Expanding Space: Just say No

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Abstract

I argue that the idea of expanding space as a way of explaining the behaviour of the universe is unhelpful, in that it not only creates misconceptions for those new to cosmology, but also has traps for those with much more experience of the subject.

1 Aristotle, Newton and Einstein

Nature and nature's laws lay hid in night;
God said "Let Newton be" and all was light. Alexander Pope

It seems obvious that to move something you need to push it, and if you stop pushing it, then it will stop. That was part of Aristotle's view of physics. However, Isaac Newton showed that it wasn't necessary to keep pushing something for it to keep moving, just to change its velocity. In mathematical terms, it is the second time derivative which is significant, not the first. This meant that not only is there no preferred position in the universe (which people eventually got used to, once the geocentric universe was disposed of), but also that there is no preferred velocity - all motion was relative. The introduction of gravity caused two problems - firstly it seemed to act instantaneously, and secondly it is hard to see how the universe could remain static if there was a universal attractive force. It is likely that Newton knew about both of these problems, and one way to lessen their seriousness was to maintain that space was absolute, in disagreement with Leibniz, who maintained the relativity of space and motion.

It did not last: the devil, shouting "Ho.
Let Einstein be," restored the status quo. J C Squires

Albert Einstein worried about the fact that although motion of objects was relative in Newton's theory, motion of electromagnetic waves in Maxwell's theory was not. In 1905 he found a way - special relativity - to get around this, but the cost was some rather counterintuitive ideas about space and time. You consider your fast moving twin to be aging slower than you, but he thinks that you are aging slower than him. And there were still the same problems with gravity. Relativity meant that Einstein couldn't appeal to absolute space. It took him ten years to deal with the seeming instantaneous propagation of gravity and devise a theory - general relativity - in which it obeyed the universal speed limit. As for the problem with a static universe, well in time it was found that the universe isn't static - it's expanding.

1.1 Expanding space

The mathematics of general relativity is difficult, however, and special relativity, on which it is built, is counterintuitive. So what has appeared is the idea of 'expanding space'. Absolute space exists, but expands with time. This is against the spirit of relativity, and introduces a spurious dependence on the first time derivative into our understanding. It's no wonder that people get confused when expanding space is mentioned. How does matter pick up the expansion of space? Well, of course, 'Brooklyn is not expanding' [1], i.e. matter doesn't just expand with space, if it did then expanding space wouldn't explain anything. So it seems that it must pick up the expansion via some sort of frictional force, i.e. one based on the first time derivative, that is Aristotelian physics. Those without a thorough grounding in physics are likely to get entirely the wrong idea from expanding space.

If you do have a thorough grounding in physics though, what then? You can use general relativity to calculate what actually happens, but that's difficult, and it's very useful to have some sort of heuristic

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idea of what's going on. A good heuristic can be invaluable in telling you whether your results are probably right, or whether you have to put more effort into checking them. For cosmological models my heuristic is to consider whether they agree with special relativity. This is actually more than a heuristic, in one case it is exactly right. Simple cosmological models are based on a homogenous, isotropic universe with the equation of state $pressure = w \times density$, where w can be given different values depending on the model. The scale factor is then given by

$$R(t) = R_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

When $w = -1/3$ it corresponds to a universe without gravity or dark energy. (this is sometimes known as the Milne universe) It is clear that this behaves according to the rules of special relativity, firstly because of the way that general relativity is built from special relativity, and secondly because you can do the calculations to show that they are equivalent.[2]

Is 'expanding space' a good heuristic? I have commented on discussions concerning expanding space in [3]. In the current paper I will look at cases where it is possible to compare the full general relativistic calculation with what is suggested by expanding space.

In [4], the authors propose a test. An object is placed a certain distance from you and they then calculate for 5 different cosmological models ($w = -2/3, -1/3, 0, 1/3, 2/3$) how it will behave subsequently. Does it:

A) Immediately join in with the expansion of space and so start to move away from you with the appropriate speed (Brooklyn is expanding)

B) Gradually start to move away from you.

C) Something else ?

In the last 4 of the 5 cases it starts to move *towards* you. It's hard to see why people still believe in expanding space.

2 'Proper' distance and the tethered galaxy?

In the last section you may have noticed a problem with what I have said. In the case $w = -1/3$, I claim an object placed a constant distance away will behave according to special relativity - so it will stay there. The calculations in [4], however, indicate that it will get closer, contradicting expanding space, but contradicting what I say as well. What's gone wrong? In cosmology there are quite few different ways of defining distance. The most popular is 'proper' distance, which is the comoving coordinate χ multiplied by the scale factor R(t). The proper distance of an object can increase faster than the speed of light, which is why you hear about superluminal recession speeds. Proper distance comes so naturally that there is a tendency for cosmologists to be blind to other ways of defining distance. Thinking in terms of special relativity means that the natural way of thinking of distance is 'radar distance', obtained from the time it takes for a light beam to travel to an object and back. This is different from 'proper distance'. An object held at constant proper distance actually has a decreasing radar distance, and if released will continue moving towards you. When $w \geqslant -1/3$ it will eventually reach you, with proper distance zero, explaining the result in [4].

This explains a counterintuitive result in [5], where the authors claim that, in the $w = -1/3$ universe, a stationary object is blueshifted. This is because they take stationary to mean 'constant proper distance', which is actually a shrinking radar distance. Note that the authors do a calculation in [5] for the $w = -1/3$ universe which seems show that proper distance is a reasonable distance measure. Their calculation indicates that an object held at a constant proper distance which is released will stay at a constant proper distance. Unfortunately they use a non-relativistic approximation. It is possible to do the calculation without the approximation and this shows that the object will approach the origin after it is released, as indicated above.

3 The Root of all Evil?

In [6] the authors look at how interatomic forces are necessary to stop an object expanding with space. They conclude 'We therefore have clear, unambiguous conditions that determine whether an object will be stretched by the expansion of space. Objects will not expand with the universe when there are sufficient internal forces to maintain the dimensions of the object.' and 'Objects are held together by forces that pull their extremities through a succession of rest frames.' The claim is clear, internal forces are necessary to pull the ends of an object together against the expansion of space. But the equation which they deduce this from is:

$$F = -mL\frac{\ddot{R}}{R}$$

This involves second derivatives, and so says something totally different to what is said in the text. In a matter dominated universe there needs to be an *outward* force, holding the two ends of the object against the gravity of the intervening matter.

3.1 A object without internal forces

Also considered in [6] is what would happen to an object which wasn't held together by interatomic forces as it moved through space. The front of the object moves away from the origin with a certain velocity v at time t_0 and the rear moves away from the origin with the same velocity at time $t_0 + \triangle t_0$. The length is then given by $L = v\triangle t_0$. After a time the object passes a galaxy, which is travelling with the Hubble flow with coordinate χ where the occupants measure the velocity and time of arrival of the two ends, and so find the length. Then

$$\chi = \int_{t_0}^{t_f} \frac{\mathrm{d}t}{R\sqrt{1 + C_0R^2}} = \int_{t_0+\triangle t_0}^{t_f+\triangle t_f} \frac{\mathrm{d}t}{R\sqrt{1 + C_1R^2}}$$

Here $C_0 = \frac{1}{R_0^2}(\frac{1}{v^2} - 1)$ where R_0 is the scale factor at time t_0 when the front of the object leaves the origin, and C_1 is the corresponding quantity for the rear of the object. The calculations in [6] said that its length expanded with the expansion of the universe. My claim is that this has to be wrong, and that in the $w = -1/3$ universe the two ends stay the same distance apart, for $w < -1/3$ they are pushed apart and for $w > -1/3$ they are pulled together.

So what is wrong with the calculation in [6]? In the course of the calculation they make the approximation $C_0 = C_1$. The difference is small if $\triangle t_0$ is small - in fact it gives an error of order $\triangle t_0$. They then subtract a large number from both sides of the equation, to leave a quantity which is of order $\triangle t_0$ So two quantities of order $\triangle t_0$ are equal to within an error of $\triangle t_0$. Their result is meaningless.

Although the above approximation is invalid, it is possible to solve the problem numerically for particular cases. We have [4]

$$\dot{\chi} = (R^2 + CR^4)^{-1/2}$$

We need to find the value of t when a given coordinate χ_f is reached, so the following differential equation needs to be solved for each end of the object:

$$\frac{dt}{d\chi} = (R^2 + CR^4)^{1/2}$$

We choose coordinates such that $t_0 = 1$ and $R(t_0) = 1$ To avoid problems with relativistic velocities we choose a relatively small value for $\chi_f = 0.1$ .The particle needs to be able to catch up with the receding galaxy and so we set $v = 0.2$. The length as measured at the start is taken to be 0.001 and so $\Delta t = 0.005$. The details of the calculation can be found at [7]. The results are as follows:

w	t_f	$t_f + \triangle t_f$	v_f	l_f	l_f/l
-2/3	-	-	-	-	-
-1/2	2.82753	2.85311	0.0509	0.001304	1.304
-1/3	1.98345	1.99337	0.10237	0.001015	1.015
0	1.72215	1.72906	0.14066	0.000971	0.971
1/3	1.64521	1.65139	0.15716	0.000972	0.972
2/3	1.60801	1.61387	0.16645	0.000975	0.975

Note that the matter and/or energy in the universe is considered to be homogenous and isotropic and so will occupy the space between the ends of the object, but that the object can move through this without friction.

$w < -1/3$ represents a universe containing 'dark energy' which accelerates the expansion and also stretches the object. This is seen for $w = -1/2$, but for $w = -2/3$ the galaxy accelerates away faster than the object can catch up. For $w = -1/3$ the length ratio is very close to 1, and the difference can be explained as a relativistic effect - the object is moving slower with respect to the galaxy at χ_f and so appears longer. For $w > -1/3$ the objects shrinks due to the effect of gravity.

4 Summary

The idea of expanding space suggests properties which are not borne out in the physics, leading to confusion of those encountering the idea. It introduces a false distinction between special relativity and general relativity. I find it much more useful to avoid any idea of absolute space - the idea concept that a point in space is something which persists through time is not useful (so questions like 'where did the big bang occur' don't have any meaning). Whenever I have seen a disagreement between the intuition obtained from expanding space and that obtained from special relativity I find that a proper general relativistic treatment of the problem shows the intuition obtained from expanding space to be wrong. The idea of expanding space has few benefits and a great many drawbacks and it should be abandoned.

References

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