It's digital but not as we know it

Stephen Lee*

February 16, 2011

Abstract

I describe some of the problems which occur when we think of the microworld as having similar properties to the macroworld of our everyday experience. In mathematics this leads to an over-reliance on approximation, and in physics to problems related to thermodynamics, problems which I see as underlying some of the weirdness of quantum theory. I explore some of the possible ways to deal with this, and look forward to a greater use of computer simulations in science, and in particular in investigating foundational issues in physics.

1 The End of Physics?

Towards the end of the 19th century many people were saying that physics was nearing completion[1]. It's rather hard to understand this now, but physics had had great successes in the 19th century. The laws of physics had been expressed in terms of differential equations, methods of dealing with those equations had been devised, and many practical applications of physics had ensued. So it might seem reasonable to assume that if the loose ends of this process were tied up then physics itself would come to a satisfactory conclusion. "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement." Those words have been ascribed to Lord Kelvin, but whether he actually said them is uncertain. He certainly saw some of the problems ahead. In a talk in 1900 [2] he saw two problems (or clouds on the horizon), the nature of the luminiferous ether, and the thermodynamics of the interaction of matter and electromagnetic radiation. It is often pointed out that these 'clouds' resulted in two revolutions in physics, relativity and quantum theory. What is less often noticed is that most of the talk dealt with the second 'cloud' and was concerned with problems with the explanation of thermodynamics in terms of statistical mechanics.

1.1 Cloud 2

Max Planck had found that in order to make sense of the spectrum of black body radiation he had to postulate that the interchange of energy between matter and radiation occured in small packets - quanta. This didn't fit in with the prevailing ethos of a physics of continuous quantities , and it was hoped that eventually it might be possible to replace it with a continuum model. However, with hindsight, we can see that this was never likely to happen. In a continuous system, it has to be assumed that there are an infinite number of modes of oscillation. Thermodynamics says that energy will become split equally between these modes, and so in the long run the energy per mode will tend to zero - in which case there will be no such thing as temperature. In his talk Kelvin saw this as one of several faults with statistical mechanics, although he wasn't particularly forthcoming as to how they might be solved. He didn't like the idea of billiard ball atoms and had earlier devised a vortex model of the atom, but that hadn't worked out [3]. I would say that there was another ghostly presence which was a problem for Kelvin as well as those who were promoting statistical mechanics. This wasn't so much of a cloud - it was a demon.

*stephen@quantropy.org

1.2 Maxwell's demon

The kinetic theory of gases,as developed by James Clerk Maxwell and Ludwig Boltzmann, had many successes in explaining the thermodynamics of macroscopic systems in terms of the motion of atoms and molecules. However there was a puzzle. If a small being - Maxwell's demon - could choose between molecules, separating out the faster ones from the slower ones, then it could defy the second law of thermodynamics. Could such a being exist? Well if you believed that "So nat'ralists observe, a flea Hath smaller fleas that on him prey, And these have smaller fleas that bite 'em, And so proceed ad infinitum."[4] then it could. But if such a being had to be made of atoms then there was a limit to how small it could be. So in order to dismiss the possibility of such a demon, you would also need to know a lot about how atoms behave. One of Kelvin's objections to the kinetic theory was that it didn't say much about the properties of atoms themselves.

What I think they were struggling with was that there has to be a limit of how small things can go, but that thermodynamics draws a veil between the macroscopic and the microscopic, so that big, hot beings like ourselves find it very hard to get a proper understanding of this limit - we tend to think things on the other side of the veil behave as they do on this side.

2 How real are the Real Numbers?

Problems with the very small weren't something new, of course. Zeno had demonstrated the paradoxes which occur if you assume that space is infinitely divisible. When calculus was invented, people worried about infinitesimals - Bishop Berkeley called them “the ghosts of departed quantities”[5]. The important thing, though, was that it worked and so it was best not to worry to much about what it meant. By the 19th century mathematicians had managed to get calculus on a rigorous footing, but this came at a price. Real numbers became defined as the limits of infinite sequences, and are now thought of as nonterminating decimals. To think in these terms, however, means accepting the reality of infinite strings of digits, that is accepting *actual* rather than *potential* infinity.

2.1 Cantor's Paradise

Georg Cantor accepted the notion of an actual infinity wholeheartedly, and showed that this lead to a hierarchy of infinities, of which the smallest, $\aleph_0$, corresponded to the idea of infinity people were used to, that of the infinity of natural numbers. Higher infinities are said to be uncountable, and an example of an uncountable set is the real numbers. But if a set is uncountable, then it is impossible to describe all of its elements. This means that thinking in terms of real numbers brings in a certain vagueness. We may know that an equation has a solution, but not be able to describe it in the way that we would like. Hence there is a rather false distinction between equations which can be solved in terms of known functions and those that cannot. The only way to deal with the latter seems to be to use some sort of approximation. David Hilbert said: "No one shall expel us from the Paradise that Cantor has created."[6][7], but I can't help wondering whether it's such a paradise after all.

3 Quantum Confusion

In 1905 Einstein decided that the quantisation of radiation wasn't something to be explained away, but instead could be used to explain other physical phenomena. The photoelectric effect was thus due to light being composed of bullet-like photons, which could knock an electron out of a metal. This was all very well, but presented a big problem - light was already known to be a wave. How were the two to be reconciled? A wave made up of photons seemed the obvious answer, but it couldn't be made to work, a further indication that explaining the very small in terms of concepts we are used to in our everyday world is likely to be misguided. (Einstein also realised that there was going to be a problem with nonlocality, but that's another story). In the 1920's quantum theory as we know it began to take shape, and it turned out to be very strange. What seemed to be forgotten, however, was its origin as a way of explaining the oddities of thermodynamics. As I see, it the weirdness of quantum theory (well at least the measurement problem) mostly originates from the inevitable problems of trying to peer through the thermodynamic veil. The Copenhagen interpretation says we should separate out the realm of quantumness and stick to understanding measurement devices (and possibly minds) in terms of classical physics. The trouble is that this is the wrong way round. We don't really understand measurement in classical physics - when we look into the microscopic details of interactions between a measuring device and what is being measured, we run into problems with thermodynamics .

3.1 Getting serious with physics

As I see it quantum theory is when we began to take the idea of 'Theory of Everything' seriously. Such a theory had to include measuring devices (and possibly minds) and the messy, microscopic details of their interactions with matter. That is why quantum theory seems mysterious in matters concerning include measuring devices (and possibly minds). There is a frequent complaint that quantum theory relies too much on abstruse mathematics, rather than presenting an intuitive model of what is going on in the microworld. But how else could it be? Some people might prefer it if the behaviour of billiard balls could be described in terms of atoms which behave, well, like billiard balls, but I would call it circular reasoning.

3.2 The puzzle of the photon

Even when the wave-particle duality of electrons and other elementary particles was accepted, there was still a problem with the photon. The electromagnetic wave isn't really the quantum wave corresponding to the photon. The coming of quantum field theory began to give a consistent view, but that had problems of it's own, being plagued with infinities, caused largely by the attempt to quantise the continuum of points in space. I would say that the best understanding of photons came with Feynman diagrams, which are essentially digital in nature, and don't try to represent what is 'really' happening in a continuous space.

4 Not as we know it

The obvious way to think of physics in digital terms is to assume that all physical quantities are quantised, and then Planck units seem to be the natural choice. I think, though, that this isn't necessarily a useful approach, at least not if it excludes other avenues. Quantum theory tells us that action is quantised, but thinking in terms of action is less natural that thinking in terms of space and time, force and energy. It's all to easy to think of space consisting of little cubes with sides of the Planck length, but this still has too much of our macroscopic thinking - we're still thinking of those cubes as being like cubes we are familiar with. We need to get away from trying to describe the smallest structures in terms of our macroscopic ideas, and begin thinking in terms of the nature of those structures in their own terms.

4.1 More than one option

In the days before computers and calculators, if you wanted to find the value of a function, such as $\sin(x)$, you looked it up in a big table. Computers don't do that, they generally calculate functions by approximating them as sums of polynomials, and generally just a few coefficients are needed for a given function, rather than a huge table. Both methods are approximate, but seeing that there is more than one method helps to move away from the idea that there is just one right way to look at a mathematical question. In quantum theory, two alternative versions, Schrödinger's wave equation and Heisenberg's matrix mechanics were developed at the same time. Think of the positive rational numbers, all squashed together, with infinitely many more between any two of them. But there's another way of looking at them, a list 1/1, 1/2, 2/1, 1/3,3/1,1/4,2/3,3,2..... The Pythagoreans found that more numbers than this were needed to describe their world, such as $\sqrt{2}$, but the algebraic numbers are also countable. Today we need more numbers still, transcendental roots of the solutions of differential equations, so it looks like we have to accept the uncountable real numbers - but do we? I can't help thinking that maybe there is a countable set of numbers somewhere which will suffice for all physical problems. For instance, π might be thought to have no finite description, because its decimal expansion goes on for ever, but this is due to thinking that infinite strings of decimals as the default way to represent numbers. Thinking of π as the root of the solution of a certain differential equation shows that it does have a finite description (just as the rational number 1/3 may be thought of as the solution of the equation 3x=1)

4.2 A note on undecidability

Of course, even if we reject the idea of actual infinities, and only deal with objects which we can describe, it doesn't mean that proofs become straightforward - Gödel's incompleteness theorem sees to that. Indeed a parallel is often drawn between undecidable statements in set theory, such as the continuum hypothesis and undecidable statements in arithmetic. So it doesn't seem that we gain anything by only dealing with computable objects. I would say that this viewpoint is wrong. Undecidability in set theory is due to the our inability to fully comprehend infinite objects, so we can never really know whether the continuum hypothesis is true or false (and if you don't believe in actual infinities then the question doesn't make much sense). In arithmetic and computing the situation is entirely different. We may never hope to have a general method to decide all possible questions, but for an individual arithmetical statement (for instance the Goldbach conjecture) we may hope to prove it one way or the other one day (even if we need to devise some new axioms in order to do so). In fact the proof of Gödel's incompleteness theorem involves finding a statement which is not provable from a given set of axioms, but which we nevertheless have shown to be true. Likewise, the halting problem in computing says that there is no general procedure for finding whether a program will halt, but for a particular program it is plausible that this can be decided.

5 Computer Simulations

Over 50 years ago, a surprising fact was discovered. Fermi, Pasta, and Ulam - and one mustn't forget Mary Tsingou who did the actual programming - simulated a system of interacting particles with spring-like bonds between them. They had expected that the system would be ergodic, that is from an organised initial state it would become more and more disorganised. But that's not what happened - instead the system looked to be moving towards disorder, but then moved back towards it's initial ordered state. This is heralded as the start of computational physics, and since then the problem has been studied in detail,[8] but to me it is surprising that it is not much more widely known. Certainly when I started trying to simulate thermodynamics on a computer I had never heard of it. I would think it should be an important part of many physics courses. For one thing, there's the quote, attributed (but without published source) to science writer Isaac Asimov (1920-1992), "The most exciting phrase to hear in science, the one that heralds new discoveries, is not *Eureka!* but *That's funny...*". But more to the point, with the wide availability of computers nowadays, it is the sort of problem which many students would be able to reproduce (and even if they couldn't, they would be able to run programs written by others). And then they could start to play with the problem: what if it were changed in a certain way, would that make the system ergodic?

More generally, computer simulations are now an important part of science, but not yet as important as I think they should be (see my website tachyos.org [9]) There often seems to be the belief that arguing via the manipulation of mathematical formulae is the gold standard, and that computer simulations are only an approximation to this. I would like to see mathematical manipulation as only one of several ways of looking at a problem, with no prior expectation of which one is the best. Indeed, if computer simulations came to the fore as a way of expressing physical theories, then it would open up a new way of judging new ideas. Expressing ideas in words can be more a matter of persuasiveness than anything else. Mathematical formalism is better, but relies on a deep knowledge of the subject to spot any problems. Computers, though, are unforgiving. If you claim to have produced a computer program which simulates some feature of the real world, then it's not that hard for others to check your claim.

5.1 Foundational issues and measuring instruments

More specifically, I believe that computer simulations have a vital part to play in investigating foundational issues. As I have said above, thermodynamics creates a veil between us and the microworld, and the way to get an understanding of what is happening on the smallest scales isn't to try to understand abstruse mathematics, but to build simulations of possible systems, and to play with them to see if they have the required properties. In particular, they should include the simulation of measuring instruments. For instance, if there is a minimum possible length then how could such a length be measured? The measuring instruments would have to be based on that minimum length. In thermodynamics there is Landauer's principle, saying that the second law and the irreversibility of macrophysics comes from the reversibility of the microphysics, and in particular the inability to 'forget' information. How do you test it - build a computer simulation! That will minimise the risk of carrying over dubious macroscopic ideas of how things work. Quantum theory - it is often said that its problems are problems of interpretation not of calculation. This isn't true, to describe the quantum details of a measuring device in terms of mathematical formulae would be hopeless. So lets see more computer simulations - then we might begin to get an idea of what is really going on.

5.2 A final thought

When the reception on a digital TV is poor, the picture tends to become blocky. I can't help thinking that if the compression methods used were really efficient, then what you were watching would turn into a cartoon instead

References

[1] W. J. Beaty. The end of science. URL: http://amasci.com/weird/end.html (url accessed 12 Feb 2011).

[2] Kelvin, "Nineteenth century clouds over the dynamical theory of heat and light." *Philosophical Magazine*, vol. 2, pp. 1–40, 1901.

[3] D. Lindley, *Degrees Kelvin: A Tale of Genius, Invention, and Tragedy.* Joseph Henry Press, 2005.

[4] J. Swift, "On poetry: A rhapsody," 1733, (Available online [14 Feb 2011]: http://shakesp.narod.ru/text/14353-9.html).

[5] G. Berkeley, *The Analyst, or A Discourse Addressed to an Infidel Mathematician*, 1734.

[6] D. Hilbert, "Über das unendliche," *Mathematische Annalen*, vol. 95, pp. 161–190, 1926.

[7] C. Reid, *Hilbert.* Springer, 1983.

[8] G. Gallavotti, Ed., *The Fermi-Pasta-Ulam Problem: A Status Report (Lecture Notes in Physics)*. Springer, 2007.

[9] S. Lee. Tachyos.org home page. URL: http://tachyos.org/home.html (page last modified May 2008, list of projects subject to change).