Radio Resource Sharing Framework for Cooperative Multi-operator Networks with Dynamic Overflow Modelling

Raouf Abozariba, Md Asaduzzaman and Mohammad Patwary

***Abstract*—Due to the exponentially growing wireless applications and services, traffic demand is increasing rapidly. To cope with such growth wireless network operators seek for radio resource cooperation strategies for their users with the highest possible grade of service (GoS). In this paper we propose a set of analytical models for dynamic spectrum access (DSA) to attain intra-network resource sharing agreements and adopt such strategies by sharing radio resources. The proposed models focus on reducing blocking probability for a secondary network to attain wireless services as a trade-off with a marginal increase of blocking probability of a primary network in return of monetary rewards. We derived the global balance equation and an explicit expression of the blocking probability for each resource sharing model. The robustness of the proposed analytical models is evaluated under different scenarios by considering varying traffic intensities, different network sizes and adding reserved resources. The results show that the blocking probabilities can be reduced significantly with the proposed DSA framework in comparison to the existing local spectrum access schemes.**

***Index Terms*—Dynamic spectrum access, overflow modelling, resource sharing, blocking probability.**

I. Introduction

As a result of increasing demand for wireless services and applications in recent years there has been a significant interest in qualitative and quantitative measurements of licensed and unlicensed spectrum use. Researchers, telecommunication companies and regulatory bodies have conducted studies to capture the overall spectrum utilisation within time and space. These studies have given a notable amount of insight on spectrum use as found in the literature [1, 2]. Most of these studies have shown that a large amount of allocated spectrum are under-utilised resulting in a waste of valuable spectrum bandwidths, so-called *spectrum holes* [3–6]. On the other hand, deterioration of grade of service is inevitable for some network operators due to the shortage of bandwidths.

Most of the current radio spectrum resource distribution are based on the static spectrum allocation principles which has been identified as a major concern of spectrum scarcity within the future generations of cellular networks [7]. Efficient spectrum sharing is considered as one of the promising approaches to enhance networks' Grade of Service (GoS). In order to cope with increasing demand of wireless services and applications and to improve the spectrum utilisation, dynamic spectrum access (DSA) and other technologies, such as spectrum aggregations, are proposed in the literature to solve these current spectrum inefficiency problems [3, 8–12].

Resource allocation in DSA systems is broadly categorised by the roles of primary networks, known as the passive and active primary network models. The passive model assumes that a primary network is unaware of the operations of secondary networks (secondary networks perform spectrum sensing to determine idle spectrum for opportunistic use) and it does not require any modification for the primary network systems. However, the passive model is considered to have high complexities due to added tasks such as spectrum sensing and control overhead. In contrast, spectrum sensing in the active model is not required by secondary networks because it is assumed that a cooperation between network operators exist. Under such cooperation, information about the allocation, occupancy and characteristics of channels and other parameters are exchanged. As a result, primary networks are benefited economically by leasing their unused spectrum resources to secondary networks at the expense of marginal performance degradation while the secondary network increases GoS to a desired level. However, the marginal performance degradation of a primary network depends on its current GoS. The current GoS of primary networks and the required GoS of secondary networks along with overall GoS requirement defines the basis of DSA agreements between concerned networks.

To legalise spectrum sharing, a number of spectrum regulators such as the Federal Communications Commission (FCC) approved the use of unlicensed devices in a number of licensed bands under restricted conditions [5]. Consequently, innovative techniques are needed that can offer exploiting the available spectrum. However, the legalising process remains to be limited in certain geographical areas and certain frequency bands.

In this paper we have proposed three different DSA models to analyse spectrum sharing specific mechanisms by embedding overflow modelling, where operators are able to acquire portions of spectrum bandwidths from coexisting network operators. We focus on the analytical generalisation and robustness of the models during the interaction between network

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operators, and investigating the potential benefits of such interactions. Our findings can be summarised as follows:

- Pre-agreed spectrum sharing with overflow modelling can be beneficial to the network operators even if it comes with certain regulatory and operational limits.
- A network with dynamic and real time overflow capabilities can improve the system performance even for limited overflow traffic such as in the uni-directional overflow model.
- Overflow mechanism in DSA is effective for reducing the overall blocking probability of the network and maximal reduction of blocking probability can be attained with reserved resources.
- Dynamic overflow modelling provides an intra-network agreement platform to gain access to the under-utilised frequency bands by using additional spectrum from co-existing operators.

The remainder of the paper is organised as follows. Related work is presented in Section II. The detailed description of the system model is given in Section III. The proposed dynamic resource sharing algorithm is presented in Section IV, while the scenario specific DSA mechanism with overflow models are studied in Subsections IV-A, IV-B, IV-C and IV-D. Analytical results are provided in Section V, followed by concluding remarks in Section VI.

II. Related Work

Resource sharing mechanisms in multi-operators networks have been studied extensively in the context of DSA and Cognitive Radio Networks (CRNs) [13–18]. In [13], the benefits of Authorised Spectrum Access are shown by considering different methods to optimise the network's resources, and simulating an LTE network where a Mobile Network Operator is allowed to use the 2300 MHz band as an ASA licensee. The authors of [14] studied a spectrum sharing problem in an unlicensed band, where multiple networks coexist and interfere with each other. A cognitive radio system based on scheduling technology has been modelled in [19]. The more recent study [15] proposed a control-free DSA algorithm for CRNs.

Although intensive research has been done on resource sharing mechanisms, only a few studies addressed the blocking probability reduction when considering dynamic pre-agreed overflow traffic in coexisting networks [19–22]. A continuous-time Markov chain model to analyse the performance of three co-located cognitive systems with various priority classes and bandwidth requirements was presented in [21]. In [22], call arrivals (demand) from primary users and secondary users in the opportunistic spectrum sharing system are modelled by a Markovian arrival process which captures correlation in the aggregate arrival process consisting of the two types of call arrivals. A Markov chain analysis for spectrum access in licensed bands for cognitive radios is presented and forced termination probability, blocking probability and traffic throughput are derived in [23].

In [24], the authors focused on performance modelling for heterogeneous wireless networks based on a hierarchical overlay infrastructure. In particular, the new traffic blocked in a network due to capacity limit can be overflowed to the networks with available capacity at the higher tiers. Such traffic overflow is considered a uni-directional overflow. While in [25], the authors considered a speed-sensitive call admission control scheme to assign overflowed calls to appropriate tiers. If the new calls of fast-speed users in a low tier network are blocked due to capacity limits, the blocked new calls are overflowed to a high-tier network for possible service. If the blocked new calls are from slow-speed users in a high tier network, they are overflowed to a low-tier network. Blocked calls from fast-speed users are overflowed to the higher tier networks with larger coverage and blocked calls from slow-speed users are overflowed to the lower tier networks with smaller coverage. Such multi-tier traffic mechanism is called bi-directional overflow which can support hierarchical heterogeneous overlay systems. In [26], a load sharing scheme was considered, an incoming voice call is preferably distributed to the cell, and overflows to the WLAN only if there is not sufficient free bandwidth for a voice call in the cell. Dynamic transfer of ongoing voice calls in the WLAN to the cell via vertical handoff whenever the cell has free bandwidth to accommodate more voice calls. Meta information of data calls that can be passed to the network layer is exploited. This scheme is also considered a bi-directional overflow model.

Five overflow policies were discussed in [27], the approach taken is to allow the new calls and handovers to compete on a first-come first-served basis. The authors developed an analytical method that treats overflow in a unified manner to allow the approximate performance of overflow strategies.

The models discussed in the aforementioned literature are specific to hierarchal admission, type of service and mobility of users. We, on the other hand, present a detailed comparisons between various possible models for DSA, in particular, we show that for an operator, the blocking probability is a non-linear function of degree of interaction within multi-service multi-operator scenarios, which can be more reliable framework to be used to attain intra-operator agreement for DSA.

Moreover, our analytical models have been derived specifically to allow for more general analysis which is crucial for the new emerging DSA applications (e.g., cognitive radio technology) and future generation of wireless telecommunications. Our investigation was conducted in order to gain a better understanding of the behaviour of the DSA networks with regard to GoS.

III. System Model and Assumptions

In the context of this investigation, we have considered an infrastructure-based wireless network architecture where the system that owns the spectrum property rights (called the primary system) willingly and actively attempts to share its spectrum with secondary systems to enhance the global spectrum utilisation within a given geographical area. We assume that the network operators own spectrum property rights of bandwidths (contiguous and/or non-contiguous) in order to supply different kinds of services. In this context, we further assume that network operators can act both as primary or secondary systems, depending on whether they lease or

TABLE I: Symbols Used for the Analytical Modelling

Notations	Descriptions
N	Number of network operators in the network
M_i	Types of services at the ith operator
n_i^j	jth number of services at the ith operator
W_i	Allocated bandwidth of the ith operator
$\mathcal{A}_i$	Set of the available services for ith operator
n_i	Number of channel requests in progress at ith operator
$P(b_i)$	Blocking probability at ith operator
λ_i	Arrival rate at ith operator
μ_i	Services Rate at ith operator
c_i	Capacity at the ith operator
$X_i(t)$	Number of channels required in ith operator at time t
Ω	State space
$\mathbf{I}_i$ and $\mathbf{I}_{ij}$	Unit vectors
$\pi(\mathbf{n})$	Steady State

borrow spectrum bandwidths, respectively. Network operators are expected to interact with each other by acquiring or leasing spectrum bandwidths owned by coexisting network operators in the same region. Secondary systems are not expected to use the infrastructure of primary system, but only acquire the right to use the incumbent spectrum of primary networks on temporal and spatial basis.

In this system model, the operators are expected to interact with each other by adjusting their actions to enhance mutual benefits. This is carried out by employing the best possible strategy for secondary and primary system with a given set of constraints to control their blocking probabilities. As shown in Figure 1, a given geographical area is covered with radio signals by a set of network operators. The operators are working in an overlapped manner to provide their respective users with a preset number of services.

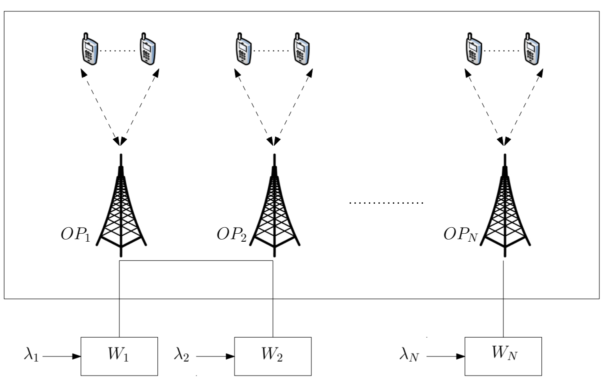


Fig. 1: System Model: Coexisting Network Operators in a DSA

We assume that each network operator supports a number of services. We denote the services as

$$\mathbf{n} = \begin{pmatrix} n_1^1 & n_1^2 & \dots & n_1^{M_1} \\ n_2^1 & n_2^2 & \dots & n_2^{M_2} \\ \vdots & \vdots & \ddots & \vdots \\ n_N^1 & n_N^2 & \dots & n_N^{M_N} \end{pmatrix}$$

where n_i^j corresponds to the type of services of the ith operator, $i = 1, 2, \dots, N,\ j = 1, 2, \dots, M_i$.

Each service supported by the network is realised by a particular data rate, which are only supportive of particular operating bands such as 791-821 MHz, 880-915 MHz, and 1920-1980 MHz. Each n_i^j has a capacity c_i^j, $i = 1, 2, \dots, N$, $j = 1, 2, \dots, M_i$. Hence, the capacity matrix can be written as follows:

$$\mathbf{C} = \begin{pmatrix} c_1^1 & c_1^2 & \dots & c_1^{M_1} \\ c_2^1 & c_2^2 & \dots & c_2^{M_2} \\ \vdots & \vdots & \ddots & \vdots \\ c_N^1 & c_N^2 & \dots & c_N^{M_N} \end{pmatrix}.$$

We assume that the network operators consider a loss model, where there are no waiting places in the system, and it blocks the arriving channel requests when all servers are busy [28]. Unlike the queueing type models, loss models are stable and the closed form analytical solution of blocking probability exists irrespective of traffic intensity. However, no closed form solution exists for infinite buffer queueing models if traffic intensity is greater than one, that is, if arrival rate is greater than departure rate.

Although multiple network operators are serving in the same geographical area, due to the variation of the service provision options among networks, there may exist a variation of services which feature specific peak time slots. Subsequently, the overall spectrum utilisation may vary from one operator to another at certain intervals. This may lead the network operators into a situation when one operator experience high demand while the resources of other coexisting operators in the region are under-utilised. This means overloaded operators may utilise the underloaded spectrum resources of adjacent operators. In this paper we present an analytical framework to enhance the overall GoS among the network operators. Such GoS enhancement is achieved by cooperative resource sharing between network operators in the form of dynamic traffic overflow modelling.

In the proposed overflow traffic modelling, a set of classification of operators are introduced on the basis of their cooperation agreements and traffic handling scenarios. Let us assume there are two types of network operators: the first one is willing to share resources when they are under-utilised, and the second one is unwilling to cooperate with other operators. The first type can be further divided into primary and secondary operators. Overflow traffic from the secondary operator to the primary operator formulate a uni-directional overflow model. In the case where the same network operator can act both as primary and secondary, then such traffic handling scenarios formulate a bi-directional overflow model. Moreover, in this paper we also consider a bi-directional overflow model with reserved capacity where additional capacity is accessible for operators. For analytical tractability, only one operator in the network is considered to have access to the reserved capacity.

The overflow mechanisms and the interactions between networks operators come with the expense of more communication overhead. Information about the extent of spatial region for spectrum use and maximum power, need to be exchanged between involved operators in order to avoid interference, and as a consequence, higher exchange of information will introduce more overhead. Moreover, the realisation of the models presented in this paper may require new technologies in the form of coordination, signalling protocols, network elements and client devices which will entail additional computational power. Measurements and analysis of such communication and computation overheads would be of great value, but are beyond the scope of this article.

A. Formulation of Agreements

One assumption in this paper is that the network operators involved in the cooperation are in some form of agreement to share their resources as predicted in the future generations of cellular networks [29, 30]. The nature of such resource sharing agreements depends on several factors such as service quality and resource availability. The agreements facilitate more control over trade-offs between GoS provision and pricing. Examples of such spectrum sharing agreements which may be motivated by monetary compensation are found in [31–34].

The level of cooperation and terms of agreements can have many forms depending on the policy of the operators. In this context, overflow traffic can be initiated from an Operator i to Operator j when the blocking probability at the Operator i is

$$P(b_i) \geq \epsilon_i, \tag{1}$$

where ϵ_i is a very small blocking probability *threshold* of the Operator i. Under an agreement, Operator j receives some monetary compensation for leasing resources to the Operator i. The amount of reward that Operator j will receive from Operator i can be written in the mathematical form given by

$$r_{ij}(t) = r_{0j} + f\Big(d(b_j)(t), r^*_{ij}(t), q_{ij}(t)\Big), \tag{2}$$

subject to

$$P(b^*_j) < \epsilon_j, \tag{3}$$

where $r_{0j} \geq 0$ is a fixed reward received by the Operator j due to the agreement, $P(b_j)(t)$ is the blocking probability of Operator j due to its own arrivals at time t, $P(b^*_j)(t)$ is the new blocking probability of Operator j as a result of its own arrivals as well as the overflow traffic from Operator i at time t, $r^*_{ij}(t)$ is the reward received from Operator i due to the admission of a unit arrival to Operator j at time t, $q_{ij}(t)$ is the amount of traffic overflowed from Operator i to j during time period t, ϵ_j is the blocking probability *threshold* for the Operator j and

$$d(b_j) = P(b^*_j)(t) - P(b_j)(t). \tag{4}$$

The second part of the reward function $f\Big(d(b_j)(t), r^*_{ij}(t), q_{ij}(t)\Big)$ may take any form (for instance, linear, exponential, etc.) agreed by both the Operator i and j during the contractual period. In the simplest case, the function may be a linear function which can be defined as

$$f(.) = r^*_{ik}(t) \cdot q_{ij}(t) \cdot \Big[1 + d(b_j)(t)\Big]. \tag{5}$$

In the event where $P(b^*_j) = \epsilon_j$, operator j could decide to block any further overflow traffic from operator i. Obviously in this case, operator j will not suffer from any further performance degradation. The monetary compensation $r_{ij}(t)$ is proportional to the performance degradation incurred by overflow traffic form operator i to operator j. In this form of agreement, both operators may have incentives to participate in spectrum sharing: either to improve the performance, represented in reducing the blocking probability, or increase in revenues at the expense of marginal performance degradation. In this agreement, Operator j charges higher rate $r_{ij}(t)$ as $P(b^*_j) \rightarrow \epsilon_j$. Note that a more realistic approach is when Operator i considers modifying the reward according to the benefit gained by overflow traffic, such that equation (2) can be written as

$$\begin{aligned} r_{ij}(t) &= r_{0j} \\ &\quad + f\Big(\alpha_i(t), P(b^*_i)(t), d(b_j)(t), r^*_{ij}(t), q_{ij}(t)\Big), \end{aligned} \tag{6}$$

subject to

$$P(b^*_j) < \epsilon_j, \tag{7}$$

and

$$r_{ij}(t) < \alpha_i(t), \tag{8}$$

where $\alpha_i(t)$ is the revenue due to overflow traffic from the Operator i to Operator j at time t and $P(b^*_i)(t)$ is the blocking probability of the Operator i at time t. Such agreements are dynamic in nature and they change at each time slot t as a function of the demands and rewards paid to Operator j. The best sharing agreement can not be determined without analysing the blocking probabilities for each network individually. In the next section, we present four possible scenarios with different overflow mechanisms in order to focus on the impact of spectrum sharing on the blocking probabilities.

IV. Proposed Dynamic Resource Sharing Algorithm

A predefined level of GoS is essential for network operators when designing or upgrading a cellular network. It constitutes one of the incentives for network operators to participate in spectrum sharing. As the number of users increase, the network operators are required to provide the users with a fixed radio resources. Cooperation among network operators in the form of dynamic resource sharing is a solution to maintain such a predefined GoS. There are two fundamental aims of such dynamic resource sharing:

- Enhanced netwsork wide GoS with efficient spectrum utilisation.
- Additional revenue generation by negotiated dynamic sub-contraction of under-utilised spectrum within each network operator.

Algorithm 1 describes a generic service selection which is used by operator i to select the accessible service, where $\mathcal{A}$ is the total number of accessible services in the network, known to every operator in advance. In this service selection algorithm, an operator continues to use its allocated resources for as long as the arrival rate is lower than the capacity of the operator (e.g., $\lambda_i < c_i$). We will show in this Section that the Algorithm 1 ensures that if operator i experiences high traffic demand, the blocking probability increases, and thus the operator i can overflow to the available spectrum of adjacent operator(s), subject to accessibility and availability.

Algorithm 1 Generic service selection

Initialisation: Number of Operators in the network = N
for $i = 1 : 1 : N$ **do**
  if ith operator is blocked and jth operator is available **then**
    $\mathcal{A}_i = \{C_i^k\} \cup \{C_j^k\}\ \forall\ j \in \{1, 2, \ldots, N\}$ and $j \neq i$
    where $\mathcal{A}_i$ is the set of accessible services for operator i
    Apply overflow Model 1 & 2.
    % If reserved capacity is available.
  else if ith operator & jth operator are blocked **then**
    $\mathcal{A}_i = \{C_i^k\} \cup \{R\}$
    Where R denotes to a reserved capacity
    Apply overflow (Model 3) with reserved capacity
  else
    Apply Non-Sharing formula
  end if
end for
return

To study the proposed algorithm, we have developed four different models based on a loss system with overflow and evaluated and compared each of these models through numerical analysis.

A. Non-Sharing Model

Consider a network consisting of two operators for a cellular communications network. We assume that the two operators are in an agreement to share the spectrum if they can both support the same services. However in this model there are no services in common in order for the operators to deploy resource sharing. Hence, we name this model a Non-Sharing Model. A state of this network is a vector $\mathbf{n} = (n_1, n_2)$, where n_i is the number of channel requests in progress in ith operator. The topology of the network is depicted in Figure 2.

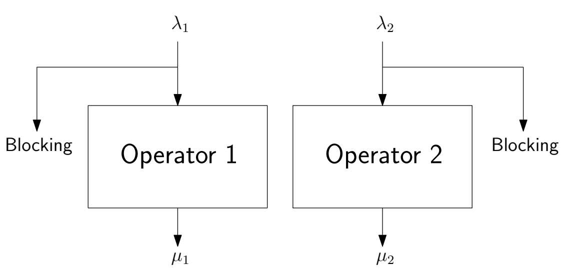


Fig. 2: Non-Sharing network with two operators

Let λ_1 and λ_2 be the arrival rates to the Operator 1 and 2 respectively, and the service rates be μ_1 and μ_2 and capacity c_1 and c_2, where both inter-arrival and service times are exponentially distributed random variables (r.v.). The blocking probability at the ith operator ($i = 1, 2$) for such an Erlang loss network can be calculated by

$$P(b_i) = \frac{1}{c_i!}\left(\frac{\lambda_i}{\mu_i}\right)^{c_i}\left[\sum_{n_i=0}^{c_i}\frac{1}{n_i!}\left(\frac{\lambda_i}{\mu_i}\right)^{n_i}\right]^{-1}. \qquad (9)$$

The blocking probability $P(b_i)$ is defined as the probability that an arrival of user at operator i is blocked because the capacity is saturated.

B. Sharing Model 1 (Uni-directional overflow)

We now consider a network with two operators with capacity c_1 and c_2 for Operator 1 and Operator 2, respectively. As assumed for the Non-Sharing Model, (discussed in subsection IV-A), here we assume that the two operators are in an agreement to share the spectrum if they can both support the same service. However, in this model, we consider a case where only Operator 1 can have access to the resources of Operator 2, while Operator 2 is not allowed to overflow to Operator 1 resources. Channel requests for Operator 1 and 2 follow Poisson processes with rate λ_1 and λ_2 for Operator 1 and 2, respectively, i.e. inter-arrival times are exponentially distributed random variables (r.v.). The service rate at Operator 1 (Operator 2) is exponentially distributed with mean μ_1^{-1} (respectively μ_2^{-1}). If all c_1 capacity are occupied at Operator 1, a channel request arriving at Operator 1 is overflowed to Operator 2 if capacity is available, and blocked otherwise. Our goal is to minimise the proportion of blocked channel requests for each operator. Figure 3 shows a detailed flow of channel requests for such network.

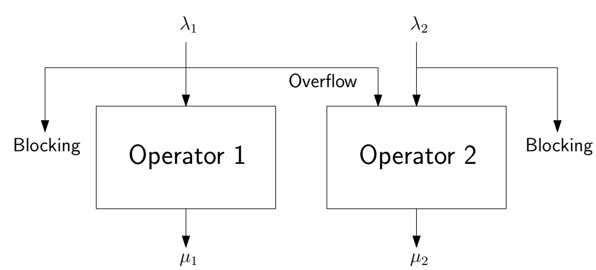


Fig. 3: A two-operator network with uni-directional overflow (Model 1)

Let $X_1(t)$ be the number of channels required in Operator 1 and $X_2(t)$ in Operator 2 at time t. Also $X_{12}(t)$ denotes the number of channels required in Operator 2 overflowed from Operator 1 at time t. The assumption of exponential distribution enables us to model the network as a continuous-time Markov chain $\mathbf{X} = (X_1(t), X_{12}(t), X_2(t), t \geq 0)$ with state space given by

$$\Omega = \{\mathbf{n} = (n_1, n_{12}, n_2) : n_1 \leq c_1, n_2 + n_{12} \leq c_2\}, \qquad (10)$$

where n_i, $i = 1, 2$, is the number of channels required at the ith operator and n_{12} is the number of channels required at Operator 2 overflowed from Operator 1. The transition rates $\mathbf{Q} = (q(\mathbf{n}, \mathbf{n}'), \mathbf{n}, \mathbf{n}' \in \Omega)$ are given by

$$q(\mathbf{n}, \mathbf{n}') = \begin{cases} \lambda_1 & \mathbf{n}' = \mathbf{n} + \mathbf{I}_1 \text{ or } \mathbf{n}' = \mathbf{n} + \mathbf{I}_{12}, \text{ if } n_1 = c_1 \\ \lambda_2 & \mathbf{n}' = \mathbf{n} + \mathbf{I}_2 \\ n_i\mu_i & \mathbf{n}' = \mathbf{n} - \mathbf{I}_i, \quad i = 1, 2 \\ n_{12}\mu_1 & \mathbf{n}' = \mathbf{n} - \mathbf{I}_{12} \\ 0 & \text{otherwise,} \end{cases} \qquad (11)$$

where $\mathbf{I}_i$ and $\mathbf{I}_{12}$ denote ith unit vectors. We are interested in deriving the blocking probability, i.e. the probability that a new channel request finds all capacities are occupied in both operators 1 and 2.

Let $\pi(\mathbf{n}) = lim_{t\to\infty}P(\mathbf{X}(t) = \mathbf{n})$ denote the equilibrium distribution that there are $\mathbf{n}$ channel requests in progress in both operators. This equilibrium distribution of $\mathbf{X}$ is the unique distribution $\pi(\mathbf{n}), \mathbf{n} \in \Omega$ that satisfies the *global balance equation* as shown in (12), where $\mathbf{1}_{\{\cdot\}}$ denotes the indicator function of the event or set of $\{\cdot\}$. We now derive the *detailed balance equations* from the *global balance equation* (12),

$$\lambda_i(\pi(\mathbf{n}-\mathbf{I}_i)+\pi(\mathbf{n}-\mathbf{I}_{12})) = (n_i\mu_i + n_{12}\mu_1)\cdot\pi(\mathbf{n}) \quad (13)$$

Equation (13) has an explicit solution which is given by

$$\pi(\mathbf{n}) = K^{-1}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{n_2}}{(n_1+n_{12})!\ n_2!}, \ \ \forall\ \mathbf{n}\in\Omega \quad (14)$$

and

$$K = \sum_{\mathbf{n}\in\Omega}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{n_2}}{(n_1+n_{12})!\ n_2!}. \quad (15)$$

This equilibrium distribution is a truncated multidimensional Poisson distribution from where blocking probability can be derived. The blocking probability for operator i, $i = 1,2$, is then given by

$$\begin{aligned} P(b_i) &= \sum_{\mathbf{n}\in T_i}\pi(\mathbf{n}) \\ &= \sum_{\mathbf{n}\in T_i}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{n_2}}{(n_1+n_{12})!\ n_2!} \\ &\quad\cdot\Big[\sum_{\mathbf{n}\in\Omega}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{n_2}}{(n_1+n_{12})!\ n_2!}\Big]^{-1}, \end{aligned} \quad (16)$$

where

$$T_1 = \{\mathbf{n}\in\Omega|(n_1 = c_1 \cap n_{12}+n_2 = c_2)\}, \quad (17)$$

and

$$T_2 = \{\mathbf{n}\in\Omega|(n_{12}+n_2 = c_2)\}. \quad (18)$$

C. Sharing Model 2 (Bi-directional overflow)

We shall now extend Sharing Model 1 by adding an overflow strategy from Operator 2 to Operator 1, see Figure 4. We assume that the two operators are in an agreement to share the spectrum and both operators can support the same services. In this model, we consider a case where Operator 1 can have access to the resources of Operator 2, and likewise, Operator 2 can have access to Operator's 1 resources. Therefore, this model is called a bi-directional overflow model. If all c_1 capacity are occupied at Operator 1 a channel request arriving at Operator 1 is overflowed to Operator 2 if capacity is available, and blocked otherwise. Similarly a channel request arriving at Operator 2 is overflowed to Operator 1 if capacity c_2 is occupied and there is a free capacity at Operator 1.

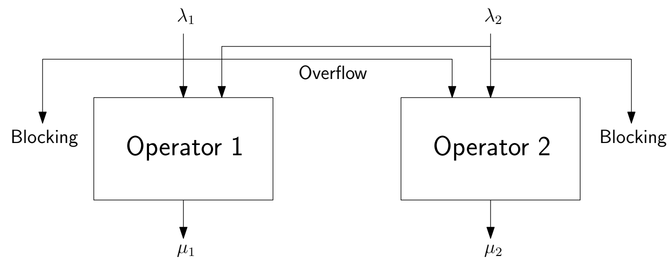


Fig. 4: A two-operator network with bi-directional overflow (Model 2)

The state space for such a process can be given by

$$\Omega = \{\mathbf{n} = (n_1, n_{12}, n_2, n_{21}) : n_1+n_{21} \le c_1, n_2+n_{12} \le c_2\}. \quad (19)$$

Deriving the *global balance equation* and *detailed balance equations* we obtain the following solution of the steady-state distribution and the expression for blocking probability calculation for each operator

$$\pi(\mathbf{n}) = K^{-1}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{(n_2+n_{21})}}{(n_1+n_{12})!\ (n_2+n_{21})!}, \ \ \forall\ \mathbf{n}\in\Omega \quad (20)$$

and

$$K = \sum_{\mathbf{n}\in\Omega}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{(n_2+n_{21})}}{(n_1+n_{12})!\ (n_2+n_{21})!}. \quad (21)$$

The blocking probability can be derived from the steady-state distribution (20). The blocking probability for operator i, $i = 1,2$, is then given by

$$\begin{aligned} P(b_i) &= \sum_{\mathbf{n}\in T_i}\pi(\mathbf{n}) \\ &= \sum_{\mathbf{n}\in T_i}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{(n_2+n_{21})}}{(n_1+n_{12})!\ (n_2+n_{21})!} \\ &\quad\cdot\Big[\sum_{\mathbf{n}\in\Omega}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{(n_2+n_{21})}}{(n_1+n_{12})!\ (n_2+n_{21})!}\Big]^{-1}, \end{aligned} \quad (22)$$

where

$$T_1 = \{\mathbf{n}\in\Omega|(n_1+n_{21} = c_1 \cap n_{12}+n_2 = c_2)\}, \quad (23)$$

and

$$T_2 = \{\mathbf{n}\in\Omega|(n_1+n_{21} = c_1 \cap n_{12}+n_2 = c_2)\}. \quad (24)$$

D. Sharing Model 3 (Bi-directional overflow with reserved capacity)

We now consider a network consisting of two operators with bi-directional overflow from Operator 1 to Operator 2 and from Operator 2 to Operator 1 (Sharing Model 2). However, in the sharing model discussed here, we assume that there is a common spectrum pool for network operators. Each network operator is considered to possess a dedicated portion of this pooled spectrum. For analytical purposes, we consider a case where only Operator 2 has such a dedicated spectrum portion with a defined capacity. This is to enable a certain predictable level of GoS for Operator 2. In this paper we denote to this spectrum portion as reserved capacity. The reserved capacity can be used to reduce blocking probability at Operator 2.

$$\Big[\lambda_1(\mathbf{1}_{\{n_1<c_1\}}(\mathbf{n}) + \mathbf{1}_{\{n_1=c_1,\ n_{12}+n_2<c_2\}}(\mathbf{n})) + \lambda_2(\mathbf{n}) + \sum_{i=1}^{2} n_i\mu_i + n_{12}\mu_1\Big]\cdot\pi(\mathbf{n}) = \lambda_1[\pi(\mathbf{n}-\mathbf{I}_1) + \pi(\mathbf{n}-\mathbf{I}_{12})\mathbf{1}_{\{n_1=c_1,\ n_{12}+n_2<c_2\}}(\mathbf{n})] + \lambda_2[\pi(\mathbf{n}-\mathbf{I}_2)] + \sum_{i=1}^{2}(n_i+1)\mu_i\pi(\mathbf{n}+\mathbf{I}_i) + (n_{12}+1)\mu_1\pi(\mathbf{n}+\mathbf{I}_{12}), \quad (12)$$

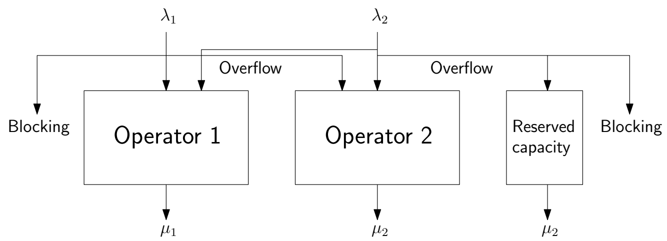


Fig. 5: A two-operators network with bi-directional overflow and reserved resources (Model 3)

Let $X_1(t)$ be the number of channel requests in Operator 1 and $X_2(t)$ be the number of channel requests in Operator 2 at time t. Also $X_{12}(t)$ denotes the number of channel requests in Operator 2 overflowed from Operator 1 and $X_{21}(t)$ denotes the number of channel requests in Operator 1 overflowed from Operator 2 at time t. Capacity at Operator 1 and 2 are denoted by c_1 and c_2 respectively. If there is no available channels to admit the new traffic in Operator 2 and Operator 1 then the request will be transferred to the reserved resource with capacity c_3. A state of the network can be written as $\mathbf{X} = (X_1(t), X_{12}(t), X_2(t), X_{21}(t), X_{23}(t), t \geq 0)$ with state space given by

$$\Omega = \{\,\mathbf{n} = (n_1, n_{12}, n_2, n_{21}, n_{23}) :\ n_1 + n_{21} \leq c_1, \quad n_2 + n_{12} \leq c_2, n_{23} \leq c_3\,\}, \qquad (25)$$

where n_i, $i = 1, 2$, is the number of channel requests at the ith operator and n_{ij} is the number of requests overflowed at operator j from operator i, $i, j \in \{1, 2, 3\}$. The transition rates $\mathbf{Q} = (q(\mathbf{n}, \mathbf{n}'), \mathbf{n}, \mathbf{n}' \in \Omega)$ are given by

$$q(\mathbf{n},\mathbf{n}') = \begin{cases} \lambda_1 & \mathbf{n}' = \mathbf{n}+\mathbf{I}_1 \text{ or } \mathbf{n}' = \mathbf{n}+\mathbf{I}_{12} \text{ if } n_1 = c_1 \\ \lambda_2 & \mathbf{n}' = \mathbf{n}+\mathbf{I}_2 \text{ or } \mathbf{n}' = \mathbf{n}+\mathbf{I}_{21} \text{ if } n_2 = c_2 \\ & \qquad \text{or } \mathbf{n}' = \mathbf{n}+\mathbf{I}_{23} \text{ if } n_2 = c_2 \\ & \qquad\qquad \text{and } n_1 + n_{21} = n_2 \\ n_i\mu_i & \mathbf{n}' = \mathbf{n}-\mathbf{I}_i,\ i = 1,2 \\ n_{ij}\mu_i & \mathbf{n}' = \mathbf{n}-\mathbf{I}_{ij},\ i,j \in \{1,2\} \\ n_{23}\mu_2 & \mathbf{n}' = \mathbf{n}-\mathbf{I}_{23} \\ 0 & \text{otherwise.} \end{cases} \qquad (26)$$

The *global balance equation* of the system can be derived as The *detailed balance equations* obtained from the *global balance equation* (27) is given by

$$\lambda_i(\pi(\mathbf{n}-\mathbf{I}_i) + \pi(\mathbf{n}-\mathbf{I}_{ij}-\mathbf{I}_{23})) = (n_i\mu_i + n_{ij}\mu_i + n_{23}\mu_2)\cdot\pi(\mathbf{n}). \qquad (28)$$

The explicit solution of the *detailed balance equations* after normalisation ($\sum \pi(\mathbf{n}) = 1$) we get

$$\pi(\mathbf{n}) = K^{-1}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{(n_2+n_{21}+n_{23})}}{(n_1+n_{12}+n_{23})!\ (n_2+n_{21})!}, \quad \forall\ \mathbf{n}\in\Omega \qquad (29)$$

and

$$K = \sum_{\mathbf{n}\in\Omega}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{(n_2+n_{21}+n_{23})}}{(n_1+n_{12})!\ (n_2+n_{21}+n_{23})!}. \qquad (30)$$

The blocking probability can be derived from the steady-state distribution (29). The blocking probability for Operator i, $i = 1, 2$, is then given by

$$\begin{aligned} P(b_i) &= \sum_{\mathbf{n}\in T_i}\pi(\mathbf{n}) \\ &= \sum_{\mathbf{n}\in T_i}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{(n_2+n_{21}+n_{23})}}{(n_1+n_{12})!\ (n_2+n_{21}+n_{23})!} \\ &\quad\cdot\Big[\sum_{\mathbf{n}\in\Omega}\frac{(\lambda_1/\mu_1)^{(n_1+n_{12})}(\lambda_2/\mu_2)^{(n_2+n_{21}+n_{23})}}{(n_1+n_{12})!\ (n_2+n_{21}+n_{23})!}\Big]^{-1}, \end{aligned} \qquad (31)$$

where

$$T_1 = \{\mathbf{n}\in\Omega | (n_1+n_{21} = c_1 \cap n_{12}+n_2 = c_2)\}, \qquad (32)$$

and

$$T_2 = \{\mathbf{n}\in\Omega | (n_1+n_{21} = c_1 \cap n_{12}+n_2 = c_2 \cap n_{23} = c_3)\}. \qquad (33)$$

The models discussed in this paper can be summarised by Figure 6. Even though the models discussed in this paper only consider the interactions between two operators, it can be extended to include more operators with added complexity, for instance, if there are more than two operators in the network, there can be a number of different interactions between operators.

V. Analysis and Results

In this section we investigate the robustness of the analytical models which are discussed in Section IV, with different offered load (0−30) assuming service rate is always 1, number of server (0−25) and reserved capacity (0, 1) across the network. The performance of the proposed resource sharing framework is examined. For the analytical results, it is reasonable that we compare the four scenario specific model configurations: Non-Sharing Model, Sharing Model 1, Sharing Model 2 and Sharing Model 3.

A. Performance comparison between Non-Sharing Model and Model 1

The comparison for Non-Sharing Model and the proposed uni-directional overflow model at Operator 1 and Operator 2 are presented in Figure 7a and 7b, respectively. The offered load at Operator 1 varies from 0 to 30 while the offered load at Operator 2 is kept fixed at 10. Figure 7a shows the blocking probabilities for the Non-Sharing Model and the proposed uni-directional overflow model. According to the analytical results in Figure 7a, it is clear that the blocking probability

$$\Big[\lambda_1(\mathbf{1}_{\{n_1+n_{21}<c_1\}} + \mathbf{1}_{\{n_1+n_{21}=c_1,\ n_{12}+n_2<c_2\}}) + \lambda_2(\mathbf{1}_{\{n_{21}+n_2<c_2\}} + \mathbf{1}_{\{n_{12}+n_2=c_2,\ n_1+n_{21}<c_1\}} + \mathbf{1}_{\{n_{12}+n_2=c_2,\ n_1+n_{21}=c_1,\ n_{23}<c_3\}})$$
$$+ \sum_{i=1}^{2} n_i\mu_i + \sum_{i,j\in\{1,2\}} n_{ij}\mu_i + n_{23}\mu_2\Big] \cdot \pi(\mathbf{n}) = \lambda_1\big[\pi(\mathbf{n}-\mathbf{I}_1) + \pi(\mathbf{n}-\mathbf{I}_{12})\mathbf{1}_{\{n_1+n_{21}=c_1,\ n_{12}+n_2<c_2\}}\big] + \lambda_2\big[\pi(\mathbf{n}-\mathbf{I}_2)$$
$$+ \pi(\mathbf{n}-\mathbf{I}_{21})\mathbf{1}_{\{n_{12}+n_2=c_2,\ n_{21}+n_2<c_2\}} + \pi(\mathbf{n}-\mathbf{I}_{23})\mathbf{1}_{\{n_{12}+n_2=c_2,\ n_{21}+n_2=c_2,\ n_{23}<c_3\}}\big]$$
$$+ \sum_{i=1}^{2}(n_i+1)\mu_i\pi(\mathbf{n}+\mathbf{I}_i) + \sum_{i,j\in\{1,2\}}(n_{ij}+1)\mu_i\pi(\mathbf{n}+\mathbf{I}_{ij}) + (n_{23}+1)\mu_2\pi(\mathbf{n}+\mathbf{I}_{23}). \tag{27}$$

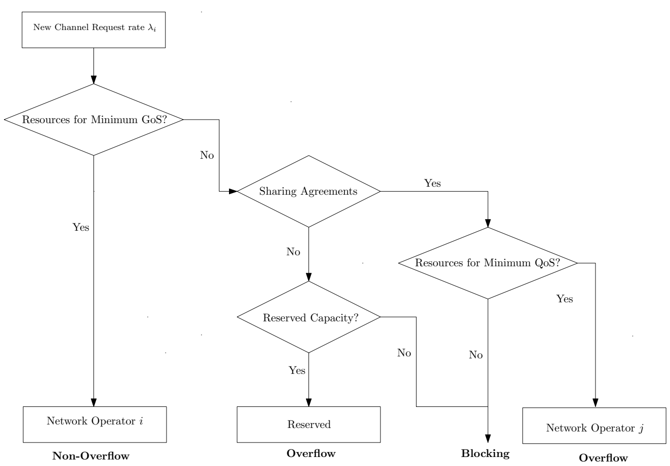


Fig. 6: The flowchart for the proposed overflow models

for the proposed overflow model for Operator 1 is reduced in comparison to the Non-Sharing Model. However, for the overflow model, the blocking probability for both Operator 1 and 2 converges as $\lambda_1 \to 30$. This is due to the fact that the uni-directional sharing model only allows overflow from Operator 1 to Operator 2. Thus, the capacity for both operators reach saturation gradually as the offered load increases. In addition, for the same offered load in Non-Sharing Model and uni-directional overflow model, it is seen that at Operator 1 with our proposed overflow model when $\lambda_1 > 10$, the blocking probability is lower than those for Non-Sharing Model. This shows superiority of our proposed model over Non-Sharing Model.

To realise the impact of our overflow model on Operator 2 with different offered load values, we have experimented with fixed offered load at Operator 1 as 10 and varied it for Operator 2 from 0 to 30, see Figure 7b. It is evident that the blocking probability of Operator 2 is higher for Model 1, except for when $\lambda_2 < 10$, because of the additional overflow load from Operator 1. Meanwhile, the blocking probability for Operator 1 has decreased as compared to when employing the Non-Sharing Model. It is evident from Figure 7b that the blocking probability for the uni-directional model at Operator 1 is lower than those for Non-Sharing Model. However, in the proposed model, the blocking probability increase with the increase of offered load. This is due to the reason that as $\lambda_2 \to 30$, the capacity gain obtained from sharing decrease with the decrease of the capacity of Operator 2.

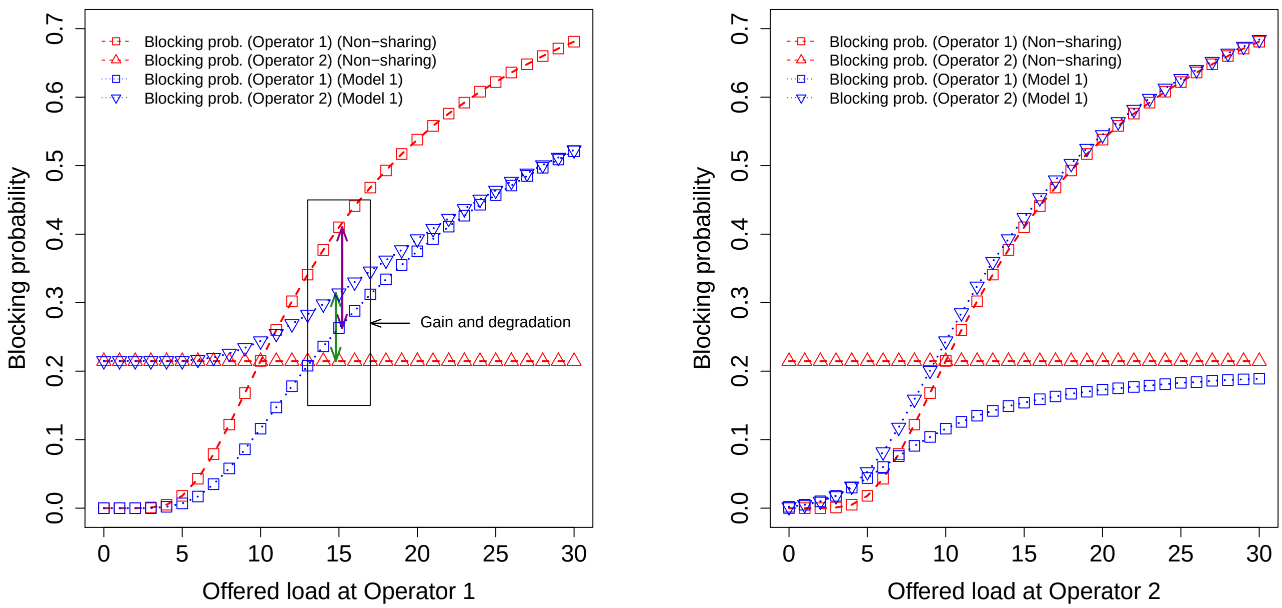
To demonstrate the trade-off agreements between operators, Figure 8 shows a zoomed region from the boxed area in Figure 7a. We show in the figure that Operator 1 improves its blocking probability by 0.174 while a degraded performance of blocking probability reduction by 0.098 for Operator 2 with offered loads 15 and 10 for operator 1 and 2, respectively, and capacity 10 for both operators. In this case, Operator 2 is expected to gain monetary reward, which may be calculated by using either equation (2) or (6) according to the agreements made during a contractual period.

In terms of performance under different number of server, we have compared the blocking probability for Non-Sharing Model with uni-directional overflow model where the number of server at Operator 1 varies from 5 to 25. The number of server is fixed at 10 for Operator 2. For simplicity, in this configuration, we set $\lambda_1 = \lambda_2 = 10$ and $\mu_1 = \mu_2 = 1$. According to the analytical results, see Figure 9a, the blocking probability at Operator 1 for our proposed model is lower than that for Non-Sharing Model. However, as $c_1 \to 25$, the advantage over the Non-Sharing Model becomes less visible due to the fact that Operator 1 increases its own capacity by overflow to Operator 2. Thus, it becomes less dependant on Operator 2, which results in lower overflow levels. In addition, it is also noticed that the blocking probability for Operator 2 with both models are almost the same when the number of server exceed 10.

In order to test the impact of varying the number of server at Operator 2, we have kept the number of server at Operator 1 fixed at 10. For this configuration, we have fixed the offered load for Operator 1 and 2 at 10. The comparison is intended to be representative of the performance in terms of blocking probability at Operator 2, see Figure 9a. It can be seen that as $c_1 \to 25$, the blocking probability of Operator 1 and 2 decreases. The overflow model performs slightly better than Non-Sharing Model, while the overflow model at Operator 1 achieves the lowest blocking probability. This analysis is used to show that a non-sharing approach where the operators do not share resources, although in certain cases might perform better than the overflow model, does not perform well when the offered load is high.

B. Performance comparison between Non-Sharing Model, Model 1 and Model 2

The results obtained in Figure 10a, 10b, 11a and 11b represent a comparison of the bi-directional model with the uni-directional and Non-Sharing Model. Figure 10a shows the



(a) (b)

Fig. 7: Comparison of blocking probability for Non-Sharing Model with Model 1 with $c_1 = c_2 = 10$ for (a) $\lambda_1 = 0 : 30$, $\lambda_2 = 10$ and (b) $\lambda_1 = 10$, $\lambda_2 = 0 : 30$

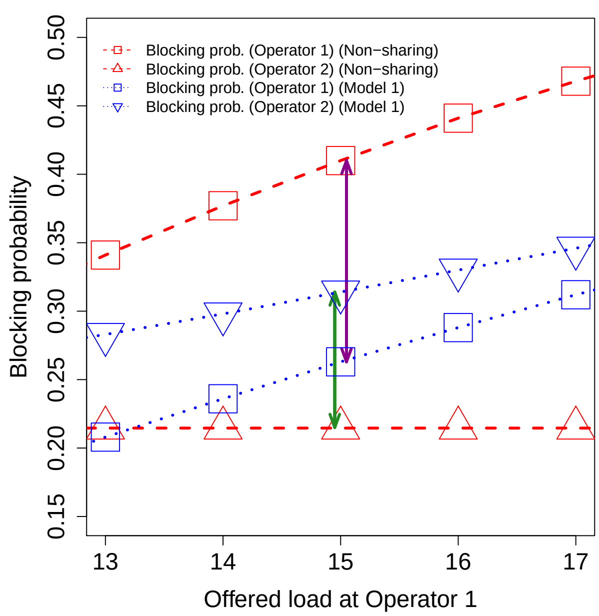


Fig. 8: Gain and degradation performance trade-off between Operator 1 and 2 for Non-sharing and Uni-directional model

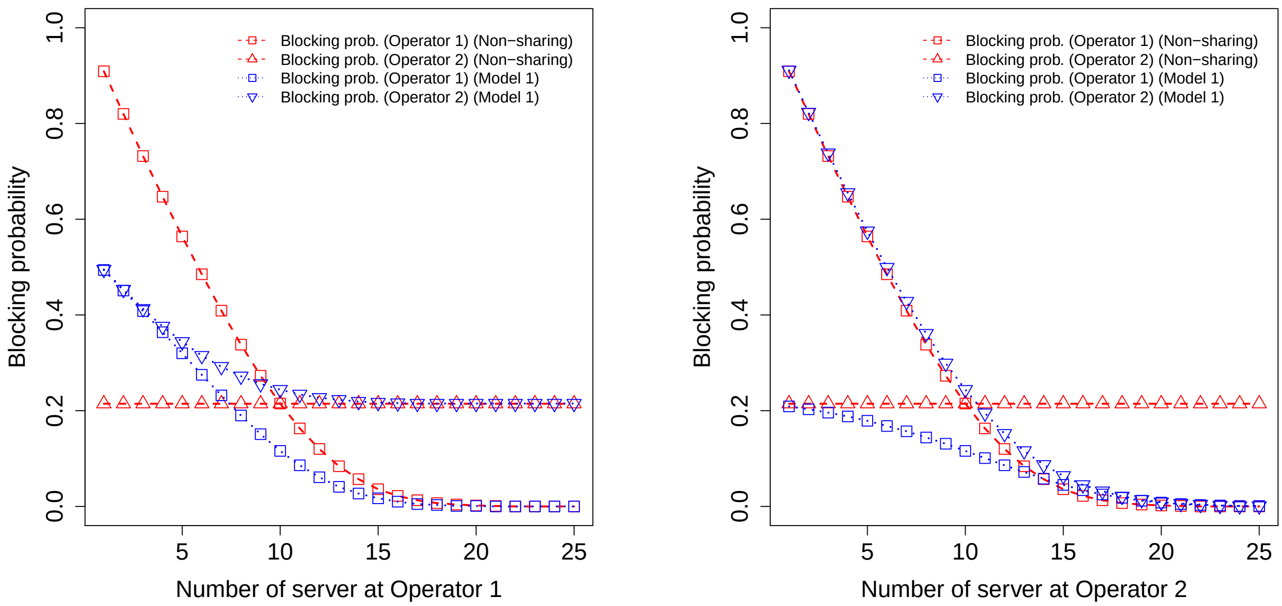
blocking probability for the case where the offered load is varied from $(0-30)$ assuming $\mu_1 = \mu_2 = 1$ and $c_1 = 10$. We see that the blocking probability for Operator 1, when considering Model 1, is lower than in Model 2, especially in the region where the offered load is between 5 and 15. The performance of Operator 2 in Figure 10b is identical to the performance in Figure 10a for Model 2 since the traffic load is always distributed uniformly over the two operators.

The other results in Figure 11a and 11b, represent a comparison of the bi-directional model with the uni-directional and Non-Sharing Model for varying number of server. When considering individual operators it is evident from the results that Model 1 present better GoS as compared to the other two models. These results show comparisons in achieving lower blocking probability for an operator using baseline assumptions for several parameters.

C. Performance comparison between Non-Sharing Model and Model 3

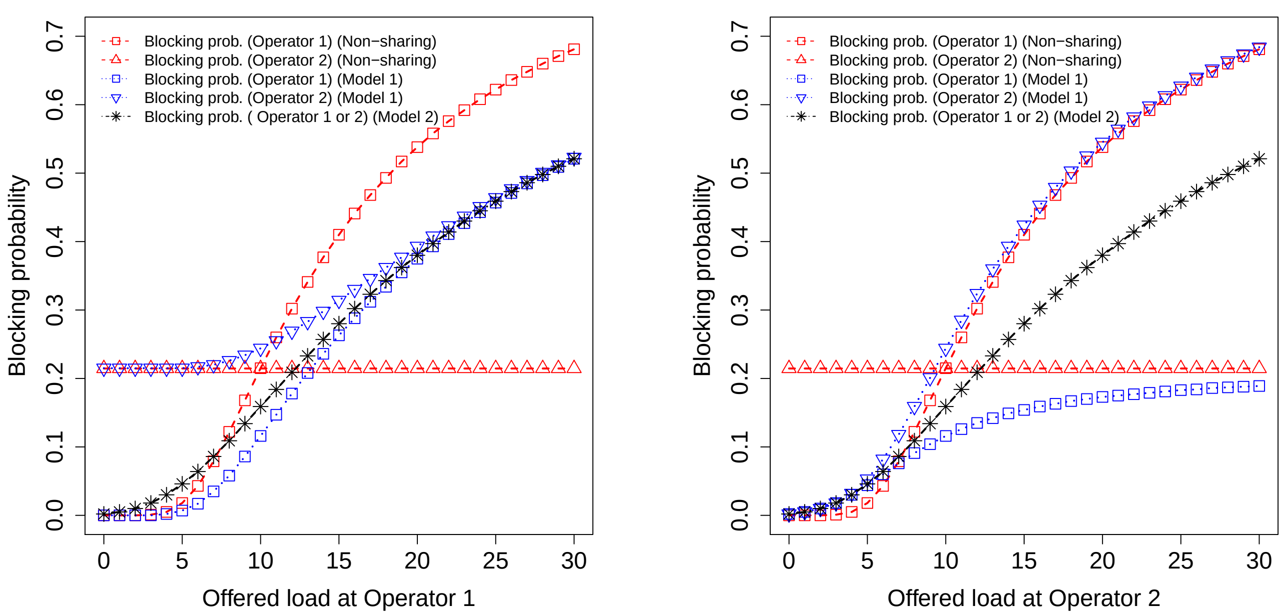
Figure 12a and Figure 12b present the comparison of blocking probabilities for Non-Sharing Model and Model 3. Figure 12a shows the effect of increasing traffic intensity at Operator 1, where we demonstrate that the blocking probability is lower when considering the Non-Sharing Model as compared to Model 3. The reason for this is that in Model 3 when the traffic at Operator 2 requires more capacity the setup allows for overflow to Operator 1 first rather than to the reserved capacity which is set to 5. This creates more traffic intensity at Operator 1, which explains the observed blocking probabilities at Operator 1 in Model 3.

In Figure 12b we have fixed the traffic intensity at Operator 1 while at Operator 2 the traffic is varied from $(0-30)$. In this example, with high traffic intensity (e.g., $\lambda_2 > 5$) Operator 2 in Model 3 shows significant blocking probability reduction in comparison to Non-Sharing Model due to available capacity from Operator 1 as well as the reserved capacity. At low traffic intensity (e.g., $\lambda_2 < 5$) at Operator 1, Model 3 performs better



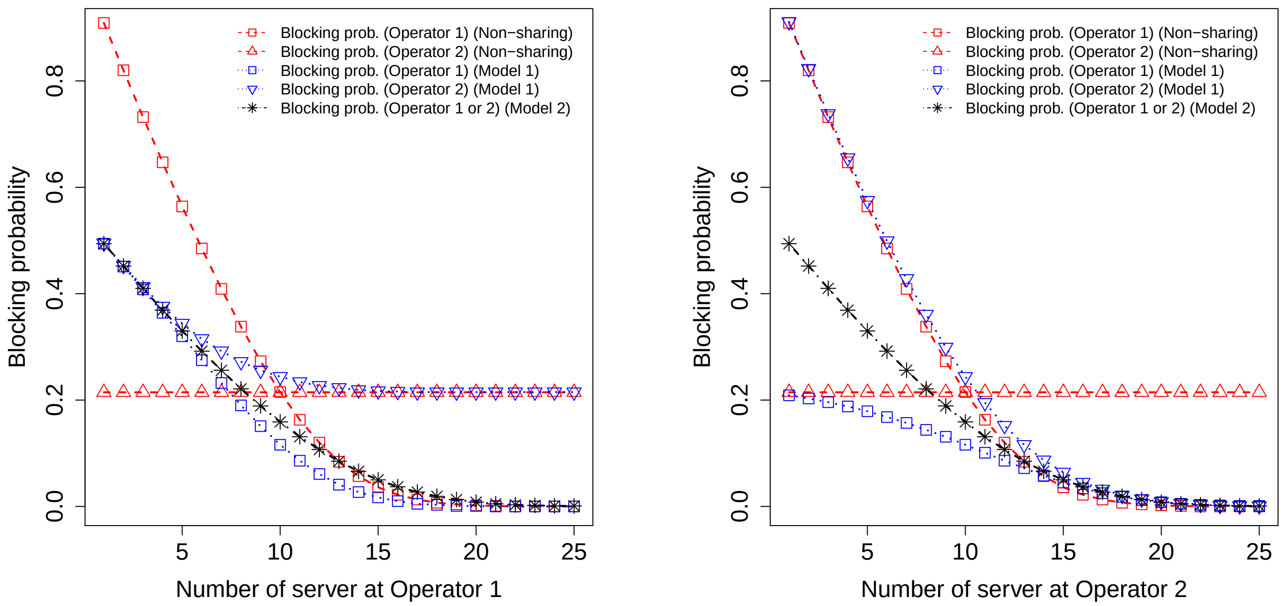
(a) (b)

Fig. 9: Comparison of blocking probability for Non-Sharing Model with Model 1 with $\lambda_1 = \lambda_2 = 10$ for (a) $c_1 = 5 : 25$, $c_2 = 10$ and (b) $c_1 = 10$, $c_2 = 1 : 25$



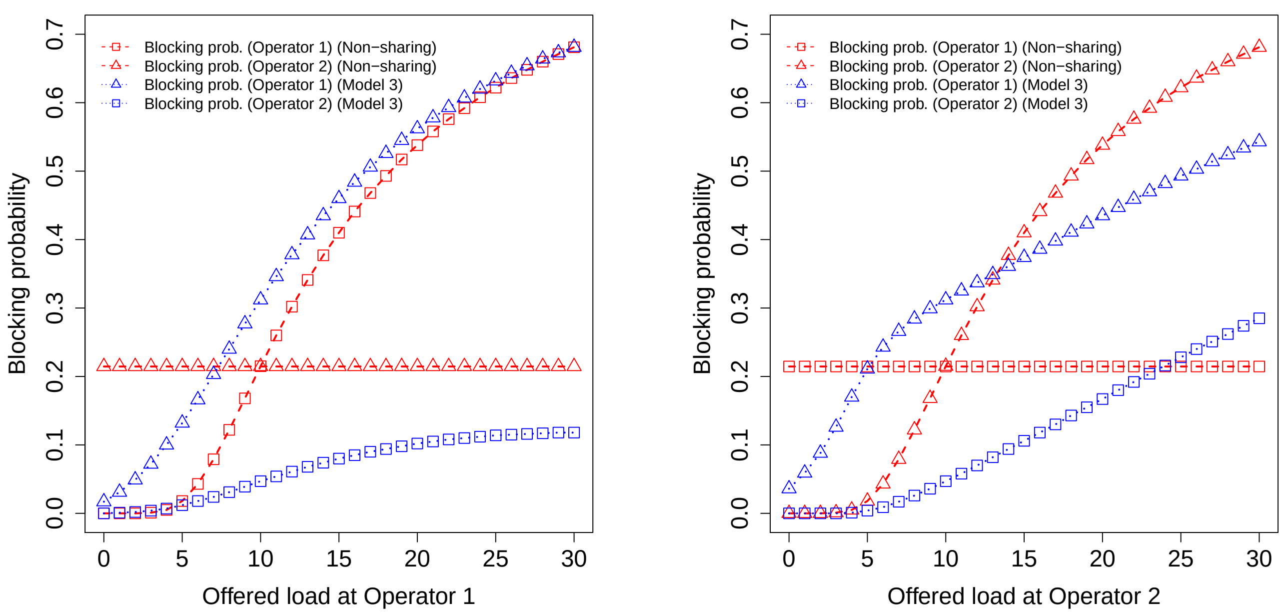
(a) (b)

Fig. 10: Comparison of blocking probability for Non-Sharing Model with Model 1 and Model 2 with $c_1 = c_2 = 10$ for (a) $\lambda_1 = 0 : 30$, $\lambda_2 = 10$ and (b) $\lambda_1 = 10$, $\lambda_2 = 10$



(a) (b)

Fig. 11: Comparison of blocking probability for Non-Sharing Model with Model 1 and Model 2 with $\lambda_1 = \lambda_2 = 10$ for (a) $c_1 = 1:25$, $c_2 = 10$ and (b) $c_1 = 10$, $c_2 = 1:25$



(a) (b)

Fig. 12: Comparison of blocking probability for Non-Sharing Model ($c_1 = c_2 = 10$) with Model 3 ($c_1 = 10, c_2 = 5$, reserved capacity = 5) for (a) $\lambda_1 = 0:30$, $\lambda_2 = 10$ and (b) $\lambda_1 = 10, \lambda_2 = 0:30$

compared to Non-Sharing Model. The number of server setup which is used for Figure 12a and Figure 12b are illustrated in Table II.

The effect of number of server on blocking probability at Operator 1 and Operator 2 for Non-Sharing Model and Model 3 is presented in Figure 13a and Figure 13b, respectively. The traffic intensity is kept fixed for both operators. The results in Figure 13a shows that the blocking probability for the Non-Sharing Model at Operator 1 is lower than Model 3. The reason is related to the traffic overflow from Operator 2, which adds an extra traffic at Operator 1. On the other hand, the blocking probability in Model 3 presents higher gain from the overflow flexibility, which benefits from the extra capacity provided by both Operator 1 and the reserved capacity. From Figure 12 and 13 we notice that for a particular operator, Model 3 does not always guarantee the enhancement of the grade of service (GoS), instead the Non-Sharing Model can serve a higher GoS. The number of server setup which is used for Figure 13a and Figure 13b are illustrated in Table III.

D. Evaluation of models under homogeneous traffic intensity

We have compared the blocking probability for Non-Sharing Model, sharing Model 1, 2 and 3, see Table IV. The table shows the overall network blocking probability for each model configuration. Note that we defined the overall blocking probability of the networks as

$$P(b) = \sum_{i=1}^{n} P(b_1)\, g(\lambda_1, \mu_1) + P(b_2)\, g(\lambda_2, \mu_2) + \dots + P(b_n)\, g(\lambda_n, \mu_n), \quad (34)$$

where $P(b_i)$ is the blocking probability at operator i and $g(\lambda_i, \mu_i)$ is a function of arrival rate and service rate for the ith operator, which give the weight for the ith operator. In our case we assumed

$$g(\lambda_i, \mu_i) = \frac{\lambda_i/\mu_i}{\lambda_1/\mu_1 + \lambda_2/\mu_2 + \dots + \lambda_n/\mu_n}. \quad (35)$$

With three different offered loads (0.25, 0.5, and 1) at Operator 1 and 2, we calculate the blocking probability for individual operators and overall network. To evaluate the models under homogeneous traffic intensity, in Table IV we present a case where the four models have equal total capacity. In Model 3, $c_1 = 2$ and $c_2 = 1$, however, Operator 2 can overflow to the reserved capacity ($c_3 = 1$) in case of no capacity is available at Operator 2 and Operator 1. Table IV shows that Model 3 has a clear advantage over Non-Sharing Model and Model 1 in terms of overall blocking probability. On the other hand, Model 3 has higher blocking probability in comparison to Model 2, this is because the overflow capacity available to Operator 1 is less in Model 3 than in Model 2 which provokes lower resource sharing efficiency.

E. Evaluation of models under heterogeneous traffic intensity

To better understand the models' behaviour, Table V shows the comparison of blocking probabilities among Non-Sharing Model, sharing Model 1, 2 and 3 for heterogeneous traffic intensity. Table V also includes the overall network blocking probability for each model configuration. It can be concluded from the table that sharing Model 2 and sharing Model 3 have superiority over Non-Sharing Model and Model 1. However, if we compare Model 2 and 3 we see that Model 2 provides the lowest overall blocking probability. This indicates that even for heterogeneous traffic intensity Model 2 provides better GoS with respect to overall network performance. Since the available capacity for both operators in Model 2 is higher, the network ensures better resource utilisation compared to sharing Model 3. Even though the total capacity at Model 3 equal to the total capacity available to Model 2, the latter performs better due to the restriction imposed on the reserved capacity which is accessible only by Operator 1. However, the results for blocking probability with respect to Operator 2 is best in Model 3 due to the reserved capacity which is available only for Operator 2.

In Summary, we have analysed and compared the performance of three different overflow models with Non-Sharing Model. As a result, the performance achievable by the operators varies according to the operator parameters (e.g. capacity, traffic intensity) and the overflow interactions between operators. It implies that operators have the incentive to participate in the proposed sharing models since they can achieve reduced blocking probability as compared to Non-Sharing Model.

VI. Conclusion

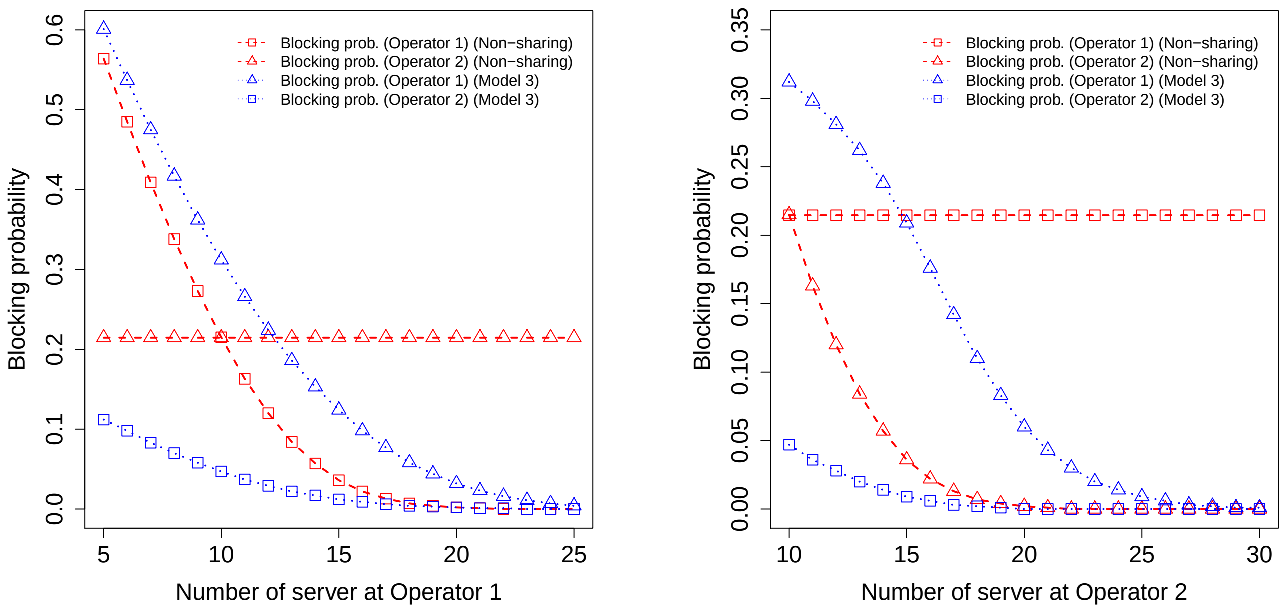
Cooperative resource sharing is considered to be one of the key challenges for future generation wireless communication networks. The problem of resource allocation under sharing environment increases as the number of cooperating network operators increase with their complex sharing agreements. Consequently, network operators has to deal with spectrum allocation for a number of service types and operators. To the best of our knowledge, there is no work that study the resource allocation problem under different resource sharing schemes which depend on many factors such as agreements between network operators and spectrum availability between coexistent network operators.

Considering a number of overflow mechanisms we addressed the resource sharing problem and presented a robust analytical framework for DSA. We have proposed four different models: Non-Sharing Model, sharing model with uni-directional overflow (Model 1), sharing model with bi-directional overflow (Model 2) and sharing model with reserved capacity for one of the operators in the network (Operator 2) and a bi-directional overflow between both operators (Model 3). We have derived the global balance equation, and found an explicit expression of the blocking probability for each resource sharing model. Blacking probabilities are calculated for each model under various traffic scenarios. The results show that the operators can achieve a notable reduction of blocking probability under the proposed models compared with the Non-Sharing Model.

Our analytical results provide a basis for further study on this type of overflow with different configurations. In addition, our proposed models can handle any type and number of

TABLE II: Number of server considered in Figure 12a and Figure 12b

Model	Figure 12a			Figure 12b		
	Number of server					
	Operator 1	Operator 2	Reserved	Operator 1	Operator 2	Reserved
Non-Sharing	10	10	−−	10	10	−−
Model 3	10	5	5	10	5	5



(a) (b)

Fig. 13: Comparison of blocking probability for Non-Sharing Model with Model 3 for $\lambda_1 = \lambda_2 = 10$. See Table III for server configurations for (a) and (b).

TABLE III: Number of server considered in Figure 13a and Figure 13b

Model	Figure 12a			Figure 12b		
	Number of server					
	Operator 1	Operator 2	Reserved	Operator 1	Operator 2	Reserved
Non-Sharing	5 : 25	10	−−	10	10 : 30	−−
Model 3	5 : 25	5	5	10	5 : 25	5

services which are provided by the operators. The results in our paper highlight the importance of resource sharing for communication networks. The analysis provided in this paper can be used to inform network operators to determine agreements terms for any future spectrum sharing cooperation with coexisting network operators.

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TABLE IV: Comparison of blocking probabilities for Non-Sharing Model with Model 1, Model 2 and Model 3 with homogeneous traffic intensity

Model	Traffic intensity		Capacity			Blocking probability		
	Operator 1	Operator 2	Operator 1	Operator 2	Reserved	Operator 1	Operator 2	Overall
Non Sharing	0.25	0.25	2	2	−−	0.024	0.024	0.024
	0.50	0.50	2	2	−−	0.077	0.077	0.077
	1	1	2	2	−−	0.200	0.200	0.200
Model 1	0.25	0.25	2	2	−−	0.001	0.025	0.013
	0.500	0.500	2	2	−−	0.011	0.081	0.046
	1	1	2	2	−−	0.069	0.220	0.145
Model 2	0.250	0.250	2	2	−−	0.002	0.002	0.002
	0.500	0.500	2	2	−−	0.015	0.015	0.015
	1	1	2	2	−−	0.095	0.095	0.095
Model 3	0.250	0.250	2	1	1	0.013	0.001	0.007
	0.500	0.500	2	1	1	0.067	0.010	0.039
	1	1	2	1	1	0.225	0.058	0.142

TABLE V: Comparison of blocking probability for Non-Sharing Model, Model 1, Model 2 and Model 3 with heterogeneous traffic intensity

Model	Traffic intensity		Capacity			Blocking probability		
	Operator 1	Operator 2	Operator 1	Operator 2	Reserved	Operator 1	Operator 2	Overall
Non Sharing	0.250	0.500	2	2	−−	0.024	0.077	0.059
	0.500	0.250	2	2	−−	0.077	0.024	0.059
	0.500	1	2	2	−−	0.077	0.200	0.159
	1	0.500	2	2	−−	0.200	0.077	0.159
Model 1	0.250	0.500	2	2	−−	0.003	0.077	0.052
	0.500	0.25	2	2	−−	0.006	0.028	0.013
	0.500	1	2	2	−−	0.021	0.204	0.143
	1	0.500	2	2	−−	0.043	0.100	0.062
Model 2	0.25	0.50	2	2	−−	0.006	0.006	0.006
	0.50	0.25	2	2	−−	0.006	0.006	0.006
	0.50	1	2	2	−−	0.048	0.048	0.048
	1	0.5	2	2	−−	0.048	0.048	0.048
Model 3	0.25	0.50	2	1	1	0.035	0.005	0.015
	0.50	0.25	2	1	1	0.035	0.002	0.024
	0.50	1	2	1	1	0.142	0.035	0.070
	1	0.500	2	1	1	0.143	0.019	0.101

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