble. However, it has an important consequence: in talking about models of quantum theory, there is always the possibility of replacing intrinsic randomness by a pseudo-random process (hidden variables) - the idea of intrinsic randomness can 'do no work'. This means that results claiming to show a model must contain intrinsic randomness for some reason must be wrong. For example von Neumann's 1932 proof of 'no hidden variables' was shown to be flawed when Bohm constructed his hidden variables model of quantum theory. [3]

What about Bell's theorem? In [4][1] there is the statement 'no model that reproduces the predictions of quantum mechanics can simultaneously satisfy the assumptions of locality and determinism', so that locality must imply randomness. My argument says that this can't be right. What did Bell say? In [5] he does seem to be arguing that locality is inconsistent with 'hidden variables', i.e. a deterministic model. However, in [6] he says pretty much the opposite, that locality implies determinism. To understand this we need to look deeper into what motivated Bell. The subtitle of [4]'What Bohr Could Have Told Einstein at Solvay Had He Known About Bell Experiments' suggests that Bell was essentially strengthening Bohr's arguments against Einstein's ideas. I see things somewhat differently - that Bell was continuing the work of Einstein. By 1909, Einstein had realised that there would be a problem with 'action at a distance' in quantum theory, and it has been suggested that many of his quantum theoretic arguments were linked to this[7]. In 1935 this was made explicit in the EPR argument. Essentially the EPR argument argues that if a random process were involved in measurement, there wouldn't be correlation between measurements at different places. Suppose you and a colleague are checking the decks of cards at a casino. You observe one table, your colleague another, for each of you the checks don't flag anything strange. However, afterwards you discover that each of you saw the same cards dealt in the same order. This would be a clear indication that something was not right with the casino.

Unfortunately there was very little interest in the EPR argument until the work of Bell. He accepted the EPR argument (which was essentially that locality

[1] I quote from[4]several times here, but I am not arguing against the results of that paper, which I find to be a useful contribution to the discussion of quantum theory. Rather I am arguing against ideas which seem to be accepted as the consensus (and indeed problems with some of these ideas are pointed out in [4]

implies determinism), but saw that it wasn't the whole story. He also didn't have the same commitment to locality that Einstein had, and so saw the argument more as one against locality than one supporting determinism. [8]

3 Realism

So what it meant by realism? In what way can you deny realism and keep locality? If one has a model of the physical world, one can certainly take an anti-realist view, that is one avoids the claim that the elements of reality correspond to the elements of the model. When applied to models of quantum theory, however, this form of anti-realism seems to say that one shouldn't claim that the elements of the model correspond to the elements of the model. So that can't be what is meant by realism here.

The arguments in section 2 of this paper show that taking realism to mean determinism doesn't work.

In [9] realism is taken to mean "all measurement outcomes depend on pre-existing properties of objects that are independent of the measurement" , which would seem to imply that a model in which the outcome depended on the measurement settings, and other properties of the environment, would be non-realistic. The trouble is that a local model in which this is the case still has to satisfy the Bell inequalities, so denying realism in this sense doesn't help you keep locality.

In [4] standard operational quantum theory (that is a collapse based model), is given as an example of a local model, but is it? In Alice and Bob's experiment, when Alice makes a measurement the collapse of the wavefunction will depend on the setting of her device. This is taken into account when predicting the result Bob will see, so it is clearly nonlocal transmission of information. In this context denying realism seems to be a way of claiming that a model is local by ignoring the nonlocal elements.

4 Local models of quantum theory.

I have argued that any local models of quantum theory must be deterministic, while other results seem to say that local models have to be nondeterministic. Is this simply because there are no local models? The answer is that local models are possible but somewhat strange. Bob's apparatus needs to know Alice's choice a to give the right result B. Superluminal signalling is one way. Another is that the universe is sufficiently deterministic that somehow the Bob's apparatus can know what setting Alice will choose. Such models are called superdeterministic, and mean that the experimenter's behaviour is part of the model, so that freewill has to be abandoned. [2] A model along these lines is given in [11] (see also [12]).

[2] In [10] it is shown that experimenters can keep 86% of their freewill.

Is it possible to have a local deterministic model of quantum theory in which experimenters are allowed to have freewill? There is one possibility, although I'm not sure whether it has been fully worked out. If one has a many-worlds model of quantum theory, but one which is local in the sense that the splitting of worlds (or minds) propagates at the speed of light, then it might be possible that the different versions of Alice and Bob meet up in such a way that they see violation of the Bell inequalities.

The above discussion indicates that abandoning determinism doesn't help in constructing a local model of quantum theory, in fact such models are in a sense *more* deterministic than those we are used to. Do they count as being realistic? Realism seems such a vague term that it is hard to tell. My conclusion is that *any local model of quantum theory must include human minds as part of the model*

Noncomputable models Local models of quantum theory have been proposes in which the behaviour is deterministic but noncomputable. (See [13]and [14], also [15]). While these are not modellable on a computer, they do not pose any problems for my arguments, as they are deterministic and involve human minds as part of the model.

Negative probabilities Classically one would expect that each of the particles has a definite state when it leaves the central source, and that this state determines the result of the measurement for a given setting. Essentially, there are 16 possible states, representing the possible ways in which each of a_1, a_2, b_1, b_2 can be set to zero or one. The challenge is to find a distribution of the proportion each of these 16 states which reproduces the observed results. What Bell's theorem says is that this is impossible. However, this naturally assumes that the proportion of each state must be no greater than 100% and no less than 0%. If one abandons this assumption, allowing a negative probability for some of the states, then it is possible to reproduce the observed results [16].

I would exclude this possibility as a sensible model - it doesn't really make sense to say that -5% of the particles have a given state. However, others might see this as no more unreasonable that splitting $100 between two people and one ending up gaining $105 and the other losing $5 - unfair maybe, but not impossible.

Note that even here, mention of probability doesn't cause a problem for the EPR argument, as here the probability represents the distribution of particles emitted from the central source, rather than being linked to the measurement.

5 Conclusion

If we assume signal locality then the above suggests that every model has elements which are not accessible to measurement by experimenters. For a local model then the inclusion of minds in the model mean that the experimenters cannot access everything. For a nonlocal model signal locality means that the experimenters cannot access the nonlocal elements. In it is shown that signal locality implies unpredictability, but this leaves open the possibility that the unpredictability could be due to intrinsic randomness. My analysis would suggest that intrinsic randomness is irrelevant, and that there will be unpredictability due to not all of the elements of the model being accessible to experimenters.

There is a more direct way to see that signal locality means that experimenters can't have access to all of the information in the model, that is entanglement swapping. Particles A and B are entangled and so are C and D. Bob takes B and D, Alice keeps A and C. Bob does a joint measurement on B and D, which makes A and C become entangled. A and C are in the same place, so there is no issue of nonlocality - but there is no way for Alice to detect the entanglement (otherwise Bob could use this to send a signal). However, the entanglement of A and C must be included in the model as there is a correlation between the result Bob gets in his joint measurement and that which Alice gets from a joint measurement of A and C.

It is hard to see why there are so many arguments against hidden variables. If they are an alternative to randomness, then as I've argued above, a model with intrinsic randomness can always be replaced by one with hidden variables. Furthermore, whether a model incorporates randomness or not, if it has signal locality then it must have hidden variables, in the sense of elements of the model which cannot be accessed by experimenters. What is not allowed is the assumption that the values of its measurable properties uniquely determined a state. A model may or may not have intrinsic randomness, but it will have indeterminacy.[17]

Claims that quantum mechanics cannot have a classical model seem to me to be incoherent. Quantum mechanical calculations can be done, and agree very well with what is observed. Calculations can be performed by a computer. Computers (in the abstract) are essentially classical entities. So the ability to do calculations implies a classical model. Of course it is possible to propose a model that involves elements which cannot be calculated, but one is not forced to do so, and it is hard to see experimental results as supporting such elements as to match the model with experiment requires calculation.

To summarise: if one thinks of the "God's eye view" then God doesn't need to play dice, even if it looks to us as if He does.

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