

A UNIFIED MATHEMATICAL MODEL FOR HIGH SPEED HYBRID (AIR AND WATER-BORNE) VEHICLES

Maurizio Collu, Minoo H Patel, Florent Trarieux, Cranfield University, United Kingdom

SUMMARY

During the last two decades, the interest in civil and military high speed marine vehicles has led to several new configurations. Some of them exploit a combination of aerodynamic and hydrodynamic forces to sustain part of the weight of the craft, leading to a hybrid vehicle (HV). This paper focuses on the study of the longitudinal high-speed dynamics of such hybrid vehicles. Since airborne and waterborne vehicles belong to two distinct areas of research, they have been investigated with a rather different approach. The authors propose a unified mathematical model to represent the kinematics suitable for hybrid vehicles, including a detailed analysis of the aerodynamic and hydrodynamic forces acting on the vehicle. Then a set of ordinary differential equations of motion is derived in the frame of small-disturbance stability theory, leading to the Cauchy standard form. An illustrative example of a hybrid vehicle (KUDU II) is analyzed with the proposed method.

1 INTRODUCTION

In order to elaborate the dynamic model for the HV, the approach used respectively for planing crafts and wing in ground effect vehicles is used and a brief background of these methodologies is presented.

Research on high speed planing started in the early twentieth century for the design of seaplanes. Later the research focused on applications to design planing boats and hydrofoil crafts. During the period between 60's and 90's, many experiments have been carried out and new theoretical formulations proposed. Savitsky [1] carried out an extensive experimental program on prismatic planing hulls and obtained some empirical equations to calculate forces and moments acting on planing vessels. He also provided simple computational procedures to calculate the running attitude of the planing craft (trim angle, draft), power requirements and also the stability characteristics of the vehicle. Martin [2] derived a set of equations of motion for the surge, pitch and heave degrees of freedom and demonstrated that surge can be decoupled from heave and pitch motion. Using the coefficients of Martin, Zarnick [3] defined a set of highly nonlinear integro-differential equations of motion, which coefficients were determined by a combination of theoretical and experimental results. Since this method proved to hide some of the physics, Zarnick built a nonlinear numerical simulator. Troesch and Falzarano ([4],[5]) studied the nonlinear integro-differential equations of motion and carried out several experiments to develop a set of coupled ordinary differential equations with constant coefficients, suitable for modern methods of dynamical systems analysis. Troesch [6] later extended his previous work and expanded the nonlinear hydrodynamic force equations of Zarnick using Taylor series up to the third order, obtaining a form of equation of motion suitable for path following or continuation methods (e.g. [7]). Modern motion simulation

and control-oriented mathematical models start from these past work to define coordinate systems, equations of motion and to calculate hydrodynamic forces (e.g. [8],[9]).

Research on WIGe vehicles has mainly been carried out in the former Soviet Union, where they were known as 'Ekranoplans'. The Central Hydrofoil Design Bureau, under the guidance of R. E. Alekseev, developed several test craft and the first line production ekranoplans: Orlyonok and Lun types [10]. In the meantime, several research programs were undertaken in the west to better understand the peculiar dynamics of the vehicles flying in ground effect (IGE). In the 60's and the 70's Kumar ([11],[12]) started research in this area in Cranfield University. He carried out several experiments with a small test craft and provided the equations of motion, the dimensionless stability derivatives and studied the stability issues of a vehicle flying IGE. Staufenbiel [13] in the 80s carried out an extensive work on the influence of the aerodynamic surface characteristics on the longitudinal stability in wing in ground effect. Several considerations about the aerofoil shape, the wing planform and other aerodynamic elements were presented, in comparison with the experimental data obtained with the experimental WIG vehicle X-114 built by Rhein-Flugzeugbau in Germany in the 70's. The equations of motion for a vehicle flying IGE were defined, including non linear effects. Hall [14], in 1994, extended the work of Kumar, modifying the equations of motion of the vehicle flying IGE, taking into account the influence of perturbations in pitch on the height above the surface. Unlike Gera [15], Hall took into account also the variation of the derivatives of C_L and C_D with respect to the height above the surface. More recently Chun and Chang [16] evaluated the stability derivatives for a 20 passenger WIG vehicle, based on wind tunnel results together with vortex lattice method code. Using the work of Kumar and Staufenbiel, the static and dynamic stability characteristics have been investigated, demonstrating the validity of the approach developed so far in the west.

2 UNIFIED APPROACH

2.1 AXIS SYSTEM

To describe the motion of the HV and the forces acting on it, a number of different axis systems are used. Starting from the axis systems used for planing crafts ([1],[6],[9]) and for WIGe vehicles ([14],[17]), an earth-axis system and two body-axis system are presented below. They are all right-handed and orthogonal as represented in Fig.1. Dashed lines represent the vehicle in a disturbed state (rotation and displacements have been emphasized for clarity).

Body-axis systems

The origin O is taken to be coincident with the center of gravity (CG) position of the HV in equilibrium state. The x and z axis lay in the longitudinal plane of symmetry, x positive forward and z positive downward. The direction of the x -axis depends on the body-axis system. Two are considered:

- **Aero-hydrodynamic axes** ($\eta_1 O \eta_3$), the direction of the x -axis η_1 being parallel to the steady forward velocity V_0 ,
- **Geometric axes** ($\xi O \zeta$), the direction of the x -axis ξ being parallel to a convenient geometric longitudinal datum (as the keel of the planing surfaces).

Aero-hydrodynamic axes are the counterpart of the aerodynamic axes (called wind or wind-body axes in UK and stability axes in USA) used for airplanes. Usually the stability derivatives are calculated in this axis system.

Earth-axis systems (xOz)

The direction of the axes are fixed in space. The z -axis is directed vertically downward, the x -axis is directed forwards and parallel to the undisturbed waterline and the origin at the undisturbed waterline level.

2.2 NOMENCLATURE

CG	Center of gravity
h	height above the waterline
HV	Hybrid Vehicle
I_{ii}	Moment of inertia with respect to η_i
I_{ij}	Product of inertia wrt η_i and η_j
IGE	In Ground Effect
m	mass of the HV
RULM	Rectilinear Uniform Level Motion
WIGe	Wing In Ground effect
wrt	with respect to
$\eta_1(t)$	surge displacement,
$\eta_3(t)$	heave displacement, positive downward,
$\eta_5(t)$	pitch rotation, positive bow up
τ	trim angle (between keel and the waterline)

<i>Superscript</i>	
a	related to aerodynamics
h	related to hydrodynamics
'	perturbation value.
\dot{x}	$\frac{\partial x}{\partial t}$
\ddot{x}	$\frac{\partial^2 x}{(\partial t)^2}$
*	derived values
**	estimated values
<i>Subscript</i>	
0	value at the equilibrium state
h	derivative with respect to h

2.3 FORCES AND MOMENTS ACTING ON THE HV

Configuration of the HV

To analyze forces and moments acting on the vehicle, a configuration is required. The general configuration has:

- a high-speed prismatic planing hull, with constant deadrise angle β (hydrodynamic surface),
- a main aerofoil and a secondary aerofoil (aerodynamic surfaces),
- hydrodynamic and aerodynamic control systems,
- an aero- or hydro-propulsion system.

The HV is supposed to have a waterborne capability at rest, therefore the hydrodynamic surface is also a hydrostatic surface. In this analysis, only the high speed equilibrium motion is analyzed (full planing mode), as explained in the following paragraph.

Main forces at a given equilibrium motion

It is possible to define the forces acting on the vehicle. In general they are:

- weight,
- hydrostatic forces, acting on the hull,
- hydrodynamic forces, acting on hydrodynamic high-speed planing hulls,
- aerodynamic forces, acting on aerodynamic surfaces,
- aerodynamic and hydrodynamic control systems' forces (supposed constants, control fixed analysis),
- aero- or hydro-propulsion forces (constant, sufficient to maintain a given steady forward speed).

Depending on the steady forward velocity of the HV, it is possible to make assumptions on the forces which are negligible. The present work concentrates on the study of a equilibrium motion characterized by a rectilinear trajectory, a constant speed and a constant altitude above the surface, which will be referred as rectilinear uniform level motion (RULM). The steady forward speed (and the geometrical configuration of the HV) is such that the main

forces are the hydrodynamic and aerodynamic ones, with a small contribution of hydrostatic forces (buoyancy) to the restoring forces.

Decoupling of Equations of Motion

The HV, represented as a rigid body in space, has 6 degrees of freedom. To describe its motion a set of six simultaneous differential equations of motion is needed. However, a decoupled system of equations of motion can be derived. For airplanes, in the frame of small perturbations approach, the lateral-longitudinal coupling is usually negligible. This is still valid for WIGe vehicle [16]. For planing craft, as demonstrated in [2], not only the lateral-longitudinal coupling is usually negligible, but also the surge motion can be decoupled from the heave and pitch motion. Therefore it is assumed that the HV has a negligible longitudinal-lateral coupling. In this work, the longitudinal motion of the HV is analyzed, then only the forces and moments acting on the longitudinal plane are taken into account: surge, heave forces and the pitch moment. Following the nomenclature used for ships and airplanes, the force in x direction is X , in z direction is Z and the moment about the y axis is M .

Forces and moments expressions

The total force acting on the HV can be expressed as:

$$\mathbf{F} = \mathbf{F}^g + \mathbf{F}^a + \mathbf{F}^h + \mathbf{F}^c + \mathbf{F}^p + \mathbf{F}^d \quad (1)$$

where the components of each force are

$$\mathbf{F}^i = [X^i \ Z^i \ M^i]^T$$

The total force is the sum of gravitational force, aerodynamic and hydrodynamic forces, control systems forces, propulsion force and environment disturbances forces.

When considering the motion of an airplane or a marine vehicle after a small perturbation from a datum motion condition, it is usual to express aerodynamic and hydrodynamic forces and moments in expansions about their values at the datum motion state. The expansion can be nonlinear and expanded up to the n-th order, but in this work a linear expansion will be used. As for airplanes and planing craft, the forces and moments are assumed to depend on the values of the state variables and their derivatives with respect to time. Then, each force and moment is the sum of its value during the equilibrium state plus its expansion to take into account the variation after the small disturbance, which is:

$$\mathbf{F} = \mathbf{F}_0 + \mathbf{F}' \quad (2)$$

$$\mathbf{F}_0 = [X_0 \ Z_0 \ M_0]^T$$

$$\mathbf{F}' = [X' \ Z' \ M']^T$$

where the subscript (0) denotes starting equilibrium state and superscript (') denotes perturbation from the datum. Initially, the HV is assumed to maintain a RULM with zero roll, pitch and yaw angle. In this particular motion, the

steady forward velocity of the HV is V_0 and its component in the aero-hydrodynamic axis system are $[\dot{\eta}_{1,0}, \dot{\eta}_{3,0}]$, with $\dot{\eta}_{1,0} = V_0$ and $\dot{\eta}_{3,0} = 0$, since this is a level motion (constant height above the surface).

Control, power and disturbances forces

In this analysis it is assumed that the controls are fixed (similar to the ‘‘fixed stick analysis’’ for airplanes). Then controls’ forces and moments variations are equal to zero. The thrust is assumed not to vary during the small perturbation motion and it is equal to the total drag of the vehicle. The effects of environmental disturbances, like waves, are beyond the scope of this work, then a stable undisturbed environment is assumed.

$$\begin{cases} \mathbf{F}^c = \mathbf{F}_0^c \\ \mathbf{F}^p = \mathbf{F}_0^p \\ \mathbf{F}^d = \mathbf{0} \end{cases} \quad (3)$$

Gravitational force

The gravitational contribution to the total force can be obtained resolving the HV weight into the axis system. Since the origin of the axis system is coincident with the CG of the HV, there is no weight moment about the y axis. Remembering that the equilibrium state pitch angle is equal to zero and the angular perturbation θ' is small, the gravitational contribution is

$$\mathbf{F}^g = \mathbf{F}_0^g + \mathbf{F}^{g'} \quad (4)$$

$$\mathbf{F}_0^g = [0 \quad mg \quad 0]^T \quad (5)$$

$$\mathbf{F}^{g'} = [-mg\theta' \quad 0 \quad 0]^T$$

Aerodynamic forces

Usually, to evaluate aerodynamic forces and moments, the state variables taken into account in their Taylor linear expansion are the velocity along the x and z axes ($\dot{\eta}_1$ and $\dot{\eta}_3$) and the angular velocity about the y axis ($\dot{\eta}_5$). Among the accelerations, only the vertical acceleration ($\ddot{\eta}_3$) is taken into account in the linear expansion. Since the dynamics of a vehicle flying IGE depends also on the height above the surface, Kumar, Irodov and Staufenbiel introduced for WIGe vehicles the derivatives with respect to height (h).

These derivatives can be evaluated knowing the geometrical and aerodynamics characteristics of the aerodynamic surfaces of the HV ([14], [17]). As shown by Chun and Chang [16], the Taylor expansion stopped at the 1st order (linear model) is a good approach to have a first evaluation of the static and dynamic stability characteristics as well as a good approximation of the dynamic motion behavior of the WIGe vehicle. Analytical formulas to have an estimation of these derivatives are presented in table 4.

The expansion of the generic aerodynamic force (moment) in the aero-hydrodynamic axis system ($\eta_1 O \eta_3$) for a HV with a longitudinal plane of symmetry is

$$\mathbf{F}^a = \mathbf{F}_0^a + \mathbf{F}^{a'} \quad (6)$$

$$\mathbf{F}_0^a = [X_0^a \quad Z_0^a \quad M_0^a]^T$$

$$\begin{aligned} \mathbf{F}^{a'} &= \begin{bmatrix} X_h^a \\ Z_h^a \\ M_h^a \end{bmatrix} h' + \\ &+ \begin{bmatrix} X_{\eta_1}^a & X_{\eta_3}^a & X_{\eta_5}^a \\ Z_{\eta_1}^a & Z_{\eta_3}^a & Z_{\eta_5}^a \\ M_{\eta_1}^a & M_{\eta_3}^a & M_{\eta_5}^a \end{bmatrix} \begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_3 \\ \dot{\eta}_5 \end{bmatrix}' + \\ &+ \begin{bmatrix} 0 & X_{\eta_3}^a & 0 \\ 0 & Z_{\eta_3}^a & 0 \\ 0 & M_{\eta_3}^a & 0 \end{bmatrix} \begin{bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_3 \\ \ddot{\eta}_5 \end{bmatrix}' \end{aligned}$$

The superscript a denotes ‘‘aerodynamic forces’’. F_j denotes the derivative of the force (or moment) F with respect to the state variable j , it corresponds to the partial differential $\partial F / \partial j$.

Hydrodynamic forces

In Hicks et al. [6] the nonlinear integro-differential expressions to calculate hydrodynamic forces and moments are expanded in a Taylor series through the third order. Therefore, equations of motion can be written as a set of ordinary differential equations with constant coefficients. Analytic expressions are available for these coefficients in the work of Hicks [18]. The planing craft dynamics is highly non-linear, but the first step in the study of the dynamics is to linearize the non-linear system of equations of motion and to calculate eigenvalues and eigenvectors, which variations are monitored with quasi-static changes of physical parameters, such as the position of the CG. This approach seems reasonable as a first step for the analysis of the HV dynamics, for which a linear system of equations is developed.

The derivatives are usually divided in restoring coefficients (derivatives with respect to displacements and rotation), damping coefficients (derivatives with respect to linear and angular velocities) and added mass coefficients (derivatives with respect to linear and angular accelerations). It has been shown that the added mass and damping coefficients are nonlinear functions of the motion but also that their nonlinearities are small compared to the restoring forces nonlinearities [5]: therefore added mass and damping coefficients are assumed to be constant at a given equilibrium motion. Their value can be extrapolated from experimental results obtained by Troesch [4]. For the restoring coefficients, the linear approximation presented in Troesch and Falzarano [5] will be followed:

$$\mathbf{F}^{h,restoring} - \mathbf{F}_0^{h,restoring} \cong -[C] \underline{\eta} \quad (7)$$

The coefficients of $[C]$ can be determined using Savitsky’s method for prismatic planing hull [1].

An approach to estimate added mass, damping and restoring coefficients is presented by Martin [2]. Furthermore, an alternative approach is to compute the added

mass and damping coefficients as presented in Faltinsen [19].

Then the expansion of the generic hydrodynamic force (moment) with respect to the aero-hydrodynamic axis system $\eta_1 O \eta_3$ for a HV is

$$\mathbf{F}^h = \mathbf{F}_0^h + \mathbf{F}^{h'} \quad (8)$$

$$\mathbf{F}_0^h = [X_0^h \quad Z_0^h \quad M_0^h]^T$$

$$\begin{aligned} \mathbf{F}^{h'} &= \begin{bmatrix} 0 & X_{\eta_3}^h & X_{\eta_5}^h \\ 0 & Z_{\eta_3}^h & Z_{\eta_5}^h \\ 0 & M_{\eta_3}^h & M_{\eta_5}^h \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_3 \\ \eta_5 \end{bmatrix}' + \\ &+ \begin{bmatrix} X_{\eta_1}^h & X_{\eta_3}^h & X_{\eta_5}^h \\ Z_{\eta_1}^h & Z_{\eta_3}^h & Z_{\eta_5}^h \\ M_{\eta_1}^h & M_{\eta_3}^h & M_{\eta_5}^h \end{bmatrix} \begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_3 \\ \dot{\eta}_5 \end{bmatrix}' + \\ &+ \begin{bmatrix} X_{\eta_1}^h & X_{\eta_3}^h & X_{\eta_5}^h \\ Z_{\eta_1}^h & Z_{\eta_3}^h & Z_{\eta_5}^h \\ M_{\eta_1}^h & M_{\eta_3}^h & M_{\eta_5}^h \end{bmatrix} \begin{bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_3 \\ \ddot{\eta}_5 \end{bmatrix}' \end{aligned}$$

The superscript h denotes ‘‘hydrodynamic forces’’. X_{η_1} , Z_{η_1} and M_{η_1} are equal to zero since surge, heave and pitch moment are not dependent on the surge position of the HV.

2.4 EQUATIONS OF MOTION

The generalized equations of motion (in 6 degrees of freedom) of a rigid body with a left/right (port/starboard) symmetry are linearized in the frame of small-disturbance stability theory. The starting equilibrium state is a RULM, with a steady forward velocity equal to V_0 . The total velocity components of the HV in the disturbed motion are (evaluated in the Earth-axis system):

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \\ \dot{\eta}_4 \\ \dot{\eta}_5 \\ \dot{\eta}_6 \end{bmatrix} = \begin{bmatrix} V_0 + \dot{\eta}'_1 \\ \dot{\eta}'_2 \\ \dot{\eta}'_3 \\ \dot{\eta}'_4 \\ \dot{\eta}'_5 \\ \dot{\eta}'_6 \end{bmatrix} \quad (9)$$

By definition of small disturbances, all the linear and the angular disturbance velocities (denoted with $'$) are small quantities: therefore, substituting (9) in the generalized 6 degrees of freedom equations of motion, and eliminating the negligible terms, the linearized equations of motion

can be expressed as

$$\begin{cases} m\ddot{\eta}'_1 = X \\ m(\ddot{\eta}'_2 + \dot{\eta}'_6 V_0) = Y \\ m(\ddot{\eta}'_3 - \dot{\eta}'_5 V_0) = Z \\ I_{44}\ddot{\eta}'_4 - I_{46}\ddot{\eta}'_6 = L \\ I_{55}\ddot{\eta}'_5 = M \\ I_{66}\ddot{\eta}'_6 - I_{64}\ddot{\eta}'_4 = N \end{cases} \quad (10)$$

If the system of equations is decoupled, the longitudinal linearized equations of motion are

$$\begin{cases} m\ddot{\eta}'_1 = X \\ m(\ddot{\eta}'_3 - \dot{\eta}'_5 V_0) = Z \\ I_{55}\ddot{\eta}'_5 = M \end{cases} \quad (11)$$

N.B. From now on the superscript $'$ representing the perturbed state will be omitted.

Equilibrium state

When an equilibrium state has reached, by definition, all the accelerations are zero as well as all the perturbations velocities and the perturbation forces and moments. Then, using (3), (4), (6) and (8) in (11):

$$\begin{cases} 0 = X_0^g + X_0^a + X_0^h + X_0^c + X_0^p + X_0^d \\ 0 = Z_0^g + Z_0^a + Z_0^h + Z_0^c + Z_0^p + Z_0^d \\ 0 = M_0^g + M_0^a + M_0^h + M_0^c + M_0^p + M_0^d \end{cases} \quad (12)$$

or

$$\begin{cases} 0 = X_0^a + X_0^h + X_0^c + X_0^p \\ 0 = mg + Z_0^a + Z_0^h + Z_0^c + Z_0^p \\ 0 = M_0^a + M_0^h + M_0^c + M_0^p \end{cases} \quad (13)$$

In this work a given equilibrium state is assumed. Alternatively the equilibrium state condition could be calculated using (13), knowing the steady state forward velocity and the geometrical and inertial characteristics of the HV.

Longitudinal linearized equations of motion

Taking into account (13), the longitudinal linearized equations of motion (11) written in the aero-hydrodynamic axis system can be rearranged as:

$$[A]\ddot{\underline{\eta}} + [B]\dot{\underline{\eta}} + [C]\underline{\eta} + [D]h = \mathbf{0} \quad (14)$$

where

$$\underline{\eta} = \begin{bmatrix} \eta_1 \\ \eta_3 \\ \eta_5 \end{bmatrix}$$

and h is the (perturbed) height above the waterline.

The matrix $[A]$ is the sum of the mass matrix, the hydrodynamic added mass derivatives and the aerodynamic “added mass” terms (usually in aerodynamics they are not called added mass terms, but simply “acceleration derivatives”).

$$[A] = \begin{bmatrix} m - X_{\eta_1}^h & -X_{\eta_3}^a - X_{\eta_3}^h & -X_{\eta_5}^h \\ -Z_{\eta_1}^h & m - Z_{\eta_3}^a - Z_{\eta_3}^h & -Z_{\eta_5}^h \\ -M_{\eta_1}^h & -M_{\eta_3}^a - M_{\eta_3}^h & I_{55} - M_{\eta_5}^h \end{bmatrix} \quad (15)$$

$[B]$ is the damping matrix and is defined as:

$$[B] = \begin{bmatrix} -X_{\eta_1}^a - X_{\eta_1}^h & -X_{\eta_3}^a - X_{\eta_3}^h & -X_{\eta_5}^a - X_{\eta_5}^h \\ -Z_{\eta_1}^a - Z_{\eta_1}^h & -Z_{\eta_3}^a - Z_{\eta_3}^h & -Z_{\eta_5}^a - Z_{\eta_5}^h \\ -M_{\eta_1}^a - M_{\eta_1}^h & -M_{\eta_3}^a - M_{\eta_3}^h & -M_{\eta_5}^a - M_{\eta_5}^h \end{bmatrix} \quad (16)$$

$[C]$ is the restoring matrix and is defined as:

$$[C] = \begin{bmatrix} 0 & -X_{\eta_3}^h & -mg - X_{\eta_5}^h \\ 0 & -Z_{\eta_3}^h & -Z_{\eta_5}^h \\ 0 & -M_{\eta_3}^h & -M_{\eta_5}^h \end{bmatrix} \quad (17)$$

The matrix $[D]$ represents the wing in ground effect, to take into account the influence of the height above the surface on the aerodynamic forces.

$$[D] = \begin{bmatrix} -X_h^a \\ -Z_h^a \\ -M_h^a \end{bmatrix} \quad (18)$$

2.5 CAUCHY STANDARD FORM OF EQUATIONS OF MOTIONS

By defining a state space vector \underline{v} as

$$\underline{v} = [\eta_1 \quad \eta_3 \quad \eta_5 \quad \dot{\eta}_1 \quad \dot{\eta}_3 \quad \dot{\eta}_5 \quad h]^T \quad (19)$$

the system of equations (14) can be transformed in the Cauchy standard form (or state-space form). The state space vector has seven variables while the system of equations (14) has only 3 equations. The remaining 4 equations

are:

$$\begin{cases} \frac{\partial(\eta_1)}{\partial t} = \dot{\eta}_1 \\ \frac{\partial(\eta_3)}{\partial t} = \dot{\eta}_3 \\ \frac{\partial(\eta_5)}{\partial t} = \dot{\eta}_5 \\ \frac{\partial(h)}{\partial t} = -\dot{\eta}_3 + V_0 \eta_5 \end{cases} \quad (20)$$

The system of equations of motion in state-space form is:

$$\dot{\underline{v}} = [H] \underline{v} \quad (21)$$

where

$$[H] = \begin{bmatrix} -[A]^{-1} [B] & -[A]^{-1} [C] & -[A]^{-1} [D] \\ [I]_{3 \times 3} & [0]_{3 \times 3} & [0]_{3 \times 1} \\ 0 & 0 & -1 & 0 & 0 & V_0 & 0 \end{bmatrix} \quad (22)$$

[I] is the identity matrix, [0] is the null (zero) matrix and $n \times n$ means n rows times n columns.

3 ILLUSTRATIVE EXAMPLE

One example of hybrid vehicle is the KUDU II [20]. It is a beam ram wing planing craft, where a wing section operating IGE is mounted between two planing sponsons. It was launched in 1974 and in September of the same year the KUDU II won the 156 mile San Francisco offshore powerboat race. It has been chosen as example because at cruise speed (in RULM motion) the main forces are aerodynamic and hydrodynamic, with a small contribution of hydrostatic forces (buoyancy) to the restoring forces.

N.B. The ideal case would be to start knowing the added mass matrix ($[A]$, eq 15), the damping matrix ($[B]$, eq 16), the restoring matrix ($[C]$, eq 17) and the WIGE matrix ($[D]$, eq 18) of the vehicle. Since these data are not available, the authors will derive, if possible, these coefficients from the data available in [20]. If not possible, a rough estimation of the coefficients will be used, since the aim of this example is to illustrate how to obtain $[H]$ in eq 21 and it is not to give the exact value of the stability derivatives of the KUDU II. Derived and values are denoted with the superscript ‘*’ and estimated values are denoted with the superscript ‘***’.

Configuration

The configuration of the KUDU II is:

- 2 planing hulls;
- 1 aerofoil, between the planing hulls;
- no controls in the longitudinal plane;
- two surface-piercing propellers.

The planing hulls are approximated with one prismatic planing hulls with beam (B^h) equal to the double of the beam of each sponson (b^h) and constant deadrise angle (β) equal to the outside deadrise angle (β_{out}). The airfoil is not taken from any “family”: it was designed specifically for the KUDU II in order to have a low pitching moment. The lower surface of the wing is set a 5 deg angle to the keel: therefore the angle of attack of the airfoil is the trim angle of the vehicle plus 5 deg. Main dimensions of the KUDU II are presented in table 1.

Equilibrium state: forces and moments

This analysis starts from an equilibrium state. In particular, a rectilinear uniform level motion is analyzed, illustrated in table 2. The equilibrium forces and moments (\mathbf{F}_0 term in eq 2) acting on the vehicle are presented in table 2. As it can be seen, aerodynamic and hydrodynamic forces and moments are of the same order of magnitude (10^3 to 10^4 in Newton and Newton-meter). Lift and drag, as usual, are defined as the component of the aerodynamic (hydrodynamic) force respectively normal (positive upward) and parallel (positive rearward) to the velocity. The propulsion force has been divided in a lift and a thrust (parallel to the velocity, positive forward) component. The moment is positive bow up and is about the CG.

Forces and moments after the disturbance

In the previous section the \mathbf{F}_0 term of the eq 2 has been analyzed. Now we need to illustrate the forces and moments arising after a perturbation, the \mathbf{F}' term.

Control, power and disturbances forces

As previously said, this is a control fixed analysis and the thrust is assumed not to vary. Furthermore environmental disturbances are assumed equal to zero.

Gravitational force

If the mass of the vehicle at the equilibrium state is known it is easy to calculate the gravitational contribution with eq 5.

Aerodynamic forces

Aerodynamic stability derivatives of the KUDU II are not available. Therefore their values have been approximated using the expressions presented by Hall and Delhaye [14] [17], illustrated in table 4. The following coefficients have to be known:

C_L	coefficient of lift
$\partial C_L / \partial V$	C_L derivative wrt the velocity
$\partial C_L / \partial \alpha$	C_L derivative wrt the angle of attack
$\partial C_L / \partial (h/c)$	derivative of C_L wrt dimensionless h
C_D	coefficient of drag
$\partial C_D / \partial V$	C_D derivative wrt the velocity
$\partial C_D / \partial \alpha$	C_D derivative wrt the angle of attack
$\partial C_D / \partial (h/c)$	derivative of C_D wrt dimensionless h

$\partial C_m / \partial V$	C_m derivative wrt the velocity
$\partial C_m / \partial \alpha$	C_m derivative wrt the angle of attack
$\partial C_m / \partial (h/c)$	derivative of C_m wrt dimensionless h

l_{cg}/b^h	Dimensionless longitudinal position of CG
vcg/b^h	Dimensionless vertical position of CG
c_Δ	Dimensionless mass

Furthermore, if a secondary wing is present, also these coefficients are needed:

$\partial C_{LT} / \partial \alpha$	C_L derivative wrt the angle of attack
S_{LT}	area of the secondary airfoil
l_T	distance of aerodynamic centre of secondary wing aft of the CG
ϵ_α	downwash derivative at secondary wing
$(C_{LT} = \text{coefficient of lift of the secondary airfoil})$	

Unfortunately only some of these coefficients are known for the KUDU II: the others have been approximated by the authors, using a NACA 6512 profile. The values of the needed coefficients are illustrated in table 3. Since this vehicle has not a secondary wing, all the related coefficients are equal to zero.

Hydrodynamic forces

No data are available on the behaviour of the KUDU II after a perturbation: no restoring, damping or added mass coefficients are available. Therefore an approximation of these values has been conducted by the authors, using the method presented by Martin [2]. The coefficients needed are:

$c_V (Fn_B)$	Froude number based on hydrodynamic beam
L_K	Keel wetted length
τ_0	Trim angle
β	Deadrise angle

and their values are in tab 3. The values of hydrodynamic stability derivatives are in tab 4.

Longitudinal Linearized Equations of Motion and Cauchy standard form

The stability derivatives values can be used now to calculate the added mass matrix $[A]$, the damping matrix $[B]$, the restoring matrix $[C]$ and the WIGe matrix $[D]$ in the eq 14.

Then the state space matrix $[H]$ can be calculated using the eq 22.

4 CONCLUSIONS

For certain configurations of high speed marine vehicles, at high speed, aerodynamic forces and moments become significant and of the same order of magnitude as hydrodynamic ones, therefore they are equally important in the analysis of the dynamics. In this work, an approach to take into account all the forces and moments acting on HVs and to calculate the state space matrix of the HVs is presented. From this matrix ($[H]$ in eq 21), the characteristic polynomial can be derived and, using a stability criterion such as the Routh-Horwitz criterion, the stability of the vehicle can be evaluated. This is a linear approach, because the added mass, damping and restoring coefficients are assumed constant: it is useful as a first step in the study of the dynamics. Further development of this work will consider only the added mass and damping coefficients constants, using a non-linear approach to estimate the restoring forces.

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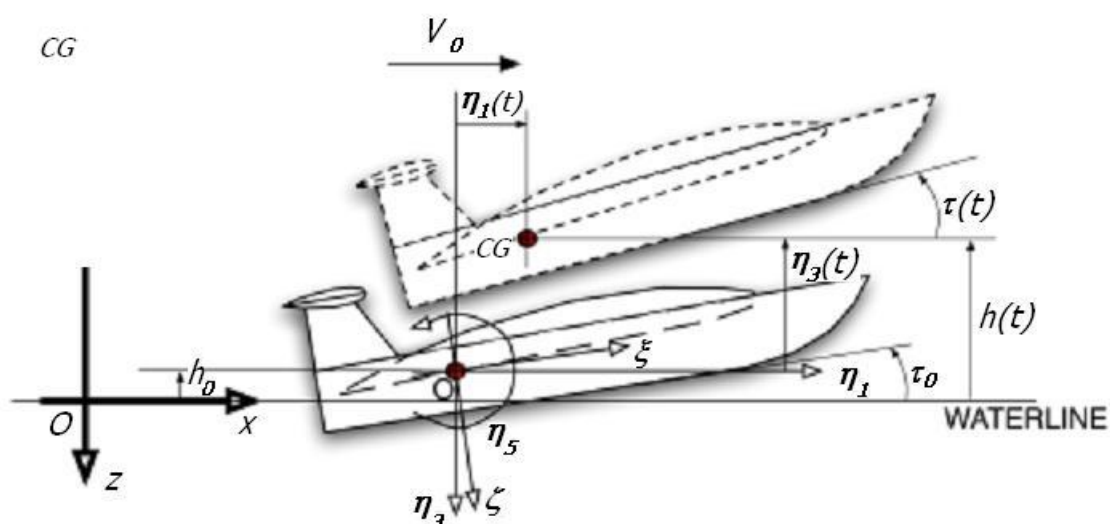


Figure 1: Hybrid vehicle axis systems

Vehicle							
LOA	length overall	34 ft	10.36 m				
B	beam	14 ft	4.27 m				
Sponsons (each)				Airfoil			
b^h	beam	3.15 ft	0.96 m	c	chord	25.9 ft	7.89 m
β_{in}	inside deadrise angle	90 deg		b^a	span	7.71 ft	2.35 m
β_{out}	outside deadrise angle	13 deg		S^a	area	199.7 ft ²	18.55 m ²

Table 1: KUDU II main dimensions

RULM characteristics			
m	mass	11820 lb	5361.6 kg
l_{cg}	longitudinal position of CG (forward from the transom)	7.5 ft	2.29 m
v_{cg}	vertical position of CG (height above the keel)	2.8 ft	0.85 m
V₀	speed	80 mph	35.76 m/s (≈ 130 km/h)
τ₀	trim angle	4.6 deg (0.0803 rad)	
L_K	keel wetted length	7 ft	2.134 m
L_C[*]	chine wetted length	1.24 ft	0.377 m

Equilibrium forces and moments						
	Aerodynamic		Hydrodynamic		Propulsion	
Lift	2631 lb	11707.5 N	10682 lb	47533 N	-1493 lb	-6643.5 N
Drag	829 lb	3688.9 N	1610 lb	7164.2 N	-2439 lb	-10853.1 N (thrust)
Moment	21854 lb-ft	29640.7 N·m	-45680 lb-ft	-61956 N·m	23826 lb-ft	32315.3 N·m

Table 2: Motion characteristics and equilibrium forces

α₀[*]	angle of attack	9.6 deg
h₀/c[*]	height of the aerodynamic center above the surface / c	0.13

C_L^a	0.845	C_D	0.266	∂C_m/∂V	0	∂C_{LT}/∂α	0
∂C_L/∂V^{**}	0	∂C_D/∂V^{**}	0	∂C_m/∂α	2.77 rad ⁻¹	S_{LT}	0
∂C_L/∂α[*]	3.62 rad ⁻¹	∂C_D/∂α[*]	1.89 rad ⁻¹	∂C_m/∂(h/c)[*]	-0.02	I_T	0
∂C_L/∂(h/c)[*]	-0.17	∂C_D/∂(h/c)[*]	-0.32			ε_α	0

B^h	approx as 2b ^h	1.92 m
β	approx as β _{out}	13 deg
c_V (Fn_B)	$\frac{V_0}{\sqrt{gB^h}}$	8.24
L_K	keel wetted length (tab 2)	0.654
τ₀	trim angle (tab 2)	4.6 deg
l_{cg}/B^h	dimensionless l _{cg} (tab 2)	1.193
v_{cg}/B^h	dimensionless v _{cg} (tab 2)	0.443
c_Δ	$\frac{m}{\rho^h(B^h)^3}$	0.737

Table 3: Coefficients for aerodynamic and hydrodynamics derivatives

Aerodynamic derivatives			
Derivative	Dimensional conversion	Dimensionless expression	Dimensionless value*
X_h^a	$1/2 \rho^a S^a V_0^2 / c$	$\left(-\frac{\partial C_D}{\partial(h/c)} \right)$	0.32
$X_{\eta_1}^a$	$1/2 \rho^a S^a V_0$	$\left(-2C_D - V_0 \frac{\partial C_D}{\partial V} \right)$	-0.532
$X_{\eta_3}^a$	$1/2 \rho^a S^a V_0$	$\left(C_L - \frac{\partial C_D}{\partial \alpha} \right)$	-1.045
$X_{\eta_5}^a$	$1/2 \rho^a S^a V_0 c$		negligible (0)
$X_{\eta_3}^a$	$1/2 \rho^a S^a c$		negligible (0)
Z_h^a	$1/2 \rho^a S^a V_0^2 / c$	$\left(-\frac{\partial C_L}{\partial(h/c)} \right)$	0.17
$Z_{\eta_1}^a$	$1/2 \rho^a S^a V_0$	$\left(-2C_L + V_0 \frac{\partial C_L}{\partial V} \right)$	-1.69
$Z_{\eta_3}^a$	$1/2 \rho^a S^a V_0$	$\left(-C_D - \frac{\partial C_L}{\partial \alpha} \right)$	-3.886
$Z_{\eta_5}^a$	$1/2 \rho^a S^a V_0 c$	$\left(-\frac{\partial C_{LT}}{\partial \alpha} \frac{S_T l_T}{S c} \right)$	0
$Z_{\eta_3}^a$	$1/2 \rho^a S^a c$	$\left(-\frac{\partial C_{LT}}{\partial \alpha} \frac{S_T l_T}{S c} \epsilon_\alpha \right)$	0
M_h^a	$1/2 \rho^a S^a V_0^2$	$\left(\frac{\partial C_m}{\partial(h/c)} \right)$	-0.02
$M_{\eta_1}^a$	$1/2 \rho^a S^a V_0 c$		negligible (0)
$M_{\eta_3}^a$	$1/2 \rho^a S^a V_0 c$	$\left(\frac{\partial C_m}{\partial \alpha} \right)$	2.77
$M_{\eta_5}^a$	$1/2 \rho^a S^a V_0 c^2$	$\left(-\frac{\partial C_{LT}}{\partial \alpha} \frac{S_T l_T^2}{S c^2} \right)$	0
$M_{\eta_3}^a$	$1/2 \rho^a S^a c^2$	$\left(-\frac{\partial C_{LT}}{\partial \alpha} \frac{S_T l_T^2}{S c^2} \epsilon_\alpha \right)$	0

Hydrodynamic derivatives		
Derivative	Dimensional conversion	Dimensionless value*
$X_{\eta_3}^h$	$\rho^h g (B^h)^2$	$\cdot(-1.510)$
$X_{\eta_5}^h$	$\rho^h g (B^h)^3$	$\cdot(-3.828)$
$X_{\eta_1}^h$	$\rho^h (B^h)^3 \sqrt{g/B^h}$	$\cdot(-0.029)$
$X_{\eta_3}^h$	$\rho^h (B^h)^3 \sqrt{g/B^h}$	$\cdot(-0.361)$
$X_{\eta_5}^h$	$\rho^h (B^h)^4 \sqrt{g/B^h}$	$\cdot(-0.432)$
$X_{\eta_1}^h$	$\rho^h (B^h)^3$	$\cdot(-0.001)$
$X_{\eta_3}^h$	$\rho^h (B^h)^3$	$\cdot(-0.017)$
$X_{\eta_5}^h$	$\rho^h (B^h)^4$	$\cdot(-0.016)$
$Z_{\eta_3}^h$	$\rho^h g (B^h)^2$	$\cdot(-18.770)$
$Z_{\eta_5}^h$	$\rho^h g (B^h)^3$	$\cdot(-29.068)$
$Z_{\eta_1}^h$	$\rho^h (B^h)^3 \sqrt{g/B^h}$	$\cdot(-0.361)$
$Z_{\eta_3}^h$	$\rho^h (B^h)^3 \sqrt{g/B^h}$	$\cdot(-4.480)$
$Z_{\eta_5}^h$	$\rho^h (B^h)^4 \sqrt{g/B^h}$	$\cdot(-5.362)$
$Z_{\eta_1}^h$	$\rho^h (B^h)^3$	$\cdot(-0.017)$
$Z_{\eta_3}^h$	$\rho^h (B^h)^3$	$\cdot(-0.213)$
$Z_{\eta_5}^h$	$\rho^h (B^h)^4$	$\cdot(-0.203)$
$M_{\eta_3}^h$	$\rho^h g (B^h)^3$	$\cdot(+0.406)$
$M_{\eta_5}^h$	$\rho^h g (B^h)^4$	$\cdot(-10.464)$
$M_{\eta_1}^h$	$\rho^h (B^h)^4 \sqrt{g/B^h}$	$\cdot(+0.149)$
$M_{\eta_3}^h$	$\rho^h (B^h)^4 \sqrt{g/B^h}$	$\cdot(+1.854)$
$M_{\eta_5}^h$	$\rho^h (B^h)^5 \sqrt{g/B^h}$	$\cdot(-3.070)$
$M_{\eta_1}^h$	$\rho^h (B^h)^4$	$\cdot(-0.016)$
$M_{\eta_3}^h$	$\rho^h (B^h)^4$	$\cdot(-0.203)$
$M_{\eta_5}^h$	$\rho^h (B^h)^5$	$\cdot(-0.201)$

Table 4: Aerodynamic and Hydrodynamic derivatives