

Bell's theorem refuted in line with Bell's hope and Einstein's ideas

"And does not any *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts?" John Bell, *Quantum mechanics for cosmologists*, 1981.¹

Gordon Watson*

13 December 2010

Abstract Supporting Einstein's advocacy for local realism and hidden-variables, we show that measurements reveal the equivalence classes to which hidden-variables belong. We show that equivalence classes are the fundamental concepts that any analysis of measurement requires. We show that the correlation of measurement outcomes equates to the pre- and post-measurement correlation of equivalence classes. We show that hidden-variables remain hidden; but measurements enable us to name their equivalence classes. We reveal the local realistic variables that alone determine measurement outcomes. We identify Bell's unrealistic assumption about measurements and refute his theorem. Responding to Bell's hope for a simple constructive model of quantum entanglement, we also deliver Einstein's wish for a classical account of EPR correlations. We thus provide a basis for understanding quantum mechanics in terms of local realism and deterministic digital outcomes.

Keywords Bell's error · deterministic digital outcomes · Einstein's wish · EPR · EPRB · EPRB-Bell · equivalence · equivalence classes · hidden-variables · local realism · measurement · quantum entanglement · unrealistic assumptions

1 Introduction

Taking quantum mechanics (QM) to be an incomplete description of physical systems, Einstein, Podolsky & Rosen (EPR)² believed that QM could be completed with variables that would restore locality and realism to the theory, with added understanding. Challenging this view, Bohr's³ influential epistemology had no place for such variables, and von Neumann,⁴ Gleason,⁵ and Jauch & Piron⁶ offered mathematical proofs that they were impossible. Examining these proofs and finding unrealistic assumptions, Bell dismissed them⁷ to present⁸ his own mathematical impossibility proof based on EPRB,⁹ Bohm's version of EPR. Bell's work (EPRB-Bell) leads to a family of mathematical relations, known as *Bell inequalities*, which are said to constrain all local realistic theories. With QM predicting Bell inequalities to be breached, and with experimental observations confirming the QM predictions, many claim that no local realistic theory can match QM.¹⁰⁻¹⁴ In fact, in supposedly requiring us to abandon *realism* or *locality* and change our consequent understanding of reality or space-time,¹⁰ Bell's impossibility theorem⁸ is widely regarded as the most profound discovery of science.¹⁵

In line with Bell's hope that such analyses might be *illuminated, perhaps harshly, by a simple constructive model*,¹⁶ we study the impact of measurements on photons and spin-half particles; sensitive contributors to the *veiled reality*¹⁷ of our world. With probability theory as our logic and highlighting its ontological implications, we take valid inference from factual data to be the necessary tool for drawing factual conclusions about sensitive hidden-variables (HVs): there being "no infinitesimals by the aid of which an observation might be made without appreciable perturbation,"¹⁸ a fact which justifies the term *hidden* when variables are sensitive, as we will show. It follows that our inferences are testable inferences; and that our conclusions, inferred from multiple observations, may be tested by additional observations. Consistent with Einstein's advocacy for local realism and HVs – but rejecting EPR's *elements of physical reality*² because we allow that a measurement perturbs the measured system^{19,20} – we reveal the *local realistic* HVs that refute Bell's theorem. We identify Bell's unrealistic assumption about measurements²¹ and deliver Einstein's wish for a classical account of EPR correlations.²² As we will show, *measurements* (a common term in QM, but see Bell,²³ or *tests*, in our terms) reveal the equivalence classes (ECs) to which HVs belong. So: Section 2 next provides a common *Framework* and notation for EPRB-Bell studies with photons or spin-half particles. Section 3, *Analysis*, reveals the local realistic variables that refute Bell's theorem; and identifies and discusses Bell's error. *Conclusions* follow in Section 4, then *References*.

* e-mail: gorstewat@gmail.com

2 Framework

Supporting *local realism*, we suppose that a local physical reality exists, independent of any test or observation, aware that any test or observation may appreciably perturb a sensitive HV. By *local* (local action) we mean that local events cannot be influenced by actions in space-like separated regions: *Locality* equates to the impossibility of any influence travelling faster than light. By *realism* (physical realism) we mean that an external reality exists and has definite (perhaps sensitive) properties, independent of observation: *Realism* requires the results of observations to be a consequence of properties carried by physical systems. *Local realism* is therefore the notion that objects have definite properties, whether tested or not, any such test or related outcome being unaffected by space-like separated events. In other words, following Einstein: The real factual situation of a system v is independent of what is done to system v' that is space-like separated from it.²⁴ Endorsing these terms, we employ a physically significant notation to sketch an idealised EPRB experiment,⁹ with every relevant element of the physical reality in the sketch and in our formalism:

$$O - [v(s, \Lambda_{a+}) \oplus v(s, \Lambda_{a-})] \Leftarrow [a] \leftarrow v(s, \lambda_k) - (S) - v'(s, \lambda_{k'}) \rightarrow [b] \Rightarrow [v'(s, \Lambda_{b+}) \oplus v'(s, \Lambda_{b-})] - z. \quad (*)$$

In (*), single-arrows (\leftarrow , \rightarrow) accompany physical inputs, preceding their interactions and transitions; double-arrows (\Leftarrow , \Rightarrow) point to the succeeding physical outputs; \oplus denotes xor, *exclusive-or*; s denotes intrinsic spin (angular momentum) in units of \hbar , and in this paper reference to a particle means $s = \frac{1}{2} \oplus s = 1$. Primes (') distinguish some elements on the right from their counterparts on the left, and k is another identifier, a number, $k = 1, 2, \dots, N$; N large. Source (S) emits N paired particles (N twins) $v(s, \lambda_k)$ and $v'(s, \lambda_{k'})$ one pair at a time, pair-wise correlated in the spherically symmetric singlet state. $v(s, \lambda_k)$ is short for $v(s, \lambda_k, \dots)$, $v'(s, \lambda_{k'})$ is short for $v'(s, \lambda_{k'}, \dots)$, the ellipsis representing properties not relevant here. Unit-vectors λ_k and $\lambda_{k'}$ – discrete sensitive random HVs over the 4π steradians of 3-space – denote the orientation of the total spin of each particle; or, where the total spin is parallel to Oz, the direction of polarization. With total spin conserved during the emission of each particle-pair, and with ρ denoting a probability distribution:

$$\lambda_k + \lambda_{k'} = 0. \text{ In the limit as } N \rightarrow \infty: \rho(\lambda_k) = 1/4\pi; \rho(\lambda_{k'}) = 1/4\pi. \quad (1)$$

The paired particles separate by counter-propagating along the line-of-flight axis Oz to interact with space-like separated test-devices [a] (with output-channels designated a^+ and a^-) and [b] (with output-channels designated b^+ and b^-). Unit-vectors orthogonal to Oz, **a** *freely or randomly*²⁵ set by Alice, **b** *freely or randomly* set by Bob, denote the principal-axis orientation (a ray or half-line) of each device; a^+ denotes an orientation parallel to **a**; a^- denotes the orientation of what we term *the secondary axis*, an orientation orthogonal to Oz and differing from **a** by the angular relation $\pi/2s$; etc. Importantly: From (1), in agreement with Bell,²⁵ λ_k and $\lambda_{k'}$ are independent of **a** and **b**. Unit-vectors Λ denote a post-test orientation defined by a subscript; i.e., $v(s, \Lambda_{a+})$ denotes a post-test particle on the left, its Λ reported by Alice's analyzer to be oriented parallel to **a**. Similarly, $v'(s, \Lambda_{b+})$ denotes a post-test particle on the right, its Λ reported by Bob's analyzer to be oriented parallel to **b**; etc. Expressed as trigonometric arguments, relevant angular differences are:

$$(\mathbf{a}, a^-) = (a^+, a^-) = (\Lambda_{a+}, \Lambda_{a-}) = (\mathbf{a}, \Lambda_{a-}) = (\mathbf{b}, b^-) = (b^+, b^-) = (\Lambda_{b+}, \Lambda_{b-}) = (\mathbf{b}, \Lambda_{b-}) = \pi/2s; \quad (2)$$

where, as is conventional, trigonometric arguments such as (\mathbf{a}, a^-) denote the angle between the orientations **a** and a^- ; etc. Then, in terms of common QM descriptors: For $s = \frac{1}{2}$, the test-devices are Stern-Gerlach magnet-analyzers; outcome Λ_{a+} is termed *spin-up* parallel to **a**, Λ_{a-} is termed *spin-down* parallel to **a**; etc. For $s = 1$, the test-devices are dichotomic linear polarizer-analyzers; outcome Λ_{a+} is termed *polarized* parallel to **a**, Λ_{a-} is termed *polarized* orthogonal to **a**; etc. Experiment (*) is thus a long run of N paired-tests on a real (concrete, sequential) ensemble of N pairs of pair-wise correlated particles, a test or measurement being a completed interaction between a particle and a relevant device.

3 Analysis

Let K be the set of HVs with a k -identifier, the set of HVs carried by the $2N$ particles measured under (*); V_s the set of particles satisfying $s = \frac{1}{2} \oplus s = 1$ and $\omega \in W$, where W is the limit of K as $N \rightarrow \infty$.

$$K = \{\lambda_k, \lambda_k' \mid k = 1, 2, \dots, N\}. \quad (3a)$$

$$V_s = \{v(s, \omega) \mid s = \frac{1}{2} \oplus s = 1; \omega \in W\}. \quad (3b)$$

$$\text{In the limit as } N \rightarrow \infty: K = W. \quad (3c)$$

Let $v(s, \omega) \rightarrow [\mathbf{a}] \Rightarrow v(s, \Lambda_{a^+})$ represent the transition of $v(s, \omega)$ to $v(s, \Lambda_{a^+})$ – during the interaction of $v(s, \omega)$ with $[\mathbf{a}]$ – where ω represents any pre-test orientation of total spin, $\omega \in W$. In other words: particle v , initially with HV ω , entered the a^+ output-channel of Alice's device $[\mathbf{a}]$ because $v(s, \omega)$ transitioned to $v(s, \Lambda_{a^+})$ during the particle's dynamic interaction with the device; etc. Let \Leftrightarrow denote material equivalence, and define an equivalence relation \sim on V_s by:

$$[v(s, \omega) \rightarrow [\mathbf{a}] \Rightarrow v(s, \Lambda_{a^+})] \Leftrightarrow v(s, \omega) \sim v(s, a^+); P(v(s, a^+) \rightarrow [\mathbf{a}] \Rightarrow v(s, \Lambda_{a^+})) = 1. \quad (3d)$$

$$[v(s, \omega) \rightarrow [\mathbf{a}] \Rightarrow v(s, \Lambda_{a^-})] \Leftrightarrow v(s, \omega) \sim v(s, a^-); P(v(s, a^-) \rightarrow [\mathbf{a}] \Rightarrow v(s, \Lambda_{a^-})) = 1. \quad (3e)$$

That is: (A) Testing *any* particle $v - v \in V_s$ – at $[\mathbf{a}]$, we find (in agreement with experiment) just two classes: those that transition $v(s, \omega) \rightarrow [\mathbf{a}] \Rightarrow v(s, \Lambda_{a^+})$ and the equiprobable class that transition $v(s, \omega) \rightarrow [\mathbf{a}] \Rightarrow v(s, \Lambda_{a^-})$ to provide just two mutually-exclusive collectively-exhaustive equiprobable transitions at $[\mathbf{a}]$. From this we infer just two mutually-exclusive collectively-exhaustive equiprobable ECs; as shown below. (B) As with Bell,^{25,26} and consistent with local realism, we supposed in (3) that particle dynamics and measurement outcomes are *locally explicable* in terms of local realistic HVs: *if only we knew them*, like Bell.²⁶ So we allowed: $P(v(s, a^+) \rightarrow [\mathbf{a}] \Rightarrow v(s, \Lambda_{a^+})) = 1$; etc. This certainty is an inference from the facts in (A) above. Testing random particles v at $[\mathbf{a}]$: half transition to $v(s, \Lambda_{a^+})$, so we allow that $v(s, a^+)$ would certainly do the same; half transition to $v(s, \Lambda_{a^-})$, so we allow that $v(s, a^-)$ would certainly do the same. (C) So in our N trials, N large, half the particles will be equivalent to $v(s, a^+)$, and half will be equivalent to $v(s, a^-)$. We now identify each EC, denoted thus $\{ \cdot \}$.

With material equivalences based on (1); (3d), (3e) and their $[\mathbf{b}]$ -based equivalents yield:

$$[v(s, a^+)] = \{v \in V_s \mid v(s, \omega) \sim v(s, a^+); v(s, \lambda_k) \in [v(s, a^+)] \Leftrightarrow v'(s, \lambda_k') \in [v'(s, -a^+)]\}. \quad (4a)$$

$$[v(s, a^-)] = \{v \in V_s \mid v(s, \omega) \sim v(s, a^-); v(s, \lambda_k) \in [v(s, a^-)] \Leftrightarrow v'(s, \lambda_k') \in [v'(s, -a^-)]\}. \quad (4b)$$

$$[v'(s, b^+)] = \{v \in V_s \mid v'(s, \omega) \sim v'(s, b^+); v'(s, \lambda_k') \in [v'(s, b^+)] \Leftrightarrow v(s, \lambda_k) \in [v(s, -b^+)]\}. \quad (4c)$$

$$[v'(s, b^-)] = \{v \in V_s \mid v'(s, \omega) \sim v'(s, b^-); v'(s, \lambda_k') \in [v'(s, b^-)] \Leftrightarrow v(s, \lambda_k) \in [v(s, -b^-)]\}. \quad (4d)$$

Note that particles may belong to more than one EC. Also note that an ω oriented a^+ is certain in W ; and that the $v(s, a^+)$ in the EC notation $[v(s, a^+)]$ is a *naming convention* derived from W : for, in our test cohort K – given that the pre-test total spin of each particle is randomly oriented over 3-space, per (1); and our tests are based on N large; not N infinite – the probability of any ω being oriented a^+ is zero; but if such an orientation were possessed then that HV would be rightly named and identified as a member of this class, with the equivalence relations still holding 50/50. Then, based on prior analysis^{19,20} nonessential here, the squared-cosine probability relations that link outcome (observable) combinations and EC/EC combinations are:

$$\begin{aligned}
P(v'(s, \Lambda_{b+}) | v(s, \Lambda_{a+})) &= P(v'(s, \lambda_k') \in [v'(s, b^+) | v(s, \lambda_k) \in [v(s, a^+)]) = \\
&P(v'(s, \lambda_k') \in [v'(s, b^+) | v'(s, \lambda_k) \in [v(s, -a^+)]) = \cos^2 s (b^+, -a^+) = \\
P(v(s, \Lambda_{a+}) | v'(s, \Lambda_{b+})) &= P(v(s, \lambda_k) \in [v(s, a^+) | v'(s, \lambda_k') \in [v'(s, b^+)]) = \\
&P(v(s, \lambda_k) \in [v(s, a^+) | v(s, \lambda_k) \in [v(s, -b^+)]) = \cos^2 s (a^+, -b^+).
\end{aligned} \tag{5a}$$

$$\begin{aligned}
P(v'(s, \Lambda_{b-}) | v(s, \Lambda_{a+})) &= P(v'(s, \lambda_k') \in [v'(s, b^-) | v(s, \lambda_k) \in [v(s, a^+)]) = \\
&P(v'(s, \lambda_k') \in [v'(s, b^-) | v'(s, \lambda_k) \in [v(s, -a^+)]) = \cos^2 s (b^-, -a^+) = \\
P(v(s, \Lambda_{a+}) | v'(s, \Lambda_{b-})) &= P(v(s, \lambda_k) \in [v(s, a^+) | v'(s, \lambda_k') \in [v'(s, b^-)]) = \\
&P(v(s, \lambda_k) \in [v(s, a^+) | v(s, \lambda_k) \in [v(s, -b^-)]) = \cos^2 s (a^+, -b^-).
\end{aligned} \tag{5b}$$

$$\begin{aligned}
P(v'(s, \Lambda_{b+}) | v(s, \Lambda_{a-})) &= P(v'(s, \lambda_k') \in [v'(s, b^+) | v(s, \lambda_k) \in [v(s, a^-)]) = \\
&P(v'(s, \lambda_k') \in [v'(s, b^+) | v'(s, \lambda_k) \in [v(s, -a^-)]) = \cos^2 s (b^+, -a^-) = \\
P(v(s, \Lambda_{a-}) | v'(s, \Lambda_{b+})) &= P(v(s, \lambda_k) \in [v(s, a^-) | v'(s, \lambda_k') \in [v'(s, b^+)]) = \\
&P(v(s, \lambda_k) \in [v(s, a^-) | v(s, \lambda_k) \in [v(s, -b^+)]) = \cos^2 s (a^-, -b^+).
\end{aligned} \tag{5c}$$

$$\begin{aligned}
P(v'(s, \Lambda_{b-}) | v(s, \Lambda_{a-})) &= P(v'(s, \lambda_k') \in [v'(s, b^-) | v(s, \lambda_k) \in [v(s, a^-)]) = \\
&P(v'(s, \lambda_k') \in [v'(s, b^-) | v'(s, \lambda_k) \in [v(s, -a^-)]) = \cos^2 s (b^-, -a^-) = \\
P(v(s, \Lambda_{a-}) | v'(s, \Lambda_{b-})) &= P(v(s, \lambda_k) \in [v(s, a^-) | v'(s, \lambda_k') \in [v'(s, b^-)]) = \\
&P(v(s, \lambda_k) \in [v(s, a^-) | v(s, \lambda_k) \in [v(s, -b^-)]) = \cos^2 s (a^-, -b^-).
\end{aligned} \tag{5d}$$

Note the probabilistic symmetries between correlated outcomes; i.e., the equality of $P(v'(s, \Lambda_{b+}) | v(s, \Lambda_{a+}))$ and $P(v(s, \Lambda_{a+}) | v'(s, \Lambda_{b+}))$; etc. Also note that the correlation of ECs is one function – $\cos^2 s$ – of the orientations that define them; e.g., from (5a):

$$P(v'(s, \lambda_k') \in [v'(s, b^+) | v'(s, \lambda_k) \in [v(s, -a^+)]) = \cos^2 s (b^+, -a^+). \tag{5e}$$

Thus the correlation of measurement outcomes reflects the pre-test correlation of the related ECs. That is: Measurement interactions perturb HVs but not their ECs; on the contrary, measurement outcomes reveal the equivalence of pre-test and post-test ECs. That is, measurements – yielding deterministic digital outcomes $0 \oplus 1$ – confirm these correlations via jumps (in our terms) from particles with unknown HVs in knowable ECs to observable outcomes that are members of the same ECs: from $v(s, \lambda_k)$ in $[v(s, a^+) \oplus v(s, a^-)]$ to $v(s, \Lambda_{a+}) \oplus v(s, \Lambda_{a-})$ in $v(s, a^+ \oplus [v(s, a^-)]$ respectively; from $v'(s, \lambda_k')$ in $[v'(s, b^+) \oplus v'(s, b^-)]$ to $\Lambda_{b+} \oplus \Lambda_{b-}$ in $[v(s, b^+) \oplus [v(s, b^-)]$ respectively. Thus, seeking physical precision, we have the so-called *quantum jumps* in our equations as dynamical processes in dynamically defined conditions, after Bell.²² So our measurement outcomes reflect the pre-test correlations of the relevant ECs, such ECs remaining unaffected by measurement-induced perturbation. Then, for a specific spin s , (5) yields the required QM results:

$$\text{For } s = \frac{1}{2}: (5a) = (5d) = (5e) = \sin^2 \frac{1}{2} (\mathbf{a}, \mathbf{b}); (5b) = (5c) = \cos^2 \frac{1}{2} (\mathbf{a}, \mathbf{b}). \tag{6a}$$

$$\text{For } s = 1: (5a) = (5d) = (5e) = \cos^2 (\mathbf{a}, \mathbf{b}); (5b) = (5c) = \sin^2 (\mathbf{a}, \mathbf{b}). \tag{6b}$$

These *local realistic* results, in full accord with QM predictions, refute Bell's theorem and any related claim against local realism; e.g., Greenberger, Horne and Zeilinger (GHZ),²⁷ Mermin.²⁸ Bell's theorem – not recognizing the interrelation between measurement settings, measurement outcomes and ECs (see next paragraph) – states: "In a theory in which parameters are added to QM to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz-invariant."⁸ On the contrary, as shown here, the setting of a measuring device influences – via interactions with HVs in its locale – the ECs that will be revealed by the associated measurement outcomes. Moreover, in our Lorentz-invariant theory, the setting of a measuring device has no influence whatsoever on the reading of another space-like separated

instrument.

We conclude with an explanation of Bell's error, first recalling Bell's challenge: "And does not any *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts?"¹ Our theory takes these questions seriously and responds positively: ECs are core concepts in mathematics and in any analysis of measurement; and though HVs remain hidden, their ECs do not, despite the perturbative interactions associated with any measurement. And thus it is that we find Bell's unrealistic assumption in the area of measurement. Here's Bell: "... the result of the measurement [say $v(s, \Lambda_{a+})$, in our terms] does not actually tell us about some property previously possessed by the system" ²¹ (Equivalent assumptions are required to derive Bell inequalities.) To the contrary, as presented here: The test result $v(s, \Lambda_{a+})$ tells us which of $[v(s, a^+)] \oplus [v(s, a^-)]$ is applicable *as a property previously possessed* – in Bell's terms²¹ – by $v \in V_s$: for to be a member of a particular EC is a property; and without this discrimination among the relevant ECs $[v(s, a^+)]$ and $[v(s, a^-)]$, $v(s, \Lambda_{a+})$ would not be a relevant test result. Here's Bell again, not fully understanding: "While imagining that I understand the position of Einstein ... as regards the EPR correlations, I have very little understanding of the position of his principal opponent, Bohr."²⁶ But as Bohr emphasized, viewed in the light of our ECs: At the last critical stage of the test procedure – as the test setting is finalized, before the measurement interaction takes place between particle and device – there is "no question of a mechanical disturbance of the system under investigation ... But ... there is a question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of system*,"^{3,29} "... closer examination reveals that the procedure of measurement has an essential influence on the conditions on which the very definition of the physical quantities in question rests."³⁰ "And just as the choice of a different frame of reference in relativity affects the result of a particular measurement, so also in quantum mechanics the choice of a different experimental setup has its effects on measurements, for it determines what is measurable."³¹

4 Conclusions

In full accord with the experimentally confirmed predictions of QM, and with local realism rigorously maintained, (6) refutes Bell's theorem. Revealed by our existence proof, λ_k and $\lambda_{k'}$ represent separable sensitive local realistic HVs; their remarkable pre-test correlation arising from their twinned emission with total spin conserved, per (1). And though HVs are transformed in perturbative particle/device interactions, such transitions do not prevent the associated measurement from yielding the requisite EC: for a measurement reveals an EC to which the HV belongs. Thus, as the following schematic shows, extending (3), ECs are the fundamental concepts that any analysis of measurement requires:

$$[v(s, \omega) \rightarrow [\mathbf{a}] \Rightarrow v(s, \Lambda_{a+}) \rightarrow [\mathbf{a}] \Rightarrow v(s, \Lambda_{a+})] \Leftrightarrow v(s, \omega) \sim v(s, a^+); v(s, \Lambda_{a+}) \in v(s, a^+); \text{ etc.} \quad (7)$$

We have also shown that the correlation of measurement outcomes equates to the pre-measurement correlation of ECs; and that, apart from our naming their ECs – i.e., in that we did not identify the specific orientation of λ_k and $\lambda_{k'}$ – HVs remain hidden. Nevertheless, particle properties λ_k or $\lambda_{k'}$ (with their ECs), and a device variable in the same locale (with which they interact), are the local realistic variables that alone determine measurement outcomes. Relatedly, we identified the unrealistic assumption about measurements that undermines Bell's theorem and provides the basis for its refutation. Further, our analysis, per (5) – identifying ECs as elements of physical reality more fundamental than measurements – cannot be negated by any experiment that accords with QM. So, with appeal to experiment ever remaining our best defense, we eliminate much mystery from QM and scotch all Bell-based claims that local realists must abandon local-action or physical-realism or both. Endorsing Einstein's advocacy for local realism and hidden-variables, we provide a basis for understanding quantum mechanics in terms of local realism and deterministic digital outcomes. In line with Bell's hope for a simple constructive model of quantum entanglement, we deliver Einstein's wish for a classical account of EPR correlations. In short: The new theory – wholistic mechanics, with its deterministic digital outcomes and lessons from the theory of relativity – paves the way for a local realistic model of the world; with added understanding.

References

Bell's essays are collected in J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*. 2nd edn, (Cambridge University Press, 2004).

- 1 Bell, J. S. in *Quantum Gravity 2* (eds C Isham, R Penrose, & D Sciama) 611-637 (Clarendon Press, 1981).
- 2 Einstein, A., Podolsky, B. & Rosen, N. Can quantum-mechanical description of physical reality be considered complete? *Physical Review* **47**, 777-780 (1935).
- 3 Bohr, N. Can quantum-mechanical description of physical reality be considered complete? *Physical Review* **48**, 696-702 (1935).
- 4 von Neumann, J. *Mathematische Grundlagen der Quantenmechanik [English translation: Princeton University Press, Princeton, 1955.]*. (Julius Springer-Verlag, 1932).
- 5 Gleason, A. M. Measures on the closed subspaces of a Hilbert space. *Journal of Mathematical Mechanics* **6**, 885-893 (1957).
- 6 Jauch, J. & Piron, C. Can hidden variables be excluded in quantum mechanics? *Helv. Phys. Acta* **36**, 827-837 (1963).
- 7 Bell, J. S. On the problem of hidden variables in quantum mechanics. *Rev. Mod. Phys.*, 447-452 (1966).
- 8 Bell, J. S. On the Einstein Podolsky Rosen paradox. *Physics* **1**, 195-200 (1964).
- 9 Bohm, D. *Quantum Theory*. (Prentice Hall, 1951).
- 10 Clauser, J. F. & Shimony, A. Bell's theorem: experimental tests and implications. *Reports on Progress in Physics* **41**, 1881-1927 (1978).
- 11 Aspect, A. Bell's theorem: The naive view of an experimentalist. <http://arxiv.org/abs/quant-ph/0402001v1> (2001).
- 12 Aspect, A. To be or not to be local. *Nature* **446**, 866-867 (2007).
- 13 Gröblacher, S. *et al.* An experimental test of non-local realism. *Nature* **446**, 871-875 (2007).
- 14 Scheidl, T. *et al.* Violation of local realism with freedom of choice. <http://arxiv.org/abs/0811.3129> v1, 1-15 (2008).
- 15 Stapp, H. P. Bell's theorem and world process. *Il Nuovo Cimento* **29B**, 270-276 (1975).
- 16 Bell, J. S. On the impossible pilot wave. *Foundations of Physics* **12**, 989-999 (1982).
- 17 d'Espagnat, B. *In Search of Reality*. (Springer, 1983).
- 18 Heisenberg, W. *The Physical Principles of the Quantum Theory*. (Dover Publications Inc (1949 republication), 1930).
- 19 Watson, G. Bell's theorem refuted: Real physics and philosophy for quantum mechanics. *Physics Essays* **11**, 413-421 (1998).
- 20 Watson, G. Erratum for some typesetting corrections in: Bell's theorem refuted: Real physics and philosophy for quantum mechanics (1998) - v11, n3, pp 413-421. *Physics Essays* **12**, 191 (1999).
- 21 Bell, J. S. Introduction to the hidden-variable question. *Proceedings of the International School of Physics 'Enrico Fermi'. Course IL: Foundations of Quantum Mechanics [New York, Academic]*, 171-181 (1971).
- 22 Bell, J. S. Einstein-Podolsky-Rosen experiments. *Proceedings of the symposium on Frontier Problems in High Energy Physics, Pisa, June 1976*, 33-45 (1976).
- 23 Bell, J. S. Against 'measurement'. *Physics World* **3**, 33-40 (1990).
- 24 Einstein, A. in *Albert Einstein: Philosopher-Scientist* Vol. 1 (ed Paul Arthur Schilpp) 1-95 (Tudor Publishing, 1949).
- 25 Bell, J. S. in *Between Science and Technology* (eds A Sarlemijn & P Kroes) 97-115 (Elsevier Science, 1989).
- 26 Bell, J. S. Bertlmann's socks and the nature of reality. 0-25 (CERN, Geneva, 1980).
- 27 Greenberger, D. M., Horne, M. & Zeilinger, A. in *Bell's Theorem, Quantum Theory and Conceptions of the Universe* (ed M Kafatos) 69-72 (Kluwer Academic, 1989).
- 28 Mermin, N. D. Quantum mysteries revisited. *American Journal of Physics* **58**, 731-734 (1990).
- 29 Bohr, N. in *Albert Einstein: Philosopher-Scientist* Vol. 1 (ed Paul Arthur Schilpp) 199-241 (Tudor Publishing, 1949).
- 30 Bohr, N. Quantum mechanics and physical reality. *Nature* **136**, 65 (1935).
- 31 Jammer, M. *The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective*. (John Wiley & Sons, 1974).