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AUTONOMOUS ATTITUDE CONTROL USING POTENTIAL FUNCTION METHOD UNDER CONTROL INPUT SATURATION

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ABSTRACT

The potential function method has been used extensively in nonlinear control for the development of feedback laws which result in global asymptotic stability for a certain prescribed operating point of the closed-loop system. It is a variation of the Lyapunov direct method in the sense that here the Lyapunov function, also called potential function, is constructed in such a way that the undesired points of the system state space are avoided. The method has been considered for the space applications where the systems involved are usually composed of the cascaded subsystems of kinematics and dynamics and the kinematic states are mapped onto an appropriate potential function which is augmented for the overall system by the use of the method of integrator backstepping. The conventional backstepping controls, however, may result in an excessive control effort that may be beyond the saturation bound of the actuators. The present paper, while remaining within the framework of conventional backstepping control design, proposes analytical formulation for the control torque bound being a function of the tracking error and the control gains. The said formulation can be used to tune to the control gains to bound the control torque to a prescribed saturation bound of the control actuators.

1. INTRODUCTION

BACKSTEPPING is a popular nonlinear control design technique [1]. The basic idea is to use a part of the system states as virtual controls to control the other states. Generating a family of globally asymptotically stabilizing control laws is the main advantage of it which can be exploited for addressing robustness issues and solving adaptive problems. The name backstepping refers to the recursive nature of the control design procedure where a control law as well as a control Lyapunov function (CLF) is recursively constructed to guarantee stability. Backstepping is among the various control methodologies which have been considered for the spacecraft attitude maneuver problem [2,3]. The cascaded structure of the spacecraft kinematics and dynamics makes the method of integrator backstepping a preferred approach for the said problem. However, the control actuators used for the attitude maneuver problem, e.g. reaction wheels, control moment gyros or thrusters, have an upper bound on the torque they can exert onto the system and the simple or conventional backstepping control method may result in excessive control input beyond that saturation bound of the actuators. Ref. 4, while remaining within the framework of conventional backstepping control design, has formulated the analytical bound for the control torque and has exploited the family of augmented Lyapunov functions proposed by [5] to introduce a constant gain which can be helpful for lowering the said bound. The present paper extends the work of [4] for autonomous attitude manoeuvres in the presence of constraints on the admissible attitudes [6,7].

The paper is organized as follows. Next section describes the model for the rigid spacecraft. Then, there comes the section giving the details of the analytical estimates of the bounds for the control torque components and the final section summarizes the work of the paper.

2. RIGID SPACECRAFT MODEL

The spacecraft is assumed to be a rigid body with actuators that provide torques about three mutually perpendicular axes that define a body-fixed frame with origin at the center of mass of the spacecraft. The equations of rotational motion of the spacecraft are given by [8]

$$\dot{\boldsymbol{q}}_v = \frac{1}{2}(q_4\boldsymbol{\omega} - \boldsymbol{\omega}\times\boldsymbol{q}_v), \quad \dot{q}_4 = -\frac{1}{2}\boldsymbol{\omega}^T\boldsymbol{q}_v \tag{1}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}\times[\mathbf{J}\boldsymbol{\omega}] = \boldsymbol{T} \tag{2}$$

where $\boldsymbol{q}_v \in \Re^3$ and $q_4 \in \Re$ satisfy $\boldsymbol{q}_v^T\boldsymbol{q}_v + q_4^2 = 1$, $\boldsymbol{q} = [\boldsymbol{q}_v^T,\ q_4]^T = [q_1, q_2, q_3, q_4]^T$ denotes the unit quaternion that represents the orientation of the spacecraft with respect to an inertial frame, $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T$ denotes the angular velocity of the spacecraft with respect to the inertial frame expressed in the body frame, $\mathbf{J} = \mathbf{J}^T$ denotes the body frame referenced positive definite inertia matrix of the spacecraft, $\boldsymbol{T} = [T_1, T_2, T_3]^T \in \Re^3$ denotes the control torque with components in the body frame. We define the three subscripts *i*, *j* and *k* as $(i, j, k) \in \{(1,2,3),\ (2,3,1),\ (3,1,2)\}$ and Eq. (1) can be written as

$$\dot{q}_i = \tfrac{1}{2}(q_4\omega_i - q_k\omega_j + q_j\omega_k) \tag{3}$$

and choosing $\mathbf{J} = diag(J_1, J_2, J_3)$ Eq. (2) becomes

$$\dot{\omega}_i = p_i\omega_j\omega_k + u_i \tag{4}$$

where $p_i = (J_j - J_k) / J_i$ and $u_i = T_i / J_i$.

3. ANALYTICAL CONTROL TORQUE BOUND

This section first describes the design of the backstepping controller then there comes the analytical estimate of the control torque bound. The candidate Lyapunov function for the kinematics subsystem stabilization is

$$V = V_a + V_r \tag{5}$$

where V_a is the attractive part taken as

$$V_a = \tfrac{1}{2}\left[q_1^2 + q_2^2 + q_3^2 + (1-q_4)^2\right] \tag{6}$$

and the repulsive part is represented by V_r chosen as

$$V_r = Ae^{-\frac{1}{2}B\left[b_1^2+b_2^2+b_3^2+(1-b_4)^2\right]} \tag{7}$$

where A and B are the positive constants shaping the repulsive potential topology and $\boldsymbol{b} = [\boldsymbol{b}_v^T,\ b_4]^T = [b_1, b_2, b_3, b_4]^T$ denotes the unit quaternion that represents the orientation of the spacecraft with respect to the inadmissible attitude $\boldsymbol{q}_a = [\boldsymbol{q}_{v,a}^T,\ q_{4,a}]^T = [q_{1,a}, q_{2,a}, q_{3,a}, q_{4,a}]^T$ which is fixed relative to the inertial frame. The separation of spacecraft attitude from the inadmissible one is given by the angle $\Delta\theta = 2\sin^{-1}[(\boldsymbol{b}_v^T\boldsymbol{b})^{1/2}]$. The pseudo control input for the kinematics subsystem stabilization ω_i^s is written as

$$\omega_i^s = -s\left[\operatorname{sgn}(q_4)q_i - \operatorname{sgn}(b_4)BV_r b_i\right] \tag{8}$$

where s is a positive constant, sgn(.) denotes the sign function defined for this study as [3]

$$\operatorname{sgn}(x) = \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases} \quad \text{or} \quad \operatorname{sgn}(x) = \begin{cases} -1, & x \leq 0 \\ 1, & x > 0 \end{cases} \tag{9}$$

The Lyapunov function of Eq. (5) is augmented for the overall system as [5]

$$U = V + \tfrac{1}{2}\sum_{i=1}^{3}\left[\Omega(\omega_i) - \Omega(\omega_i^s)\right]^2 \tag{10}$$

where $\Omega(\cdot)$ is a function of class κ_∞ i.e. it is zero at zero, strictly increasing and becomes unbounded when its argument does so [5]. The time derivative of the above equation becomes

$$\begin{aligned} \dot{U} &= -\tfrac{1}{2}s\sum_{i=1}^{3}\left[\operatorname{sgn}(q_4)q_i - \operatorname{sgn}(b_4)BV_r b_i\right]^2 + \\ &\sum_{i=1}^{3}\Bigg(\tfrac{1}{2}\left[\operatorname{sgn}(q_4)q_i - \operatorname{sgn}(b_4)BV_r b_i\right] + \\ &\frac{\Omega(\omega_i) - \Omega(\omega_i^s)}{\omega_i - \omega_i^s}\Big[\Omega'(\omega_i)(u_i + p_i\omega_j\omega_k) - \\ &\Omega'(\omega_i^s)\dot{\omega}_i^s\Big]\Bigg)(\omega_i - \omega_i^s) \end{aligned} \tag{11}$$

where $\Omega'(x)$ defines the derivative of $\Omega(x)$ with respect to x. Taking

$$\begin{aligned} \dot{U} &= -\tfrac{1}{2}s\sum_{i=1}^{3}\left[\operatorname{sgn}(q_4)q_i - \operatorname{sgn}(b_4)BV_r b_i\right]^2 - \\ &g\sum_{i=1}^{3}(\omega_i - \omega_i^s)^2 \end{aligned} \tag{12}$$

we obtain the backstepping controller

$$u_i = \frac{1}{\Omega'(\omega_i)}\left(-\frac{\omega_i - \omega_i^s}{\Omega(\omega_i)-\Omega(\omega_i^s)}\left[\tfrac{1}{2}\left(\operatorname{sgn}(q_4)q_i - \operatorname{sgn}(b_4)BV_r b_i\right) + g(\omega_i - \omega_i^s)\right] + \Omega'(\omega_i^s)\dot{\omega}_i^s \right) - p_i\omega_j\omega_k \quad (13)$$

where g is a positive constant. In an effort of achieving the boundedness of u_i one may expect taking $\Omega'(\omega_i) > 1$ to be useful so we consider the simple case of $\Omega(\omega_i) = \eta\omega_i$ with $\eta > 1$ and the above control law is written as

$$\begin{aligned}u_i =& -\frac{1}{\eta^2}\left[\tfrac{1}{2}\left(\operatorname{sgn}(q_4)q_i - \operatorname{sgn}(b_4)BV_r b_i\right) + g(\omega_i - \omega_i^s)\right] - \\ &\tfrac{1}{2}s\Big[\operatorname{sgn}(q_4)(q_4\omega_i - q_k\omega_j + q_j\omega_k) - \\ &BV_r\operatorname{sgn}(b_4)(b_4\omega_i - b_k\omega_j + b_j\omega_k) + \\ &B^2V_r(b_i\omega_i + b_j\omega_j + b_k\omega_k)b_i\Big] - p_i\omega_j\omega_k\end{aligned} \quad (14)$$

Defining $e_i = \omega_i - \omega_i^s$ the above equation can be written as

$$\begin{aligned}u_i =& \frac{1}{\eta^2}\left(\omega_i^s/(2s) - ge_i\right) - \\ &\tfrac{1}{2}s\Big[\operatorname{sgn}(q_4)(q_4e_i - q_ke_j + q_je_k + q_4\omega_i^s - q_k\omega_j^s + q_j\omega_k^s) - \\ &BV_r\operatorname{sgn}(b_4)(b_4e_i - b_ke_j + b_je_k + b_4\omega_i^s - b_k\omega_j^s + b_j\omega_k^s) + \\ &B^2V_r(b_ie_i + b_je_j + b_ke_k + b_i\omega_i^s + b_j\omega_j^s + b_k\omega_k^s)b_i\Big] - \\ &p_i(e_j + \omega_j^s)(e_k + \omega_k^s)\end{aligned} \quad (15)$$

Because $|q_i| \le 1$ and $|b_i| \le 1$ so the absolute value of the control inputs $|u_i|$ is bounded by

$$\begin{aligned}|u_i| \le& \frac{1}{\eta^2}\left(\frac{|\omega_i^s|}{2s} + g|e_i|\right) + \\ &\tfrac{1}{2}s\Big(\left|q_4e_i - q_ke_j + q_je_k + q_4\omega_i^s - q_k\omega_j^s + q_j\omega_k^s\right| + \\ &BV_r\left|b_4e_i - b_ke_j + b_je_k + b_4\omega_i^s - b_k\omega_j^s + b_j\omega_k^s\right| + \\ &B^2V_r\left|(b_ie_i + b_je_j + b_ke_k + b_i\omega_i^s + b_j\omega_j^s + b_k\omega_k^s)b_i\right|\Big) + \\ &\left|p_i(e_j + \omega_j^s)(e_k + \omega_k^s)\right| \\ \le& \frac{1}{\eta^2}\left(\tfrac{1}{2}(1 + B\overline{V}_r) + g|e_i|\right) + \\ &\tfrac{1}{2}s\Big[|q_4e_i| + |q_ke_j| + |q_je_k| + |q_4\omega_i^s| + |q_k\omega_j^s| + |q_j\omega_k^s| + \\ &B\overline{V}_r\left(|b_4e_i| + |b_ke_j| + |b_je_k| + |b_4\omega_i^s| + |b_k\omega_j^s| + |b_j\omega_k^s|\right) + \\ &B^2\overline{V}_r\left(|b_ie_i| + |b_je_j| + |b_ke_k| + |b_i\omega_i^s| + |b_j\omega_j^s| + |b_k\omega_k^s|\right)|b_i|\Big] \\ &+ |p_i|\left|e_je_k + \omega_j^se_k + \omega_k^se_j + \omega_j^s\omega_k^s\right| \\ \le& \frac{1}{\eta^2}\left[\tfrac{1}{2}(1 + B\overline{V}_r) + g|e_i|\right] + \\ &\tfrac{1}{2}s\left[1 + (1+B)B\overline{V}_r\right]\left(|e_i| + |e_j| + |e_k| + |\omega_i^s| + |\omega_j^s| + |\omega_k^s|\right) + \\ &|p_i|\left(|e_j||e_k| + |\omega_j^s||e_k| + |\omega_k^s||e_j| + |\omega_j^s||\omega_k^s|\right) \\ \le& \frac{1}{\eta^2}\left[\tfrac{1}{2}(1 + B\overline{V}_r) + g|e_i|\right] + \\ &\tfrac{1}{2}s\left[1 + (1+B)B\overline{V}_r\right]\left[|e_i| + |e_j| + |e_k| + 3s(1 + B\overline{V}_r)\right] + \\ &|p_i|\left[|e_j||e_k| + s(1 + B\overline{V}_r)\left(|e_k| + |e_j|\right) + s^2(1 + B\overline{V}_r)^2\right]\end{aligned} \quad (16)$$

where $\overline{V}_r$ is the bound for the repulsive potential V_r and it depends on the minimum permissible separation angle $\Delta\theta$. Rearranging the terms, the above inequality becomes

$$|u_i| \le k_1 + k_2|e_i| + k_3\left(|e_j| + |e_k|\right) + |p_i||e_j||e_k| \quad (17)$$

where the constants k_1, k_2 and k_3 are

$$k_1 = \frac{1}{2\eta^2}(1+B\overline{V}_r) + \tfrac{3}{2}s^2\left[1+(1+B)B\overline{V}_r\right](1+B\overline{V}_r) + \left|p_i\right| s^2 (1+B\overline{V}_r)^2$$

$$k_2 = \frac{g}{\eta^2} + \tfrac{1}{2}s\left[1+(1+B)B\overline{V}_r\right]$$

$$k_3 = \tfrac{1}{2}s\left[1+(1+B)B\overline{V}_r\right] + \left|p_i\right| s(1+B\overline{V}_r)$$

Using Eqs. (11) and (12) we can write

$$\frac{d}{dt}\left(\tfrac{1}{2}\left[\Omega(\omega_i) - \Omega(\omega_i^s)\right]^2\right) = -ge_i^2 - \tfrac{1}{2}\left[\operatorname{sgn}(q_4)q_i - \operatorname{sgn}(b_4)BV_r b_i\right]e_i \quad (18)$$

Using $\Omega(\omega_i) = \eta\omega_i$ we get

$$\eta^2 e_i \dot{e}_i = -ge_i^2 - \tfrac{1}{2}\left[\operatorname{sgn}(q_4)q_i - \operatorname{sgn}(b_4)BV_r b_i\right]e_i \quad (19)$$

The above two equations show that if $\left|e_i\right| > \left(\left|q_i\right| + B\overline{V}_r\left|b_i\right|\right)/(2g)$ or, being more conservative, $\left|e_i\right| > \left(1+B\overline{V}_r\right)/(2g)$ then $\left|\Omega(\omega_i) - \Omega(\omega_i^s)\right|$ or $\left|e_i\right|$ will be decreasing until it reaches a value of $\left(1+B\overline{V}_r\right)/(2g)$ after which it will remain bounded by $\left(1+B\overline{V}_r\right)/(2g)$ and, hence, Eq. (17) can be used to calculate the bounds of the controls u_i. Moreover, the minimum bounds identifiable by Eq. (17) are

$$\left|u_i\right| \le k_1 + \frac{1+B\overline{V}_r}{g}\left(\tfrac{1}{2}k_2 + k_3 + \frac{1+B\overline{V}_r}{4g}\left|p_i\right|\right) \quad (20)$$

4. CONCLUSION

The problem of spacecraft constrained attitude manoeuvres under control input saturation has been addressed by expressing the bounds for the control torque components analytically as a function of the tracking error and the typical conventional backstepping control gains. The said expressions can be used to tune the control gains for lowering the control torque bound to be within the control actuators saturation limit.

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