Schütze, O. and Vasile, M. and Coello, C.A. (2008) *Approximate solutions in space mission design.* Lecture Notes in Computer Science, 5199 . pp. 805-814. ISSN 0302-9743

Approximate Solutions in Space Mission Design

Oliver Schütze[1], Massimiliano Vasile[2], and Carlos A. Coello Coello[1]

[1] CINVESTAV-IPN, Computer Science Department, Mexico City, Mexico
{schuetze,ccoello}@cs.cinvestav.mx,
[2] University of Glasgow, Department of Aerospace Engineering, Glasgow, Scotland
m.vasile@aero.gla.ac.uk

Abstract. In this paper, we address multi-objective space mission design problems. From a practical point of view, it is often the case that, during the preliminary phase of the design of a space mission, the solutions that are actually considered are not 'optimal' (in the Pareto sense) but belong to the basin of attraction of optimal ones (i.e. they are nearly optimal). This choice is motivated either by additional requirements that the decision maker has to take into account or, more often, by robustness considerations. For this, we suggest a novel MOEA which is a modification of the well-known NSGA-II algorithm equipped with a recently proposed archiving strategy which aims at storing the set of approximate solutions of a given MOP. Using this algorithm we will examine some space trajectory design problems and demonstrate the benefit of the novel approach.

1 Introduction

In a variety of applications in industry and finance a problem arises that several objective functions have to be optimized concurrently leading to *multi-objective optimization problems* (MOPs). For instance, in space mission design, which we address here, there are two crucial aims for the realization of a transfer: minimization of flight time and fuel consumption of the spacecraft ([1], [12], [11], [10]). The former objective is related to the cost of operations which could account for roughly 50% of the cost of an interplanetary space mission. The latter objective is directly related to the cost of the launch and on the mass of the payload. The scope of this paper is (a) to show that it makes sense to consider in addition to the 'optimal' trajectories also approximate solutions since by this the decision maker (DM) is offered a much larger variety of possibilities, and (b) to present one way to compute this enlarged set of interest with reasonable effort. As a motivating example for (a) we consider the MOP in Section 4.2 which is a model for the sequence Earth – Venus – Mercury, and the following two points x_i with images $F(x_i), i = 1, 2$:

$$\begin{aligned} x_1 &= (782,\, 1288,\, 1788)\ , & F(x_1) &= (0.462,\, 1001.7) \\ x_2 &= (1222,\, 1642,\, 2224), & F(x_2) &= (0.463,\, 1005{,}3) \end{aligned}$$

The two objectives are the propellant mass fraction—i.e., the portion of the vehicle's mass which does not reach the destination—and the flight time (in days).

In the domain, the first parameter is of particular interest: it determines the departure time from the Earth (in days after 01.01.2000). $F(x_1)$ is less than $F(x_2)$ in both components, and thus, x_1 can be considered to be 'better' than x_2. However, the difference in image space is small: the mass fraction of the two solutions differs by 0.001 which makes 0.1% of the total mass, and the flight time differs by four days for a transfer which takes almost three years. In case the DM is willing to accept this deterioration, it will offer him/her a second choice in addition to x_1 for the realization of the transfer: while the two solutions offer 'similar' characteristics in image space this is not the case in the design space since the starting times for the two transfers differ by 440 days.

The identification of the two solutions would be a fundamental requirement during the preliminary design of a space mission. In fact, in order to increase the reliability of the design, the mission analysts would need to identify one or more back-up solutions, possibly with identical cost, for each baseline solution. Furthermore, for each mission opportunity (i.e. each launch date) rather than an optimal solution, it is generally required to identify a set of nearly optimal ones, possibly all with similar cost. Such a set would represent a so called *launch window*, since for each solution in the set a launch would be possible. Designing for the suboptimal points further increases the reliability of the mission since it gives the freedom to deviate from the chosen design point with little or no penalty. This holds true also for Pareto optimal solutions. It is therefore desirable to have a whole range of nearly Pareto optimal solutions for each Pareto point.

The field of evolutionary multi-objective optimization is well-studied and MOEAs have been successfully applied in a number of domains, most notably engineering applications ([**?**]). Approximate solutions in multi-objective optimization have been studied by many researchers so far (e.g., [6], [13], [5]). A first attempt to investigate the benefit of considering approximate solutions in space mission design has been done in [**?**], albeit for the single-objective case.
The additional consideration of approximate solutions in multi-objective space mission design problems is new and will be addressed in this paper. Crucial for this approach is the efficient computation of the enlarged set of 'optimal' points (the second scope of this paper) since in many cases the 'classical' multi-objective approach is a challenge itself. For this, we will propose an algorithm which is based on the well-known NSGA-II ([2]) but equipped with an arching strategy which was designed for the current purpose. Note that 'classical' archiving/selection strategies—e.g., the ones in [3], [7], [5], [4], or the one NSGA-II uses—store sets of mutually non-dominating points (which means that e.g. the points x_1 and x_2 in the above example will never be stored *jointly*). That is, these selection mechanisms—though they accomplish an excellent job in approximating the efficient set—can not be taken for our purpose.
The remainder of this paper is organized as follows: in Section 2, we give the required background which includes the statement of the space mission design problem under consideration. In Section 3, we propose a new genetic algorithm

for the computation of the set of approximate solutions and present further on in Section 4 some numerical results. Finally, we conclude in Section 5.

2 Background

Multi-Objective Optimization In the following we consider continuous multi-objective optimization problems

$$\min_{x\in Q}\{F(x)\}, \tag{MOP}$$

where $Q \subset \mathbb{R}^n$ is compact and F is defined as the vector of the objective functions $F : Q \to \mathbb{R}^k, \quad F(x) = (f_1(x), \ldots, f_k(x))$, and where each $f_i : Q \to \mathbb{R}$ is continuous.

Definition 1. *Let $v, w \in Q$. Then the vector v is* less than *w ($v <_p w$), if $v_i < w_i$ for all $i \in \{1, \ldots, k\}$. The relation $\leq_p$ is defined analogously. $y \in Q$ is* dominated *by a point $x \in Q$ ($x \prec y$) with respect to (3) if $F(x) \leq_p F(y)$ and $F(x) \neq F(y)$. $x \in Q$ is called a* Pareto optimal point *or* Pareto point *if there is no $y \in Q$ which dominates x.*

The set of all Pareto optimal solutions is called the *Pareto set* (denoted by P_Q). The image of the Pareto set is called the *Pareto front*. We now define another notion of dominance which we use to define approximate solutions.

Definition 2. *Let $\epsilon = (\epsilon_1, \ldots, \epsilon_k) \in \mathbb{R}^k_+$ and $x, y \in Q$. x is said to ϵ-dominate y ($x \prec_\epsilon y$) with respect to (3) if $F(x) - \epsilon \leq_p F(y)$ and $F(x) - \epsilon \neq F(y)$. x is said to $-\epsilon$-dominate y ($x \prec_{-\epsilon} y$) with respect to (3) if $F(x) + \epsilon \leq_p F(y)$ and $F(x) + \epsilon \neq F(y)$.*

The notion of $-\epsilon$-dominance is of course analogous to the 'classical' ϵ-dominance relation but with a value $\tilde{\epsilon} \in \mathbb{R}^k_-$. However, we highlight it here since we use it to define our set of interest:

Definition 3. *Denote by $P_{Q,\epsilon}$ the set of points in $Q \subset \mathbb{R}^n$ which are not $-\epsilon$-dominated by any other point in Q, i.e., $P_{Q,\epsilon} := \{x \in Q |\ \nexists y \in Q : y \prec_{-\epsilon} x\}$.*

The set $P_{Q,\epsilon}$ contains all ϵ-efficient solutions, i.e., solutions which are optimal up to a given (small) value of ϵ. Fig. 1 gives two examples.

Archiving strategy for approx. soluttions: [9], [10]

The Design Problem In the following we will analyze few examples taken from two classes of typical problems in space trajectory design: a bi-impulsive transfer from the Earth to and the asteroid Apophis, and two low-thrust multi-gravity assist transfers from the Earth to a planet.

Bi-impulse Problem

For the bi-impulsive case, the propellant consumption is a function of the velocity change, or Δv[**?**], required to depart from the Earth and to rendezvous with a given celestial body. Both the Earth and the target celestial body are point

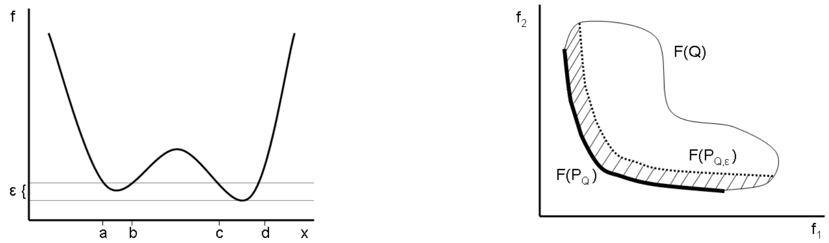


Fig. 1. Two different examples for sets $P_{Q,\epsilon}$. Left for $k = 1$ and in parameter space with $P_{Q,\epsilon} = [a, b] \cup [c, d]$. Right an example for $k = 2$ in image space.

Algorithm 1 $A := ArchiveUpdateP_{Q,\epsilon}\ (P, A_0, \Delta)$

Require: population P, archive A_0, $\Delta \in \mathbb{R}_+$, $\Delta^* \in (0, \Delta)$
Ensure: updated archive A

$A := A_0$
for all $p \in P$ **do**
  if $\nexists a_1 \in A:\ a_2 \prec_{-\epsilon} p$ and $\nexists a_2 \in A:\ d_\infty(F(a_2), F(p)) \leq \Delta^*$ **then**
    $A := A \cup \{p\}$
    for all $a \in A$ **do**
      if $p \prec_{-(\epsilon+1\Delta)} a$ **then**
        $A := A \backslash \{a\}$
      end if
    end for
  end if
end for

masses with the only source of gravity attraction being the Sun. Therefore, the spacecraft is assumed to be initially at the Earth, flying along its orbit. The first velocity change, or Δv_1, is used to leave the orbit of the Earth and put the spacecraft into a transfer orbit to the target. The second change in velocity, or Δv_2, is then used to inject the spacecraft into target's orbit.

The two Δv's are a function of the positions of the Earth and the target celestial body at the time of departure t_0 and at the time of arrival $t_f = t_0 + T$, where T is the time of flight. Thus, the MOP under consideration reads as follows:

$$\text{minimise:} \quad \begin{cases} \Delta v_1 + \Delta v_2 \\ T \end{cases} \tag{1}$$

MLTGA Problem

It is here proposed to use a particular model for multiple gravity assist low-thrust trajectories (MLTGA). Low-thrust arcs are modeled through a shaping approach based on the exponential sinusoid proposed by Petropoulos et al.[**?**]. The spacecraft is assumed to be moving in a plane subject to the gravity attraction of the Sun and to the control acceleration of a low-thrust propulsion engine[**?**]. Gravity manoeuvres are modeled through a powered swing-bys approximation[**?**]: a pair of low-thrust arcs are linked through a Δv manoeuvre when the gravity of the swing-by planet is not strong enough to gain the required change in velocity. As for the bi-impulsive case, we are interested in the minimization of two objectives: the propellant mass fraction and the flight time. The first objective is given as a function of the velocity change due to the low-thrust propulsion system, the impulsive correction at the swing-by planets (i.e. the powered swing-bys) and the departure increment in velocity given by the launcher:

$$J = 1 - e^{-\left(\frac{\Delta V_{GA} + \Delta V_0}{g_0 I_{sp1}} + \frac{\Delta V_{LT}}{g_0 I_{sp2}}\right)} \tag{2}$$

with the solution vector[**?**] $y = [t_0, T_1, k_{2,1}, n_1, ..., T_i, k_{2,i}, n_i, ..., T_N, k_{2,N}, n_N]^T$. Where ΔV_{GA} is the sum of all the ΔVs (variation in velocity) required to correct every gravity assist manoeuvre, ΔV_0 is the departure manoeuvre, while ΔV_{LT} is the sum of the total ΔV of each low-thrust leg. Then, $k_{2,i}$ is the $i-th$ shaping parameter for the exponential sinusoid and n_i the number of revolutions around the Sun, t_0 is the departure time and T_i the transfer time from planet i to planet i+1. The two specific impulses I_{sp1} and I_{sp2} are respectively for a chemical engine and for a low-thrust engine and g_0 is the gravity acceleration on the surface of the Earth. For the tests in this paper, we used $I_{sp1} = 315$s and $I_{sp2} = 2500$s. Thus, the MOP under consideration reads as follows:

$$\begin{aligned} \text{minimise:} &\quad \begin{cases} J(y) \\ t_N - t_0 \end{cases} \\ \text{subject to:} &\quad r_p \geq r_{min} \end{aligned} \tag{3}$$

where r_p is the vector of the minimum admissible distances from each of the swing-by planets and t_N is the time of arrival at destination.

3 A Genetic Algorithm for the Computation of $P_{Q,\epsilon}$

In this section we propose a MOEA which aims for the computation of the set of approximate soluttions, $P_{Q,\epsilon}$-NSGA-II, which is a hybrid of NSGA-II ([2]) and the archiver $ArchiveUpdateP_{Q,\epsilon}$. Further, in order to be able to compare the obtained solutions with an other strategy, we introduce a perfomance metric.

The Algorithm The algorithm we propose in the following is based on NSGA-II. We have decided to take this one as base algorithm by two reasons. First, this algorithm is well-known and proven to be very efficient. Second, we think that the elements which constitute NSGA-II fit nicely to our context: a (finite) archive A containing points which are mutually non-$(-\epsilon)$-dominating can be viewed as a set of Pareto fronts with different ranks, and also in the current setting the first front (i.e., the non-dominated front) should be given the priority since (i) improvement of the current set is clearly an objective and—in case the solutions are already near to P_Q—a local search around P_Q (e.g., via mutation) is a search within $P_{Q,\epsilon}$. Thus, we have decided to adopt the ranking from NSGA-II, as well as the crowding distance in order to maintain diversity. Finally, we also adopt the genetic operators since they are proven to be well-suited for continuous problems.
The algorithm $P_{Q,\epsilon}$-NSGA-II reads as follows: the initial offspring $\mathcal{O} \subset Q$ is chosen at random, and the first archiver is set to $\mathcal{A}_0 := ArchiveUpdateP_{Q,\epsilon}(\emptyset, \mathcal{O}_0, \Delta)$. Alg. 2 describes how to obtain the subsequent archives $\mathcal{A}_{l+1}$ from $\mathcal{A}_l$. Hereby the function $Select()$ picks $n_p/2$ elements from $\mathcal{A}$ at random, if $|\mathcal{A}| \leq n_p/2$ then $\mathcal{C} := \mathcal{A}$ is chosen (n_p denotes the population size). The next three operators are as in NSGA-II: $DominationSort()$ assigns rank and crowding distance to $\mathcal{C}$, $TournamentSelection()$ performs the tournament selection, and $GeneticOperator()$ performs simulated binary crossover and polynomial mutation on $\mathcal{P}$. Finally, the archive $\mathcal{A}_l$ is updated by $\mathcal{O}$ using $ArchiveUpdateP_{Q,\epsilon}$ leading to the new archive $\mathcal{A}_{l+1}$.
The new algorithm is in fact very close to NSGA-II, merely the selection strategy to keep the 'promising' points of the search has changed (by adding an archive to NSGA-II). Recall that the motivation for the storage of approximate solutions is to obtain in addition to the 'optimal' points also points which are close to these points in image space but which differ significantly in parameter space. Thus, it is desired to maintain a certain diversity in parameter space, and that is why the chromosomes $\mathcal{C}$ are chosen randomly from the current archive by $Select()$.

Performance Metric In order to be able to compare the results of different algorithms, or just two sets A and B, we propose to use the following metric:

$$\mathcal{C}_{-\epsilon}(A, B) := |\{b \in B \;:\; \exists a \in A : a \prec_{-\epsilon} b\}|/|B|, \tag{4}$$

which is a straightforward extension of the *set coverage metric* suggested in [14]. Analogue to the original metric, $\mathcal{C}_{-\epsilon}(A, B)$ is an unsymmetric operator which aims to get an idea of the relative spread of the two solution sets.

Algorithm 2 Iteration step of $P_{Q,\epsilon}$-NSGA-II

Require: archive $\mathcal{A}_l$, $\Delta \in \mathbb{R}_+$, population size n_p
Ensure: updated archive $\mathcal{A}_{l+1}$
1: $\mathcal{C} := Select(\mathcal{A}_l, n_p/2)$
2: $\mathcal{C}' := DominationSort(\mathcal{C})$
3: $\mathcal{P} := TournamentSelection(\mathcal{C}')$
4: $\mathcal{O} := GeneticOperator(\mathcal{P})$
5: $\mathcal{A}_{l+1} := ArchiveUpdateP_{Q,\epsilon}(\mathcal{A}_l, \mathcal{O}, \Delta)$

4 Numerical Results

Here we present some numerical results coming from three different settings. For the internal parameters (e.g., mutation probability) of NSGA-II we have followed the suggestions made in [2], and have taken the same values for $P_{Q,\epsilon}$-NSGA-II.

4.1 Two Impulse Transfer to Asteroid Apophis

For the bi-impulse problem we analyze an apparently simple case: the direct transfer from the Earth to the asteroid Apophis. The contour lines of the sum of the two Δv's is represented in Fig.2a) for $t_0 \in [3675, 10500]^T$ MJD2000 and $T \in [50, 900]$days. The intervals for t_0 and T were chosen in such a way that a wide range of launch opportunities are included. The solution space presents

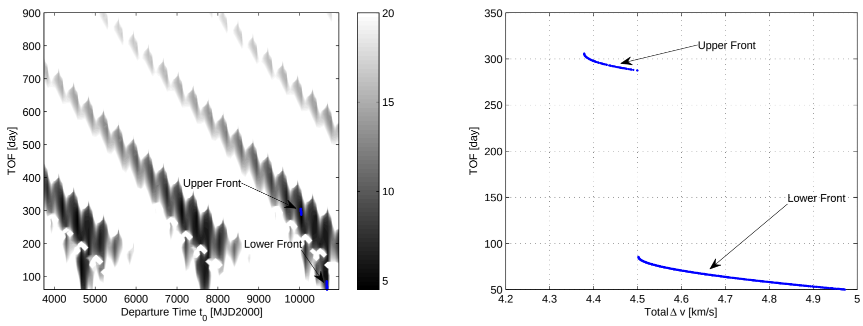


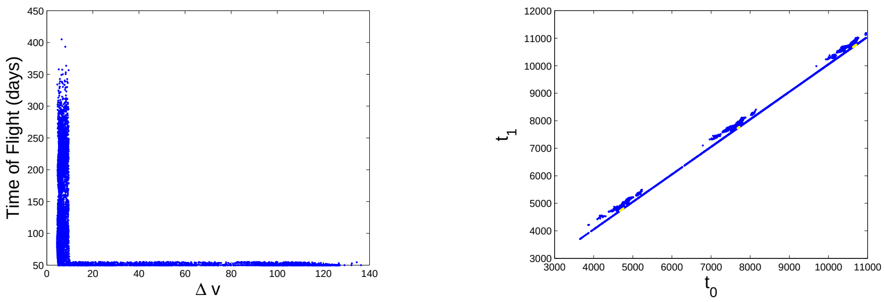
Fig. 2. a) Earth-Apophis search space, b) pareto front

a large number of local minima. Many of them are nested, very close to each other and with similar values. For each local minimum, there can be a different front of locally Pareto optimal solutions. The global Pareto front contains the best transfer with minimum total Δv and the fastest transfer with minimum T. The best known approximation of the global Pareto front is represented in Fig. 2b) and was obtained with an extension to MOO problems of the algorithm described in [?]. It is a disjoint front corresponding to two basins of attraction of two minima as can be seen if Fig. 2a).

The two basins of attraction present similar values of the first objective function. Converging to the upper front is therefore quite a challenge since the lower front has a significantly lower value of the second objective function. It is only when the optimizer converges to the a vicinity of the local minimum of the upper front that the latter becomes not dominated by the lower front. The upper front contains the global minimum with a total $\Delta v = 4.3786$ k/s while the lower front contains only a local minimum. It should be noted that, though the front in Fig. 2b) is the global one, it represents only two launch opportunities. Furthermore for each launch opportunities we would need to characterize the space around each of the Pareto optimal point.

Figure 3 shows a result for $P_{Q,\epsilon}$-NSGA-II and using $\epsilon = (5, 5)$. Since apparently the transfer of 50 days can be reached from any starting date the bi-objective problem shrinks in practice down to a monobjective problem with optimal image value around $y_0 = (5, 50)$. A search within $N(y_0, \epsilon, A)$ revealed different possible starting times which fall into three cluster: the preimages of $N(y_0, \epsilon, A)$ are all located around the points $c_1 = (4700, 50)$, $c_2 = (7700, 50)$, and $c_3 = (10700, 50)$. That is, the starting time t_0 differs by 3000 days for neighboring solutions, and by 6000 in total.
Note that, compared to the accurate solution of the global Pareto front, the extended $\epsilon-$pareto set offers, as required, not only more launch opportunities but also the whole neighboring solutions for each one of them.

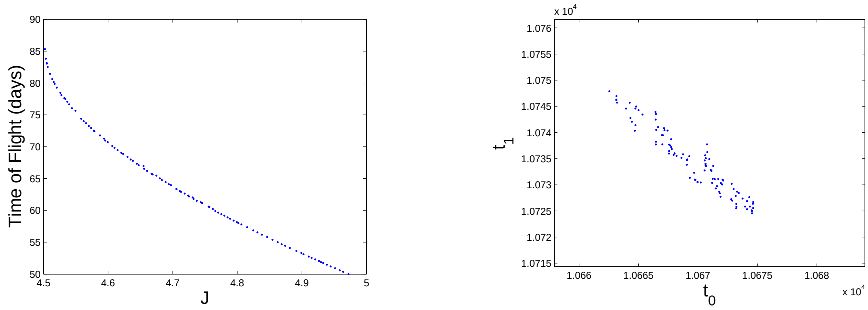
As a comparison we have used the classical NSGA-II to attack the problem (see Figure 4, note the different scale of these figures to Figure 3). As anticipated, NSGA-II computes a good approximation of the (very narrow) Pareto set, but in fact generates point only around c_3, the maximal difference according to t_0 is given by 35 days.



(a) Image space

(b) Parameter space

Fig. 3. Numerical result for Example 2 using $P_{Q,\epsilon}$-NSGA-II.



(a) Image space (b) Parameter space

Fig. 4. Numerical result for Example 2 using NSGA-II.

4.2 Sequence EVMe

For the MLTGA problem we first consider a relatively simple but significant case: the sequence Earth – Venus – Mercury (EVMe).

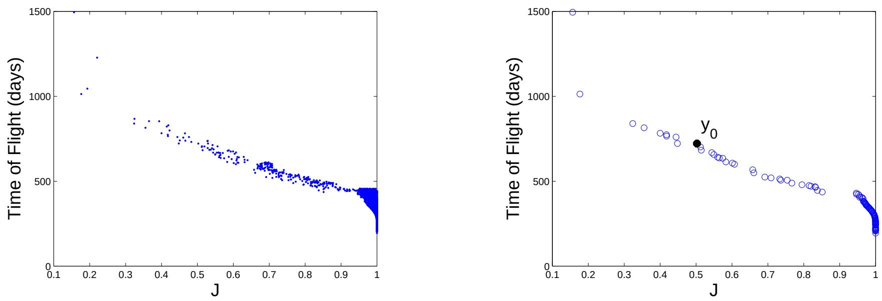
For such a mission we have chosen to allow a deterioration of 5% of the mass fraction and of 20 days transfer time compared to an optimal trajectory which leads to $\epsilon = (0.05, 20)$. Figure 5 shows a numerical result of $P_{Q,\epsilon}$-NSGA-II for 100 generations with population size 100 (i.e., the size of $\mathcal{P}$ in Alg. 2) and $\Delta = \epsilon/3$, which took several minutes on a standard PC. To compare the result and since so far no such algorithm exists we have taken a random search procedure coupled with $ArchiveUpdateP_{Q,\epsilon}$. For $N_R = 10,000$ randomly chosen points we obtain (averaged of 20 test runs) $\mathcal{C}_{-\epsilon}(A_N, A_R) = 0.4739$ and $\mathcal{C}_{-\epsilon}(A_R, A_N) = 0$, where A_N denotes the result from $P_{Q,\epsilon}$-NSGA-II and A_R the result coming from the random search procedure. For $N_R = 100,000$ the result of the random search procedure can still not compete with the same MOEA result: $\mathcal{C}_{-\epsilon}(A_N, A_R) = 0.4261$, $\mathcal{C}_{-\epsilon}(A_R, A_N) = 0$.

Interesting for every non-dominated point x_0 with $F(x_0) = y_0$ of an archive A is the set

$$N(y_0, \epsilon, A) := \{a \in A \ : \ F(a) \in B(y_0, \epsilon)\}, \tag{5}$$

where $B(y, \epsilon) := \{x \in \mathbb{R}^k \ : \ |x_i - y_i| \leq \epsilon_i, \ i = 1, .., k\}$, i.e., the set of solutions in A those images are 'close' to y_0. Since in this design problem the starting date t_0 of the transfer is of particular interest one can e.g. distinguish the entries in $N(y_0, \epsilon, A)$ by the value of t_0. For instance, the final archive displayed in Figure 5 (a) consists of 3650 solutions whereof 106 are non-dominated. The maximal difference of the value of t_0 for a point y_0 inside $N(y_0, \epsilon, A)$ is 449 days, and for 23 solutions this maximal difference is larger than one year (including also values Δt_0 of several days or months which can be also highly interesting for the decision making process). Hence, the number of options for the DM is enlarged significantly in this example.

The consideration above leads to a natural way of presenting the large amount of data to the DM: it is sufficient to present non-dominated front as in the 'classical' multi-objective case. When the DM selects one solution y_0 the set $N(y_0, \epsilon, A)$ can be displayed, ordered by the value of t_0 (see Figure 5 (b)).



(a) ϵ-efficient front

(b) non-dominated front

Fig. 5. Numerical result for sequence EVMe. Left the final archive and right the set of non-dominated solutions which is in this case sufficient to display for the decision making process.

4.3 Sequence EVEJ

Finally we consider the more complex sequence Earth – Venus – Earth – Jupiter (EVEJ) which consists of 7 parameters.
This design problem—as well as other problems of this kind—is characterized by a disconnected feasible domain, and the fraction of this set compared to the entire search space is tiny. In order to increase the performance of the search procedure, we followed [12] and have applied a space pruning algorithm on the search space leading to set $\mathcal{B}$ such this set covers the feasible domain and where the volume of $\mathcal{B}$ is much less than the volume of the entire search space. Next, we have run $P_{Q,\epsilon}$-NSGA-II on these eight domains separately (population size 100, 30 generations) and have merged the results afterwards. We have chosen again $\epsilon = (0.05, 20)$, but $\Delta = (0, 0)$ since the feasible solutions are not too easy to find, and thus, every 'good' solution should be captured. Figure 6 shows one numerical result, which has been obtained within one hour. As one example we assume that the point y_0 has been selected by the DM (see Fig. 6). The set $N(y_0, \epsilon, A_{final})$ consists of in this case of three solutions with starting times $t_{0,1} = 5085$, $t_{0,2} = 5562$, and $t_{0,3} = 6792$. Assuming that the values of y_0 have been chosen for the transfer and the archive A is the basis for the DM, then there are three possibilities: to launch the spacecraft in December 2013 (i.e., 5085 days after 01.01.2000), to launch it 16 months later, or to wait another 3.5 years after $t_{0,2}$. Similar statements hold in this example for all 15 non-dominated solutions

x with $J = f_1(x) \geq 0.45$, and thus, also in this case the DM's decision space has been augmented by allowing approximate solutions.

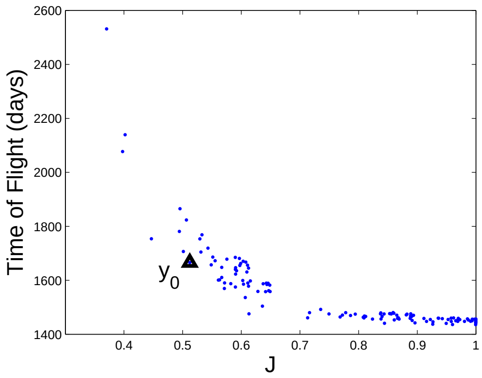


Fig. 6. Numerical result for sequence EVEJ using $P_{Q,\epsilon}$-NSGA-II.

5 Conclusion and Future Work

We have considered multi-objective space mission design problems. For this kind of problems, it is desirable to identify not only the global pareto set, but also a number of neighboring solutions. In particular, it was shown that each part of the pareto set belongs to a different launch window. In order to increase the reliability of the mission design, it is required to have a wide launch window (i.e. a large number of solutions with similar cost) and one or more back-up launch windows (i.e. one or more locally pareto optimal sets).

In order to address this problem, we have proposed a new variant of an existing MOEA which aims for the computation of $P_{Q,\epsilon}$. As an example of its effectivness, we have considered three design problem. The results indicate that the novel approach accomplishes its task within reasonable time and that the idea to include approximate solutions is indeed beneficial since in all cases the enlarged set of solutions offered a much larger variety to the DM.

Though the algorithm proposed in this paper seems to be well-suited for the design problems under consideration, the authors think that its performance can be increased in general, and in particular a variant for disconnected domains has to be developed (see Section 4.3). Further, since the underlying idea of the novel approach is to obtain large variety of the solutions in parameter space, it would be desirable to have a performance metric which measures this. The one proposed here merely consideres the approximation quality in objective space.

References

1. V. Coverstone-Caroll, J.W. Hartmann, and W.M. Mason. Optimal multi-objective low-thrust spacecraft trajectories. *Computer Methods in Applied Mechanics and Engineering*, 186:387–402, 2000.
2. Kalyanmoy Deb, Amrit Pratap, Sameer Agarwal, and T. Meyarivan. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA–II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, April 2002.
3. T. Hanne. On the convergence of multiobjective evolutionary algorithms. *European Journal Of Operational Research*, 117(3):553–564, 1999.
4. J. D. Knowles and D. W. Corne. *Metaheuristics for Multiobjective Optimisation*, volume 535 of *Lecture Notes in Economics and Mathematical Systems*, chapter Bounded Pareto Archiving: Theory and Practice, pages 39–64. Springer, 2004.
5. M. Laumanns, L. Thiele, K. Deb, and E. Zitzler. Combining convergence and diversity in evolutionary multiobjective optimization. *Evolutionary Computation*, 10(3):263–282, 2002.
6. P. Loridan. ϵ-solutions in vector minimization problems. *Journal of Optimization, Theory and Application*, 42:265–276, 1984.
7. Günter Rudolph and Alexandru Agapie. Convergence Properties of Some Multi-Objective Evolutionary Algorithms. In *Proceedings of the 2000 Conference on Evolutionary Computation*, volume 2, pages 1010–1016, Piscataway, New Jersey, July 2000. IEEE Press.
8. G. Ruhe and B. Fruhwirt. ϵ-optimality for bicriteria programs and its application to minimum cost flows. *Computing*, 44:21–34, 1990.
9. O. Schütze, C. A. Coello Coello, and E.-G. Talbi. Approximating the ϵ-efficient set of an MOP with stochastic search algorithms. In A. Gelbukh and A. F. Kuri Morales, editors, *Mexican International Conference on Artificial Intelligence (MICAI-2007)*, pages 128–138. Springer-Verlag Berlin Heidelberg, 2007.
10. O. Schütze, C. A. Coello Coello, E. Tantar, and E.-G. Talbi. Computing finite size representations of the set of approximate solutions of an MOP wiht stochastic search algorithms. To appear in the Proceedings of the Genetic and Evolutionary Computation Conference (GECCO'08), 2008.
11. O. Schütze, M. Vasile, O. Junge, M. Dellnitz, and D. Izzo. Designing optimal low thrust gravity assist trajectories using space pruning and a multi-objective approach. To appear in Engineering Optimization, 2008.
12. M. Vasile, O. Schütze, O. Junge, G. Radice, and M. Dellnitz. Spiral trajectories in global optimisation of interplanetary and orbital transfers. Research report, European Space Agency, 2006.
13. D. J. White. Epsilon efficiency. *Journal of Optimization Theory and Applications*, 49(2):319–337, 1986.
14. E. Zitzler. *Evolutionary Algorithms for Multiobjective Optimization: Methods and Applications*. PhD thesis, ETH Zurich, Switzerland, 1999.