Applications of Inverse Simulation Methods to a Nonlinear Model of an Underwater Vehicle

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Abstract

Inverse simulation provides an important alternative to conventional simulation and to more formal mathematical techniques of model inversion. The application of inverse simulation methods to a nonlinear dynamic model of an unmanned underwater vehicle with actuator limits is found to give rise to a number of challenging problems. It is shown that this particular problem requires, in common with other applications that include hard nonlinearities in the model or discontinuities in the required trajectory, can best be approached using a search-based optimization algorithm for inverse simulation in place of the more conventional Newton-Raphson approach. Results show that meaningful inverse simulation results can be obtained but that multi-solution responses exist. Although the inverse solutions are not unique they are shown to generate the required trajectories when tested using conventional forward simulation methods.

1. INTRODUCTION

Methods of inverse simulation have been developed and used successfully for many years in aircraft flight mechanics investigations, handling qualities studies and for the assessment of aircraft manoeuvrability. These techniques provide a convenient basis for model inversion and are applicable to any form of mathematical or computer-based model. The most widely used approaches (e.g. [1-4]) involve a form of Newton-Raphson algorithm but other approaches have been successfully applied including methods based on differentiation [5, 6] and an a more recently developed approach based on sensitivity functions [7]. Thomson and Bradley have provided two useful reviews of inverse simulation techniques in the context of aircraft and helicopter applications [8, 9].

A recent development [10] involves an approach that uses search-based optimisation methods such as the Nelder-Mead algorithm. Methods such as this are derivative-free and have particular advantages in situations involving saturation constraints or discontinuities in the model or in the manoeuvres to be considered. Surface ships have hard limits in terms of rudder deflection, rudder deflection rate and propeller characteristics. Underwater vehicles also have these limits and with additional control surfaces such as stern-planes and bow-planes such vessels have hard limits associated with all the control inputs.

One of the factors commonly encountered in applying inverse simulation methods to complex multi-input multi-output nonlinear systems is that there can be a lack of uniqueness in the inverse solutions. In other words, a number of input combinations can produce outputs that match the required output trajectory to the required accuracy. In practical engineering applications this is not necessarily a major difficulty and for many situations what is important is that an inverse solution can be found that gives the required set of outputs when tested through subsequent forward simulation.

2. OUTLINE OF THE UUV MODEL

The underwater vehicle model considered in this work is based on the nonlinear state space description by Healey and Lienard in their 1993 paper on multivariable sliding mode control of unmanned underwater vehicles [11]. The model involves twelve state variables – the surge, sway and heave velocities, the roll, pitch and yaw angular velocities, positions in the x, y and z directions and the roll, pitch and yaw angles. There are six input variables – the rudder angle (δ_r), the port and starboard stern plane angles (δ_s), the top and bottom bow plane angles (δ_b), the port bow plane angle (δ_{bp}), the starboard bow plane angle (δ_{bs}), and the propeller shaft speed (n). There are four output variables – positions in x, y and z-directions and the yaw angle (Ψ). The model presented by Healey and Lienard has been applied by Fossen [12] and by McGookin who has introduced a bilinear thruster sub-model and has modified some aspects of the model to give a better representation of the surge characteristics [13]. This modified description has also been the basis of the underwater vehicle model employed in recent collaborative work on multi-rate simulation involving California State University Chico, the University of South Carolina and the University of Glasgow [14].

3. ESSENTIAL FEATURES OF THE NELDER-MEAD APPROACH

In a review of direct search methods Lewis *et al.* [15] point out that these techniques remain popular because they are simple, flexible, and reliable. The downhill simplex method of Nelder and Mead (NM) [16] is one of the most widely used of these direct-search algorithms. It is suitable for minimizing a scalar-valued nonlinear function of q real variables using only function values, without any explicit or implicit derivative information since it avoids any need to determine the elements of Jacobian or Hessian matrices that are a feature of the more conventional methods of inverse simulation. Developments of this search-based approach [17-19] allow it to be used to tackle multimodal, discontinuous, and constrained optimization problems.

For the purposes of inverse simulation, as with the NR method, the NM approach is applied over the interval [t_k, t_{k+1}]. One of the main differences is that the NM method relies exclusively on values of a cost function to find the optimal solution [15] and it is thus very important to use an appropriate form of cost function. One suitable cost function may be described by the following:

$$\begin{cases} \min\limits_{\boldsymbol{u}\in \mathrm{R}^q} L[\boldsymbol{u}(t_k)], \quad \text{where} \\ L[\boldsymbol{u}(t_k)] = \sum\limits_{i=1}^{p} \{\boldsymbol{h}_i[\boldsymbol{u}(t_k), \boldsymbol{x}(t_{k+1})] - \boldsymbol{y}_{d_i}(t_{k+1})\}^2 \end{cases} \tag{1}$$

subject to

$$\begin{cases} u_{min,j} \le u_j(t_k) \le u_{max,j} \qquad j = 1,2,\ldots,q \\ \dot{\boldsymbol{x}} = \boldsymbol{f}[\boldsymbol{x}(t_k), \boldsymbol{u}(t_k)] \end{cases} \tag{2}$$

where $L[\cdot]$ is the cost function. If the NM algorithm fails for the quadratic cost-function form of Eq. (1), the following equation based on the absolute value provides a potentially useful alternative:

$$\begin{cases} \min\limits_{\boldsymbol{u}\in \mathrm{R}^q} L[\boldsymbol{u}(t_k)], \quad \text{where} \\ L[\boldsymbol{u}(t_k)] = \sum\limits_{\boldsymbol{i}=1}^{\boldsymbol{p}} \left| \boldsymbol{h}_{\boldsymbol{i}}[\boldsymbol{u}(t_k), \boldsymbol{x}(t_{k+1})] - \boldsymbol{y}_{d_i}(t_{k+1}) \right| \end{cases} \tag{3}$$

The process of finding solutions is divided into two sub-processes: one-forward simulation to obtain $\boldsymbol{x}(t_{k+1})$ and then calculation of the solution $\boldsymbol{u}(t_k)$ from Eq. (1) or Eq. (3) with the available values $\boldsymbol{x}(t_{k+1})$. For the case where only input saturations are of interest the inequalities in Eq. (2) may be solved by means of two transformations before the solution process of Eq. (1) or Eq. (3).is applied These transformations are based on concepts outlined by Errico [20].

The first transformation changes the original domain of the input variables into a new space. The unconstrained input variables will be left alone. In cases where both upper and lower bounds are required, which commonly arises with actuators used in engineering control systems, a *sin* transformation is used as follows:

$$\begin{aligned} &\text{if } u_j(t_k) \le u_{\min,j} \quad \text{then } u_{a,j} = -\pi/2 \\ &\text{or if } u_j(t_k) \ge u_{\max,j} \quad \text{then } u_{a,j} = \pi/2 \\ &\text{otherwise} \\ &u_{a,j} = \arcsin[-1, (1, 2\frac{u_j(t_k) - u_{\min,j}}{u_{\max,j} - u_{\min,j}} - 1)_{min}]_{max} \end{aligned}$$

where $\boldsymbol{u}_a$ is the transformed input vector.

The second step is used to transform the new input domain back into the original domain, again leaving unconstrained input variables unchanged but with the transformed values appropriately bounded. The transformation for the particular case where both the lower and upper bounds are required again involves a *sin* function and can be defined as follows:

For the lower and upper bounds:

$$u_{b,j} = \frac{1}{2}\{\sin[u_{a,j}(t_k)] + 1\}\{u_{\max,j} - u_{\min,j}\} + u_{\min,j}$$

where $\boldsymbol{u}_b$ is now the bounded input vector needed for the application of the next stage in this process which involves evaluation of the cost function and the search-based optimisation. Errico [20] provides full details of this transformation and transformations for other cases, such as those where an input variable is constrained only by a lower or upper bound.

The particular form of the NM algorithm used is the modified version described by Lagarias [21]. The algorithm first characterises a simplex in q dimensional space using $q + 1$ vertices. Then, based on four rules that involve simple processes of reflection, expansion, contraction and shrinkage, a new point in or near the current simplex is generated. Then a new simplex can be found by replacing a vertex in the original simplex after the function value from Eq. (1) or Eq (3) is compared with the function values at the original vertices. This process is carried out repeatedly until the diameter of the simplex is smaller than some specified value. Optimum solutions are thus found for the step under consideration. If each step converges successfully the complete set of input .time histories can be formed by combining together the solutions obtained over each interval.

The final stage in this four-step process involves transformation of the final solutions back to the original domain. The whole process outlined above is illustrated by the following flow chart:

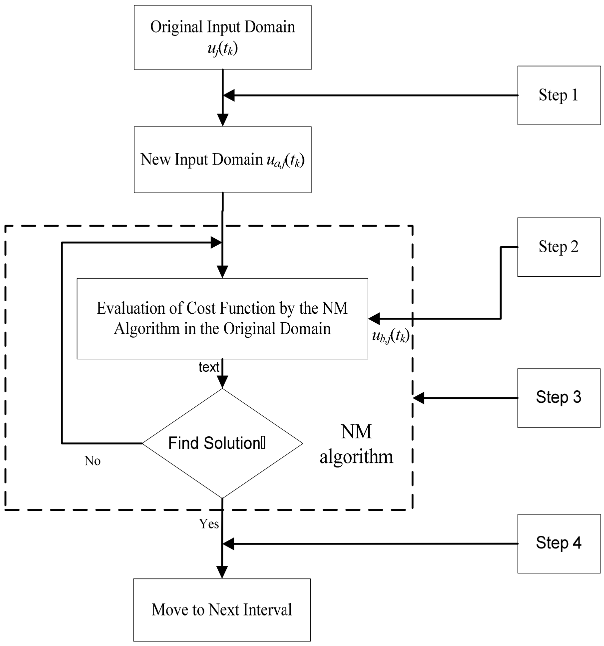

Figure1. Flow chart for the k_{th} interval of inverse simulation with the constrained NM algorithm

The initial guess values for the input at any new step are the calculated values from the previous step and, if the manoeuvre is smooth and continuous, this is certainly a good starting point. Even in the presence of discontinuous points, the probability of the NM algorithm finding a global solution is still high because a piece-wise constant input is assumed within the time interval between t_k and t_{k+1}, rather than a set of time-varying signals $\boldsymbol{u}(t)$, which would span a higher dimensional space. Hence, problems associated with discontinuous points in the manoeuvre, which may harm convergence of standard NR schemes are successfully handled by the NM method.

4. APPLICATION TO THE NONLINEAR UUV MODEL

The constrained NM method has been applied to the nonlinear UUV model and results presented here relate to two specific types of manoeuvre The first of these is a turning circle manoeuvre and the second is a zig-zag manoeuvre.

The parameters configured to generate the manoeuvres are as follows: the time point at which rudder movement is executed is 5 s for both the turning circle and the zigzag. Hence, the application discussed in this section involves a redundant situation in that the number of inputs (six) is larger than the number of outputs (four). The cost function is defined by Eq. (1) with dimension equal to four.

Figure 2 shows the results obtained from inverse simulation of the UUV performing a turning circle type manoeuvre using the Nelder-Mead method. In this case the desired manoeuvre was defined by carrying out a conventional forward simulation. The demanded rudder angle which was applied for the turning-circle manoeuvre was 25 deg. (applied in a stepwise fashion) which involves a magnitude of step that actually exceeds the saturation level of 20deg. The propeller speed for the manoeuvre was a step input from zero to a demanded level of 1200 rpm. All the other control surfaces were held constant at demanded values of zero. Therefore, the only inputs used for the manoeuvre were the propeller and the rudder and clearly the actual rudder input for the UUV model was the saturation value (20 deg).

As shown in Figure 3, the main calculated input values – δ_r, and n, found from the inverse simulation process comply well with the expected values of 20 deg. and 1200rpm. However, results for other input channels differ significantly from the ideal values of zero. It should be noted, however, that these other inputs are within the saturation limits for each of those input channels and thus well within the range of feasible input values.

Although the calculated inputs shown in Figure 3 do not correspond exactly to the inputs that might have been expected, the responses found when these six calculated inputs are applied to a forward simulation for the UUV model are still consistent with the ideal manoeuvre. This may be seen in Figure 3 by comparing the continuous response curves in terms of the *x, y* and *z* position variables and also the heading variable with the corresponding pattern of points on the ideal trajectory shown by * symbols. These points correspond to points on the demanded manoeuvre generated using the initial forward simulation and thus used within the inverse simulation process. Minor differences exist in terms of the heading variable but these are not significant. This difference between the expected and calculated set of inputs is believed to be a multi-solution phenomenon.

For the case of a zigzag manoeuvre the input is more complex and involved manipulation of all five control surfaces and propeller. In this case demanded manoeuvre was generated using a square wave sequence of input values for the rudder which was switched periodically between 15 deg and -15 deg. The other four control surfaces were moved to 15 deg and held at that value throughout the manoeuvre. The propeller speed was changed from zero to 1200 rpm in a step-like fashion at the start of the manoeuvre. It should be noted that all of these input values are within the saturation limits.

Inverse simulation using the demanded manoeuvres generated from the preliminary forward simulation was applied to the UUV model using the Nelder-Mead method. The results in terms of the calculated inputs were, as shown in Figure 4, completely different from the set of inputs used to generate the demanded trajectory. Instead of being based on periodic inputs switching between 15deg. and -15 deg. the input sequences were more complex and differed in terms of the inputs calculated for the different input channels.

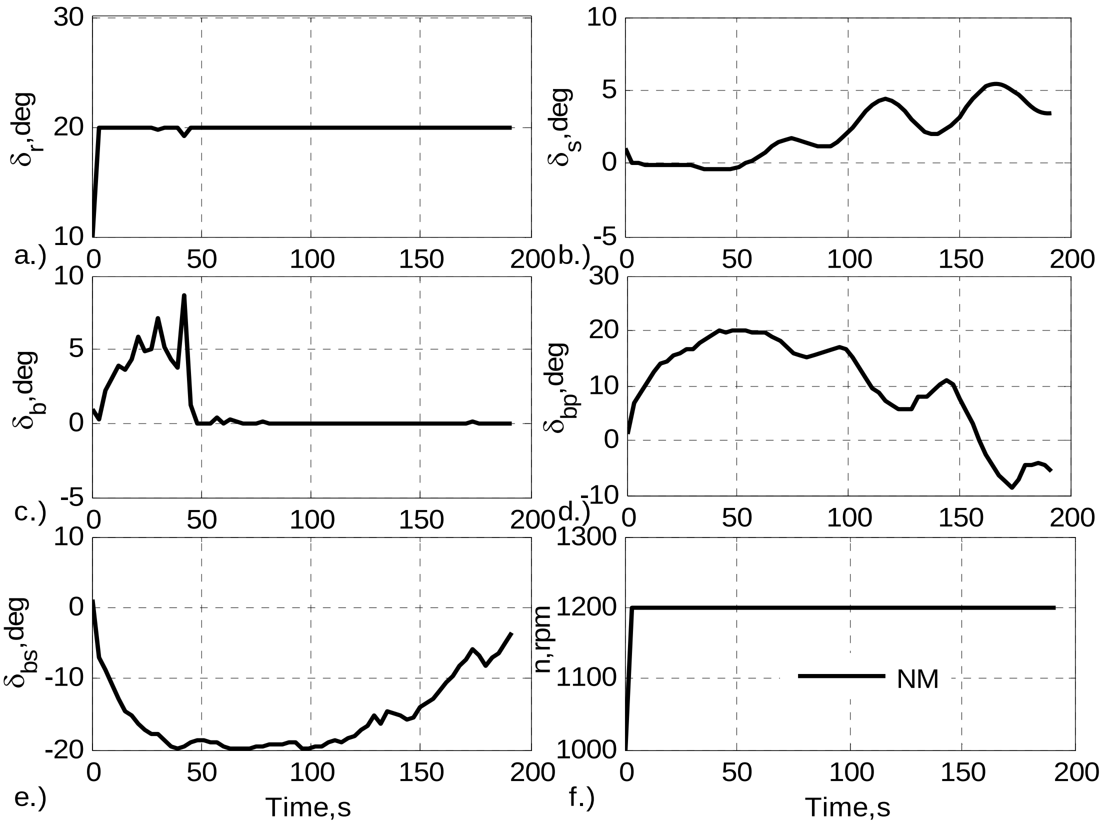

Figure 2. Inputs obtained from inverse simulation for a turning circle manoeuvre for the UUV model with saturation limits, using on the Nelder-Mead method (Δt = 3 s).

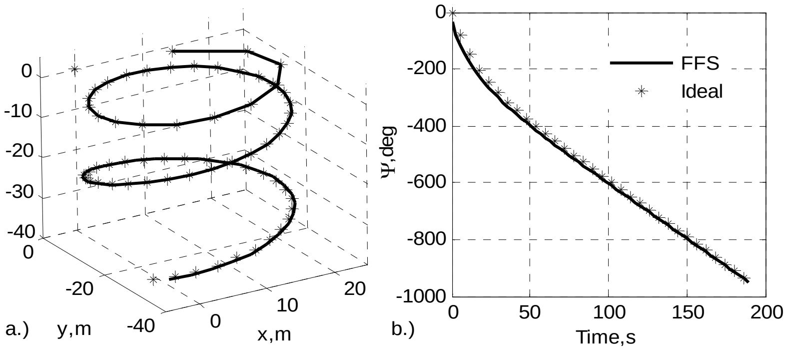

Figure 3 Results obtained from forward simulation of the UUV using the inputs shown in Figure 2 (continuous lines) for the turning circle manoeuvre compared with the ideal manoeuvre (points indicated by * symbol).

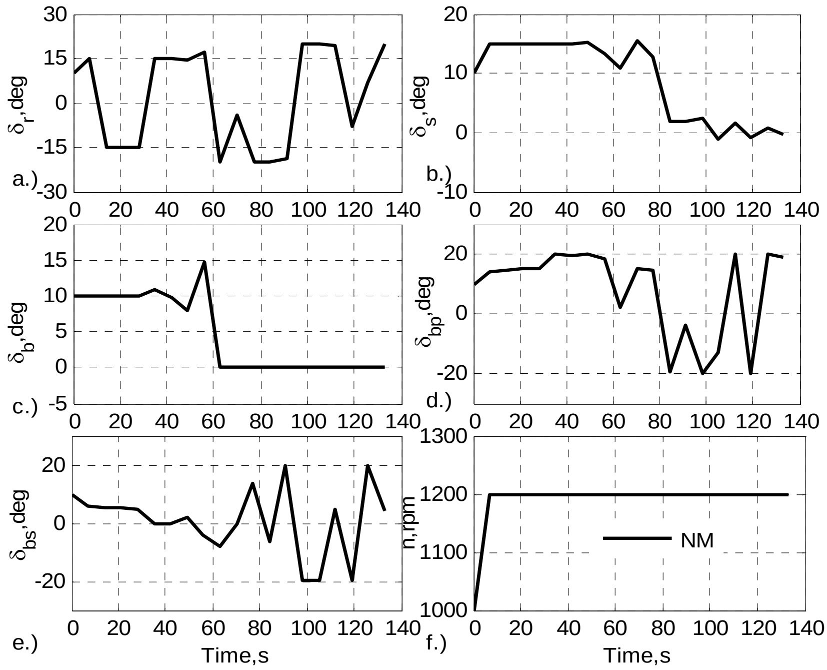

Figure 4. Input variables obtained from the application of inverse simulation to the UUV model with saturation limits for the zigzag manoeuvre using the Nelder-Mead approach (Δt = 7 s)

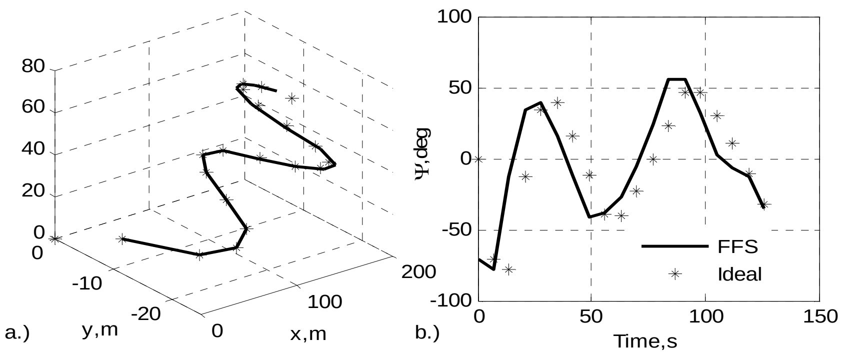

Figure 5. Results obtained from forward simulation (FFS) of the UUV using model with saturation limits showing comparison with the ideal manoeuvre in terms of *x, y* and *z* axis displacements (Fig. 5a) and heading versus time (Fig. 5b) (Δt = 7s)**.**

However, Figure 5 shows that the output results calculated from a conventional forward simulation with the calculated inputs applied to the UUV model agree well with the ideal zigzag manoeuvre except for the case of the yaw angle Ψ which diverges slightly from the ideal. This same phenomenon also appears in Figure 3, as noted previously.

As with the turning circle manoeuvre this apparent difficulty with the inputs for with the zigzag manoeuvre appears to be a multi-solution phenomenon. The outputs of the forward simulation for the case with these calculated inputs still comply, in general terms, with the ideal trajectories. It is believed that the slight divergence in the yaw angle channel in Figures 3 and 5 may arise from the relatively large Δt value used in these two examples.

5. DISCUSSION

In general terms, the outputs of the forward simulation found using the inputs determined by the Nelder-Mead inverse simulation method comply well with the ideal zigzag manoeuvre and with the circle manoeuvre. In contrast, it should be noted that the standard Newton Raphson method for inverse simulation fails to converge for all situations involving this model, even for the case of the smooth turning-circle manoeuvre where there are no discontinuities within the trajectory.

In addition, it should be noted that the Newton Raphson algorithm also fails for this UUV model for situations without input saturation. This is interesting and unexpected as the Newton-Raphson algorithm has been applied with success in other applications, such as helicopters, which also involve multi-input multi-output descriptions with significant nonlinearities. Further research is required to gain a fuller understanding of the reasons for these difficulties and the range of situations in which the NR algorithm presents difficulties of this kind but for which the search-based Nelder-Mead approach converges satisfactorily.

The results obtained with both the turning circle and the zig-zag manoeuvres suggest that the control efforts required to perform such manoeuvres are not unique. Different patterns of input applied to the available input channels can produce identical, or very similar, responses in terms of the output variables. Similar problems have been encountered in the course of investigations involving inverse simulation methods applied to complex models of surface ships [10] and this multi-solution phenomenon has also been mentioned by Gao and Hess [2]. Therefore, in addition to providing information that is potentially useful for providing enhanced physical insight relating to practical problems such as the sizing of actuators, inverse simulation also may possibly provide a tool for control allocation [18] or may facilitate finding an optimal trajectory from the set of possible trajectories [19].

6. CONCLUSIONS

A new, completely derivative-free, procedure has been presented for inverse simulation, based on the constrained Nelder-Mead algorithm. Some issues of inverse simulation associated with input saturation and discontinuous manoeuvres have been explored and discussed. The method presented solves the constrained problem by one-step forward simulation and the application of input transformations.

Simulation studies involving a relatively complex nonlinear model of an unmanned underwater vehicle have shown that the new method of inverse simulation provides better convergence and numerical stability for cases involving input saturation or discontinuous manoeuvres. However, for severe manoeuvres, a multi-solution phenomenon may appear in the results.

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