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3 **Theoretical Analysis of the Spatial Variability in Tillage Forces**
4 **for Fatigue Analysis of Tillage Machines**

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12
13 **Abstract**

14 This paper presents a new theoretical model to describe the spatial variability in tillage forces for
15 the purpose of fatigue analysis of tillage machines. The proposed model took into account both
16 the variability in tillage system parameters (soil engineering properties, tool design parameters
17 and operational conditions) and the cyclic effects of mechanical behaviour of the soil during
18 failure ahead of tillage tools on the spatial variability in tillage forces. The stress-based fatigue
19 life approach was used to determine the life time of tillage machines, based on the fact that the
20 applied stress on tillage machines is primarily within the elastic range of the material. Stress
21 cycles with their mean values and amplitudes were determined by the rainflow algorithm. The
22 damage friction caused by each cycle of stress was computed according to the Soderberg criterion
23 and the total damage was calculated by the Miner's law.

24 The proposed model was applied to determine the spatial variability in tillage forces on the shank
25 of a chisel plough. The equivalent stress history resulted from these forces were calculated by
26 means of a finite element model and the Von misses criterion. The histograms of mean stress and
27 stress amplitude obtained by the rainflow algorithm showed significant dispersions. Although the
28 equivalent stress is smaller than the yield stress of the material, the failure by fatigue will occur
29 after a certain travel distance. The expected distance to failure was found to be $= 0.825 \times$
30 10 . It is concluded that the spatial variability in tillage forces has significant effect on the
31 life time of tillage machines and should be considered in the design analysis of tillage machines
32 to predict the life time. Further investigations are required to correlate the results achieved by the
33 proposed model with field tests and to validate the proposed assumptions to model the spatial
34 variability in tillage forces.

35 **Keywords:** spatial variability; tillage forces; tillage machines; fatigue analysis; life time.

36

37 **1- Introduction**

38 Mechanical loads on tillage machines show considerable variability due to the variability in
39 tillage system parameters and the mechanical behaviour of soil during failure. The variability in
40 tillage system parameters reflects the variability in soil engineering properties and the variations
41 in tool design parameters and operational conditions. The variability in soil engineering
42 properties is resulted from the heterogeneity of agricultural soils. The variations in tool design
43 parameters are due to the manufacturing processes, while the variations in operational conditions
44 are due to the fact that these parameters are not completely controlled during tillage operation.

45 The crescent manner of soil failure for wide and narrow tines with depth/width ratio less than 5
46 tend to push the soil upward and forward [1]. This failure pattern involves the development of
47 successive shear planes in front and at the side of tillage tools, which leads to distinct soil failure

48 blocks (crescents) as the tine moves forward through the soil. The repeated formation of soil
49 crescents creates cyclic loading on the tillage tool. Stafford [2] identified two types of soil failure,
50 namely the brittle failure and the flow failure. Rajaram and Erbach [3] reported the following four
51 soil failure patterns: 1) collapse failure, 2) fracture failure, 3) chip-forming failure and 4) plastic
52 and frictional flow failures. However, they attributed the changes of soil failure patterns to two
53 main factors: the soil type and the moisture content. In other work, Rajaram and Erbach [4]
54 reported that the variations in soil failure patterns are mostly affected by the soil moisture content,
55 soil type, tillage implement type and tillage speed. Factors controlling the soil mechanical
56 behaviour during failure under load are not completely understood/known. In addition, there are
57 no available models that can estimate the effects of soil failure on tillage forces.

58 Several analytical (e.g. McKyes and Ali [5]; Grisso et al. [6]) and numerical models (e.g. Shen
59 and Kushwaha [7]; Mouazen and Neményi [8]) of soil-tool interaction have been developed to
60 predict tillage forces for assigned tillage system parameters. Furthermore, the effects of tillage
61 system parameters on tillage forces have been investigated [1,9]. Although, these models have
62 shown good agreements with experimental observations for specific test conditions, there is still
63 no well-defined, generalised theoretical model to predict the behaviour of soil-tool interactions
64 [10]. In order to take into consideration the fact that tillage system parameters are variable during
65 tillage operations, Abo Al-kheer et al. [11] proposed a methodology for modeling the variability
66 in tillage forces derived from the variability in tillage system parameters. In another report, they
67 integrated this variability into the reliability analysis for achieving reliable tillage machines [12].

68 The variability in tillage forces due to the soil failure has been observed in many works in the
69 literature. James and Shrini [13] reported that the large variations in the horizontal force on a
70 plough can be attributed to two factors. These are related to the soil failure and to the within-field
71 spatial variability in soil properties. They confirmed that both of these factors lead to relatively
72 large changes in the horizontal force within a short time period. Makanga et al. [14] studied the

73 effects of the tine rake angle and the aspect ratio in a laboratory glass-sided soil bin with a dry
74 compacted loam soil with 5.2 % (d.b.) moisture content. They concluded that the horizontal and
75 vertical soil reactions under dry soil conditions were cyclic in nature and in phase. The cyclic
76 variations in the soil reactions were due to the soil failure patterns being repetitive and cyclic in
77 nature throughout the tine travel. Due to variability in soil properties and operational conditions
78 the frequency and amplitude of these cycles vary significantly.

79 From a fatigue analysis viewpoint, it is essential to account for the effects of the variability in
80 tillage forces on the resulted stress on tillage machines. The current state of knowledge suggests
81 that there are only experimental works available to estimate the spatial variability in tillage forces
82 [15,16,17]. These methods do not provide a tool for estimating the life time of tillage tools due to
83 fatigue as it cannot account for all affecting parameters.

84 The main objective of this work is to propose a model for estimating the variability in tillage
85 forces for the purpose of fatigue analysis of tillage machines. The proposed model is applied to
86 estimate the spatial variability in tillage forces on the shank of a chisel plough and the expected
87 distance to failure is calculated. This model is presented in the first part of this work. The second
88 part presents the stress-based fatigue life approach used to estimate the life time of tillage
89 machines.

90

91 **2- Materials and methods**

92 **2-1 Modeling the spatial variability of tillage forces**

93 **2-1-1 Basic assumptions for the proposed model**

94 The basic assumptions behind the proposed model are that 1) the spatial variability of tillage
95 forces derived from the variability in tillage system parameters is random, reflecting the
96 heterogeneity in agricultural soils, the uncontrolled field operational conditions during tillage and

97 the manufacturing tolerances of tool design parameters and 2) the spatial variability of tillage
 98 forces derived from the mechanical behavior of soil failure is cyclic, reflecting the repeated
 99 formation of soil crescents in front of the tool. In addition, we suppose that the total tillage force
 100 is the sum of two types of forces, namely the global tillage force and the local tillage force. The
 101 global tillage force is due to the tillage system parameters (soil engineering properties, tool design
 102 parameters and operational conditions) and the local tillage force is due to the soil failure of
 103 cyclic nature.

104 Conventionally, a tillage force is determined by its horizontal and vertical components.
 105 Therefore, the horizontal and vertical forces can be calculated, according the earlier assumption,
 106 by Equations (1) and (2).

$$= + \quad (1)$$

$$= + \quad (2)$$

107 where is the global horizontal force in kN, is the local horizontal force in kN, is the
 108 global vertical force in kN and is the local vertical force in kN.

109 The variability in the global tillage forces (,) can be modelled using the methodology
 110 proposed by Abo Al-kheer et al. [11]. This methodology is based on the estimation of tillage
 111 forces according to the McKyes-Ali model accounting for the variability in tillage system
 112 parameters. Abo Al-kheer et al. [11] subdivided the tillage system parameters contributing to the
 113 global tillage forces into three main categories: 1) soil engineering properties including soil bulk
 114 density, soil cohesion, internal friction angle, soil-metal friction angle and soil-tool adhesion), 2)
 115 tool design parameters including tool width and rake angle and 3) operational conditions
 116 including tool working depth, surcharge pressure and tool speed. A combination of graphical and
 117 quantitative techniques was proposed for modeling the variability in soil engineering properties
 118 and two statistical tests were used to select the probability distributions of these properties,

119 namely the chi-square test and the Kolmogorov-Smirnov test. A total of 57 variations of soil
 120 engineering properties, representing 57 different soil samples were considered for implementing
 121 our mixed technique approach [11]. **The probabilistic characteristics of these properties are given**
 122 **in Table 1.**

123 The local tillage forces (F_x , F_y) have been observed in many works in the literature but there
 124 are no available models can be used to estimate these forces. However, the majority of reports are
 125 attributing these forces to nearly the same parameters contributing to the global tillage forces [4].
 126 Therefore, we assume that the local tillage force components can be estimated as a percentage of
 127 the global tillage force components as shown in Equations (3) and (4).

$$F_x = \alpha F_{gx} \quad (3)$$

$$F_y = \beta F_{gy} \quad (4)$$

128 where α is the percentage of the local tillage force to the global tillage force. According to the
 129 assumptions in Equations 3 and 4 global and local forces are related by the tillage system
 130 parameters (soil engineering properties, tool design parameters and operational conditions). The
 131 high values of α corresponding to a brittle soil failure and the little values of α corresponding to a
 132 flow soil failure. In other words, the values of the local tillage forces (F_x , F_y) are important for
 133 the brittle soil failure since the force cyclic pattern is much more pronounced that with flow
 134 failure, while the values of these forces are nearly zero when the soil failure is of flow type [10].

135 The linear correlation between the global and local tillage forces may not be accurate for all soil
 136 texture types and all operational conditions. Thus, more work should be done to investigate the
 137 relationship between the global and local tillage forces.

138 Based on the earlier assumptions, the spatial variability in tillage forces can be represented by the
 139 spatial variability in the global and local tillage forces, as shown in Equations (5) and (6).

$$F_x = F_{gx} + F_{lx} \quad (5)$$

$$F_y = F_{gy} + F_{ly} \quad (6)$$

140 where σ_{Gx} represents the spatial variability in the global horizontal force in kN, σ_{Lx} represents
 141 the spatial variability in the local horizontal force in kN, σ_{Gz} represents the spatial variability in
 142 the global vertical force in kN, σ_{Lz} represents the spatial variability in the local vertical force in
 143 kN and L is the distance travelled in m.

144 **2-1-2 Modeling the spatial variability in the global tillage forces**

145 The spatial variability in the global tillage forces (F_{Gx} , F_{Gz}) is resulted from the spatial variability
 146 of soil resistance and uncontrolled operational conditions. This spatial variability can be
 147 attributed to several factors, e.g., the characteristics of the field, the geography and topography of
 148 the field and the soil management system (no-till, reduced till or conventional tillage). Therefore,
 149 the spatial variability in the global tillage forces changes from one location to another within the
 150 same field and from field to field.

151 To take the earlier observations into account, we modelled the spatial variability in the global
 152 tillage forces with the following assumptions: 1) the spatial variability in the global tillage forces
 153 is linear and 2) the distance L between two successive changes of the values of global tillage
 154 forces is random. The linearity of the spatial variability in the global tillage forces between the
 155 global tillage force samples may not be an accurate assumption. However, the increase of the
 156 global tillage force samples improves the accuracy of this model. Taking the distance L as a
 157 random variable allows considering the variability in the field characteristics over the distance L .
 158 Based on these assumptions, the spatial variability in the global tillage forces can be expressed as
 159 in Equations (7) and (8). An illustration of the spatial variability in the global tillage forces over
 160 the distance L is shown in Figure 1.

$$F_{Gx}(i) = F_{Gx}(i-1) + \sigma_{Gx} \left(\frac{L(i) - L(i-1)}{L} \right) \quad = 1, \dots, \quad (7)$$

$$F_{Gz}(i) = F_{Gz}(i-1) + \sigma_{Gz} \left(\frac{L(i) - L(i-1)}{L} \right) \quad = 1, \dots, \quad (8)$$

161 where F_{ih} is the i th global horizontal force sample in kN, F_{iv} is the i th global vertical
 162 force sample in kN, Δx is the distance between two successive changes of the global tillage force
 163 values in m, $\sum_{i=1}^{n-1} F_{ih}$ is the cumulative sum of F_{ih} for $i = 1$ to $n - 1$ and n is the number of
 164 the global tillage force samples.

165 2-1-3 Modeling the spatial variability in the local tillage forces

166 As mentioned before, the soil failure creates cyclic loading on tillage tools by the repeated
 167 formation of soil crescents. The global tillage forces are calculated at failure when the tillage
 168 forces achieve their maximum values. The local tillage forces reach their maximum values at
 169 failure and then drop down after the first soil block has formed and these forces will increase to
 170 form the second soil block until achieve failure and so on. Therefore, we can imagine that the
 171 total tillage forces fluctuate below the global tillage forces.

172 Based on the fact that the effect of the soil failure in the tillage forces is cyclic, the sinusoid
 173 function was used to describe the spatial variability in the local tillage forces with the
 174 amplitude F_{jmax} and cycle length L_j . Therefore, the spatial variability of tillage forces can be
 175 expressed as in Equations (9) and (10) and illustrated as in Figure 2. The terms $-F_{jmax}$ and
 176 $+F_{jmax}$ are added to these Equations to keep the values of total tillage forces fluctuate below the
 177 values of global tillage forces.

$$F_{jh} = F_{jmax} \sin 2\pi \frac{-\sum_{i=1}^{n-1} F_{ih} - \sum_{i=1}^{n-1} F_{iv}}{L_j} - F_{jmax} \quad = 1, \dots, \quad (9)$$

$$F_{jv} = F_{jmax} \sin 2\pi \frac{-\sum_{i=1}^{n-1} F_{ih} - \sum_{i=1}^{n-1} F_{iv}}{L_j} + F_{jmax} \quad = 1, \dots, \quad (10)$$

178 where F_{jh} is the j th local horizontal force in kN, F_{jv} is the j th local vertical force in kN,
 179 L_j is the cycle length of the j th cycle in m, n is the number of calculated values in a cycle and
 180 n_j is the number of cycles between two successive changes of the global tillage forces.

181 **2-1-4 Modeling the spatial variability in the total tillage forces**

182 By combining the spatial variability in the global and local tillage forces and taking into account
183 the assumption that the local tillage force components can be estimated as a percentage of the
184 global tillage force components, it concluded that the spatial variability in tillage forces can be
185 represented by the following five parameters:

$$(\) = (P_{HG}, S_1, S_2, \) \quad (11)$$

$$(\) = (P_{VG}, S_1, S_2, \) \quad (12)$$

186 All of these parameters can be considered as variables to represent the variability in the forces on
187 the tillage tool during the tillage operation, as shown in Figure 3.

188 **2-1-5 Special cases**

189 Two special cases of spatial variability in tillage forces, namely, at constant global tillage forces
190 and at insignificant local tillage forces are shown in Figures 4 and 5, respectively. The first case
191 supposes that all tillage system parameters do not vary during tillage. This assumption may be
192 suitable for quasi-homogeneous soils and when the variations in the operational conditions are
193 not important. The second case can be used to represent the spatial variability in tillage forces
194 when the soil failure is of flow type. However, in most cases, both the global and local tillage
195 forces should be taken into consideration in the estimation of the spatial variability in tillage
196 forces.

197

198 **2-2 Stress-based fatigue life**

199 The stress-based fatigue life approach is generally characterized by a high-cycle fatigue
200 methodology, and is widely used in design applications where the applied stress is primarily
201 within the elastic range of the material and the resulting fatigue life is long. The basis of the

202 stress-based fatigue life approach is the stress (S)-number of cycles to failure (N) curve, also
 203 known as a Wöhler curve. The S-N curve is a graph of the amplitude of a cyclical stress against
 204 the logarithmic scale of cycles to failure. In some materials, particularly ferrous alloys, the S-N
 205 curve flattens out eventually, so that below a certain limit, called the fatigue limit or the
 206 endurance limit (typically $> 10^7$ cycles), the material may not fail and can be cycled infinitely
 207 [18] (curve a in Figure 6). For some other materials such as aluminum and copper alloys, no
 208 fatigue limit exists. In such cases, the fatigue strength for a given number of cycles (e.g. 1×10^7
 209 cycles) must be specified [19] (curve b in Figure 6).

210 The S-N curve for a material, that has a fatigue limit such as steel, can be expressed as in
 211 Equation (13).

$$S = \sigma_f \cdot N^{\frac{1}{m}} \quad \begin{matrix} < \\ \geq \end{matrix} \quad (13)$$

212 where S is the stress amplitude in MPa, σ_f is the regression intercept (also called the fatigue
 213 strength coefficient) in MPa, m is the regression slope (also called the fatigue strength exponent),
 214 N is the number of cycles and N_f is the number of cycles corresponding to the fatigue limit.

215 The most basic S-N curves are generated using a fully-reversed stress, where the ratio (R)
 216 between the maximum and minimum stress is equal to -1. When the stress applied on a structure
 217 is constant over the structure life and the ratio (R) is equal to -1, the Equation (13) can be used
 218 directly to determine the number of cycles to failure i.e. the fatigue life. If the number of cycles to
 219 failure N is greater than the number of cycles corresponding to the fatigue limit N_f , it can be
 220 accepted that the structure has an infinite life.

221 When the ratio R is not equal to -1, a Haigh diagram is usually used to estimate the fatigue life.
 222 This diagram plots the mean stress along the x-axis and the stress amplitude along the y-axis and
 223 the lines of constant life are drawn through the data points. A very substantial amount of testing is
 224 required to generate a Haigh diagram, and it is usually impractical to develop curves for all

225 combinations of mean and amplitude stresses. Therefore, several empirical criteria that relate the
 226 stress amplitude to the mean stress have been developed to address this difficulty. These criteria
 227 define various curves to connect the fatigue limit on the stress amplitude axis to either the yield
 228 strength or the ultimate strength on the mean stress axis [20]. The zone under the curves defined
 229 the safe zone against fatigue while the zone above the curves represents the failure zone. Figure 7
 230 illustrates three of these criteria, namely the criterions of Goodman, Gerber and Soderberg.

231 Kwofie [21] proposed a function, presented in Equation (14), to take into account the effect of
 232 mean stress. This function allows determining the stress amplitude according to the material
 233 constant, material properties, number of cycles to failure and to different fatigue criteria.

$$= 1 - \frac{\sigma_m}{\sigma_u} \quad (14)$$

234 where k is a numerical constant, representing the mean stress sensitivity of the material, σ_m is the
 235 mean stress in MPa and σ_u is the ultimate strength in MPa. The value of the numerical constant
 236 depends on the fatigue criterion (Goodman, Gerber, Soderberg, etc.).

237 In practice, a structure (e.g. tillage machines) is exposed to a random stress. In such cases, the
 238 random stress should be reduced to a series of simple cyclic stresses using counting methods, e.g.,
 239 range pair method and rainflow method [22]. The rainflow method is the most popular and widely
 240 used in practice [23]. It allows one to determine the amplitude and mean value (σ_a , σ_m) for each
 241 stress cycle at a fixed time interval t or a distance interval s . The damage friction caused by the
 242 i th cycle of stress can be computed by Equation (15).

$$= \frac{1}{N_f(\sigma_a, \sigma_m)} \quad (15)$$

243 where $N_f(\sigma_a, \sigma_m)$ is the number of cycles to failure according to the amplitude σ_a and the
 244 mean value σ_m . In this work, the Soderberg criterion was used to determine N_f , based on the

245 fact that the applied stress is primarily within the elastic range of the material. According to this
 246 criterion, $\sigma = \sigma_{el}$ [21] and ϵ was calculated as in Equation (16).

$$\epsilon = \frac{\sigma_{el}}{E} \quad (16)$$

247 The total damage, caused by all cycles, can be computed by a cumulative damage model. More
 248 than 60 fatigue damage models have been proposed for this purpose. However, the linear damage
 249 rule (Miner's law) is still dominantly used because of its simplicity in addition to its sufficient
 250 accuracy [24]. For these reasons the Miner's law was used in this work. The Miner's law assumes
 251 that the total damage can be expressed as the sum of damage frictions, as shown in Equation (17).
 252 Failure is assumed to occur when $D \geq 1$.

$$D = \sum_{i=1}^n \frac{n_i}{N_i} \quad (17)$$

253 where n_i is the number of cycles determined by the rainflow algorithm for the time interval t_i or
 254 the distance interval L_i . If the total damage is calculated for a distance interval L , which is the
 255 case of our work, the expected travel distance to failure can be calculated by dividing the distance
 256 interval by the total damage, i.e. $L_f = L/D$, to fulfill the assumption that the failure will occur
 257 when $D \geq 1$.

258

259 3- Numerical application

260 3-1 Modeling the spatial variability in tillage forces

261 The proposed model, presented in Section 2.1, is implanted in MATLAB program (Mathworks
 262 INC. 2008) to model the spatial variability in tillage forces on the shank of a chisel plough shown
 263 in Figure 8. The shank cross-section is rectangular of height $h = 58$ mm and width $b = 32$ mm,
 264 as shown in Figure 9.

265 Abo Al-kheer et al. [11] found that the variability in the global horizontal and vertical forces
 266 followed lognormal distributions. The distribution parameters of these forces were $\sigma = 0.872$,
 267 $\mu = 0.449$, $\sigma = 0.004$ and $\mu = 0.447$, where σ and μ are the scale and shape
 268 parameters of a lognormal distribution, respectively. The correlation coefficient between F_x and
 269 F_y was found to be $(\rho_{F_x, F_y}) = 0.93$. Therefore, the variability in the local horizontal and
 270 vertical forces should have lognormal distributions with the following distribution
 271 parameters $\mu = \ln(\mu) + \frac{\sigma^2}{2}$, $\sigma = \sigma$, $\mu = \ln(\mu) + \frac{\sigma^2}{2}$ and $\sigma = \sigma$. In this
 272 work, ρ was selected to be equal 0.2 for brittle failure. Furthermore, we assumed
 273 that F_x and F_y have normal distributions with the following distribution parameters $\mu =$
 274 10 , $\sigma = 0.1$, $\mu = 0.05$ and $\sigma = 0.1$, where μ and σ are the location and scale
 275 parameters of a normal distribution, respectively. The distribution parameters of the main model
 276 parameters, as described in Equations (11) and (12), are listed in Table 2.

277 To generate correlated tillage forces F_x and F_y , two non-correlated normalized variables
 278 U_1 and U_2 were generated by the MATLAB function “normrnd” and then the random values of
 279 U_1 and U_2 were calculated using two transformations [25,26]. The first one transforms non-
 280 correlated normalized variables U_1 and U_2 to correlated normalized variables V_1 and V_2 and the
 281 second one transforms correlated normalized variables to correlated tillage forces F_x and F_y .
 282 The spatial variability of the horizontal and vertical forces can then be shown in Figure 10 for a
 283 distance of 1000 m.

284 From Figure 10, it can be observed that a clear correlation exists in the spatial variability between
 285 the horizontal and vertical forces. This is resulted from the correlation between the global
 286 horizontal and vertical forces ($(\rho_{F_x, F_y}) = 0.93$). Another observation is the correlation
 287 between the global tillage forces (F_x, F_y) and the local tillage forces (F_{x1}, F_{y1}). The increase
 288 of global tillage forces increases the amplitudes of local tillage forces and vice-versa. These

289 increases in force amplitudes are marked in cycles in Figure 10. This is caused by the calculation
290 of the local tillage forces as a percentage of the global tillage forces.

291 **3-2 Equivalent stress history**

292 The equivalent stress, resulted from the tillage forces, was calculated according to the Von misses
293 criterion. Firstly, the point of the maximum equivalent stress was determined by means of the
294 finite element (FE) method and ANSYS program (ANSYS INC. V11). Figure 11 shows the
295 meshed model, boundary conditions and the point of maximum equivalent stress (in MPa)
296 determined for the mean values of tillage forces $\sigma = 2.641$ and $\tau = 1.106$. Then, the
297 equivalent stress, presented in Figure 12, was calculated at the point of maximum equivalent
298 stress using the finite element model, implemented in the CALFEM toolbox of MATLAB [27].

299 The rainflow algorithm [23] was used to extract the stress cycles with their amplitude and mean
300 values. The histograms of stress amplitude and mean stress are shown in Figure 13. Both the
301 histograms indicate that the dispersions of mean stress and stress amplitude are significant. This
302 reflects the high dispersions of the spatial variability in tillage forces.

303 **3-3 Expected distance to failure**

304 The material constants (the regression intercept and the regression slop) used to calculated the
305 expected distance to failure are $a = 754$ MPa and $b = 0.121$. The yield stress of the material
306 is $\sigma_y = 250$ MPa. The total damage was calculated over the distance ($L = 980.902$)
307 according to Equation (17). The total damage is equal to $D = 1.189 \times 10^{-3}$. By dividing the
308 distance L by the total damage D , the expected distance to failure is $L_f = 0.825 \times 10^3$.
309 Despite the fact that the equivalent stress is smaller than the yield stress (Figure 10), the failure
310 will occur after a certain distance L_f . This example shows the significant effect of the spatial
311 variability in tillage forces on the life time of tillage machines. Since agricultural soils are

312 characterized to be high spatial variability [17], it is expected that this variability will reduce the
313 life time of tillage tools.

314 The expected distance to failure is plotted as a function of the shank cross-section dimensions
315 (b , h) in Figure 14. For all combinations of b and h , the equivalent stress is smaller than the yield
316 stress. The minimum distance to failure $L_f = 2.004 \times 10^6$ occurs with $b = 25$
317 and $h = 55$, the maximum distance to failure $L_f = 9.213 \times 10^6$ occurs with
318 $b = 35$ and $h = 70$. It is noted that the augmentation of the shank cross-section
319 dimensions increases the distance to failure. This is due to the fact that the augmentation of the
320 shank cross-section dimensions augments the resistance to failure by fatigue and by consequence
321 augments the distance to failure. The Figure 14 allows one determining the shank dimensions
322 according to the required distance to failure, e.g., for $L_f = 10 \times 10^6$ the shank dimensions
323 are $b = 25$ and $h = 65$.

324 To investigate the effect of the percentage of the local tillage forces to the global tillage forces (α)
325 on the expected distance to failure (L_f), the percentage α is plotted against the logarithmic scale
326 of L_f in Figure 15. It is observed that with an increase of α from 0.1 to 0.4, a reduction of
327 of 1.2×10^6 will take place, meaning that the reduction of L_f due to the augmentation of α is
328 very significant. Indeed, the increase of the value of the percentage α augments the amplitudes of
329 the fatigue cycles and as a result reduces the distance to failure (Equations 15, 16 and 17).
330 Therefore, to reduce the L_f values, (by consequence, the values of the local tillage forces) it is
331 recommended to perform the tillage operation when the moisture content is closed to the liquid
332 limits, where the soil conditions became most favorable for soil-working. This can improve
333 significantly the expected distance to failure and by consequence the life time of tillage machines.

334

335

336 **4- Conclusions**

337 A new model to describe the spatial variability in tillage forces and the methods used to calculate
338 the life time of tillage machines are presented in this paper. The proposed model was applied to
339 determine the spatial variability in tillage forces for the shank of a chisel plough and the expected
340 distance to failure was caculated. The expected distance to failure for the shank cross section
341 of $h = 58$ and $b = 32$ was found to be $L = 0.825 \times 10^6$. In addition, different values
342 of h and b could be used to calculate the expected distance to failure. Based on this work the
343 following conclusions can be drawn:

- 344 • The spatial variability in tillage forces can be modelled by only five random parametes
345 ($P_{HG}, P_{VG}, S_1, S_2, \tau$).
- 346 • The spatial variability in tillage forces has singificant effect on the equivalent stress history.
- 347 • The dispersions of mean stress and stress amplitude of the stress cycles are important.
- 348 • The failure by fatigue will occur even the equivalent stress is smaller than the yield stress of
349 the material.
- 350 • The shank cross-section dimensions have a significant impact on the distance to failure
351 ($L = 2.004 \times 10^6$, $L = 9.213 \times 10^6$).
- 352 • The effect of the percentage of the local tillage forces to the global tillage forces (α) on the
353 expected distance to failure (L) is very significant.

354 A further research is needed to investigate the relationships between the global and local tillage
355 forces (P_{HG}, P_{VG}) and the variability in the others parameters (S_1, S_2, τ). In addition, a further
356 study is needed to validate the results achieved in this paper with experiment.

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360

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431 **Tables**

432

433 Table 1: Probabilistic characteristic of soil engineering properties (Abo Al-kheer et al. [11])

Soil engineering properties	Type of distribution	Distribution parameters
Soil density, kN. m	Lognormal	$\mu = 0.13, \sigma = 2.7$
Soil cohesion, kPa	Weibull (2P)	$\lambda = 15.51, k = 1.66$
Internal friction angle, deg	Normal	$\mu = 32, \sigma = 3.96$
Soil-tool friction angle, deg	Weibull (3P)	$\lambda = -64.08, k = 87.14, \theta = 31.52$
Soil-tool adhesion, kPa	Exponential	$\lambda = 0.76$

434 μ and σ are the shape and scale parameters of a lognormal distribution; λ , k and θ are, respectively, the
435 location, scale and shape parameters of a Weibull distribution; μ , σ are, respectively, the location and scale
436 parameters of a normal distribution; λ is the scale parameter of an exponential distribution.

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450 Table 2: Distribution parameters of the model's parameters

Model's parameters	Distribution type	Distribution parameters	
[]	Lognormal	= 0.872,	= 0.449
[]	Lognormal	= 0.004,	= 0.447
[]	Normal	= 10,	= 1
[]	Normal	= 50,	= 5
	Deterministic	= 0.2	

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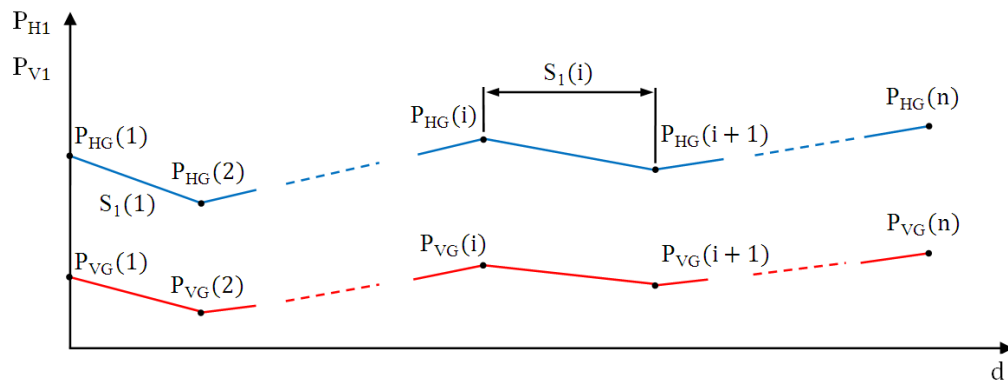
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472 **Figures**

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Figure 1: Illustration of the spatial variability in the global tillage forces

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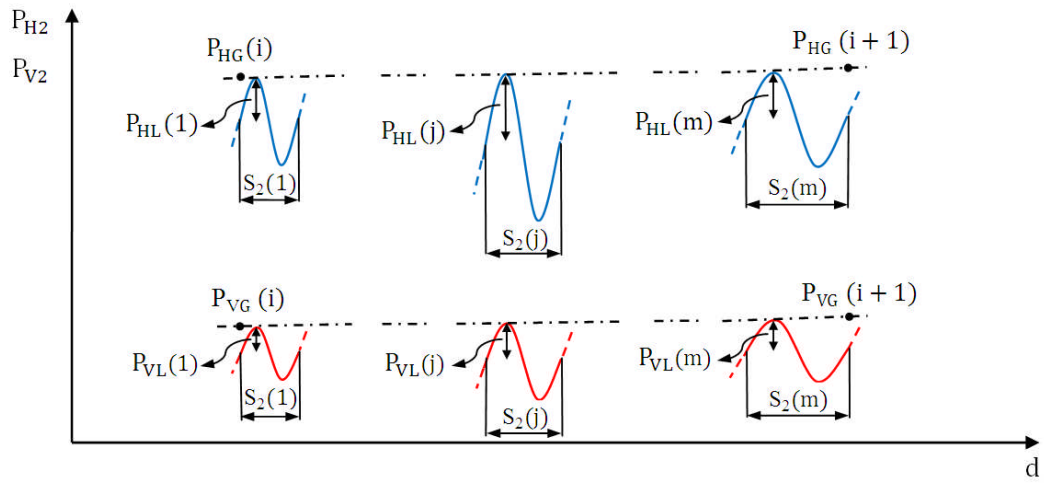
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Figure 2: Illustration of the spatial variability in the local tillage forces

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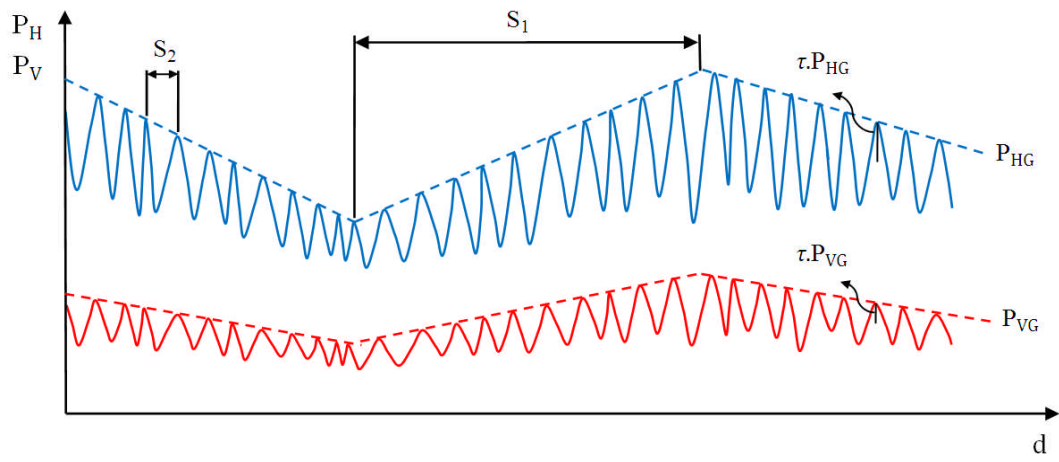
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Figure 3: Illustration of the spatial variability in the horizontal and vertical forces

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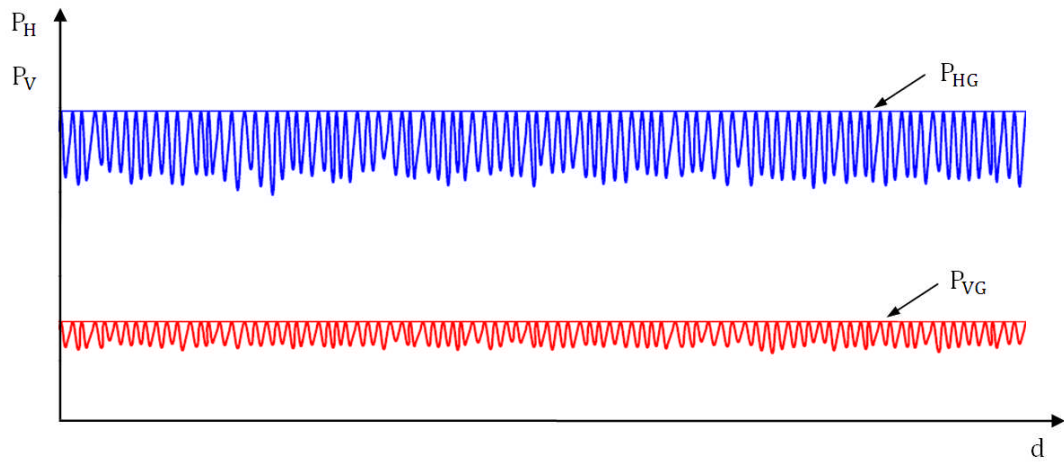
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Figure 4: Spatial variability in tillage forces for constant global tillage forces

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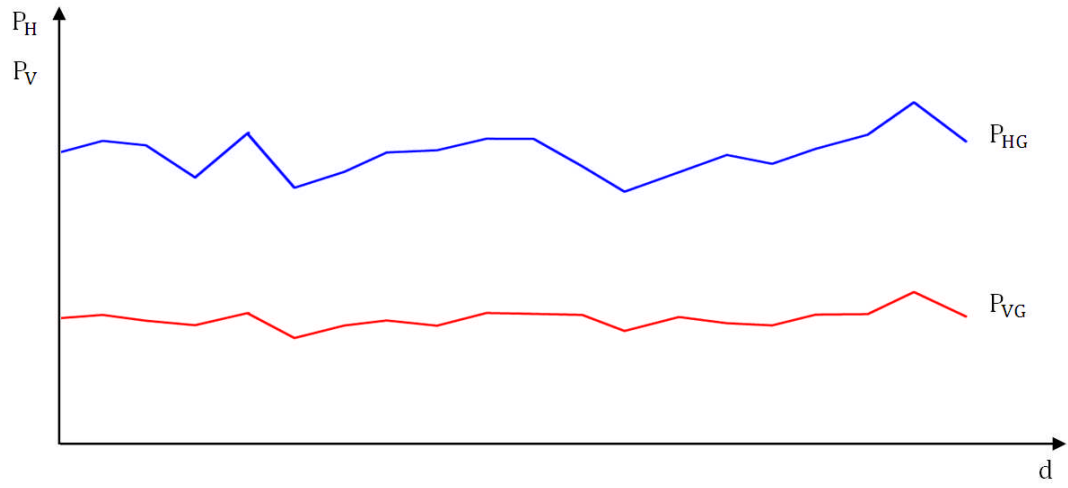
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Figure 5: Spatial variability in tillage forces when omitting local tillage forces

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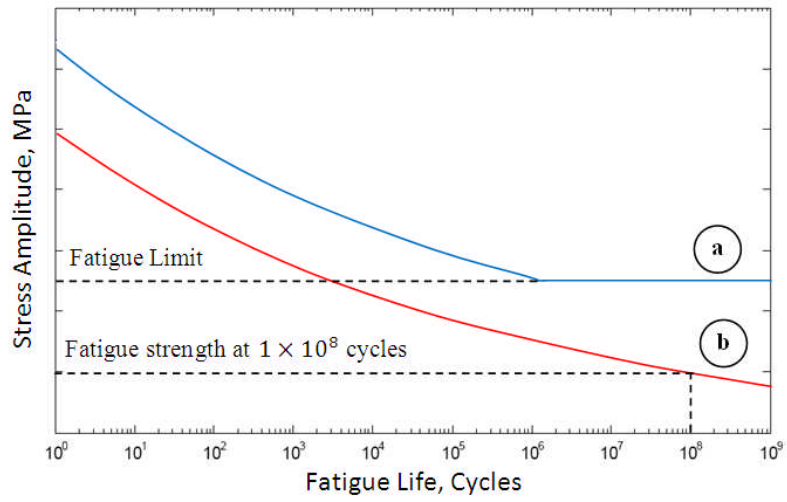
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Figure 6: Typical S-N curves

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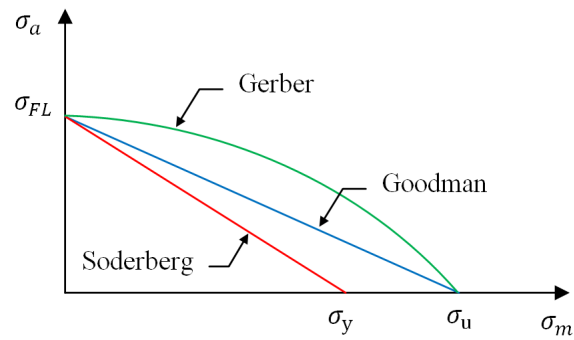
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Figure 7: Comparison of mean stress - stress amplitude relationships

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(where σ_{FL} is the fatigue limit, σ_u is the ultimate strength and σ_y is the yield strength)

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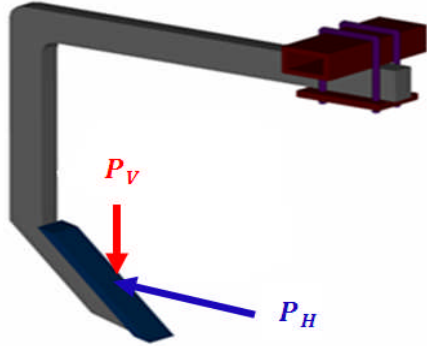
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Figure 8: Illustration of the shank of a chisel plough with tillage forces

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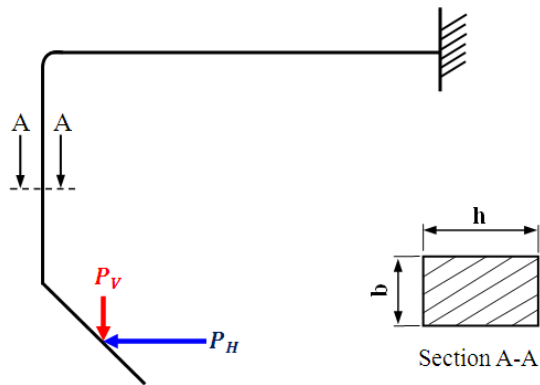
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Figure 9: A schematic drawing of the chisel plough shank ($h = 58$, $b = 32$)

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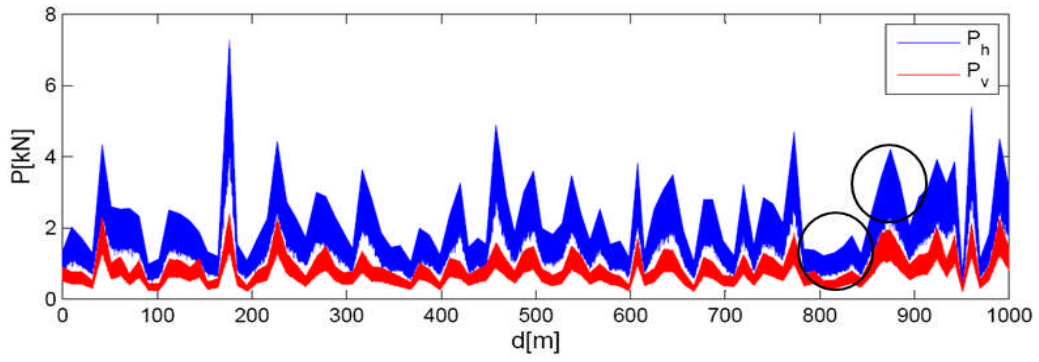
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618 Figure 10: The spatial variability of the horizontal and vertical forces across proposed 1000 m
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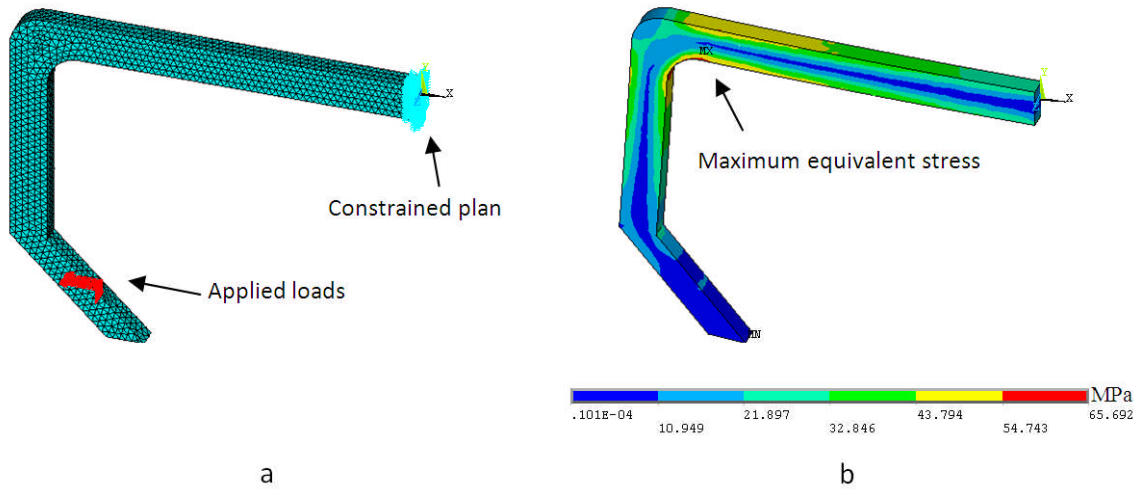
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634 Figure 11: a- Meshed model and boundary conditions; b- Point of maximum equivalent stress

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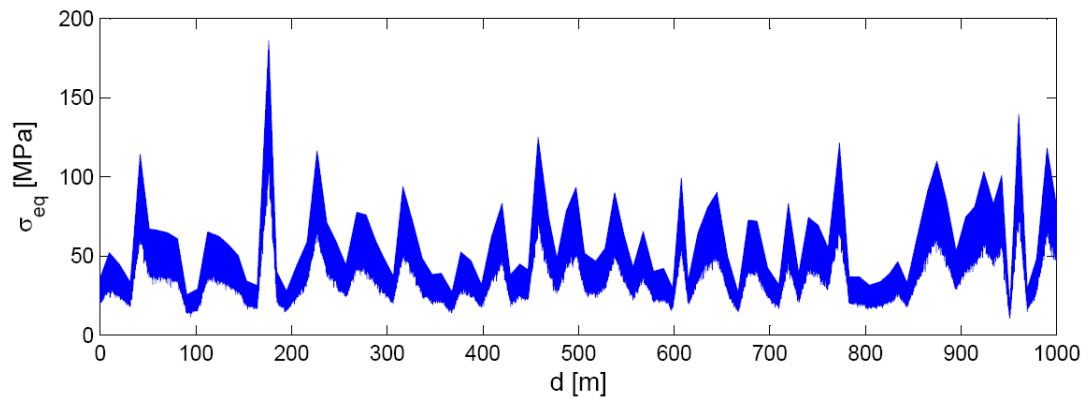
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Figure 12: Equivalent stress history across proposed 1000 m distance

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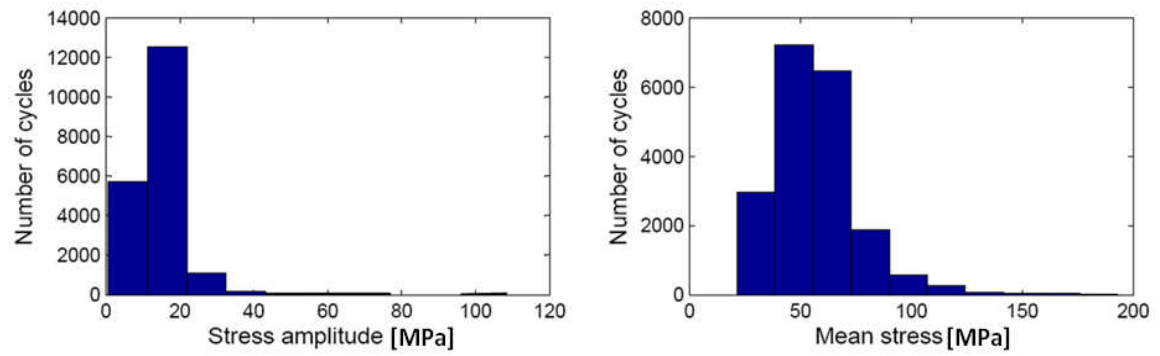
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Figure 13: Histograms of stress amplitude and mean stress

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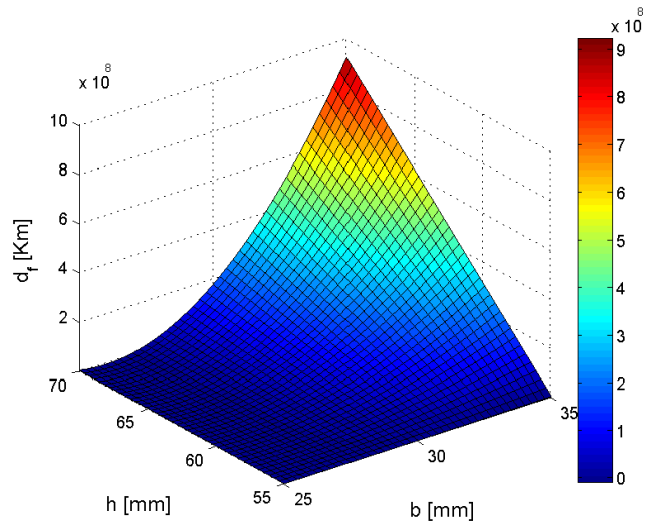
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Figure 14: Distance to failure-Shank dimensions plot

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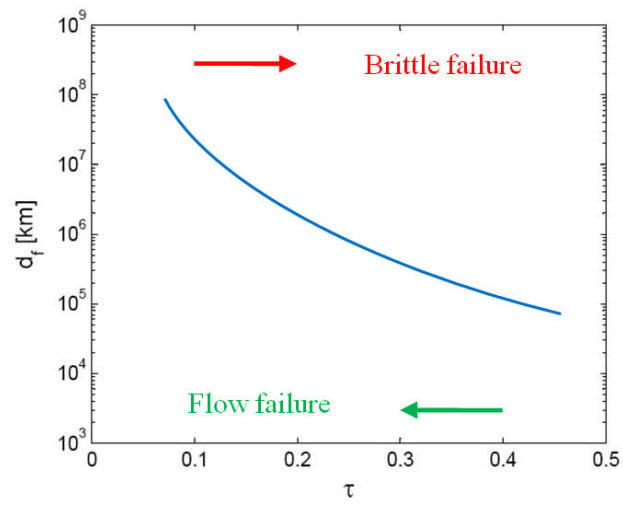
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Figure 15: — relationship

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