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Adaptive diagnosis of the bilinear mechanical systems

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Abstract.

A generic adaptive approach is proposed for diagnosis of the bilinear mechanical systems. The approach adapts the free oscillation method for bilinearity diagnosis of mechanical systems. The expediency of the adaptation is proved for a recognition feature, the decrement of the free oscillations. The developed adaptation consists of variation of the adaptive likelihood ratio of the decrement with variation of the resonance frequency of the bilinear system. It is shown that in the cases of the frequency-independent and the frequency-dependent internal damping the adaptation is expedient. To investigate effectiveness of the adaptation in these cases, a numerical simulation was carried out. The simulation results show that use of the adaptation increases the total probability of the correct diagnosis of system bi-linearity.

1. Introduction

Bilinear mechanical systems abound in many settings and applications. Some examples of bilinear mechanical systems discussed in literature include:

- oscillating mechanical systems with clearances and motion limiting stops [1]
- offshore structures: free-hanging risers, tension leg platforms and suspended loads [2-4], articulated loading towers, constrained by a connection to a massive tanker or vessels moored against fenders [5]
- damaged (e.g. cracked, pitted, etc.) mechanical systems [6-10]
- un-damaged gearboxes [11]

Among the most widely used approaches for vibro-acoustical diagnosis of bilinearity of mechanical systems is the free oscillation method [7-8, 12]. The method consists of the impact excitation of the free mechanical oscillations of the bilinear system and evaluation of a diagnosis feature from the vibro-acoustical signals radiated by these oscillations. As a diagnosis feature, the decrement of the free oscillations has widely been used [8-9].

Normally, diagnosis of the bilinear mechanical systems is carried out under presence of variable nuisance parameters [13-14] of the systems. Changes of nuisance parameters lead to the deterioration of recognition effectiveness [13-14]. Typical examples of the variable nuisance parameters for mechanical systems are the performance parameters: e. g., the shaft speed, a load, etc.

The problem is to preserve diagnosis effectiveness of the bilinear mechanical systems in presence of the variable nuisance parameters. It is important to solve this problem for various bilinear mechanical systems: systems with clearances and motion limiting stops, offshore structures, damaged and un-damaged machinery. This problem has not been investigated in literature.

To preserve the recognition effectiveness, a new generic adaptive approach for system diagnosis is proposed here based on an adaptive likelihood ratio. In general, the proposed adaptive likelihood ratio depends on diagnosis feature vector and also on vector of measurable variable nuisance parameters. This is an improvement over most published applications concerning recognition and diagnostics of various systems; normally the classical likelihood ratio [15] averaged over ranges of variable nuisance parameters is used for system recognition and diagnostics.

The purposes of this paper are to:

- propose a generic adaptive approach for system diagnosis
- apply an adaptive approach for the free oscillation method of diagnosis of system bilinearity with the decrement as a recognition feature
- compare the proposed and traditional approaches for diagnosis of the bilinear mechanical systems

2. The Bilinear System and Decrement of the Free Oscillations

Let's consider a generic bilinear mechanical system, a single degree of freedom oscillator, in which the stiffness is bilinear:

$$\begin{cases} \ddot{X} + 2h_S\,\dot{X} + \omega_S^{\,2} X = 0, & X \ge 0, \\ \ddot{X} + 2h_C\,\dot{X} + \omega_C^{\,2} X = 0, & \mathrm{X} < 0, \end{cases} \tag{1}$$

where X is the displacement, m is the mass, $h_S = \zeta_S \omega_S$, $h_C = \zeta_C\,\omega_C$, ζ_S and ζ_C are the damping ratios at the positive and negative displacements, $\zeta_S = \dfrac{c}{2\sqrt{k_S m}}$, $\zeta_C = \dfrac{c}{2\sqrt{k_C m}}$, $\omega_S = \sqrt{\dfrac{k_S}{m}}$, $\omega_C = \sqrt{\dfrac{k_C}{m}}$, c is the damping, k_S and k_C are the stiffness at the positive and negative displacements.

At the positive displacement the stiffness decreases with the quantity $\Delta k = k_C - k_S$ [1-11].

Using Equation (1), the resonance frequencies at the positive and negative displacements can be written as follows respectively:

$$\omega_{Sd} = \omega_S \sqrt{1 - \zeta_S^{\,2}} \ ,$$

``

$$\omega_{Cd} = \omega_C \sqrt{1-\zeta_C{}^2} \tag{2}$$

Using Equations (1-2), the resonance frequency of the bilinear system after transformations can be written as follows:

$$\omega_0 = \sqrt{4\frac{\omega_{Cd}{}^2 \omega_{Sd}{}^2}{(\omega_{Cd}+\omega_{Sd})^2} + \frac{c^2}{4m^2}} \tag{3}$$

Equation (3) is generic. In the important case of the low damped system (i.e., the damping ratios are less than 0.1) substituting Equations (2) into Equation (3) yields:

$$\omega_0 = \sqrt{4\frac{\omega_C{}^2 \omega_S{}^2}{(\omega_C+\omega_S)^2} + \frac{c^2}{4m^2}} \tag{4}$$

Finally, Equation for resonance frequency can be written as follows (see Appendix) [16]:

$$\omega_0 = \omega_{\mathrm{Cd}} \frac{2\cdot\sqrt{1-k^*}}{1+\sqrt{1-k^*}}, \tag{5}$$

where k^* is the stiffness ratio, $k^* = \frac{\Delta k}{k_C}$.

The logarithmic decrement of the free oscillations of the bilinear system under impact initial conditions [$x(0)=0, \dot{x}(0)=v_0$] is obtained from Equations (1) after transformations as:

$$d = \frac{2\pi \zeta_C \omega_{Cd}}{\omega_0 \sqrt{1-\zeta_C{}^2}\left[1+\left(1-k^*\right)^{1/2}\right]}\left[1-h^*+\left(1-k^*\right)^{1/2}\right], \tag{6}$$

where $h^\bullet$ is the normalized damping ratio [7],

$$h^* = \frac{h_C - h_S}{h_C} \tag{7}$$

3. Adaptation of System Diagnosis

Procedures of proposed generic adaptive approach are to

- find a vector of measurable variable nuisance parameters for the selected recognition feature vector
- measure these parameters
- use an adaptation of the likelihood ratio (i.e., the adaptive likelihood ratio): to vary the likelihood ratio of diagnosis feature vector with variation of a vector of measurable variable nuisance parameters in order to decrease influence of these parameters and thus, preserve the diagnosis effectiveness

The adaptation of the free oscillation method using the logarithmic decrement as a recognition feature could be achieved with use of measurable variable nuisance parameters on which the decrement depends. As shown by Equation (6), the decrement depends on the damping ratio ζ_C, the resonance frequency ω_{Cd} at the negative displacement of the bilinear system, the resonance frequency ω_0 of the bilinear system, the stiffness ratio k^* and the normalized damping ratio h^*.

The stiffness ratio k^* is a basic diagnosis parameter [1-11]; the considered diagnosis is based on the difference in the stiffness ratios between the linear and the bilinear systems. According to Equation (7), the normalized damping ratio h^* depends on the basic recognition parameter k^*. The damping ratio ζ_C at the negative displacements (i. e., the linear system), is a nuisance parameter.

One can see from Equation (5) that the resonance frequency ω_0 of the bilinear system depends on a basic diagnosis parameter, the stiffness ratio k^*, and the resonance frequency at the negative displacement ω_{Cd} (i.e., the resonance frequency of the linear system). Therefore, the resonance frequency ω_0 can be used as a recognition feature [6] when the resonance frequency at the negative displacement ω_{Cd} *is known*. Otherwise, the resonance frequencies ω_0 and ω_{Cd} also become nuisance parameters.

An important case is considered here: the resonance frequency at the negative displacement ω_{Cd} is the *unknown* measurable nuisance parameter. Thus, the resonance frequency of the bilinear system ω_0 is also the *unknown* measurable nuisance parameter. Normally, the basic parameters of the bilinear system: mass m, damping c and stiffness k are variable random parameters due to manufacturing tolerances of bilinear systems. Therefore, the resonance frequencies of the linear and bilinear systems are also variable random parameters.

For these cases, the adaptation depending on the measurable variable random resonance frequency of the bilinear system is proposed and investigated.

The adaptive method consists of estimating the decrement d, the resonance frequency ω_0 and the adaptive likelihood ratio L_a which depends on the decrement estimate $\widehat{d}$ and the estimate $\hat{\omega}_0$ of the resonance frequency:

$$L_a(\hat{d}, \hat{\omega}_0) = \frac{W(\hat{d} | \hat{\omega}_0, S_1)}{W(\hat{d} | \hat{\omega}_0, S_0)}, \quad (8)$$

where $W(\hat{d} | \hat{\omega}_0, S_j)$ is the one-dimensional conditional probability density function (pdf) of the decrement estimate $\hat{d}$ which depends on status (class) S_j of the system and the estimate $\hat{\omega}_0$ of the resonance frequency, $j = 0,1$; class S_0 corresponds to the linear system with resonance frequency ω_{Cd}, damping ratio ζ_C and the zero stiffness ratio; class S_1 corresponds to the bilinear system with the bilinear stiffness and non-zero stiffness ratio.

In contrast, the classical non-adaptive method consists of estimating the decrement d and the classical likelihood ratio L_c which depends *only* on the decrement estimate $\hat{d}$:

$$L_c(\hat{d}) = \frac{W(\hat{d} | S_1)}{W(\hat{d} | S_0)}, \quad (9)$$

where $\hat{W}(\hat{d} | S_j)$ is the one-dimensional conditional pdf of the decrement estimate $\hat{d}$ which depends on class S_j of the system.

The pdf $\hat{W}(\hat{d} | S_j)$ of the classical likelihood ratio *is averaged over a range of the variable resonance frequency* for class S_j. In contrast, the pdf $W(\hat{d} | \hat{\omega}_0, S_j)$ of the adaptive likelihood ratio *is not averaged* over a range of the variable resonance frequency.

Then the standard decision-making procedure [15] should be used: the likelihood ratio L_a or L_c is compared with one or several thresholds depending on the selected effectiveness criterion. For example, if the maximum likelihood criterion [15] is used, then a threshold for the likelihood ratios is unity.

It should be highlighted that the proposed adaptation is expedient only if the resonance frequency of the bilinear system is variable.

To prove expediency of the adaptation, dependencies between the decrement d and the stiffness ratio k^* were studied for various values of the resonance frequency ω_0 and two different proportional internal dampings [7].

4. The Decrement of the Free Oscillations for Different Internal Damping

4. 1. The frequency-independent internal damping

For the frequency-independent internal damping, the normalized damping ratio h^* can be obtained from Equation (7) after transformations as:

$$h^* = 1 - \sqrt{1 - k^*} \tag{10}$$

Substituting Equation (10) into Equation (6) yields, after transformations:

$$d = \frac{2\pi\, h_C}{\omega_0}\left[\frac{2\left(1 - k^*\right)}{1 + \left(1 - k^*\right)^{1/2}}\right] \tag{11}$$

The dependencies of the decrement (11) on the stiffness ratio are shown in Fig. 1 for various values of the resonance frequency. As follows from Equation (11) and Fig. 1, the decrement depends on the stiffness ratio k^* and the resonance frequency ω_0. The adaptation is expedient since, for constant values of the nuisance parameter ω_0, the decrement depends on the stiffness ratio k^*.

4. 2. The frequency-dependent internal damping

For the frequency-dependent internal damping, the normalized damping ratio, h^* can be obtained from Equation (7) after transformations as:

$$h^* = k^* \tag{12}$$

Substituting Equation (12) into Equation (6) yields, after transformations:

$$d = \frac{2\pi\, h_C}{\omega_0}\left[\frac{1 - k^\bullet + \left(1 - k^*\right)^{1/2}}{1 + \left(1 - k^*\right)^{1/2}}\right] \tag{13}$$

The dependencies of the decrement (13) on the stiffness ratio k^* are shown in Fig. 2 for various values of the resonance frequency ω_0. As follows from Equation (13) and Fig. 2, the decrement depends on the stiffness ratio k^* and the resonance frequency ω_0. As in the previous case, the adaptation is expedient since, for constant values of the nuisance parameter ω_0, the decrement depends on the stiffness ratio k^*.

5. Numerical Simulation

To estimate the effectiveness of the adaptation a numerical simulation was carried out for cases 4.1-4.2 and classes S_0 and S_1. The variable random resonance frequency of the linear system ω_{Cd} is assumed to be uniformly distributed in the range (1000-1200) Hz, i.e., 20% variation. The random stiffness ratio of the bilinear system is uniformly distributed in the range (0-0.4]. The considered variation values correspond to real cases occurring in the bilinear systems listed in the chapter 1.

It is assumed without loss of generality that the decrement and the resonance frequency were measured without errors for both classes. The damping parameter is $h_C = 0.05$ rad/s.

5. 1. Non-adaptive approach

Estimates of the conditional pdfs of the decrement *averaged over the resonance frequency ranges* for classes S_0 and S_1 for different damping models were obtained using the Monte-Carlo procedure and Equations (5, 11, 13). To estimate these pdfs, 950 runs were simulated for each combination of the class and the damping model. Then the classical likelihood ratios (9) were estimated on the basis of the averaged pdfs of the decrement.

Estimates P of the total probabilities of the correct diagnosis for the non-adaptive and the adaptive approaches are calculated as follows: $P = \mu_l P_l + \mu_b P_b$, where P_l and P_b are estimates of the probability of the correct diagnosis of the linear and the bi-linear systems respectively; μ_l, and μ_b are a priori probabilities of the linear and the bi-linear systems respectively. Taking into account the equal number of the simulated runs for the linear and the bi-linear systems, a priori probabilities are $\mu_l = \mu_b = 0.5$.

These estimates are obtained by the simulation using the classical likelihood ratios and the maximum likelihood criterion [15] and presented in Table.

5. 2. Adaptive approach

According to the assumption that the decrement and the resonance frequency were measured without errors for both classes and due to the monotonous character of the dependencies presented in Fig. 1-2, the decrement values of the class S_0 do not overlap with the decrement values of the class S_1. Therefore, when using the adaptive likelihood ratio (8) and measuring the decrement and the resonance frequency without errors, the *estimates* of the total probability of the correct diagnosis are equal to unity.

Taking into account these estimates, one can see from Table that when using the adaptive approach, the estimates of the total probability of the correct diagnosis increase by (19-20%). This increase highlights the efficiency of the proposed adaptive approach for bi-linearity diagnosis with the logarithmic decrement as a diagnosis feature.

6. Conclusions

1. A generic adaptive approach was proposed for system diagnosis. The main idea of the approach is a variation of the adaptive likelihood ratio with variation of vector of measurable variable nuisance parameters. The approach decreases the influence of these parameters on diagnostic features and thus preserves diagnosis effectiveness in the presence of variable nuisance parameters.

2. The proposed approach was applied for the free oscillation method for bilinearity diagnosis of the bilinear mechanical systems. Expediency of the adaptation was proved for the following diagnosis feature: the decrement of the free oscillations of the bilinear system. The resonance frequency of the bilinear system was selected as the measurable variable random nuisance parameter. The new generic analytical Equation for the resonance frequency of the bilinear system was obtained.

3. The adaptation consists of a variation of the adaptive likelihood ratio of the decrement of the free oscillations with variation of the variable random resonance frequency of the bilinear system. It was shown that in the cases of the frequency-independent and the frequency-dependent internal dampings the proposed adaptation is expedient.

Generally, the proposed adaptation is expedient if a diagnostic feature depends on a variable nuisance parameter.

4. To investigate the adaptation effectiveness in the above mentioned cases, a numerical simulation was carried out. The simulation results have shown that, when the adaptive approach was used, the estimates of the total probability of the correct recognition increased by (19-20)%. This increase in probabilities is obtained for the considered numerical example.

Generally, increase depends mainly on dependency between a diagnostic feature and a nuisance parameter and an error in estimation of a nuisance parameter.

5. The paper results indicate that use of the proposed adaptation for the decrement of the free oscillations improves the effectiveness of bilinearity diagnosis for bilinear mechanical systems in presence of the variable nuisance parameter, the resonance frequency of the system.

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Table Estimates of the total probabilities of bilinearity diagnosis

Damping model	Non-adaptive approach	Adaptive approach
The frequency-dependent internal damping	0.8	1.0
The frequency-independent internal damping	0.81	1.0

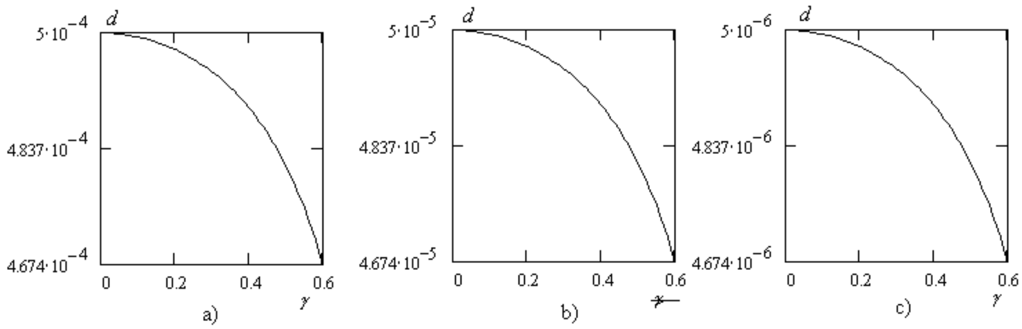

Fig.1. Dependencies of the decrement of the free oscillation on the stiffness ratio k^* and the resonance frequency ω_0 for the frequency-independent internal damping (h_C =0.05 rad/s): a) ω_0 = 2π 100 rad/s; b) ω_0 = 2π 1000 rad/s; c) ω_0 = 2π 10000 rad/s

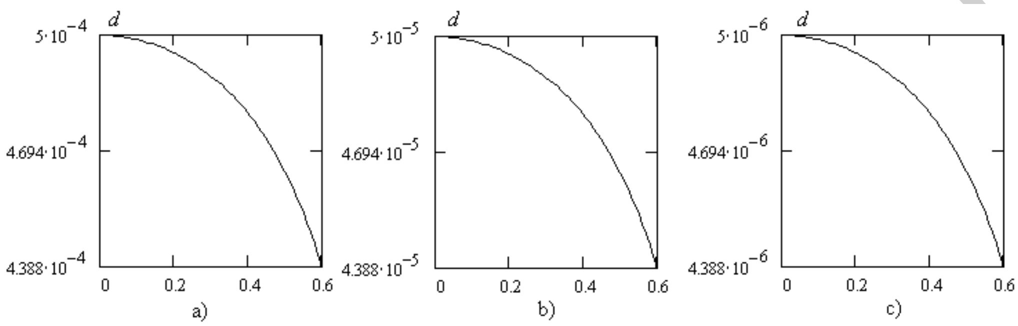

Fig. 2. Dependencies of the decrement of the free oscillation on the stiffness ratio k^* and the resonance frequency ω_0 for the frequency-dependent internal damping (h_C =0.05 rad/s): a) ω_0 = 2π 100 rad/s; b) ω_0 = 2π 1000rad/s; c) ω_0 = 2π 10000rad/s

Appendix

Equation (4) after transformations can be written as follows:

$$\omega_0 = \sqrt{4 \frac{\omega_C{}^2 \omega_S{}^2 \left[1 + \frac{(\omega_C + \omega_S)^2 \zeta_S{}^2}{\omega_C{}^2} \right]}{(\omega_C + \omega_S)^2}} \quad \text{(A1)}$$

Equation (A1) after additional transformation can be presented as follows:

$$\omega_0 = \sqrt{4 \frac{\omega_C{}^2 \omega_S{}^2 \left[1 + \left(1 + \sqrt{k_S / k_C} \right)^2 \zeta_S{}^2 \right]}{(\omega_C + \omega_S)^2}} \quad \text{(A2)}$$

Taking into account that $k_S \le k_C$ [1-11], Equations (2) and expression for the stiffness ratio, Equation (A2) for low damped system finally can be presented as follows:

$$\omega_0 = \omega_{\mathrm{Cd}} \frac{2 \cdot \sqrt{1 - k^*}}{1 + \sqrt{1 - k^*}} \quad \text{(A3)}$$