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On the Benefit of ϵ-Efficient Solutions in Multi-Objective Space Mission Design

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Abstract. In this work we consider multi-objective space mission design problems. We will show that it makes sense from the practical point of view to consider in addition to the (Pareto) optimal solutions also nearly optimal ones since this increases significantly the number of options for the decision maker, whereas the possible loss of such approximate solutions compared to optimal—and possibly even 'better'—ones is dispensable. For this, we will examine several typical problems in space trajectory design—a bi-impulsive transfer from the Earth to the asteroid Apophis and several low-thrust multi-gravity assist transfers—and demonstrate the possible benefit of the novel approach. Further, we will present an evolutionary multi-objective algorithm which is designed for this purpose.

In most real world engineering problems several aims have to be taken into account simultaneously which leads mathematically speaking to *multi-objective optimization problems* (MOPs):

$$\min_{x\in Q} F:\mathbb{R}^n\to\mathbb{R}^k, \tag{1}$$

where $Q\subset\mathbb{R}^n$ denotes the set of feasible solutions. Such problems also naturally arise in space mission problems, where the task is to find an 'optimal' transfer between several celestial bodies. It turned out that the most important aims are the minimization of both the flight time and the fuel consumption of the spacecraft ([1], [7], [6]). The former objective is related to the cost of operations which could account for roughly 50% of the cost of an interplanetary space mission. The latter objective is directly related to the cost of the launch and on the mass of the payload.
In this work we want to show that for such problems the consideration of approximate or ϵ-efficient solutions (based on the notion of ϵ-dominance [3]) is beneficial since by doing so the variety of possibilities offered to the decision maker (DM) can be increased significantly while the loss of such a solution compared to an 'optimal' one is negligable (and can be adjusted a priory by choosing the value of ϵ). Further, we present one way to compute the set of approximate solutions using an evolutionary multi-objective (EMO) algorithm. The main difficulty of this approach is to handle the large amount of nearly optimal solutions: it is well known that the set of optimal solutions forms a $(k-1)$-dimensional object (where k is the number of objectives, see (1)), and thus, the computation of this set is already challenging. Moreover, the set of ϵ-approximate solutions—denote by $P_{Q,\epsilon}$—is even n-dimensional. Hence, it is clear that an efficient approach requires a suitable discretization strategy of the set of interest. For this, we use the archiving strategy proposed in ([4], [5]). It has been shown that an application of the archiver (in combination with a stochastic search procedure as an EMO algorithm) leads under certain (mild) assumptions to a sequence of archives which (a) converges to a discretization of the set of approximate solutions, and (b) where the order of the magnitude of the limit archive is $O(\frac{1}{\Delta^{k-1}})$ for $\Delta\to 0$, where Δ is the discretization parameter of the archiver. Hence, the cost for the approximation of $P_{Q,\epsilon}$ is from a theoretical viewpoint as high as for the 'classical' multi-objective case, and thus, low enough to attack typical design problems. For the computation we propose the EMO algorithm $P_{Q,\epsilon}$-NSGA-II. This algorithm is a hybrid of the well known EMO algorithm NSGA-II ([2]) and the archiving strategy mentioned above, and is capable of computing satisfying approximations of $P_{Q,\epsilon}$ for all space mission problems we have considered within reasonable time (i.e., within several minutes).

As one general example we consider the sequence Earth – Venus – Earth – Jupiter (EVEJ). The model consists of seven parameters, the most important one for our purpose is the launch date t_0

of the transfer. Figure 1 shows one numerical result for $\epsilon = (0.05, 20)$ and discretization parameter $\Delta = 0$ (i.e., all ϵ-efficient solutions are stored in the archive). $\epsilon = (0.05, 20)$ corresponds to a possible loss of 5% of the fuel consumption, and 20 days of additional flight time compared to optimal solutions are tolerated (note that the flight times in Figure 1 vary from three to seven years). The result contains of 15 non-dominated solutions and further 120 ϵ-efficient solutions (compared to the non-dominated solutions). Assuming that the result is the basis for the DM and that the values of y_0 have been chosen for the transfer (Figure 1), then there are three possibilities: to launch the spacecraft in December 2013, to launch it 16 months later (April 2015), or to wait another 3.5 years (October 2018). In case only non-dominated solutions are stored only one option (y_0) would be given to the DM. The reason for this difference is that points which are 'near' in objective space do not have to be near in parameter space, as it is here the case for the parameter value t_0. Similar statements hold in this example for all 15 non-dominated solutions x with $f_1(x) \geq 0.45$, and hence, the DM's decision space is augmented significantly by allowing approximate solutions.

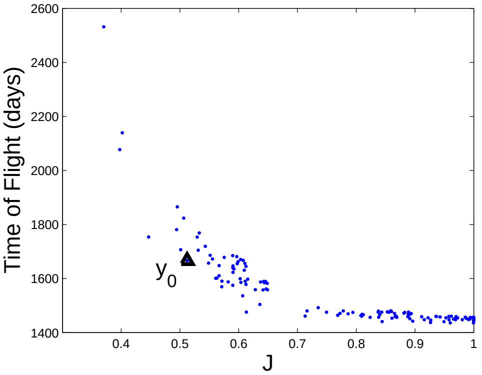

Fig. 1. Numerical result for sequence EVEJ using $P_{Q,\epsilon}$-NSGA-II consisting of 135 elements whereas 15 are non-dominated.

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