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A New Problem of Descriptive Power*

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> The woman stared at the fruit. ...
> She wanted the wisdom that it would give her,
> and she ate some of the fruit. ...
> So the Lord sent them out of the Garden of Eden.
> Genesis 3

1 Actualism and the Possibilist Paradise

Analytic philosophers appear to quantify over unactualized possibilities all the time, as if they believed in a plurality of full-blooded worlds, spatio-temporal arrangements of individuals, properties and relations. For example, metaphysicians discuss whether individuals have some properties in all worlds in which they exist, or whether any pair of properties can co-exist in one world. A domain of discourse that includes possibilia allows to sharpen up questions, claims and arguments. Its benefits are so great that David Lewis called it "Philosopher's Paradise".[1] However, many philosophers are actualists, denying that there are any unactualized entities. Can they nevertheless partake of paradise, i.e. make sense of questions phrased in terms of

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[1]This is the title of part I of *On the Plurality of Worlds*, Oxford: Blackwell, 1986.

possible worlds and individuals? *Prima facie* it is not clear what actualists are committing themselves to when they affirm an existential quantification over possibilia. They are thus under the obligation to give a story about what they are doing. If they cannot discharge it, modal realists like Lewis will mercilessly expel them from paradise.

Linguistic Ersatzism, as characterized by Lewis, seems to offer such a story: the worlds quantified over are not concrete universes, but maximally consistent classes of sentences.[2] Linguistic Ersatzism gives truth-values of possibilist idioms as a function of the consistency of classes of sentences in a non-possibilist language. For example, 'There is a possible world in which there is a talking donkey.' comes out true because 'There is a talking donkey.' is a member of a maximally consistent class of sentences.

Does Linguistic Ersatzism offer a reduction of the possibilist language to a meta-language, with a consistency predicate, of a non-possibilist language? Lewis has shown that it does not. What is supposed to be the reducing language is too poor in descriptive power to determine the truth-values of all sentences of the possibilist languages. Lewis's well-known example involves *aliens*, properties not existing in the actual world. He raises the question whether there could be a pair of *role-swappers*: alien properties such that two worlds only differ from each other with respect to which of these two properties plays which role. Linguistic Ersatzism, depending on how exactly it is understood, either cannot make sense of this question, or answers it trivially in the negative.[3] This is its so-called "problem of descriptive power".

[2]Lewis's influential discussion is in "Paradise on the Cheap?", which is Part III of *On the Plurality of Worlds*, loc.cit. What he calls "Linguistic Ersatzism" is a technical elaboration of proposals by R.M. Adams, Carnap and Jeffrey.

[3]Unamended, it cannot make sense of the question. It gives a negative answer if "there is a property X such that there is a world in which $\Phi(X)$" is taken as a substitutional quantification and translated as "there is a property name N such that '$\Phi(N/X)$' belongs to some maximally consistent class of sentences", where '$\Phi(N)$' names the result of substituting N for X in $\Phi(X)$, and where the quantification ranges only over actual names. In that version, Linguistic Ersatzism implies that there are no alien properties, and *a fortiori*

Thus if an actualist aspires to use the full range of the possibilist language, she needs to develop a different theory.

In "The Ersatz Pluriverse", Theodore Sider attempts to do that:

> Like many, I turn to reduction. An adequate reduction of talk of possibilia—worlds and their inhabitants—must be materially adequate, but must not appeal to objectionable entities (like Lewisian worlds). Given such a reduction, the contemporary possible-worlds theorist's tools may be freely used with a clear ontological conscience.[4]

For Linguistic Ersatzism, the basis of the (partial) reduction is a metalinguistic consistency predicate. It can thus respect the traditional empiricist constraint that all necessity is linguistic, or conceptual. In Quine's terms, Linguistic Ersatzism is modally involved to the first grade. Sider's theory, in contrast, is modally involved to the third grade. It assumes that not only iterating modal operators, but also quantifying into modal contexts makes sense, and produces sentences which have truth-values.[5] The question whether this cost in ideology outweighs the gain in descriptive power is likely to get different answers from different philosophers, depending on how comfortable they are with irreducible *de re* modality and essentialism. I am not trying to adjudicate it here.

Sider is explicit that his theory is reductive specifically about the possibilist language, and does not attempt a reduction of the modal notion of possibility as well. If the reduction is successful, any dispute in the possibilist language is meaningful in the reducing language. Sider claims that the theory he presents, which I call "Pluriversal Ersatzism", achieves such a reduction.

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no alien role-swappers.

[4] This JOURNAL, XCIX (2002), pp. 279-315. Quotation from p. 281.

[5] Since Sider advocates counterpart-theory, he does not assume that for individual variables, but it is crucial for his solution to the problem of alien role-swappers that quantification over property variables into modal contexts is allowed.

He lists four examples of possibilist sentences that actualists might want to use, and argues:

> The reduction does indeed assign truth conditions to these sentences... Thus my reduction allows one to partake fully of modal metaphysics and semantics. (p. 306)

However, the last statement is a *non sequitur*; the reduction might succeed with these four examples but fail with others. Thus it has not been shown that Sider's theory gives actualists a licence to use the full possibilist language. I will show that in fact, it does not give such a licence, by presenting an example of a possibilist distinction Sider's theory cannot cope with. The theory thus fails to do the job it is designed for.

2 Pluriversal Ersatzism: Exposition

Pluriversal Ersatzism uses a modal language, which I call *ML*, as its reducing language. A reduction does not need to provide a recursive translation between two languages. It can make a detour through model-theoretic semantics, if the same models are used to define truth of sentences of the two languages. In the case at hand, so-called "modal models" are the relevant ones, set-theoretic structures modelling the space of possible worlds.[6] Possibilist sentences can straightforwardly be interpreted as true or false in one of these models. Sentences of *ML* are evaluated in the way familiar from the possible-worlds-semantics for modal logic. If every *ML*-sentence that is true (*simpliciter*) is true in a modal model $\mathcal{M}$, then $\mathcal{M}$ is *realistic*. Sider stipulates:

(1) A possibilist sentence is *true* iff it is true in every realistic model; it is *false* iff its negation is true.

[6] See pp. 292-293.

A simple example may illustrate how the reduction is supposed to work. Suppose that 'It is possible that there is a talking donkey.' is a true *ML*-sentence. This sentence is true in all and only the modal models in which for some w, the extension of 'talking' in w and the extension of 'donkey' in w overlap. Since the sentence is true, every realistic model fulfills that condition. Hence the possibilist sentence 'There is a world in which there is a talking donkey.' is true in every realistic model. By (1), it is true. If we suppose, on the other hand, that 'It is possible that there is a talking donkey.' is false, we can conclude by similar reasoning that 'There is a world in which there is a talking donkey.' is false. In either case, the theory succeeds in giving a truth-value to that particular possibilist sentence.

Does it likewise succeed for all other possibilist sentences? This crucial question about the expressive power of *ML* is not addressed in full generality in Sider's article. As I noted above, he claims in one place that the reduction works, based on the successful handling of a few examples. However, in another place he acknowledges that he has not proved this: "It is unclear to me at present whether a harmful multiplicity arises from CONSTRAINTS failing to constrain realistic models up to isomorphism."[7] Before showing that such a multiplicity may arise, I want to clarify what exactly a reduction would have to achieve.

It is crucial that for any sentence in the possibilist language, it can be shown that it has a truth-value, independently of what the *ML*-truths are. In the above example, this condition is fulfilled; we do not have to assume either the truth or the falsity of 'It is possible that there are talking donkeys'. It is a language that is to be reduced, not a particular theory formulated in that language. Sider rightly points out in several places that the reduction should not be hostage to substantive claims.[8] It is supposed to be neutral

[7]Footnote 27, p. 295. 'CONSTRAINTS' denotes the class of true *ML*-sentences that are *de dicto* with respect to quantifiers binding individual variables.

[8]For example, on p. 286 and pp. 314-315.

among competing metaphysical views, since it only provides a framework in which they can be debated. For all we know, Linguistic Ersatzism might give the right verdict on alien role-swappers, since for all we know there might be no alien properties at all.[9] But for all we know, it gets it wrong, and that is enough to disqualify it as a reduction.

In logician's terminology, two models are *elementarily equivalent* relative to a language L if exactly the same sentences of L are true in both models. Given that isomorphism between two models is defined in such a way that no possibilist sentence has different truth-values in isomorphic models, reducibility would be established if it could be shown that only isomorphic modal models are elementarily equivalent relative to ML. On the other hand, reduciblity can be shown to fail by describing, in the possibilist language, a pair of non-isomorphic modal models that are elementarily equivalent. This is, in effect, what Lewis did for Linguistic Ersatzism, and what I will do for Pluriversal Ersatzism.[10]

Whether the possibilist language is reducible in the sense explained depends on the expressive power of ML. Sider makes this language, which has a possibility- and an actuality-operator as modal primitives, very powerful. It has a name for every actual individual and property. Since every actual entity is its own name, it is a so-called *Lagadonian* language.[11] Moreover, ML allows infinite disjunctions and conjunctions. The quantifiers are world-restricted, or actualist: when the modal-model-semantics evaluates a formula at w, it interprets the quantifiers as ranging only over the domain of w (see pp. 292 and 293).

[9]Linguistic Ersatzism can be taken to say that there are no alien properties. See footnote 3.

[10]While the next section gives an informal argument, the appendix presents a proof.

[11]A qualification is needed here: if every actual entity were its own name, then some symbol would have to be ambiguous, for we need logical expressions besides names in the language. The problem can be solved by a code for assigning sets as non-Lagadonian names for other sets, thereby making some small sets available to serve as logical expressions.

ML is supposed to be the reducing language. I have so far only vaguely characterized the language to be reduced as the "possibilist language" that philosophers are using. It is ostensibly talking about individuals instantiating properties at worlds.[12] Fortunately, we do not need to specify a precise syntax for it. To ask whether it is reducible, it suffices in any case to know its expressive power in terms of modal models. The question then becomes whether the modal language can express the same distinctions among these models.

When arguing that there are non-isomorphic modal models that are elementarily equivalent, I will use our possibilist language, which is a fragment of what is sometimes called 'philosopher's English', i.e. ordinary English plus logic and set theory. Thereby I will show that the two models are distinguishable in the possibilist language, and hence that the reduction fails.

We can thus ignore in the following the second Lagadonian language that Sider's paper introduces. It is an infinitary possibilist language, which is also given a semantics in terms of modal models. For the purpose of reduction, its details are irrelevant. The reducing language *ML* may certainly be a language we cannot speak. But that some language can be reduced is relevant only for those who want to speak that language. Hence a reduction of Sider's Lagadonian possibilist language would be relevant only for the philosophers of Lagado, of whom Captain Lemuel Gulliver reports the following:

> [M]any of the most learned and wise adhere to the new scheme of expressing themselves by *things*, which hath only this inconvenience attending it, that if a man's business be very great, and of various kinds, he must be obliged in proportion to carry a greater bundle of *things* upon his back, unless he can afford one or two strong servants to attend him. I have often beheld two of these

[12] Only ostensibly for an actualist, of course. A realist about possible worlds, like David Lewis, interprets it literally.

sages almost sinking under the weight of their pack.[13]

But along with "the women in conjunction with the vulgar and illiterate"[14] of Lagado, non-Lagadonian philosophers will refuse to adopt the new scheme, and the question whether Sider's Lagadonian possibilist language is reducible is of no direct interest to them.[15] While the language to be reduced should be one that we speak, but may not be able to understand independently of any translation, the reducing should be one that we understand independently, although it may not be one that we speak. This last point will be important later, in the assessment of proposals to get around the problem of *ML*'s lack of expressive power.

3 Paradise Regained?

Sider's theory fails to reduce the possibilist language to a non-possibilist modal language. A reduction needs to answer all questions that can be asked in the possibilist language, not just those which Lewis used as examples in his critique of Linguistic Ersatzism. The case of role-swapping has a feature that makes it easily amenable to be settled by *ML*: the pair of worlds whose existence is in question is stipulated to have the same domain of properties.

[13]Jonathan Swift, *Gulliver's Travels*, New York: Barnes and Noble Classics, 2003, p. 199. Originally published 1726.

[14]*Loc.cit.*, pp. 198-9.

[15]It can be relevant only indirectly, via some relation between its expressive power to the expressive power of a language we do use. By Sider's Theorem on page 296, the Lagadonian possibilist language is reducible if no two non-isomorphic modal models are elementarily equivalent relative to *ML*. This leaves the question how our possibilist language compares in expressive power to the Lagadonian possibilist language. Surely, there are things expressible in that language that are inexpressible in our finitary language, where many actual things do not have names. On the other hand, it seems safe to assume that everything expressible in our language can be expressed in the Lagadonian possibilist language. However, my argument will not depend on settling this question.

Since in *ML*, the quantifiers in each worlds range only over entities existing at that world, we might suspect that trouble arises when we ask whether there are worlds with structurally identical descriptions but different domains of properties.

Like the old problem of descriptive power, the new problem arises from distinct entities playing the same role. I say that properties X and Y are *alternatives* of each other iff for every world w there is a world v that has exactly the same pattern of property instantiation, except that w has Y in all places where v has X.[16] If we wish, we may think of properties having an essence which determines what roles they possibly play. On that picture, properties are alternatives if they have the same essence. Among alternatives, there are both role-swappers and *substitutes*. Role-swappers co-exist in a world and stand in a certain relation to each other in that world. Since they are alternatives, there is a different world where the converse relation holds between them. But if alternatives are not compossible, not coexisting in any world, they are substitutes.[17] Two team-mates who can both play either central midfield or right midfield are role-swappers, and two others who can only play as goal-keepers are substitutes of each other, unable to play together in the team.[18] Sider's theory can deal with role-swappers, even if they come in infinite numbers, thanks to its use of an infinitary language. It can deal with finitely many substitutes as well. But as I now show, it cannot reduce sentences about an infinite cardinality of substitutes, which are very easily formulated in a possibilist language, for example by the sentence 'There are uncountably many substitutes'.[19]

[16]If X does not exist in w, then w and v are identical.

[17]Alternatives can be role-swappers with respect to some roles and substitutes with respect to other roles, but this does not matter to my argument.

[18]We can think of two worlds with substitutes as differing from each other merely quiddistically, by a "brute" non-identity of properties. Alternatively, we might think of them as qualitatively different from each other, but in ways that are ineffable from the vantage point of this world.

[19]Like Lewis, Sider takes individuals to be world-bound, and he builds this assumption

How can it be expressed in *ML* that there are at least two alien substitutes? Let the open sentence $\Phi(X)$ be a *repertoire* for a property, specifying all the complete roles a property can play, and saying that they are all the roles it can play. If C is a class of complete world-descriptions W in one free variable, $\Phi(X)$ is of the following form ($\bigwedge$ and $\bigvee$ take classes of formulas as arguments and form the conjunction or disjunction, respectively, of all their members):

$$(2) \quad \bigwedge\{\Diamond W(X) : W \in C\} \wedge \Box[\exists Y(Y = X) \rightarrow \bigvee\{W(X) : W \in C\}]$$

For any property, it is non-contingent whether it satisfies $\Phi(X)$.[20] Suppose the repertoire is such that two properties having it cannot co-exist. Thus the following is a true sentence of *ML*, implying that whatever Φ-alternatives there are will be substitues, not role-swappers:

$$(3) \quad \Box\forall X\forall Y(\Phi(X) \wedge \Phi(Y) \rightarrow Y = X)$$

This assumption about Φ is dispensable for my argument, but it makes its formulation easier. For example, Φ might say that whenever X is instantiated, it is the only property of a loner, an individual not co-existing with anything contingent except its own parts. (4) expresses that some possible property with repertoire Φ has at least one alternative:

$$(4) \quad \Diamond\exists X\Diamond\exists Y(X \neq Y \wedge \Phi(X) \wedge \Phi(Y))$$

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into the definition of a modal model. World-bound individuals cannot be role-swappers. However, they may be substitutes, and the ensuing argument might as well be put in terms of individuals instead of properties.

As an anti-haecceitist, Lewis classifies worlds that differ only through substitution of individuals as indiscernible, but as a quidditist, he classifies worlds that differ only through substitution of properties as discernible. (I presume that Sider agrees with Lewis on this issue.) I put the argument in terms of properties rather than individuals to get the stronger conclusion that *ML* fails to distinguish between different cardinalities of discernible worlds.

[20]Following Sider, I am assuming S5 here. Otherwise a conjunction of sentences that start with a modal operator, such as (2), may be true in one world and false in another.

Following that pattern, we can express that some possible property with repertoire Φ has at least two alternatives:

(5) $\Diamond\exists X\Diamond\exists Y\Diamond\exists Z(X \neq Y \wedge X \neq Z \wedge Y \neq Z \wedge \Phi(X) \wedge \Phi(Y) \wedge \Phi(Z))$

It is now obvious how to express, for a given natural number n, that some property with repertoire Φ has at least n alternatives. By conjoining (3), we can express that there are at least n substitutes. Let the sentence expressing that be Φ_n. A class of sentences that contains Φ_n, for every n, only has models with infinitely many Φ-substitutes. Thus *ML* can express that there are infinitely many substitutes. So far, all is well with *ML*.

But there is no way for this language to distinguish among infinite cardinalities of substitutes. To be sure, *ML* allows infinite blocks of quantifiers and infinite conjunctions. But this does not yield infinite alternations of existential quantifiers with other symbols. In standard infinitary languages, there are no infinite alternations of existential and universal quantifiers, or of quantifiers and negations. Likewise, there are no infinite alternations of diamonds and quantifiers in *ML*; the pattern at the beginning of (5) can only be repeated finitely many times. Thus a sentence built after this template can distinguish only between models with finite numbers of alternatives. The position of the Φ's and the negated identities may be different than in (5), but any sentence expressing that there are n alternatives must alternate quantifiers and modal operators no fewer than n times. Since sentences of that type are the only candidates, no *ML*-sentence can make any distinctions among infinite cardinalities of them.

Suppose now there are infinitely many substitutes for some possible property. Then for any infinite cardinal κ, there will be a realistic modal model in which there are exactly κ substitutes. Hence all sentences in the possibilist language about infinite cardinalities of substitutes will lack a truth-value, given (1). As a consequence, *ML* cannot tell us how many individuals, properties or worlds there are in modal space. There will be realistic modal models of different cardinalities, and some claims about the cardinality of the class of worlds or of the class of properties will lack a truth-value.

I anticipate the response that the new problem of expressive power is not serious. In practice, it may be argued, philosophers do not make claims whose truth depends on how many substitutes there are.[21] In my view, it would be good if this were true, but I am not sure it is. For once one is thinking in the possible-worlds-framework, such claims appear natural. This can be illustrated by an example involving worlds that are structurally like ours, i.e. have the same pattern of distribution of properties in regions of spacetime, but possibly with different properties in corresponding places. In some world that is structurally alike, schmass may be instantiated wherever mass is in the actual world.

Suppose a hypothetical philosopher Louise thinks there are worlds structurally like ours where mass, but no other property is replaced. Since any finite upper bound on the number of such replacement worlds would be arbitrary, Louise claims that there are infinitely many of them. She holds that all these replacements have the same repertoire. Slightly heretically, she denies that mass and its replacement properties are freely recombinable in such a way as to be able to co-exist in a world. Hence she is committed to there being infinitely many substitutes of mass. Further, she holds that we cannot epistemically rule out any world that is structurally like our world, and that our credence in a world that we cannot epistemically rule out is non-zero. From these claims it follows that at least as many worlds are given non-zero credence as there are substitutes of mass. Since at most denumerably many worlds can be given non-zero credence,[22] Louise is committed to affirm the

[21]This response lowers the standards for a successful reduction. It is then not at all clear any more whether Linguistic Ersatzism does not meet the lower standards as well, for philosophers do not too frequently make claims whose truth-value depends on the possibility of alien role-swappers either.

[22]A world has positive probability iff for some n, $P(w) > \frac{1}{2^n}$. Let $W_n = \{w : P(w) > \frac{1}{2^n}\}$; then the set of all worlds with positive probability is $\bigcup_{n \in N} W_n$. But for a given n, W_n has at most 2^n elements, since the probabilities cannot sum to more than one. Hence

two following possibilist sentences: 'There are infinitely many substitutes of mass.' and 'There are not uncountably many substitutes of mass'. But according to Pluriversal Ersatzism, if the first of these is true, the second lacks a truth-value, and if the second is true, then the first is false.[23] Hence it Pluriversal Ersatzism is right, these sentences are not both true, and thus neither is Louise's theory, taken as a set of sentences formulated in the possibilist language. By issuing this verdict, Pluriversal Ersatzism clearly violates its professed neutrality among philosophical theories that have a consistent formulation in the possibilist language.

4 Considering Patches

My argument shows that *ML*, the language Sider specified, is not powerful enough to reduce the possibilist language. The question remains whether something in the spirit of his proposal does work. Is there some technical trick that could save the project? I offer reasons to think that this is unlikely. The limitations of *ML* may well be the limitations of any approach that takes the modal operators as primitive.

To begin with, it is important to be clear what the rules of the game are. In one sense, it is trivial to express that there are uncountably many substitutes in a language of boxes and diamond. For we can express it in English, and we could easily set up a coding of sentences of English in a

the set of worlds with positive probability is a countable union of finite sets, and thus itself countable.

[23]Suppose 'There are infinitely many substitutes of mass.' is true. By the argument above, there is a realistic model with countably and a realistic modal with uncountably many substitutes of mass. By (1), 'There are not uncountably many substitutes of mass.' is neither true nor false.

Suppose that latter sentence is true. Then there are at most countably many substitutes of mass in every realistic modal model. By the argument above, there are at most finitely many; hence 'There are infinitely many substitutes of mass.' is false.

language of boxes and diamonds (or, for that matter, any other language with a sufficiently rich syntax). We could then stipulate that the sentences of that language mean what their English decodings mean. Of course, though, this is of no significance to the question whether the possibilist language can be reduced. It is a constraint on the reduction that the box and diamond have, at least roughly, the meaning of our familiar 'necessarily' and 'possibly'.

Still, we need to ask whether the power of the language can be increased while being faithful to what the symbols mean. In this section, I examine two pertinent proposals. One is technically attractive, but does not afford a reduction in a philosophically interesting sense, as I argue in the next section. The other can be dismissed because it surreptitiously introduces a possibilist quantifier.

The first proposal involves a radical departure from the usual way of specifying the syntax of infinitary languages: a modal language with non-well-founded, or infinitely deep, sentences. To introduce this notion, it is useful to think of a sentence as represented by a tree. For example, $\exists x(Fx \wedge Gx)$ can be represented by a tree of length 3; its bottom node is labelled by $\exists x$; above it, there is a node labelled by $\wedge$; on level 3, there are two branches, their nodes labelled by Fx and Gx, respectively. Standard languages only allow trees that are both of finite width and length. Infinitary languages of the type Sider employs make room for infinitely wide trees. Their formation rules allow $\bigwedge$ and $\bigvee$ to apply to infinitely many formulas, and the quantifiers to a formula and infinitely many variables. Infinitely deep, or non-well-founded sentences can be represented by trees that are infinitely long.[24] Since my argument above relies crucially on *ML* not being able to alternate $\Diamond$ and $\exists$ infinitely many times, infinitely deep languages promise to remedy the

[24]Clearly, such sentences cannot be formed in *ML*, as specified by Sider: "The language is infinitary, so the usual truth conditions for infinite conjunctions and infinite blocks of quantifiers must be adapted to the modal case in the natural way." (p.293). Sider refers us to M.A. Dickmann, *Large Infinitary Languages: Model Theory*, (Amsterdam: North-Holland, 1975), a work that does not discuss non-well-founded languages.

problem of expressive power. In the attempt to give a semantics for infinitely deep languages, concepts and methods from game theory have proved to be fruitul.[25]

I am now sketching a game-theoretic semantics of an extension *IDML* of *ML* that allows infinitely deep formulas.[26] A sentence and a modal model together define a game with two players, Myself (the initial verifier) and Nature (the initial falsifier). If the game reaches a true atomic formula, I win. If the game reaches a false atomic formula, Nature wins. A sentence is true in a model if I have a winning strategy, and false if Nature has a winning strategy.[27] A game starts at the bottom node and then moves up the tree. *IDML* has the standard rules of game-theoretic semantics: at a conjunction node, the falsifier chooses a branch; at a disjunction, the verifier chooses a branch; at a negation, the roles of the verifier and the falsifier are reversed; at an existential quantification node, the verifier chooses an element of the domain of the world of evaluation. At $\diamond$, the verifier chooses a world in the modal model.

The infinitely deep sentence S, represented by the figure, is only true in modal models with uncountably many alternatives with repertoire Φ. It is a tree of length $\omega_1 + 4$, where ω_1 is the first uncountable ordinal.[28]

[25]The original and most widely used example of an infinitely deep construction is provided by the *Henkin quantifier* or *game quantifier*, which is equivalent to an infinite alternation of universal and existential quantifiers. It is given a game-theoretic interpretation in Yiannis N. Moschovakis, "The Game Quantifier", *Proceedings of the American Mathematical Society, 31*, no. 1 (Jan. 1972): 245-50. Infinitely deep languages also allowing infinite alternations of $\vee$ and $\wedge$ have been explored by Jaakko Hintikka and Veikko Rantala, "A New Approach to Infinitary Languages", *Annals of Mathematical Logic, 10* (1976): 95-115.

[26]For an introduction to game-theoretic semantics, see for example Jaakko Hintikka and Gabriel Sandu, "Game-Theoretical Semantics", in Johan van Benthem and Alice ter Meulen (eds.), *Handbook of Logic and Language*, (Cambridge: MIT Press, 1997): pp. 361-410.

[27]The notion of a strategy may be understood in an intuitive way; formally, it is a function mapping any initial segment of the game onto a next move.

[28]To save space, I draw one node with $X_1 \neq X_2$ instead of one node with a negation

Sentence S: (appended)

sign below a node with $X_1 = X_2$; and I do not draw separate nodes for $\exists$ and $\diamond$.

I choose a sentence of the length of a successor ordinal, to avoid discussion of what constitutes winning a game that ends at a limit ordinal stage; see Hintikka and Rantala, loc.cit.

Suppose the game S is played over a modal model $\mathcal{M}$ with uncountably many Φ-substitutes. Let it be my strategy that at each diamond, I choose a different world, and at the existential quantifier I choose a property with repertoire Φ. If Nature at some point chooses a finite branch, leading to an atomic formula, she loses. But eventually, she must choose a finite branch. Hence on this strategy, I always win, and consequently S is true in $\mathcal{M}$.

Suppose now that S is played over a modal model $\mathcal{M}'$ with at most countably many Φ-substitutes. Then it cannot be the case that I choose a world and a property that both has repertoire Φ and is different from any previously chosen property uncountably many times. As soon as I fail to do so, Nature may choose the appropriate finite branch. Hence Nature has a winning strategy, and S is false in $\mathcal{M}'$.

A suitable elaboration of this sketch of *IDML* may well technically yield a reduction of the possibilist language. However, given the story of punctures and patches so far, it seems reasonable to remain skeptical until we have a proof that such a language can characterize modal models up to isomorphism. However, I argue in the next section that even if such a proof were forthcoming, its philosophical significance would be limited. First, though, I want to show that the second proposal, requiring a less drastic revision of syntax and semantics, does not help.

In my argument above, I exploit the inability of *ML* to alternate diamonds and existential quantifiers infinitely many times. Thus there is a temptation to modify the language to allow an infinite repetition of the pattern $\Diamond\exists$, maybe using the notation $[\Diamond\exists]$.[29] Thus if Φ is a formula and τ a sequence of variables, $[\Diamond\exists]\tau\Phi$ should be read as $\Diamond\exists v_1 \Diamond\exists v_2 ... \Phi$.

Since *ML* already allows infinite repetitions of $\exists$, this might seem a small step. However, there is a crucial difference between the ways $[\Diamond\exists]$ and $\exists$ operate on infinitely many arguments. $\exists$ takes in a set, and the result of applying it can be evaluated in one step. $[\Diamond\exists]$, on the other hand, takes in

[29]Thanks to Ted Sider and an anomymous referee for suggestions on this point.

a sequence, and the result of applying it needs to be evaluated in as many steps as there are elements in the well-ordering. For every occurrence of $\Diamond$ takes us to another world in the model. As long as there are only finitely many variables, this presents no problem, since then the result of pre-fixing $[\Diamond\exists]$ is equivalent to a formula in the language without that new symbol. But in order to increase the expressive power, $[\Diamond\exists]$ needs to operate on an infinite sequence. Semantically, the resulting sentence is infinitely deep, or non-well-founded; there is an infinitely descending chain under the relation of being a proper subformula. In the syntax, this is merely disguised by the use of the notation $[\Diamond\exists]$. We are used to evaluate a formula in finitely many steps until we reach atomic formulas. Infinitely deep formulas do not allow this. To interpret them, we may still proceed in steps indexed by the ordinals; but since infinitely many steps are required, we need a new rule for the limit ordinal step.

If this proposal is to be also semantically and not just syntactically different from the game-theoretic one, another rule for the ordinal step is needed. It is tempting to simply stipulate that after $[\Diamond\exists]\sigma$, where σ is a sequence of variables whose length is a limit ordinal, Φ is to be evaluated at the actual world. Thus $[\Diamond\exists]\sigma\Phi$ is true in a modal model if there is a world w_1 where a σ_1 exists and a world w_2 (that may or may not be distinct from w_1) where a σ_2 exists and ... such that Φ is true of σ_1, σ_2, etc. at the actual world. Now the language does indeed have the power to express that there are uncountably many substitutes of a given property. But this has no tendency to help Sider's project of a reduction that is acceptable for actualists. For $[\Diamond\exists]$ with these stipulated truth-conditions is just a cumbersome notational variant of an infinitary possibilist quantifier, i.e. a quantifier that ranges at any world over all possibilia, whether they exist in that world or not. It is a quantifier of the sort that an actualist is seeking a reduction for.

I do not know how to show in general that no other such move will help, but I am extremely skeptical. We have to recognize that the diamond and the existential quantifier already have their meaning given by their recursive clauses. It is not open to us to stipulate that they have special meanings when they happen to occur in a certain syntactic configuration. In the above example, the notation [$\Diamond\exists$] tried to suggest that we have a combination of familiar symbols, but failed to conceal that it was a new primitive operator in the language. I cannot pre-empt every actualist attempt to introduce a new operator and argue that we understand it independently of the possibilist language. But before such a proposal is made and adequately defended, possibilist discourse cannot count as reducible in a way that is acceptable to actualists.

5 Renouncing Parts of Paradise?

My argument shows that Pluriversal Ersatzism fails on Sider's standards. A reduction is not supposed to rule on substantive questions in modal metaphysics. In particular, it should not rule that there are most finitely many alternatives, in order to prevent embarassing questions from arising. Thus it needs to make sense of all questions asked in the possibilist language.

By adopting *IDML*, that problem might be overcome. This language has a deviant syntax, though, and calling it a "language" at all might be stretching the notion. But even if a solution to the technical problem can be found, there remains a philosophical worry. A reduction is only philosophically (as opposed to technically) significant if the reducing language is understood independently of the language to be reduced. There is a serious worry that already *ML* cannot be understood independently of the possibilist language. The problem is not that we do not speak a Lagadonian language, but rather that it is not clear that we have a conception of what it is for sentences about role-swappers and substitutes to be true or false, if we are not thinking in terms of possibilia. For the sake of the argument, however, I grant that we do. But I want to insist that *IDML* would be a philosophically ill-motivated choice of a reducing language. A language with sentences that cannot be produced by any finite number of applications of rules is essentially beyond what finite creatures like us can comprehend. If we cannot comprehend it, there is no reason to assume that it is indeed a language, in the sense of a class whose members are true or false, and not just some non-representing mathematical structure. As far as I can see, the only way in which we could specify an interpretation for *IDML* is by using the possibilist language, which would defeat the purpose of the reduction.

I recommend that actualists brace themselves for the discovery that possibilist discourse is not fully reducible to modal sentential operators. This need not tell against actualism, but rather against some instances of possibilist discourse. Maybe our hypothetical Louise should stop worrying about how many substitutes mass has. The path of wisdom is to start with the modal distinctions that do make sense, and tailor the use of the possibilist language accordingly. By going that path, we might well renounce parts of paradise, but *eo ipso* rid ourselves of *Scheinprobleme* raised by some unanswerable questions that can be asked in the possibilist language. As for giving up the comforts of paradise, philosophers should be ready to pay that price for attaining wisdom.

A Proof of Elementary Equivalence

This appendix proves the elementary equivalence of two non-isomorphic modal models relative to the modal language. $\mathcal{M}$ has non-denumerably many worlds. In each of them, exactly one individual, one monadic property, and the two-place identity relations for individuals and for properties exist (there are no higher-order properties and relations), and the individual instantiates the property. The individuals and properties of all these worlds are distinct. The worlds of $\mathcal{M}'$ are a denumerable subset of those of $\mathcal{M}$; the actual world @ is the same one in both models. (It would be easy to represent $\mathcal{M}$ and $\mathcal{M}'$ as 6-tuples corresponding to Sider's definitions on pp. 292/293.) The language, here called L, has names a for the actual individual, P for the actual property, and $=$ for the identity relation.

Let a mapping * of formulas of L into formulas be defined recursively as follows:[30] (i) For atomic ϕ, $\phi = \phi^*$. (ii) $(\neg\phi)^* = \neg(\phi^*)$. (iii) $(A\phi)^* = A(\phi)^*$. (iv) $(\bigwedge \Phi)^* = \bigwedge(\Phi^*)$. (v) $(\Diamond\phi)^* = \Diamond(\phi^*)$. (vi) $(\exists \vec{x}\ \phi(\vec{x}))^* = \exists x_0 \phi^*(\vec{x_0})$. (vii) $(\exists \vec{X}\ \phi(\vec{X}))^* = \exists X_0 \phi^*(\vec{X_0})$. ($\Phi^* = \{\phi^*\colon \phi \in \Phi\}$; $\vec{x}$ is a (possibly infinite) sequence of individual variables; $\vec{x_0}$ is a sequence of the same length, each of whose entries is x_0; and analogously for the upper-case property variables.)

Lemma 1. a) For every L-formula ϕ, world w and assignment σ, $\mathcal{M}_{w\sigma} \models \phi$ iff $\mathcal{M}_{w\sigma} \models \phi^*$. b) The same for $\mathcal{M}'$.

Proof. This is trivial for atomic formulas. The inductive steps for negation, conjunction, the actuality-operator and the diamond are straightforward. Existential quantification: Suppose $\mathcal{M}_{w,\sigma} \models \exists \vec{x}\ \phi(\vec{x}))$. Since $\exists \vec{x}\ \phi(\vec{x}))$ is logically equivalent to $\exists \vec{x} \bigvee\{x_0 \neq x_i : i \neq 0$ and x_i occurs in $\vec{x}\} \vee \exists \vec{x_0}\ \phi(\vec{x_0})$, $\mathcal{M}_{w,\sigma} \models \exists \vec{x} \bigvee\{x_0 \neq x_i : i \neq 0$ and x_i occurs in $\vec{x}\}$ or $\mathcal{M}_{w,\sigma} \models \exists \vec{x_0}\ \phi(\vec{x_0})$. Since the first disjunct is false for any world in $\mathcal{M}$,

[30] For the idea to use that mapping and Lemma 1 in the proof, I am indebted to John P. Burgess.

$\mathcal{M}_{w,\sigma} \models \exists \vec{x_0}\ \phi(\vec{x_0})$. Let $\sigma(x/w)$ denote the assignment that differs from σ at most in that it assigns the individual in w to the variable x. Hence $\mathcal{M}_{w,\sigma(x_0,w)} \models \phi(\vec{x_0})$, and by the induction hypothesis, $\mathcal{M}_{w,\sigma(x_0,w)} \models \phi^*(\vec{x_0})$. Hence $\mathcal{M}_{w,\sigma} \models \exists x_0 \phi^*(\vec{x_0})$. The argument for the property quantifier is exactly analogous; and so is the induction to prove b).$\blacksquare$

The language L^* is defined like Sider's L, except for the quantifier- and the conjunction-rule: If ϕ is a formula of L^* and x, X variables, then $\exists x\phi$ and $\exists X\phi$ are formulas of L^* (however, infinite blocks of quantifiers cannot be introduced); and if Φ is a (possibly infinite) set of formulas *with only finitely many distinct variables occuring in it*, then $\bigwedge \Phi$ is a formula of L^*. It is easy to check that for every sentence ϕ of L, ϕ^* is a sentence of L^*.

An assignment σ for L^* is an ordered pair $<\sigma_I, \sigma_P>$, where σ_I is an Ω-sequence of finitely many individuals, and σ_P analogously for properties. An *L^*-isomorphism* between world-assignment pairs is defined as follows: $<w, \sigma_I, \sigma_P>$ is L^*-isomorphic to $<w', \sigma'_I, \sigma'_P>$ iff all of the following conditions hold: (i) Actuality-Preservation: a) If $w = @$ or $w' = @$, then $w = w'$. b) If $\sigma_I(n) = a$ or $\sigma'_I(n) = a$, then $\sigma_I(n) = \sigma'_(n)$. c) Likewise for the property assignment. (ii) Identity-Preservation: a) $\sigma_I(n) = \sigma_I(m)$ iff $\sigma'_I(n) = \sigma'_I(m)$. b) Likewise for the property assignment. (iii) Instantiation-Preservation: $\sigma_I(n)$ and $\sigma_P(m)$ are worldmates iff $\sigma'_I(n)$ and $\sigma'_P(m)$ are worldmates. (iv) a) $\sigma_I(n)$ is in w iff $\sigma'_I(n)$ is in w'. b) $\sigma_P(n)$ is in w iff $\sigma'_P(n)$ is in w'.

Lemma 2. For any formula ϕ of L^*, worlds w and w' and assignments σ and σ' such that $<w, \sigma>$ and $<w', \sigma'>$ are isomorphic, $\mathcal{M}_{w\sigma} \models \phi$ iff $\mathcal{M}_{w'\sigma'} \models \phi$.

Proof. It is routine to check that the conditons on L^*-isomorphisms ensure that this holds for atomic formulas. The inductive clauses for negation and conjunction are straightforward. Actuality-operator: $\mathcal{M}_{w\sigma} \models A\psi$ iff $\mathcal{M}_{@\sigma} \models A\psi$. Since $<@, \sigma>$ and $<@, \sigma'>$ are isomorphic, $\mathcal{M}_{@\sigma'} \models A\psi$ by the induction hypothesis. Hence $\mathcal{M}_{w\sigma'} \models A\psi$. Diamond: Suppose $\mathcal{M}_{w\sigma} \models \Diamond\psi$. By definition, $\mathcal{M}_{w*\sigma} \models \psi$ for some world w^*. Then there is a world $w^{*\prime}$

such that $<w^*, \sigma>$ and $<w^{*\prime}, \sigma'>$ are isomorphic. Thus $\mathcal{M}_{w*'\sigma'} \models \psi$ by the induction hypothesis. By definition, $\mathcal{M}_{w'\sigma'} \models \Diamond\psi$. Existential quantification: Suppose $\mathcal{M}_{w\sigma} \models \exists x\psi(x)$. Then $\mathcal{M}_{w\sigma(x/w)} \models \psi(x)$. Since $<w, \sigma(x/w)>$ and $<w', \sigma'(x/w')>$ are isomorphic, $\mathcal{M}_{w'\sigma'(x/w')} \models \psi(x)$ by the induction hypothesis, and thus $\mathcal{M}_{w'\sigma'} \models \exists x\psi(x)$. The clause for the property quantifier is exactly analagous. ■

A *shared* world is one that belongs to both models; equivalently, one that belongs to $\mathcal{M}'$. A shared assignment assigns only individuals and properties from shared worlds.

Lemma 3. For any formula ϕ of L^*, shared assignment σ and shared world w, $\mathcal{M}_{w\sigma} \models \phi$ iff $\mathcal{M}'_{w\sigma} \models \phi$.

Proof. For atomic formulas, this is immediate from the construction of the models. The clauses for negation, conjunction, and the actuality-operator are straightforward. $\phi = \Diamond\psi$: Suppose $\mathcal{M}_{w\sigma} \models \Diamond\psi$. Then there is a world w' such that $\mathcal{M}_{w'\sigma} \models \psi$. For some shared world w'', $<w', \sigma>$ and $<w'', \sigma>$ are isomorphic. (Either w' is shared, in which case $w' = w''$; or w' is not shared, then since σ is shared, w'' may be any shared world not involved in σ.) By Lemma 2 and the induction hypothesis, $\mathcal{M}'_{w''\sigma} \models \psi$. Hence $\mathcal{M}'_{w\sigma} \models \Diamond\psi$. The other direction is straightforward. $\phi = \exists x\psi(x)$: Suppose $\mathcal{M}_{w\sigma} \models \exists x\psi(x)$. Then $\mathcal{M}_{w\sigma(x/w)} \models \psi(x)$. Since $\sigma(x/w)$ is obviously a shared assignment, it follows that $\mathcal{M}'_{w\sigma(x/w)} \models \psi(x)$ by the induction hypothesis, and hence $\mathcal{M}'_{w\sigma} \models \exists x\psi(x)$. Again, the other direction is straightforward. The clause for the property quantifier is exactly analagous. ■

Proposition. The non-isomorphic modal models $\mathcal{M}$ and $\mathcal{M}'$ are elementarily equivalent relative to L; i.e. for every sentence ϕ of L, $\mathcal{M} \models \phi$ iff $\mathcal{M}' \models \phi$.

Proof. The harder direction is right-left: Suppose $\mathcal{M}' \models \phi$, i.e. $\mathcal{M}'_{@\sigma} \models \phi$ for every $\mathcal{M}'$-assignment (shared assignment) σ. By Lemma 1.b), $\mathcal{M}'_{@\sigma} \models \phi^*$ for every shared σ. By Lemma 3, and since @ a shared world, $\mathcal{M}_{@\sigma} \models \phi^*$

for every shared assignment. By Lemma 2, and since for every σ, there is some shared σ' such that $<@, \sigma>$ and $<@, \sigma>$ are isomorphic, $\mathcal{M}_{@\sigma} \models \phi^*$ for every σ. By Lemma 1.a), $\mathcal{M}_{@\sigma} \models \phi$. ■