A MATHEMATICAL MODEL TO ANALYZE THE STATIC STABILITY OF HYBRID (AERO-HYDRODYNAMICALLY SUPPORTED) VEHICLES

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SUMMARY

Among the new concepts developed for high speed marine vehicles over the last two decades [1], the 'aerodynamic alleviation' [2] approach consists in using an aerodynamic surface to 'alleviate' the weight sustained by the hydrodynamic lift. Such vehicle experiences aerodynamic and hydrodynamic forces of the same order of magnitude, therefore the dynamic models developed for airborne and waterborne vehicles are not suitable.

Considering a vehicle having a high-speed prismatic planing hull and one or more aerodynamic surfaces, the authors propose two mathematical methods. The first one calculates the equilibrium attitude of the vehicle at a given speed and its numerical implementation has been used to undertake a parametric analysis of the influence of some configuration characteristics on performances. The second method analyzes the static stability of the HV. Starting from the dynamic analysis previously proposed by the authors [3], the characteristic polynomial of the HV dynamics is derived and a static stability criterion is proposed.

NOMENCLATURE

a	pitch moment arm of D_F
[A]	aero- and hydrodynamic added mass matrix
a_{ah}	pitch moment arm of D_{ah}
a_{ws}	pitch moment arm of D_{ws}
A_{ij}	$\partial F_i / \partial \ddot{\eta}_j$ (kg), (kg m)
AC_i	aerodynamic center of ith-aerofoil
[B]	aero- and hydrodynamic damping matrix
B_{ij}	$\partial F_i / \partial \dot{\eta}_j$ (kg s^{-1}), (kg s^{-1} m)
c	pitch moment arm of N
$c_{L,ai}$	Lift coefficient of the i-th aerodynamic surface
$c_{D,ai}$	Drag coefficient of the i-th aerodynamic surface
$c_{m,ai}$	Pitch moment coefficient of the i-th aerodynamic surface
[C]	hydrodynamic restoring matrix
C_{ij}	$\partial F_i / \partial \eta_j$ (kg s^{-2}), (kg s^{-2} m)
CG	center of gravity
[D]	aerodynamic WIGe matrix
D_{ah}	planing hull aerodynamic drag
D_{ai}	i-th surface aerodynamic drag
D_F	hydrodynamic friction drag
D_{i0}	$\partial F_i / \partial \eta_0$ (kg s^{-2}), (kg s^{-2} m)
D_{ws}	hydrodynamic whisker spray drag
F_i	surge (i=1), heave (i=3) force or pitch (i=5) moment
HV	Hybrid Vehicle
L_{ai}	i-th surface aerodynamic lift
lcg	CG longitudinal position, from transom

m	mass of the HV at equilibrium (kg)
M_{ai}	i-th surface aerodynamic moment
mac_{ai}	mean aerodynamic chord of the i-th aerofoil
N	hydrodynamic potential force (N)
R/W	total resistance-to-weight ratio
RULM	Rectilinear Uniform Level Motion
S_{ai}	Reference area of the i-th aerodynamic surface
T	thrust force (N)
TP	thrust force point of action
V_0	velocity at equilibrium state (m s^{-1})
vcg	CG vertical position, from the keel
W	weight of the vehicle (N)
WIGe	Wing In Ground effect
wrt	with respect to
β	deadrise angle of the planing hull
ε	angle between the direction of T and the keel
ζ_i	coordinate of the i-th point in the body-fixed axes system, *z* axis
η_0	height above the surface, pos. upward (m)
η_1	surge displacement (m)
η_3	heave displacement, positive downward (m)
η_5	pitch rotation, positive bow up (rad)
$\dot{\eta}_i$	$\partial \eta_i / \partial t$
$\ddot{\eta}_i$	$\partial^2 \eta_i / (\partial t)^2$
η^{ai}	angle between the i-th wing mac and the keel
ρ^i	density (kg m^{-3}), ρ^a air, ρ^h seawater
τ	trim angle (deg), angle between the keel of the planing hull and the waterline
ξ_i	coordinate of the i-th point in the body-fixed axes system, *x* axis

INTRODUCTION

The hybrid vehicle (HV) configuration analyzed here is composed by a high speed prismatic planing hull and one or more wings. In particular, being at low height above the surface, the wing experiences the 'wing in ground effect (WIGe)', a positive effect that influence the lift, drag and moment. Therefore, in order to develop a model which studies the dynamic of the HV, research on dynamic models of prismatic planing hull and WIGe vehicles has been carried out.

1.1 WING IN GROUND EFFECT VEHICLES

Research on WIGe vehicles has mainly been carried out in the former Soviet Union, where they were known as 'Ekranoplans'. The Central Hydrofoil Design Bureau, under the guidance of R. E. Alekseev, developed several test craft and the first line production ekranoplans: Orlyonok and Lun types [4]. In the meantime, several research programs were undertaken in the west to better understand the peculiar dynamics of the vehicles flying in ground effect. Irodov [5] and Rozhdestvensky [6] [7] made important contributions to the development of WIGe vehicles dynamic models. In the 60's and the 70's, Kumar [8], [9] started research in this area in Cranfield University. He carried out several experiments and studied the stability issues of a vehicle flying in ground effect. Staufenbiel [10] in the 80s carried out an extensive work on the influence of the aerodynamic surface characteristics on the longitudinal stability in wing in ground effect. He obtained experimental data with the WIGe vehicle X-114 built by Rhein-Flugzeugbau in Germany in the 70's. The equations of motion for a vehicle flying IGE were defined, including non linear effects. In the USA, Gera [11] used the Staufenbiel's work to investigate the stability of a Russian ekranoplan, using the available data for the F-104 aircraft, a vehicle with an aerodynamic layout similar to the Russian vehicle. The approach is similar to that used for conventional airplane, even if the model is not so accurate (changes of stability derivatives with height not taken into account). Hall [12], in 1994, extended the work of Kumar, modifying the equations of motion of the vehicle flying in ground effect, taking into account the influence of perturbations in pitch on the height above the surface.

1.2 PLANING CRAFT

Research on high speed planing started in the early twentieth century for the design of seaplanes. Later, the research focused on applications to design planing boats and hydrofoil crafts. During the period 1960 to 1990, many experiments have been carried out and new theoretical formulations proposed. Savitsky [13] carried out an extensive experimental program on prismatic planing hulls and developed a mathematical method to estimate the equilibrium attitude of a planing craft. Martin [14] derived a set of equations of motion for the surge, pitch and heave degrees of freedom and demonstrated that surge can be decoupled from heave and pitch motion. The model of Savitsky is still under development, as showed by a recent article of Savitsky, DeLorme and Datla [15]. It is still one of the reference methods used for the preliminary design of planing craft.

1.3 'HYBRID VEHICLES' CONCEPT

In 1976, Shipps [16] analyzed a new kind of race boats, known as 'tunnel hull' race boats. The advantages of this new configuration come from the aerodynamic lift. In 1978, Ward et al. [17] published an article on the design and performance of a ram wing planing craft: the KUDU II. This vehicle, which consists in two planing sponsons separated by a wing section, was able to run at 78 kts (almost 145 km/h), thanks to the aerodynamic lift alleviation. In 1978, Kallio [18] performed comparative tests between the KUDU II and the KAAMA. The KAAMA is a conventional mono hull planing craft. The data obtained during comparative trials showed that the KUDU II pitch motion, in sea state 2, at about 40 to 60 knots, was about 30% to 60% lower than the conventional planing hull KAAMA. In 1997, Doctors [2] proposed a new configuration called 'Ekranocat' for which he mentioned the 'aerodynamic alleviation concept'. The weight of the catamaran was alleviated by aerodynamic lift, thanks to a more streamlined superstructure than in traditional catamarans. The theoretical analysis and computed results showed that a reduction in the total drag of around 50 % can be obtained at very high speed. Russell [19] developed an analysis of aerodynamic and hydrodynamic forces and moments acting on tunnel race boat. This approach is tailored only to the race tunnel boat, therefore cannot be used to design and evaluate a different configuration.

In the following references [16][17][18] some experimental data, theoretical and computed results on vehicles which can be classified as 'HV' are presented, but none of them presents an analysis of the static stability of a vehicle having both hydrodynamic and aerodynamic surfaces.

AXIS SYSTEM

In this work the same axis systems proposed by the author in [3] will be used. Briefly they are: one earth-axis system and two body-axis system, right-handed and orthogonal.

1.4   EARTH-AXIS SYSTEM (*xOz*)

The directions of the axis are fixed in space. The *z*-axis is oriented vertically downward; the *x*-axis forwards and parallel to the undisturbed waterline and the origin is fixed at the undisturbed waterline level.

1.5   BODY AXIS SYSTEMS

The origin O is coincident with the CG of the HV. The *x* and *z* axis lay in the longitudinal plane of symmetry, *x* positive forward and *z* positive downward. Two systems are used:

- <u>aero-hydrodynamic axes</u> ($\eta_1 O \eta_3$), the *x*-axis being parallel to the steady forward velocity V_0,

- <u>geometric axes</u> ($\xi O \varsigma$), the *x*-axis ξ being parallel to the keel of the planing hull.

Aero-hydrodynamic axes are the counterpart of the aerodynamic axes (called also wind or body-wind axes (UK) or stability axes (US) used for airplanes.

CONFIGURATION

The general configuration used to represent the HV has to be intended as modular. The elements presented are all the possible elements that can be taken into account in this analysis:

- a high-speed prismatic planing hull (<u>hydrodynamic surface</u>), with constant deadrise angle β,

- one or more airfoils (<u>aerodynamic surfaces</u>),

- an aero- or hydro-propulsion system.

This analysis focuses on the high speed equilibrium state, but the HV is supposed to have waterborne capability at rest. Hydrodynamic and aerodynamic surfaces can be fitted with control systems, but this work is limited to a control-fixed analysis.

EQUILIBRIUM STATE CALCULATION

This work analyses the static stability of a HV configuration. The stability analysis starts from an equilibrium state and studies how the vehicle reacts after a disturbance. The static stability studies how it reacts immediately after the disturbance whereas the dynamic stability studies if and how the vehicle, eventually, returns back to the initial equilibrium state.

The author proposes a mathematical method that can calculate the equilibrium attitude of the HV, starting from geometric, inertial, aerodynamic and hydrodynamic characteristics of the vehicle.

1.6   HYPOTHESES

The present work concentrates on the analysis of an equilibrium state characterized by a rectilinear trajectory, a constant speed and a constant altitude above the surface, which will be referred as Rectilinear Uniform Level Motion (RULM). The vehicle is supposed to be always in contact with the water, and in a calm water situation. Waves are not taken into account.

1.7   FORCES AND MOMENTS

The forces and moments acting on the vehicle are illustrated in Figure 2. They can be divided in four groups:

- gravitational (weight, W),

- thrust (propulsion force, T),

- aerodynamic (lift, drag and moment from the 1st and 2nd aerodynamic surface, L_{ai}, D_{ai}, M_{ai} and aerodynamic drag of the hull above the surface, D_{ah}),

- hydrodynamic (potential force, N, frictional force, D_F, whisker spray drag, D_{ws}).

1.7.a)   Gravitational force

Since the equilibrium motion analyzed is a level motion, the height above the surface is constant; therefore the direction of the velocity at equilibrium is normal to the weight direction. The weight (W) acts at the center of gravity (CG), which is also the origin of the body-fixed axis system. The coordinates of the CG are (0,0).

1.7.b)   Power force

The thrust can be provided by an aero-propulsion system or a hydro-propulsion system. The thrust acts at the thrust point (TP), in a direction determined by the angle ε, the angle between the direction of the thrust and the keel, positive for an anticlockwise movement (view from the starboard side of the vehicle).

1.7.c) Aerodynamic forces

Aerodynamic forces can be divided in two groups: forces acting on aerofoils and force acting on the portion of the hull above the water.

The aerodynamic force acting at the center of pressure of the aerofoil, in the longitudinal plane, is usually represented with an equivalent system of two forces and one moment acting at the aerodynamic center of the aerofoil (AC): lift (L), defined as perpendicular to the velocity, drag (D), defined as parallel to the velocity, and a pitch moment (M), positive for a bow up movement.

$$L_i = \frac{1}{2}\rho_a V_0^2 S_{ai} c_{L,ai}$$

$$D_i = \frac{1}{2}\rho_a V_0^2 S_{ai} c_{D,ai}$$

$$M_i = \frac{1}{2}\rho_a V_0^2 S_{ai} c_{m,ai} mac_{ai}$$

$$(0)$$

Usually the aerodynamic coefficients $c_{L,ai}$, $c_{D,ai}$ and $c_{m,ai}$, are functions only of the angle of attack, but in this analysis, also the influence of the height above the surface has to be taken into account. The angle of attack is the sum of the trim angle and η_{ai}, which is the angle between the chord of the wing and the keel of the hull. The height above the surface to evaluate the aerodynamic coefficients is that of the aerodynamic center.

The dry section of the hull experiences an aerodynamic drag force (D_{ah}). To evaluate its contribution, Savitsky [15] proposes the expression:

$$D_{ah} = \frac{1}{2}\rho_a V_0^2 A_h c_{D,ah}$$

$$(0)$$

where

- A_h is the frontal area of the planing hull,
- $C_{D,ah}$ is the aerodynamic drag coefficient of the hull (approximated as 0.70).

Since it is not known where the hull aerodynamic drag acts, D_{ah} is supposed to be acting on the CG. Therefore no moment is generated by this force.

1.7.d) Hydrodynamic forces

Referring to the work developed by Savitsky et al. from 1964 through 2007 [13], [15], the hydrodynamic forces are:

- potential force N,
- friction force D_F,
- whisker spray drag D_{WS}.

The potential force direction is supposed to be normal to the keel and acting at the hydrodynamic center HC. The friction force acts parallel to the keel line, half-height between the keel and the chine line. The whisker spray drag D_{WS} has been analyzed in particular in [15]. Also D_{WS} is assumed to act through the CG of the HV.

1.8 SYSTEM OF EQUATIONS OF EQUILIBRIUM

Once all the forces and moments are known, a system of equations of equilibrium can be developed. The vehicle, in the longitudinal plane, has three degrees of freedom, and a system of three equations of equilibrium is needed. The system is:

- surge equation: sum of the vertical forces = 0,
- heave equation: sum of horizontal forces = 0,
- pitch equation: sum of pitch moments = 0.

The CG of the HV has been chosen to be the point of reference for the moments.

1.8.a) Surge Equation

It states that the sum of the aerodynamic drags, the component of potential and friction hydrodynamic forces parallel to the velocity, and the whisker spray drag has to be equal to the component of the thrust parallel to the velocity.

$$-D_{a1} - D_{a2} - D_{ah} +$$
$$- N\sin(\tau) - D_F \cos(\tau) - D_{WS} +$$
$$+ T\cos(\tau + \varepsilon) = 0$$

$$(0)$$

1.8.b) Heave Equation

The sum of aerodynamic lift, vertical components of the potential and friction hydrodynamic force and the vertical component of the thrust has to be equal to the weight of the HV:

$$L_{a1} + L_{a2} + N\cos(\tau) - D_F\sin(\tau) +$$
$$- W + T\sin(\tau + \varepsilon) = 0$$

(0)

1.8.c)   Pitch Moment Equation

The sum of the aerodynamic moments, hydrodynamic moments and the moment generated by the thrust force has to be equal to zero.

$$L_{a1}\left[\xi_{AC1}\cos(\tau) + \zeta_{AC1}\sin(\tau)\right] +$$
$$+ D_{a1}\left[\xi_{AC1}\cos(\tau) + \zeta_{AC1}\sin(\tau)\right] + M_{a1} +$$
$$+ L_{a2}\left[\xi_{AC2}\cos(\tau) + \zeta_{AC2}\sin(\tau)\right] +$$
$$+ D_{a2}\left[\xi_{AC2}\cos(\tau) + \zeta_{AC2}\sin(\tau)\right] + M_{a2} +$$
$$D_{ah}a_{ah} + D_{ws}a_{ws} - N\cdot c - D_F\cdot a +$$
$$T\left[\xi_{TP}\sin(\varepsilon) + \zeta_{TP}\cos(\varepsilon)\right] = 0$$

(0)

with the conditions

$$a_{ah} = a_{ws} = 0$$

1.9   SOLUTION OF THE EQUATIONS OF EQUILIBRIUM

The method to solve the system of the three equations of equilibrium is a modified version of the 'Savitsky long-form method' illustrated in [20].

In the Savitsky method, the weight is sustained by hydrostatic and hydrodynamic forces, while in this work the weight is sustained by both aerodynamic and hydrostatic-hydrodynamic forces. Obviously, as it can be seen in equations (0), (0) and (0), the aerodynamic drag and moments also have to be taken into account.

In the Savitsky method, the trim angle is not known at the start, therefore a trim angle has to be assumed and, through a cycle, the right trim angle that fulfill equations (0), (0) and (0) is eventually found.

In this work an additional assumption has to be taken, since the aerodynamic forces depend on:

* trim angle (τ) (the angle of attack is the sum of the trim angle τ and the angle between the mac and the keel η),

* height above the surface.

This leads to a <u>trim angle (τ) cycle</u> nested into the <u>height above the surface (h) cycle</u>, as illustrated in Figure 3. Assuming a value for the height above the

surface of the CG (h_i) and a trim angle τ_i, the aerodynamic forces can be calculated. Then the weight sustained by the hydrodynamic forces is equal to the difference between the total weight and the sum of the aerodynamic lifts. At this point the 'long-form method' of Savitsky can be followed, taking into account also aerodynamic drags and moments, and the equilibrium trim angle can be derived. The height above the surface of the vehicle h_{i+1} can then be calculated. If this h_{i+1} is equal to the h_i assumed, then the equilibrium attitude of the vehicle has been found. If not, a new h cycle is performed.

1.9.a)   Validation of the mathematical model

As far as the authors are aware, no experimental data on HV trials are available in the public domain, but the model proposed can analyze also planing craft configurations.

Analyzing the configuration B of Table 1, the comparison between the data obtained by the authors and the results presented in [15] are presented in Figure 4. The data are in good agreement through the whole speed range, both for the trim angle and for the resistance to weight ratio.

STATIC STABILITY

Analyzing the forces and moments under the small disturbances hypothesis, the static stability of the HV is derived using the Routh-Hurwitz criterion.

1.10   ROUTH-HURWITZ CRITERION

In general, given a polynomial equation in s,

$$A_n s^n + A_{n-1} s^{n-1} + \ldots + A_1 s + A_0 = 0$$

(0)

the Routh-Hurwitz method determines how many roots will have positive real parts. If the polynomial equation is the characteristic polynomial of the dynamics of a vehicle, this method can be used to determine the stability of the vehicle. In fact, if all the roots have a negative real part, the system is stable, statically and dynamically.

Furthermore, Staufenbiel [10] showed how the last coefficient of the polynomial (A_0) can be used to determine the static stability of the system. If the condition

$$A_0 / A_n > 0$$

(0)

is fulfilled, then the system can be considered statically stable.

1.11 HV CHARACTERISTIC POLYNOMIAL AND STATIC STABILITY CONDITION

In [3], the authors developed a mathematical model to study the longitudinal dynamics of the HV. A system of ordinary differential equations of motion was derived for the longitudinal plane in the frame of small-disturbance stability theory. The same mathematical model has been adopted for this work.

1.11.a) Complete Order System

By defining a state space vector $\underline{\upsilon}$ as:

$$\underline{\upsilon} = \begin{bmatrix} \dot{\eta}_1 & \dot{\eta}_3 & \dot{\eta}_5 & \eta_3 & \eta_5 & \eta_0 \end{bmatrix}$$

(0)

the system of equations of motion can be rearranged in the Cauchy standard form (or state-space form). The characteristic polynomial of the complete order system can be derived:

$$A_6 s^6 + A_5 s^5 + A_4 s^4 + A_3 s^3 +$$
$$+ A_2 s^2 + A_1 s + A_0 = 0$$

(0)

With $A_6 = 1$, the static stability is assured when

$$A_0 = num_0 / \Delta > 0$$

(0)

where num_0 is equal to

$$V_0 \left[D_{10} \left(B_{31} C_{53} - B_{51} C_{33} \right) - B_{11} \left(C_{35} D_{50} - C_{53} D_{30} \right) \right]$$

(0)

and Δ is equal to

$$\left(I_{55} + A_{55} \right) \left[m^2 + m \left(A_{11} + A_{33} \right) + A_{11} A_{33} - A_{31} A_{13} \right] +$$
$$- \left(m + A_{11} \right) A_{53} A_{35} - \left(m + A_{33} \right) A_{51} A_{15} +$$
$$+ A_{53} A_{31} A_{15} + A_{51} A_{13} A_{35}$$

(0)

1.11.b) Reduced Order System

This mathematical method requires to be validated against experimental data. Unfortunately, no experimental data on the static stability of a HV configuration is available in the public domain.

To plan such experiments, it is necessary to have a physical insight of the condition stated in equation (0). This condition, applied to the complete order system (0), is relatively complex. Assuming that the surge degree of freedom (η_1) can be decoupled from heave (η_3) and (η_5) pitch degrees of freedom, a simplified version of the condition (0) can be obtained, leading to a better physical insight.

In [3] the author derived a mathematical model of the dynamics of the HV starting from the systems of equations of motion of WIGe vehicles and planing craft. As regard the dynamics of a planing craft, Martin [14] demonstrated that the surge motion can be decoupled from the heave and pitch motion. For the dynamics of WIGe vehicles, Rozhdestvensky [6] proposed a reduced order system where the surge motion is decoupled from heave and pitch motion. This hypothesis has been confirmed by, among others, Delhaye [21].

By defining the reduced order state space vector $\underline{\upsilon}$ as

$$\underline{\upsilon} = \begin{bmatrix} \dot{\eta}_3 & \dot{\eta}_5 & \eta_3 & \eta_5 & \eta_0 \end{bmatrix}$$

(0)

the Cauchy standard form (or state-space form) of the reduced order system is obtained. The characteristic polynomial can be derived:

$$A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0$$

(0)

With $A_5 = 1$, the static stability is assured when

$$A_0 = \frac{V_0 \left(C_{33} D_{50} - C_{53} D_{30} \right)}{\left(A_{55} + I_{55} \right) \left(m + A_{33} \right) - A_{53} A_{35}} > 0$$

(0)

1.12 REDUCED ORDER STATIC STABILITY: PHYSICAL INSIGHT

Each coefficient in equation (0) is the derivative with respect to:

- accelerations (A_{ij}),

- heave position(C_{ij})

- height above the surface (D_{ij})

of the sum of aerodynamic and hydrodynamic forces (and moments). Referring to [3], remembering that the superscript 'a' stands for aerodynamic and 'h' for hydrodynamic and that Z is the heave force (positive

downward) and M the pitch moment (positive bow up), the coefficients are equal to:

$$A_{33} = A_{33}^a + A_{33}^h = -Z_{\dot{\eta}\,3}^a - Z_{\dot{\eta}\,3}^h$$

$$A_{35} = A_{35}^a + A_{35}^h = -Z_{\dot{\eta}\,5}^a - Z_{\dot{\eta}\,5}^h$$

$$A_{53} = A_{53}^a + A_{53}^h = -M_{\dot{\eta}\,3}^a - M_{\dot{\eta}\,3}^h$$

$$A_{55} = A_{55}^a + A_{55}^h = -M_{\dot{\eta}\,5}^a - M_{\dot{\eta}\,5}^h$$

(0)

$$C_{33} = C_{33}^h = -Z_{\eta\,3}^h, C_{33}^a = 0$$

$$C_{53} = C_{53}^h = -M_{\eta\,3}^h, C_{53}^a = 0$$

$$D_{30} = D_{30}^a = -Z_{\eta\,0}^a, D_{30}^h = 0$$

$$D_{50} = D_{50}^a = -M_{\eta\,0}^a, D_{50}^h = 0$$

(0)

The aerodynamic derivatives can be estimated with, for example, [21][22] and the hydrodynamic derivatives with the approaches presented in [14] or [23]. Using these expressions for the configuration presented in section 1.5 we have

$$(A_{55} + I_{55})(m + A_{33}) - A_{53}A_{35} > 0$$

(0)

Therefore the static stability condition of the reduced order becomes

$$\frac{D_{50}}{D_{30}} - \frac{C_{53}}{C_{33}} > 0$$

(0)

1.12.a)   Similarity with WIGe vehicles

A parallel with WIGe vehicles static stability criteria is illustrated. The static stability condition derived by Staufenbiel [10] and Irodov [5] is:

$$M_w / Z_w - M_h / Z_h < 0$$

(0)

where M_w and M_h are the derivatives of pitch moment with respect to the heave velocity and the height above the surface, Z_w and Z_h are the heave force same derivatives (M_h/Z_h is the same as D_{50}/D_{30}). M_w/Z_w is defined also as the aerodynamic center of pitch and M_h/Z_h as the aerodynamic center in height. Remembering positive abscissa means ahead the CG, the condition (0) can be expressed as in [7]:

"the (aerodynamic) center in height should be located upstream of the (aerodynamic) center in pitch"

Dividing the lift due to a variation of the pitch angle (L_{alpha}) from the lift due to a variation of the height above the surface (L_{height}), condition (0) states the point of action of force L_{height} should be located upstream the point of action of force L_{alpha}.

1.12.b)   HV Static Stability Criterion (reduced order)

For the HV, using the expressions (0), the static stability condition $\frac{D_{50}}{D_{30}} - \frac{C_{53}}{C_{33}} > 0$ (0) is:

$$M_{\eta\,0}^a / Z_{\eta\,0}^a - M_{\eta\,3}^h / Z_{\eta\,3}^h > 0$$

(0)

The first term $M_{\eta 0}^a/Z_{\eta 0}^a$ is the analogue of the aerodynamic center in height of WIGe vehicles. The author proposes for the second term the name 'hydrodynamic center in heave', so that the condition (0) can be also expressed as:

"the hydrodynamic center in heave should be located downstream of the aerodynamic center in height"

As before, dividing the hydrodynamic lift due to a heave variation (L_{hyd}) from the lift due to a variation of the height above the surface (L_{height}), the point of action of L_{height} should be located upstream the point of action of L_{hyd}.

PARAMETRIC ANALYSIS

A parametric analysis is required in the preliminary design phase of a new vehicle. Using the method illustrated in section 1.9, the influence of some configuration parameters on the resistance to weight ratio (R/W) of the HV is investigated.

The Resistance-to-Weight ratio is defined as:

$$R/W = \frac{Aer.Drag + Hyd.Drag}{Weight}$$

1.13   PARAMETERS

Considering the hybrid configuration of the HV, many parameters have to be taken into account (Figure 1). The aerodynamic surfaces parameters are:

- aerodynamic profile type,

- mean aerodynamic chord (mac),

- surface area,

- angle between the keel and the mac (η),
- position of the wing relative to the hull.

The profile shape determines the aerodynamic coefficients; therefore the choice of the profile is very important. The investigation of the optimum profile for a given configuration is beyond the scope of this work, since it requires a thorough analysis of the available profiles and CFD simulations. The author has chosen a modified Glenn Martin 21 section. This profile has a very poor efficiency (lift-to-drag ratio), but it has been adopted only to have experimental validated values, presented in [24]. The influence of the length of the mean aerodynamic chord (and the surface area) has been analyzed in section 1.13.a, the influence of η in section 1.13.b.

As regard the planing hull the parameters are:

- beam length,
- deadrise angle β.

To have a direct comparison with a planing craft configuration (no wing), one of the configurations tested by Savitsky in [15] has been adopted: its characteristics are presented in Table 1, Case B. Its beam and deadrise angle have been used also for the HV configurations of these analysis.

Some important characteristics of the vehicle are also:

- mass,
- longitudinal position of the CG,
- vertical position of the CG.

The mass of the vehicle in all the analysis has been kept constant: the author supposes that the aerodynamic surface or surfaces can be exploited to, for example, carry part of the fuel (like airplanes) or other systems of the vehicle.

The vertical position of the vehicle has been kept fixed, since its position is designed to fulfill the hydrostatic stability criteria. An analysis of the influence of the longitudinal position of the CG (lcg) is presented.

1.13.a)   Wing surface area analysis

Three configurations have been analyzed: A, B and C. They are identical unless for the mac length:

- config. B, mac = 0 meter (no wing)
- config. A, mac = 14.14 meters,

- config. C, mac = 20 meters.

In Figure 5, the resistance to weight ratio curves of the three configurations are presented. The speed range can be divided in two zones by the speed at which the curves cross each other (about Froude number 2.9, 40 knots), called V_X. In the speed range $V_0 < V_X$, the total drag of B (planing craft) is lower than the total resistance of A and C. For $V_0 > V_X$, the HV with wing experiences less drag.

This is because at low speed the hydrodynamic forces experienced by the HV and the planing craft are almost the same, since the aerodynamic forces are still low compared to hydrodynamic forces. Nonetheless the aerodynamic forces lead to an increase of the trim angle, therefore the horizontal component of the hydrodynamic potential force (hydrodynamic potential drag) acting on the HV is bigger (in module) than the same component acting on the HV without any wing. The vehicle with wing/s experiences a higher hydrodynamic drag.

As the speed increases, the aerodynamic forces grow and the required hydrodynamic lift force becomes lower and lower and, at the same time, the hydrodynamic drag diminishes. Therefore, also if the trim angle of the HV with wing is still bigger than the trim angle of the HV without wing, at a certain speed the total drag experienced by the HV with wing is lower than the total drag experienced by the HV without wing.

If the requirement of the vehicle is to reach a speed $V > V_X$, the configurations with wing will require a power propulsion lower than the planing craft. Remembering that the profile used has a low efficiency, with a more efficient profile the speed V_X can be lowered and the power propulsion can be further reduced.

1.13.b)   Angle between the mac and the keel (η) analysis

The configurations analyzed are A, D and E. In order:

- η_D = 0 degrees,
- η_E = 5 degrees,
- η_A = 10 degrees.

The resistance to weight ratio of the three configurations is presented in Figure 6. The curves are similar to that of Figure 5. This is because the aerodynamic lift is increased both if the mac length (wing area) is increased and if the angle of attack is

increased (augmenting η the angle of attack is augmented). Therefore the same physical insight proposed for the analysis of section 1.13.b applies here. The behavior is less accentuated because, while in Figure 5 there is a configuration without wing, in this analysis all the three configurations have a wing.

1.13.c) Longitudinal position of the CG (lcg) analysis

The configurations analyzed are A, F and G and the resistance to weight ratio is represented in Figure 7.

- $lcg_F = lcg_A*0.85$ (-15%),

- $lcg_A = 8.656$ meters,

- $lcg_G = lcg_A*1.15$ (+15%).

In order to keep the position of the aerodynamic center fixed, the coordinates have been changed accordingly (since the point of origin is the CG).

As it can be seen, lcg seems to have the strongest influence on the performance of the HV. The position of the CG strongly influences the trim angle of the HV, and a rearward shift of the CG leads, at equal speed, to a bigger trim angle variation than the increase of the wing area or the increase of angle η. Comparing the trim angle of configuration A with configuration F:

- $Fn_B= 1$, $\tau_F = \tau_A + 1.5$ deg,

- $Fn_B=1.8$, $\tau_F = \tau_A + 2$ deg,

- $Fn_B=3.5$, $\tau_F = \tau_A + 0.5$ deg.

As explained in section 1.13.a and 1.13.b, an increase of the trim angle corresponds to an increase of the angle of attack of the wing, therefore to an increase of the aerodynamic lift. Since the increase of the trim angle, shifting the CG rearward, is bigger than increasing the wing surface S or the angle η of the same percentage, the positive effect on the resistance to weight ratio is enhanced. Vice versa, the forward shift of the CG has a negative effect.

CONCLUSIONS

The authors developed a mathematical framework to calculate the performance of a hybrid vehicle, a vehicle having a prismatic planing hull and one or more wing(s).

Using the mathematical model developed to estimate the HV equilibrium attitude across a range of speed, a parametric analysis has been conducted. The results are that:

- diminishing the longitudinal distance between the transom of the hull and the CG or

- increasing the surface of the wing (S) or

- increasing the angle between the wing mean aerodynamic chord and the keel (η)

lead to the resistance to weight ratio of the HV to diminish. In particular, the positive effect of shifting the CG rearward is more significant than increasing the surface of the wing. The increase of η angle has the smallest positive effect.

Finally, following the approach of Irodov [5] and Staufenbiel [10] for WIGe vehicles, a criterion to estimate the static stability of HV has been developed.

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Maurizio Collu GMRINA graduated in Aerospace Engineering (mark 100/100) in Milan, Italy. He then joined Cranfield University and the Offshore Engineering & Naval Architecture Group as a PhD researcher under the supervision of Prof. Minoo Patel. The project is a preliminary study on Marine Unmanned Aerial Vehicles (MUAVs) for British Aerospace. He is responsible for the study of the dynamics of this HV, a hybrid vehicle between a wing in ground effect vehicle and a high speed marine vehicle. He presented a paper on the dynamic stability of HV at the *International Conference on Marine Research and Transportation 2007*.

Prof. Minoo Patel   FREng, BSc, PhD, CEng, FIMechE,   FRINA,   Hon   RCNC After graduation from a PhD in 1973, Minoo Patel started his career as an aerodynamicist and subsequently worked on the determination of aerodynamic gust loads on aircraft wings before opting for a change of career to offshore mechanics. He established a large group working in this field and has an output of over 110 research papers, 2 books and 8 Patents. His current research and work for industry is on combined air and water borne high speed vehicles and on aspects of maritime Unmanned Air and Surface Vehicles.

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VEHICLE CHARACTERISTICS		BASIC CONFIG.	COMPARISON CONFIGURATIONS					
			Wing area analysis		η analysis		lcg analysis	
Geometry	unit	conf. A	conf. B	conf. C	conf. D	conf. E	conf. F	conf. G
Propulsion								
(ξ, ζ) TP	[m]	(0, 0) m						
ε [deg]	[deg]	12 deg						
Aerodynamic surface (one wing)								
mac	[m]	14.1	**0**	**20**	14.1		14.1	
S	[m²]	200	**0**	**400**	200		200	
η	[deg]	10	\	10	**0**	**5**	10	
(ξ, ζ) AC₁	[m]	(20, 0)	\	(20, 0)	(20, 0)		(21.3,0)	(18.7,0)
profile	\	G.M. 21	\	G. M. 21	G.M. 21		G.M. 21	
Hydrodynamic surface (prismatic planing hull)								
beam	[m]	5.547 m						
β (deadrise)	[deg]	14 deg						
A_h (frontal area)	[m²]	20.067 m²						
Inertial								
lcg (from transom)	[m]	8.656					**7.358** (-15%)	**9.954** (+15 %)
vcg (from keel)	[m]	1.387						
mass	[kg]	52160 kg						

Table 1: Characteristics of the analyzed configurations
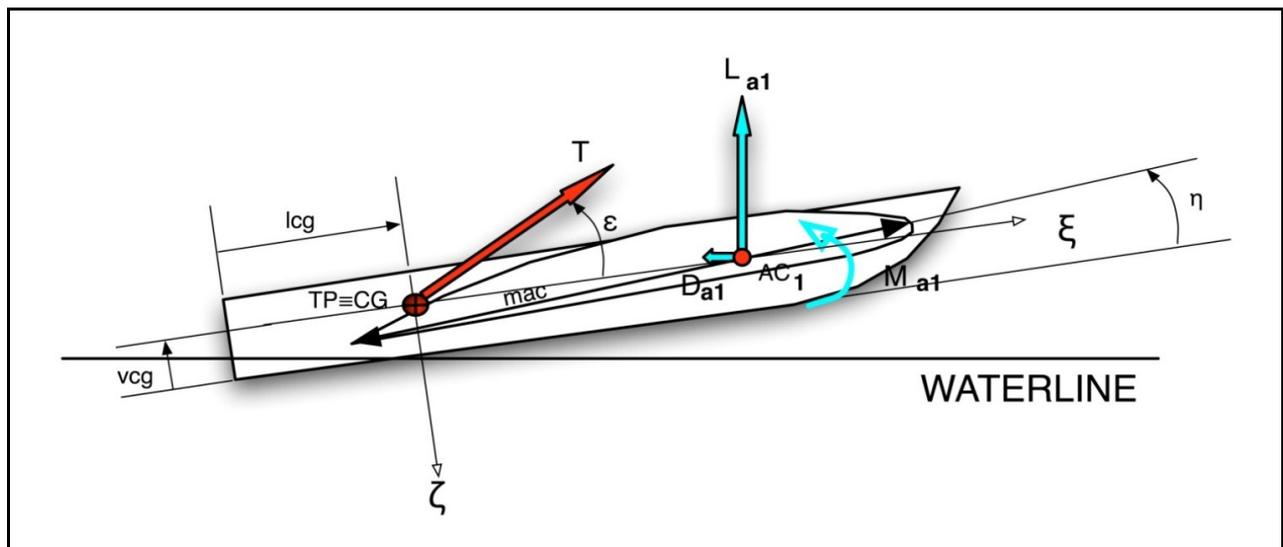

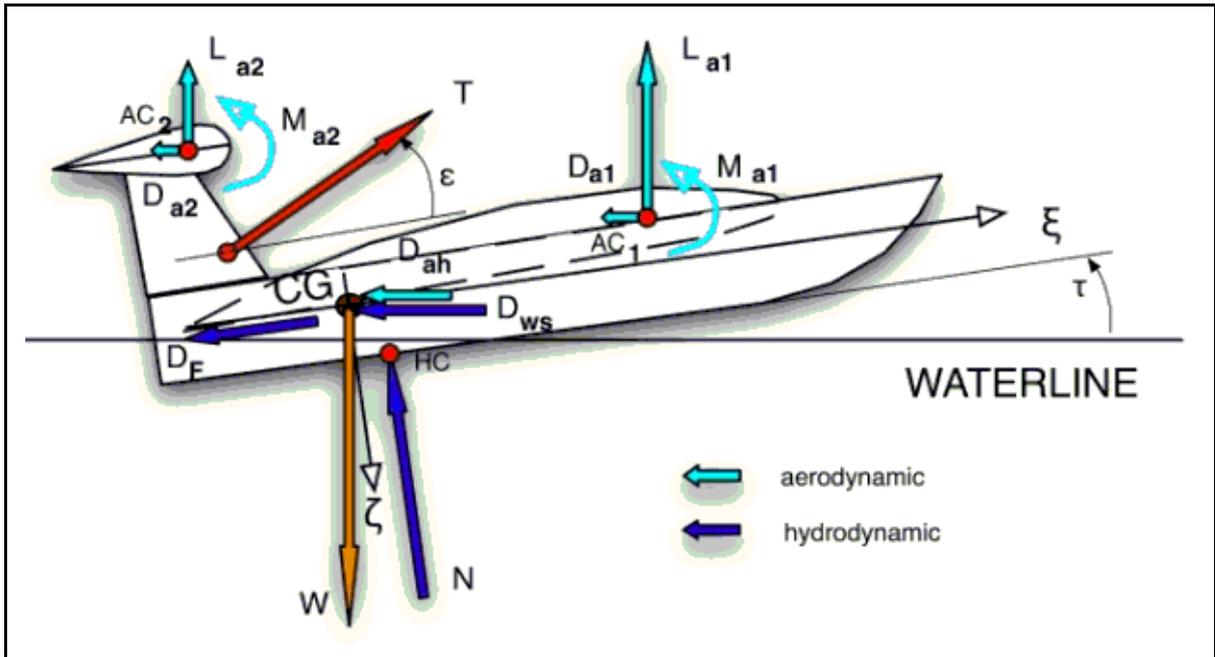**Figure 1: characteristics of the analyzed configurations**

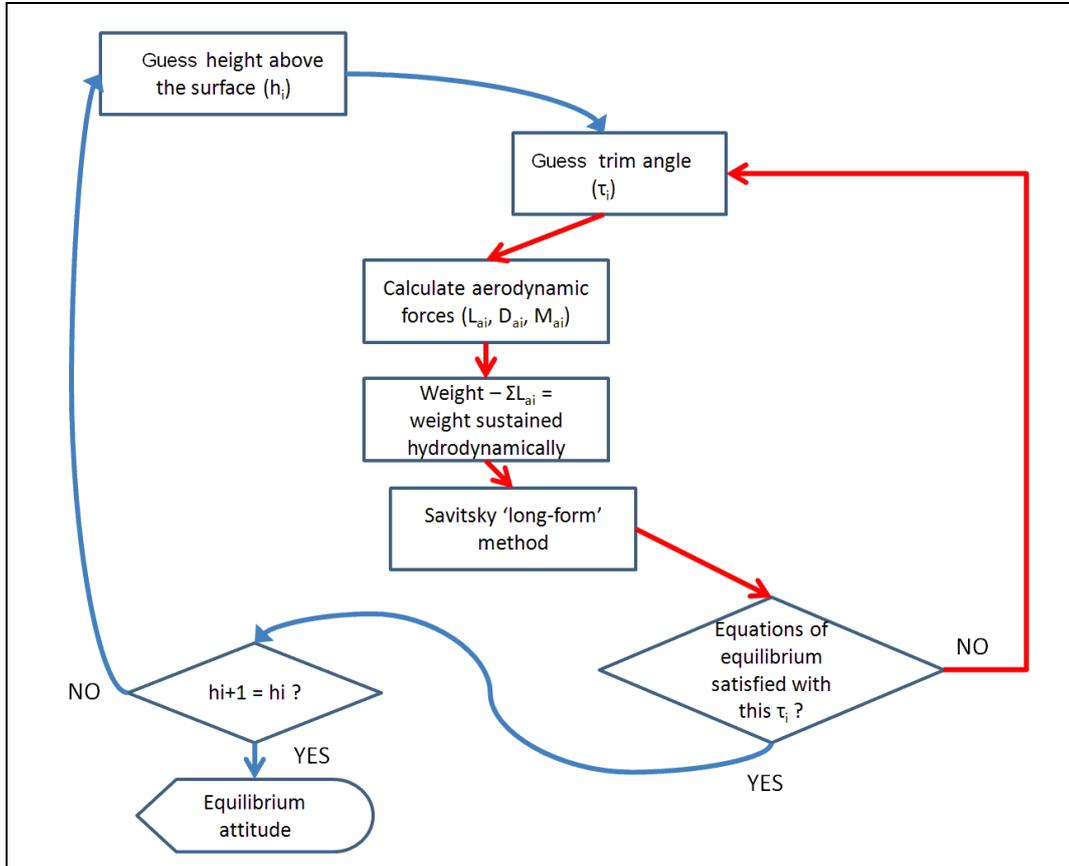**Figure 2: forces and moments acting on the hybrid vehicle**

Figure 3: flow chart of the method to find the equilibrium attitude of the HV
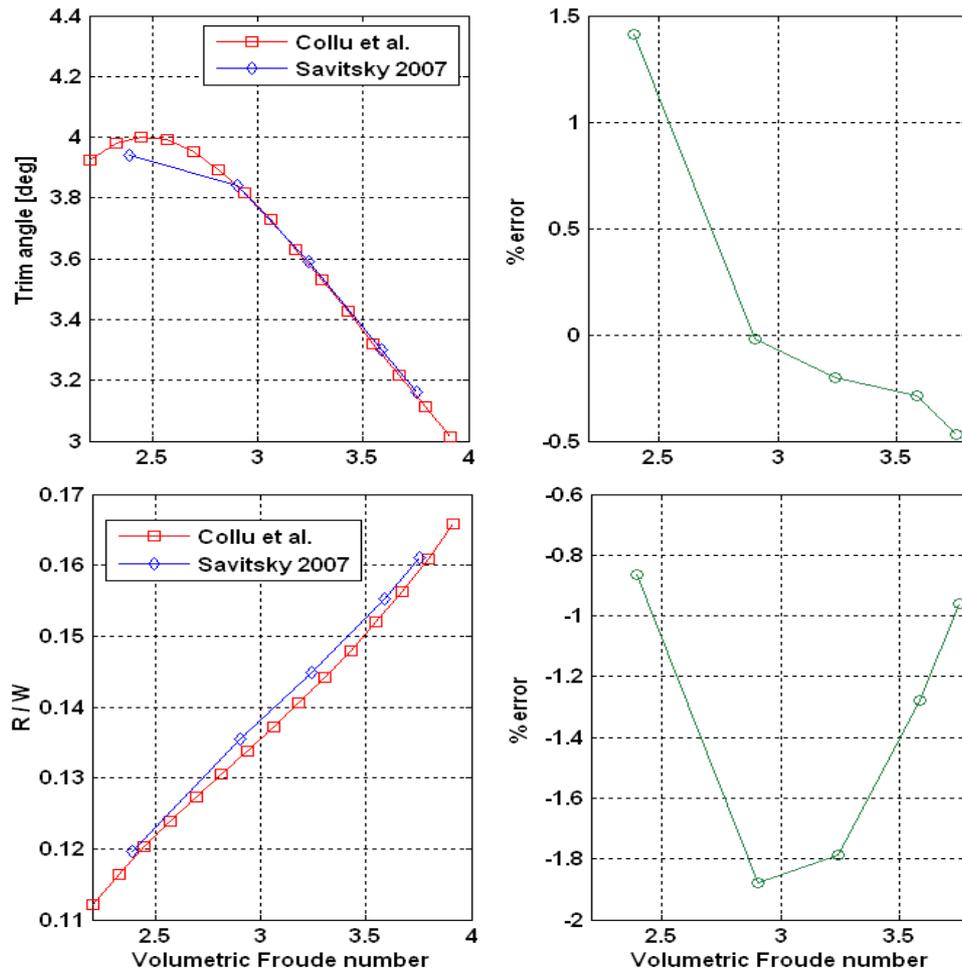
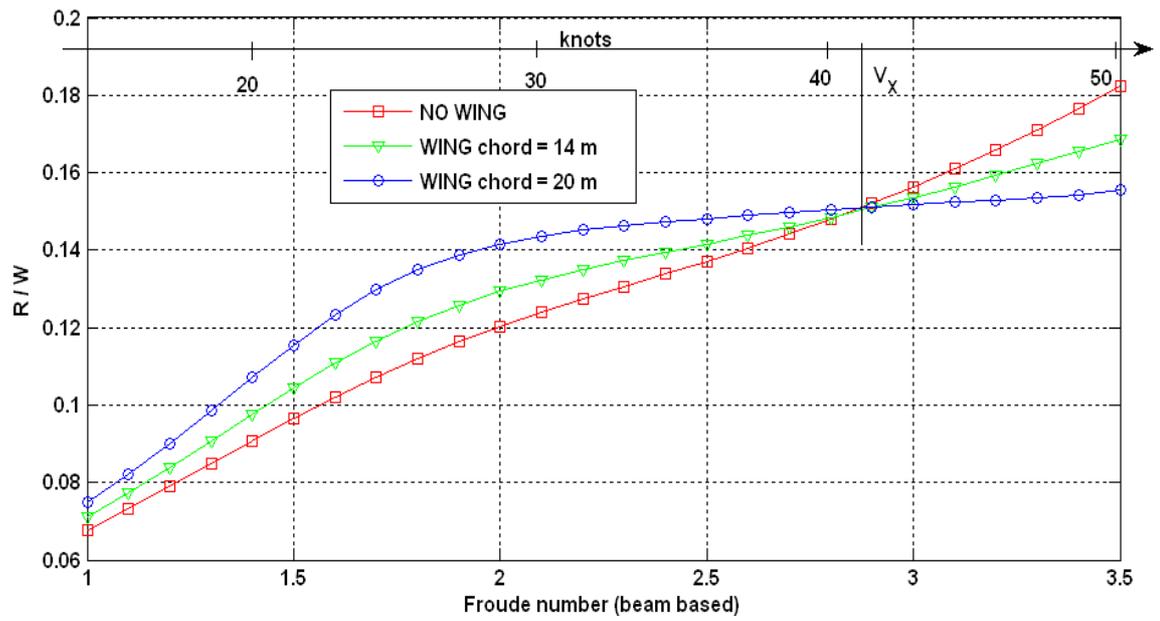Figure 4: Savitsky 2007 (15) vs. Collu et al. – Trim angle and Resistance-to-Weight ratio

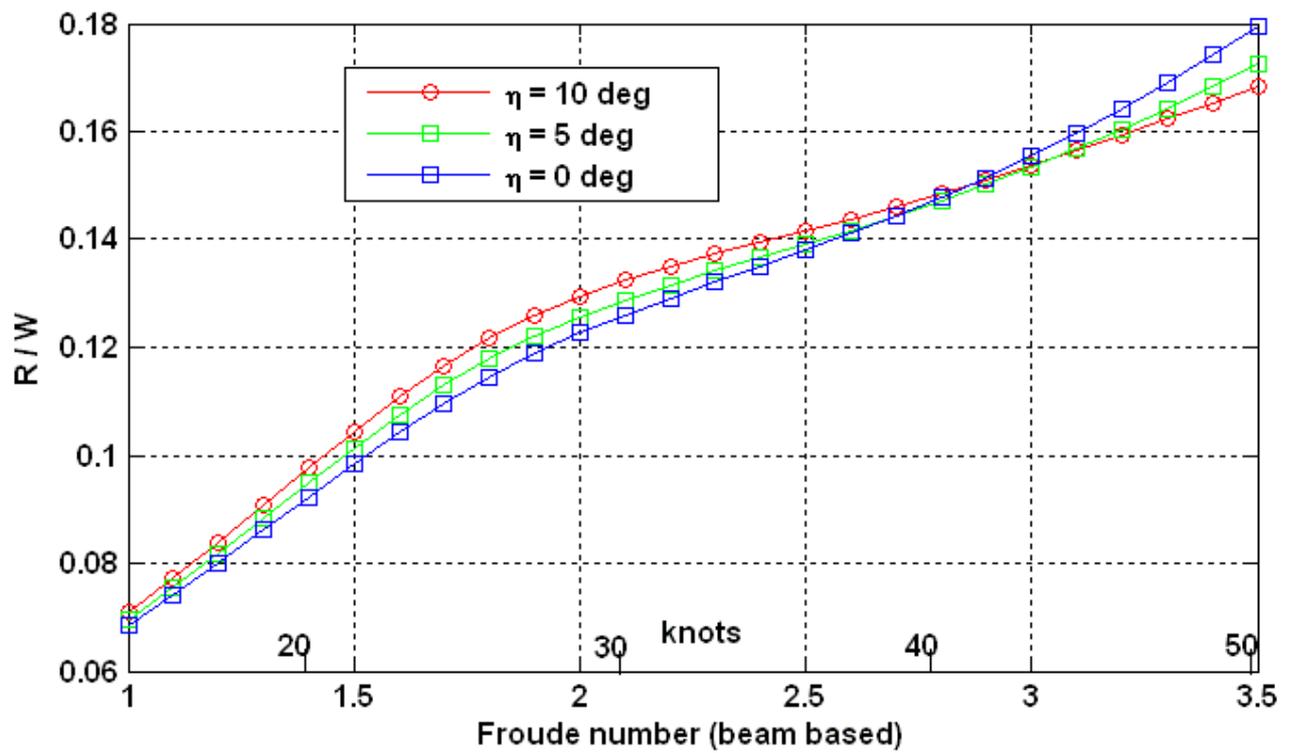**Figure 5: influence of the wing area considering the Resistance-to-Weight ratio**

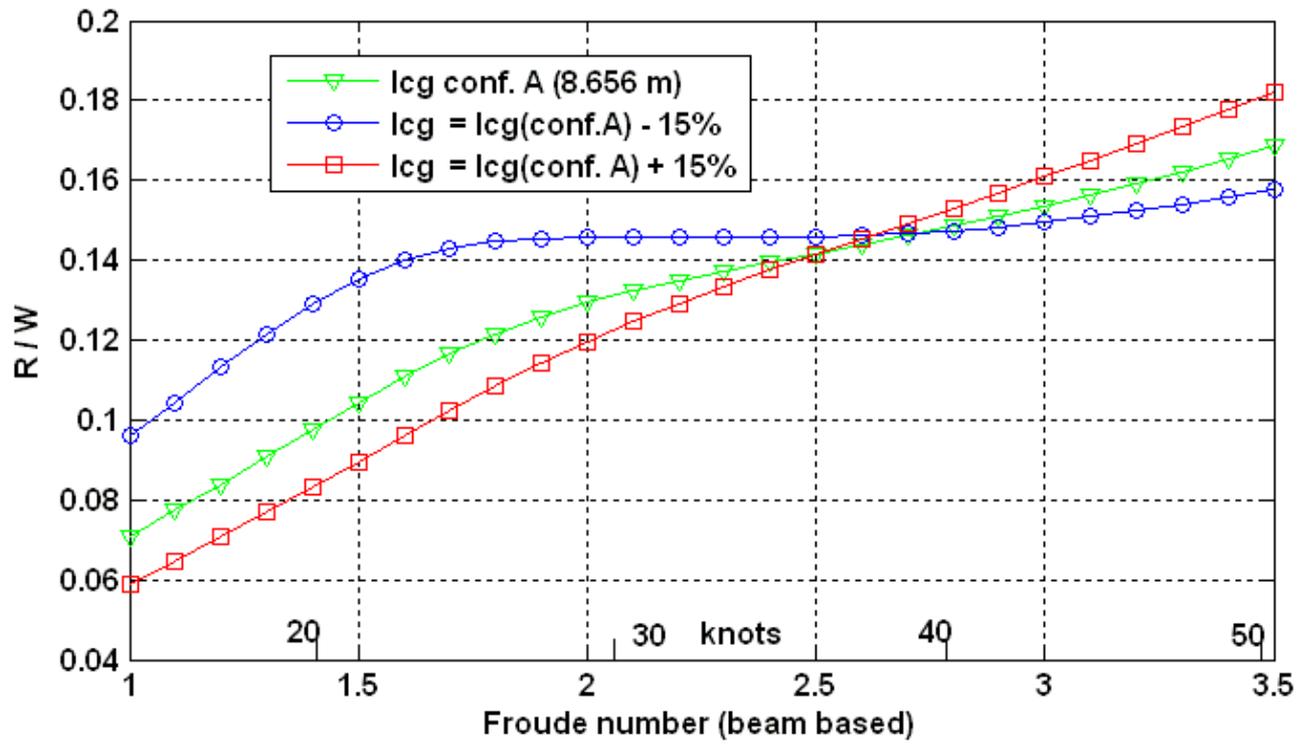**Figure 6: influence of η considering the Resistance-to-Weight ratio**

Figure 7: Influence of the longitudinal position of CG considering the Resistance-to-Weight ratio