Harrison, P. and Wiggers, J. and Long, A.C. (2008) Normalisation of shear test data for rate-independent compressible fabrics. *Journal of Composite Materials* 42(22):pp. 2315-2344.

Normalisation of shear test data for rate-independent compressible fabrics

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Abstract

This paper describes a method of using both Picture Frame (PF) and Bias Extension (BE) tests together to characterise accurately the trellis shearing resistance of engineering fabrics under low in-plane tension conditions. Automated image analysis software has been developed to reduce the amount of laborious manual analysis required to interpret BE data accurately. Normalisation methods for both PF and BE tests on rate-independent compressible fabrics are presented. Normalisation of PF test results is relatively straightforward while normalisation of BE test results for direct comparison with PF data is more complicated. The normalisation method uses a number of simple assumptions to account for the non-uniform shear strain field induced across BE samples during testing. Normalised results from BE tests on samples of different aspect ratios are compared and provide validation of the theory.

Keywords: A. Textile Composites B. Mechanical Properties C. Deformation D. Shear Resistance

1 INTRODUCTION

The automated draping and subsequent resin infusion of dry reinforcing woven and non-crimp fabrics offers a viable route to manufacturing three dimensional Continuous Fibre Reinforced Composite (CFRC) components. Potential decreases in manufacture costs through simulation technology are currently driving the development of both macro [1-7] and meso-scale material models [5, 8-11] and associated characterisation experiments for dry reinforcing-fabrics. Two such characterisation experiments, specifically designed for measurements of large in-plane shear and wrinkling, include the Picture Frame (PF) [8, 10-19] and Bias Extension (BE) [6, 10, 12, 13, 20-25] test methods. The original concept behind these and other similar in-plane shear tests can be traced back to research in textile and fabric forming [26-28].

An important criterion of a material characterisation experiment is that measured properties should be independent of the test method or sample dimensions. In this paper normalisation methods for treating experimental data from both PF and BE test methods are described for shear strain rate independent materials, such as dry fabrics, undergoing so-called trellis shear. Normalisation of PF data may depend on specimen shape [29-32] though becomes relatively straightforward when using square samples that completely fill the area of the PF. In this case the measured force is normalised simply by dividing by the side length of the sample specimen. This can be justified using simple energy arguments presented both here and elsewhere [33, 34].

Normalisation of BE test data is complicated by the non-uniform strain profile occurring in the sample. Here we show how this complication can lead to errors in the measured shear force. The amount of error is quantified and a method of correcting the force data before applying, for example, the gauge method of determining stiffness properties [7, 13] is presented. It is also shown that the amount by which results must be corrected depends not just on sample dimensions but also the form of the material's force versus shear angle displacement curve. Other advantages of the normalisation method are also apparent: BE tests with a minimum initial length / width ratio of just two can be treated, i.e. with no gauge section. This is advantageous since tests on specimens with large length / width ratios can increase difficulties associated with handling the fabric, particularly when dealing with loose fabrics that tend to disintegrate easily [13]. Use of smaller length / width ratios can also decrease the amount of material required for testing.

The structure of this paper is as follows. Brief descriptions of the PF and BE tests are given in Section 2 and the method of applying each of these tests to dry fabric materials is described. Normalisation methods for both the PF and BE tests and an associated numerical algorithm are proposed in Section 3 and possible limitations of the methods are outlined. To reduce laborious manual image analysis associated with the BE test, image analysis software has been developed, and analysis reveals unexpected deformation in the central region of the BE samples that can be explained simply using pin-jointed net kinematics. In Section 4 materials used in this investigation are described and normalisation procedures are applied to results of PF and BE tests. Conclusions of the investigation are given in Section 5.

2 CHARACTERISATION TESTS

2.1 Picture frame test

A schematic of the PF test is presented in Figure 1. A tensile force is applied across diagonally opposing corners of the PF rig causing the PF to move from an initially square configuration into a rhombus. Consequently the sample held within the frame experiences trellis shear. Fibre misalignment within the PF rig can lead to large errors in the measured results [12, 17-18]. Depending on the type of misalignment, the reinforcement fibres can be forced to undergo either tensile or compressive strain. Tensile strain tends to inhibit wrinkling of the specimen but can produce large overestimates in the measured force. Compressive strains tend to promote buckling of the sample at low shear angles and decrease the amount of required fabric shear, producing a decrease in the measured force. Throughout this investigation results from PF tests in which samples buckled at low shear angles were discarded.

For dry fabrics, fibre alignment within the PF test can be improved using pre-tensioning apparatus [16-18]. Such apparatus can be used to investigate effects of in-plane tension on the shear behaviour of the material [13, 17-18]. However, measuring this tension during testing is extremely difficult. A recent attempt to do so has been made by mounting load cells along the side-bars of a PF rig [18]. The current investigation is restricted to low in-plane tension, and as such all PF tests are started with zero pre-tension. However, it is thought unlikely that this zero-tension state remains throughout the course of each PF test due to the severe boundary constraints [18]. Improved clamping of dry fabrics was achieved using thin rubber sheet placed between the fabric and clamps.

The force required to pull the crosshead of the testing machine is recorded and the trellis shear force per unit length is subsequently calculated using,

$$N_s = \frac{F_1}{2L_1 \cos\Phi} \qquad (1)$$

where Φ is the frame angle (see Figure 1) F_1 is the measured axial PF force and L_1 is the distance between the centres of the bearings of the PF rig (in this investigation L_1 = 145 mm). Test data are often analysed to produce graphs of shear force against shear angle, where the shear angle is defined as:

$$\theta = \pi / 2 - 2\Phi \qquad (2)$$

Throughout the current investigation axial force is plotted rather than the shear force. This is to facilitate comparison with BE results, which are not necessarily the result of trellis shearing, as discussed in Section 2. Consideration of the PF geometry shows that the shear angle in the material can be related directly to the displacement of the crosshead, d_1, by Equation (3)

$$\theta = \frac{\pi}{2} - 2\arccos\left[\frac{1}{\sqrt{2}} + \frac{d_1}{2L_1}\right] \qquad (3)$$

where d_1 is the displacement of the crosshead mounting (see Figure 1). This shear angle is defined from the PF geometry. In practice the shear angle measured in the material during testing may deviate by several degrees from this ideal shear angle depending on the shape of the specimen [9, 14, 17]. However, for most purposes the calculated ideal angle is a sufficiently accurate approximation.

Figure 1

The PF test procedure is simple to perform. Since the deformation of the material is essentially homogeneous throughout the deforming sample (edge effects being ignored – i.e. in-plane bending of yarns at the clamps), the kinematics of the material deformation are readily calculated, facilitating quantitative analysis of the results. A major benefit of the test is that the shear angle and current angular shear rate of the fabric can be assumed to relate directly to the crosshead displacement and displacement rate of the rig. However, one of the main concerns with the test is the boundary condition imposed on the sample. Loose pinning of the sample edges in the side clamps may fail to induce the required kinematics, whereas tight clamping of the sample edges can cause spurious results if the sample is even slightly misaligned [12, 17-18].

2.2 Uniaxial Bias-Extension test

The bias-extension test involves clamping a piece of biaxial material such that the warp and weft tows are orientated initially at +/- 45^o to the direction of the applied tensile force. Note that the clamping areas of the test specimen are cut wider than the test area in order to reduce slippage of yarns from underneath the clamps, as noted in [22, 23]. The sample dimensions can be characterised by the sample's length / width ratio, $\lambda = L_o / w_o$, where the total length of the material sample, L_o, must be at least twice the width, w_o. The reason for this is associated with 'end effects' due to the clamping constraint imposed at the two ends of the fabric. This can be seen when analysing the idealised deformation of a material sample in a BE test.

Figure 2

Figure 2 shows an idealised BE test sample with $\lambda = 2$, in which the material is divided into three regions. If the tows within the sample are considered inextensible and no intra-ply slip occurs within the sample (see Figure 3) then one can show that the shear angle in Region A is always twice that in Region B, while Region C remains un-deformed. The deformation in Region A can be considered equivalent to the deformation produced by the pure shear of a PF test. The length of the material sample must be at least twice its width in order for the three different deformation regions to exist. Increasing the length/width ratio, λ, to higher values serves to increase the length of Region A.

Like the PF test, BE tests are simple to perform and provide reasonably repeatable results. The test provides an excellent method of estimating a material's locking angle; the angle at which the material's deformation kinematics begin to deviate from trellis shear to a combination of trellis shear and intra-ply slip [35, 36]. Unlike the PF test, as long as the material sample is tightly clamped, the boundary conditions are much less relevant to the test result. However, the test does suffer from certain drawbacks. First, the shear angle in the material must be measured by time-consuming visual analysis, which can be complicated further if the sample is to be heated during testing. Second, the deformation field within the material is not homogenous, complicating analysis of the results. Finally, the test induces a combination of both pure shear and intra-ply slip, see for example, Figure 3 from Harrison et al. [35]. Figure 3 is computer-generated using idealised slip kinematics derived previously [35] and represents the sample undergoing both trellis shear as well as intra-ply slip, i.e. the sample pulls itself apart at higher shear angles with tows sliding past one another rather than being pinned at the crossovers. In terms of analysis this presents extra difficulty, though conversely, this deformation may be used to advantage as a means to investigate intra-ply slip as a potential deformation mechanism of woven fabrics, e.g. see ref [36].

Figure 3

2.2.1 Image Analysis

One of the main drawbacks of the BE test compared with the PF test is the manual effort involved in analysing recorded images of the test sample during deformation. The aim of the visual analysis is to determine shear angle throughout the test. To decrease the time and effort expended in this process, and also to improve accuracy, image analysis software has been developed by the authors.

During testing a digital video camera is used to record the deformation of the BE sample. The digital video images are stored as files on a computer. Lines are drawn on the sample along the tow directions of the sample prior to testing. The software fits linear equations to the lines drawn on the sample using a simple search algorithm (see Figure 4). Using the fitted equations the inter-fibre angle is readily calculated at several points within Region A of the specimen and Equation (2) is used to provide the shear angle. Due to the finite thickness and varying contrast of the drawn lines error is inevitable in the fitting procedure. This is manifest as 'noise' in the shear angle output. However, this error is small compared to the error involved in a manual visual analysis and the automated method allows the collection of a much larger number of data points.

Figure 4

The software also allows accurate determination of the fabric's initial state of deformation prior to testing. While an initial inter-tow angle of 90^{o} is the ideal scenario, loading and handling the specimens invariably leads to some small degree of fabric shear prior to testing. Experience shows that this initial shear can dramatically influence the repeatability of the test data. Thus, careful image analysis following testing allows shifting of the test results which leads to significant improvements in the quality of the measured data, demonstrated in Section 4. All results presented here use image analysis.

3 NORMALISATION PROCEDURE

Ideally, determination of material properties in a characterisation test should be independent of test method and sample size. For example, the shear modulus or shear viscosity (depending on material and modelling approach) and consequently the shear force produced during testing of a material should be independent of both sample dimensions (PF and BE tests) and length / width ratio (BE test). Appropriate normalisation techniques must be used before results from different tests can be compared directly.

3.1 Normalisation of Picture Frame force

A simple argument is used to justify normalisation of PF test results by the side length of the PF rig. A similar argument was presented in Harrison et al. [33, 34] and also Peng et al. [31, 32]. For clarity and continuity in later sections, the derivation is summarised here.

Figure 5

Figure 5a shows two PF experiments of different size. The stress-power in extending the PF is:

$$P_i = F_i \dot{d}_i \tag{5}$$

where i = 1 or 2 corresponding to Figure 5a, F_i are the measured forces and $\dot{d}_i$ are the crosshead displacement rates. Note that for the PF geometry:

$$\dot{d}_i = \dot{\theta} \sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right) L_i \tag{6}$$

If the angular shear rate, $\dot{\theta}$, is the same in both (a) and (b) in Figure 5a and the material is sheared to the same shear angle, θ, then Equation (6) can be written as:

$$\dot{d}_i = k_1 L_i \tag{7}$$

where k_1 is the same constant in both equations. Equation (7) can be substituted in Equation (5) to find:

$$P_i = k_1 F_i L_i \tag{8}$$

If the power dissipated by a given material (compressible or incompressible) at a given deformation and deformation rate increases linearly with the initial volume of material deformed, then for a given initial material area, the stress-power generated in shearing the picture frame at a specified angle and angular shear rate will increase linearly with the initial area of the sample. This argument assumes that the material properties of the sample are homogeneous throughout, irrespective of sample size (and therefore the tension in the tows). Thus,

$$P \propto V_A \propto A_A \propto L^2 \tag{9}$$

where V_A is the initial volume of the material and A_A is the initial area of the material. This can be written as $P_i = m_A V_A$ where m_A is the power storage/dissipation per unit initial volume of the material and is a function of θ. For compressible materials the volume changes with shear angle. The instantaneous volume can be written as thickness multiplied by current area

$$P_i = c_A T_A L_i^2 \cos\theta \tag{10}$$

where T_A is the instantaneous thickness of the sheet for a given θ. c_A is the power storage/dissipation per unit *current* volume at a given θ and $\dot{\theta}$. The form of c_A versus θ

will depend on the thickness behaviour as a function of shear angle, $T(\theta)$. For a given thickness behaviour, c_A will be a constant at any given θ and $\dot{\theta}$. Equation (10) can be substituted into Equation (8) to find:

$$c_A T_A L_i \cos\theta = k_1 F_i \qquad (11)$$

given

$$k_2 = k_1 / \cos\theta \qquad (12)$$

Equation (11) can be used to show:

$$\frac{c_A T_A}{k_2} = \frac{F_1}{L_1} = \frac{F_2}{L_2} \qquad (13)$$

where $c_A T_A / k_2$ is a constant for any given θ and $\dot{\theta}$. Thus, by considering the energy required to extend a PF, one can show that two PF tests of different size will give the same size ratio between force and side length when sheared to the same angle. In practice corners are cut from the square specimen to facilitate loading in the PF rig. For non-square or cruciform test samples where the contribution to the load force from the arm area is considered to be negligible (i.e. yarns are removed in the arm sections to produce non-woven arm regions) the argument above can be modified [32] to account for the shape of the test specimen and referring the Figure 5b, Equation (13) becomes

$$\frac{c_A T_A}{k_2} = \frac{F_1 \cdot L_1}{\left(L_{fab1}\right)^2} = \frac{F_2 \cdot L_2}{\left(L_{fab2}\right)^2} \qquad (14)$$

where $L_{fab\ i}$ is shown in Figure 5b and the measured force is now normalised by both the PF side length and the width of the arms of the cruciform specimen. If on the other hand yarns in the arm regions are not removed and these regions are considered to contribute to the measured force in the same way as the central region of the specimen then, referring to Figure 5c, it can be shown that Equation (13) becomes

$$\frac{c_A T_A}{k_2} = \frac{F_1}{L_1} = \frac{F_2 \cdot L_2}{L_2^2 - 4L_c^2} \tag{15}$$

where L_c is the side length of the corner cut-outs. Equation (15) was used in this investigation (L_2 was 145 mm and L_c was about 13 mm resulting in a small increase in the normalised force of about 3 percent compared to Equation (13)).

3.2 Normalisation of bias extension force

Energy arguments (Section 3.1) indicate that PF test results can be normalised by the side length of the PF rig (or any other characteristic length). Similar arguments apply also to the BE test, thus BE results could be normalised by a characteristic length for comparison with results from tests on different sized samples with the same length / width ratio, λ. However, a method of normalising BE data for comparison with other BE tests of different length / width ratios or with PF tests is less obvious because of the different shapes of the test specimens and also the different deformations induced by the BE and PF experiments.

The BE test is essentially a uniaxial tensile test. For most materials the usual procedure is to monitor the strain in a gauge section of the material while measuring tensile stress in order to determine the tensile modulus. This method can cause problems when applied to engineering fabrics. To illustrate the complication that can occur, consider the deformation of two test samples of different geometries as shown in Figure 6. The material behaviour is the same for both specimens. Fibre inextensibility is assumed and fibre direction is indicated by the dashed lines. Figure 6(a) shows a square specimen before (top left) and after deformation (bottom left), Figure 6(b) shows a larger (twice the area) square specimen before (top right) and after deformation (bottom right). The strain in both specimens is homogenous and equal. In this example, the material is considered to deform elastically, though the argument applies equally well to rate-independent plastic behaviour. Since the constitutive behaviour of both samples is the same the shear stress induced throughout both samples is equal and therefore the strain energy density in both samples is the same (note that inextensible fibres do not contribute to the strain energy density of the material, irrespective of the tensile stress they support since their tensile strain is zero). Since twice as much material undergoes deformation in Figure 6(b) compared with Figure 6(a), twice the total elastic energy is stored. Since the distance moved in the direction of the applied force in both cases is equal, due to the kinematic constraints imposed by the inextensible fibres, it follows that the extension force is twice as high in Figure 6(b) compared with Figure 6(a), as indicated in the diagram. The implication is that the tensile stress across the gauge section A-A' is half that across B-B'. This may seem to produce a paradox – the same type of material deformed to the same strain should produce the same stress. This is an implicit assumption of the gauge section method of determining material properties. However, the situation shown in Figure 6 is possible since greater tensile stresses are induced along the inextensible fibres across the gauge section in Figure 6(b) compared to Figure 6(a) due to the extra amount of deforming material. These fibre stresses allow the balance of forces across any given section to be maintained. The higher the E/G ratio of the fabric (where E is the tensile modulus of the reinforcement and *G* is the trellis shear modulus of the fabric), the smaller is the contribution of the fibres to the deformation energy of the sample (see Appendix A) and the greater is their ability to transmit stresses throughout the specimen. For high modulus rather than inextensible fibres, the deformation state of the two specimens described above would differ very slightly due to extension induced in the reinforcement directions. However, any fibre strain would be very difficult to detect and for practical image analysis purposes would be completely overshadowed by the trellis shear deformation of the sample. One way to avoid this problem is to take into account the shape and deformation field induced across the entire test specimen. With this in mind a normalisation method has been developed that takes these factors into account and is presented in the following section.

3.2.1 Energy Normalisation Method of BE Data

An alternative method of normalising the BE data is through the use of energy arguments. One advantage of this method is that no gauge section is required, i.e. the length / width ratio can be just two. This is particularly useful when dealing with fragile fabrics that are more difficult to handle and cut, that disintegrate easily and tend to pull apart or show intra-ply slip at relatively low strains. The argument involves determining the relative contribution to the deformation energy from Regions A and B of the sample (see Figure 2). It is based on a number of simple approximations that are clearly stated in the following derivation

Figure 7

Figure 7 shows three geometries. Figure 7(a) shows a PF geometry before and after shear, Figure 7(b) show a hypothetical geometry, initially with a square central region, before and after shear and Figure 7(c) show a typical BE test, before and after shear, with $\lambda > 2$. The hypothetical test geometry is used in developing the following normalisation argument for BE test with $\lambda \geq 2$. Examination of Figure 7(c) reveals

$$V_B = \left(\frac{2}{2\lambda - 3}\right) V_A \tag{16}$$

where V_A and V_B are the *initial* volumes of material in Regions A and B respectively and

$$\theta_B = \theta_A / 2 \tag{17}$$

where θ_A and θ_B are the shear angles in Regions A and B respectively. Thus, ideal kinematics are assumed throughout this energy argument. It follows from Equation. (17) that

$$\dot{\theta}_B = \dot{\theta}_A / 2 \tag{18}$$

where $\dot{\theta}_A$ and $\dot{\theta}_B$ are the angular shear rates in regions A and B respectively, note that Equation (17) and (18) also apply for Figure 7(b). Since Region C remains un-deformed during the course of an ideal test[1], Equation (7) applies to both Figure 7(a) and Figure 7(b) though in this case i = 3 or 4 and k_1 is a constant. As before, Equation (7) holds as long as the shear rate and shear angle of Region A in Figure 7(a) and (b) are equal. Under these conditions an equivalent to Equation (10) can be written for the geometry of Figure 7(b)

[1] Note, region C may compact due to tension in the tows, an energy contribution neglected in this analysis.

$$P_4 = F\dot{d} = c_A T_A L_4^2 \cos\theta + m_B V_B \tag{19}$$

where V_B is the *initial* volume of Region B and $c_A T_A$ is the same constant at a given θ and $\dot{\theta}$ as in Equation (13). Here m_B is the power storage / dissipation per unit *initial* volume of the material. Using Equation (16) this can be written as

$$P_4 = F\dot{d} = c_A T_A L_4^2 \cos\theta + \frac{2 c_B T_B L_4^2 \cos(\theta/2)}{(2\lambda - 3)} \tag{20}$$

where c_B is the power storage/dissipation per unit *current* volume in Region B and can be plotted as a function of θ, the shear angle in Region A. T_B is the thickness of the material in Region B. Assuming Region B will generate the same proportion of the total stress-power of the material at a given λ, θ and $\dot{\theta}$ irrespective of the size of the sample it follows that

$$\frac{c_B T_B \cos(\theta/2)}{c_A T_A \cos\theta} = X(\theta) \tag{21}$$

where in general X is purely a function of θ. Substitution of Equations (8), (11) and (21) in Equation (20) gives

$$k_1 F_4 L_4 = c_A T_A L_4^2 \cos\theta + \frac{2 c_A T_A X L_4^2 \cos\theta}{(2\lambda - 3)} \tag{22}$$

which can be rearranged to give

$$\frac{c_A T_A}{k_2} = \frac{F_4}{L_4}\left[\frac{2\lambda - 3}{2\lambda - 3 + 2X}\right] \tag{23}$$

where for a given shear angle and angular shear rate the constant $c_A T_A / k_2$ is the same as in Equation (13). Thus, if X were known, this equation could be used to normalise the force of the hypothetical test of Figure 7(b). Equation (23) can be modified further to apply to the BE test geometry shown in Figure 7(c). Assuming the volume in Region A of Figure 7(c) is equal to the volume of Region A in Figure 7(b) then it can be shown that

$$L_4 = L_5\sqrt{2\lambda - 3} \tag{24}$$

Furthermore, since Figure 7(b) and 7(c) now represent the same volume of material, then if the angular shear rates of Region A in Figure 7(b) and 7(c) are equal, the stress-power generated by the two tests must also be equal. In order to impose the same angular shear rate in Figure 7(c) as Figure 7(b), the right hand side of Equation (6) must be multiplied by a factor $(\lambda - 1)$. It follows that

$$k_2 F_4 L_4 = (\lambda - 1) k_2 F_5 L_5 \tag{25}$$

Rearranging Equation (25) gives

$$F_4 = \frac{(\lambda - 1) F_5}{\sqrt{2\lambda - 3}} \tag{26}$$

Equation (26) can be substituted into Equation (23) to give

$$\frac{c_A T_A}{k_2} = \frac{F_1}{L_1} = \frac{(\lambda - 1)}{(2\lambda - 3 + 2X)} \frac{F_5}{L_5} \tag{27}$$

The unknown factor X prevents Equation (27) being used to normalise force data from BE tests with a length / width ratio λ. In order to overcome this the following procedure is used and involves determining the relationship between T_A and T_B and also that between c_A and c_B. Assume that the thickness in Region B will become equal to the thickness in Region A when the shear angle in Region A was $\theta/2$, thus

$$T_B(\theta) = T_A(\theta/2) \tag{28}$$

where θ is the shear angle in Region A. Determining the relationship between c_A and c_B is perhaps the most complicated part of the derivation. The reasoning behind this step is explained at length in Appendix B and results in the relationship:

$$c_B(\theta) = \frac{\dot{\theta}_B(\theta)}{\dot{\theta}_A(\theta/2)} c_A(\theta/2) = \frac{\sin\left(\frac{\pi}{4} - \frac{\theta}{4}\right)}{2\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)} c_A(\theta/2) \tag{29}$$

where the factor introduced in Equation (29) accounts for the different angular shear rate experienced by Region A when the shear angle in this region is $\theta/2$ and Region B when the shear angle in region is $\theta/2$ (i.e. when the shear angle in Region A is θ). Note that Eq (29) neglects the changing shear resistance of the material as a function of in-plane tension [13]. However, given the small in-plane tensions induced in a BE test, this is considered a reasonable assumption. Using Equations (28) and (29) it follows that,

$$c_B(\theta) T_B(\theta) = \frac{\dot{\theta}_B(\theta)}{\dot{\theta}_A(\theta/2)} c_A(\theta/2) T_A(\theta/2) \tag{30}$$

or

$$\xi(\theta) = \frac{\dot{\theta}_B(\theta)}{\dot{\theta}_A(\theta/2)} \psi(\theta/2) \tag{31}$$

where ψ and ξ represent the power dissipation / storage per unit area of Regions A and B respectively. Thus, Equations (21), (27), (29) and (31), can be rearranged to find

$$\psi(\theta) = c_A T_A = \frac{(\lambda - 1)}{(2\lambda - 3)} \frac{F_5 k_2}{L_5} - \frac{\psi(\theta/2)}{(2\lambda - 3)} \left(\frac{1 + \cos\theta/2 - \sin\theta/2}{1 + \cos\theta - \sin\theta} \right) \tag{32}$$

If $\psi(\theta)$ can be found using Equation (32) then a direct comparison between the BE and PF test can be made using Equation (13). To do this, examining Equation (32), F_5 and

d_5 are measured during BE tests and λ and L_5 are known from the initial sample geometry. Assuming ideal kinematics the shear angle can be found from d_5 and L_5 using

$$\theta = \frac{\pi}{2} - 2\cos^{-1}\left[\frac{d_5}{2(\lambda - 1)L_5} + \frac{1}{\sqrt{2}}\right] \tag{33}$$

also the angular shear rate in Region A is given as

$$\dot{\theta} = \frac{\dot{d}_5}{(\lambda - 1)\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)L_5} \tag{34}$$

thus using Equations (6), (7), (12) and (34) one obtains

$$k_2 = \frac{\dot{d}_5}{(\lambda - 1)L_5 \cos\theta} \tag{35}$$

Thus, all the terms on the right hand side of Equation (32) are obtained apart from $\psi(\theta/2)$. In order to evaluate Equation (32) an iterative scheme can be implemented.

3.2.2 Implementation of iterative technique

In implementing the iterative scheme, the first iteration for $\psi(\theta)$ is calculated taking $\psi(\theta/2) = 0$. The values of $\psi(\theta)$ are then used to determine $\psi(\theta/2)$ for the next iteration. This process is continued until the average percentage change of data values between consecutive iterations is less than 0.01%. The iteration procedure can be implemented in a spreadsheet. Having calculated the power dissipation factor, $\psi(\theta)$, this can then be related to the normalised force F_i/L_i by dividing it by k_2, as given in Equation (35) to produce the same result as Equation (13), the normalised axial force. The normalised shear force per unit length, N_s is found using Equation (1). Thus,

$$N_S = \frac{\psi}{2k_2 \cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)} \tag{36}$$

At this point it is interesting to examine predictions of the normalisation method. Figure 9 shows the effect of normalising (a) a linear shaped axial force versus shear angle curve and (b) a cubic-shaped axial force versus shear angle curve. The linear-shaped curve can be assumed to be representative of a stitched fabric whereas the cubic-shaped curve corresponds more closely to a woven fabric. Two normalised curves are shown for a BE sample with λ = 2. The continuous line has been normalised by the length L_5 = 1m (see Figure 7) while the dashed line has been normalised using the energy method. The difference between the two curves reflects the size of the contribution from Region B to the axial force. As expected, the shape of the axial force versus shear angle curve has a significant effect on the results. The linear shaped curve produces a much larger difference after normalisation than the cubic-shaped curve when the curves are normalised by L_5 and the energy method.

Figure 8

The energy normalisation method can also be used to approximate the form of un-normalised axial force versus shear angle curves that would be produced by the same material when performing BE tests on samples with the same width but different λ ratios, e.g. 2, 2.5 and 3 (see Figure 9). This is done by ensuring that the normalised axial force versus shear angle curves of the different tests for a given material collapse onto a single line. Figure 9 shows these un-normalised curves predicted for both linear and cubic-shaped curves. The results suggest that the lower the λ ratio of a test sample, the lower one expects the measured axial force versus shear angle curve to be. Figure 9 suggests that the difference in sample dimensions produces only a small difference in the BE results, especially at low shear angles.

Figure 9

3.2.3 Validity of the normalisation procedure

The normalisation technique is valid only while the measured shear angle of Region A corresponds to the calculated theoretical shear angle. Once the measured shear angle deviates from the theoretical shear angle, the shear distribution within the sample no longer matches the assumed distribution illustrated in Figure 2. After this point the test is measuring the material behaviour under a mixed mode deformation, that is, the material undergoes both trellis shear and inter-tow slip as shown in Figure 3, mechanisms that may also lead to a change in dimensions of Region C, which is assumed constant in this analysis [22, 23]. Thus, measurements of shear angle in Region A versus displacement should be made during tests to determine the range of displacement under which the normalisation method remains valid. Another factor neglected in the normalisation procedure is the effect of in-plane tension on the fabric shearing behaviour. However, both experimental [33] and theoretical [8] results have suggested that shear properties of dry fabrics are only weakly affected by in-plane stresses. Thus, given the small magnitudes of the stresses present during BE tests, it is considered reasonable to neglect this point in the development of the normalisation procedure.

4 RESULTS

4.1 Materials and tests

Two woven fabrics have been tested. The same fabrics were tested by other institutions in a recent benchmarking exercise [13]. For consistency with other studies the fabrics will be termed Fabrics 1 and 3. The details of each fabric are given in Table 1.

Table 1

PF experiments (without pre-tension) were conducted as well as BE experiments for three different length/width ratios (λ) on both fabrics.

4.1.1 Shifting of BE data

Before normalising BE data, force versus displacement curves were corrected to account for fabric pre-shear caused by inherent 'off the roll' material variability [37] and deformation induced when handling and loading the specimens in the BE grips. Without shifting, the data show significant variability. A typical example of un-shifted and shifted force versus displacement data is show in Figure 10.

Figure 10

The shifting procedure involves two stages. In the first stage the amount of pre-shear for each sample is determined through automated image analysis. The measured pre-shear angle is used to calculate the size of the displacement shift required to correct the data, using Equation (33). For example, a pre-shear of one degree in a 3:1 sample of width 100 mm corresponds to a displacement shift of about 1.8 mm. Once each curve is corrected using this method, variability is decreased but can be improved further by a second phase of shifting. In this next step shifted curves are plotted together as axial force versus shear angle data using Equation (33). An arbitrary axial force is chosen and the shear angle of each curve is measured to produce an average shear angle at that force. The difference between the actual and average shear angle of each curve at that force can then be calculated and used to shift the curves individually, again by an appropriate distance along the displacement axis. The resulting force versus displacement curves show much improved agreement (e.g. Figure 10). The need for this second stage of shifting is necessary since even careful visual analysis of specimens to determine specimen pre-shear may not totally correct the data. One reason for this is that specimens may show variations in pre-shear on a local scale, preventing accurate estimation of the pre-shear angle.

Figure 11

Figure 11 shows how specimens can contain pre-shear on a local scale without being extended. The figure was produced using a simple BE trellis shear algorithm based on pin-jointed net kinematics and implemented in a spreadsheet. Local pre-shear has been verified through measurements of shear angle at different points within Region A using the image analysis software described in Section 2.2.1 (see Figure 4). Typical results produced during a test on Fabric 3 using a specimen with $\lambda = 2$ are shown in Figure 12. Clearly, initial variation in shear angle exists at different points within Region A, typically of the order of plus or minus two degrees.

Figure 12

4.1.2 PF and BE results

Figure 13 shows un-normalised BE axial force versus shear angle data for both fabrics. Good agreement is shown between un-normalised shifted BE curves for all λ ratios at low shear angles. In general BE test samples tend to undergo ideal trellis shear behaviour at low shear angles but gradually change their mode of deformation as the test progresses. As the sample displacement increases the specimens tend to deform through a combination of trellis shear and intra-ply slip, as shown in Figure 3. For this reason, visual analysis must be used to verify the shear angle at which the sample deformation kinematics deviate significantly from ideal trellis shear behaviour. Figure 12 suggests this shear angle to be around 30 degrees, similar to that noted in [23]. This is important since the assumptions of the normalisation procedure are valid only for trellis shear. It is worth noting here that wide specimen BE tests may increase the shear angle (up to 50^o in some cases) before which the ideal kinematic assumption breaks down [23]. However, altering the shape of the specimen implies modification of the normalisation equation, an issue that has been addressed elsewhere [38] but is beyond the scope of this paper.

Figures 13 & 14

Figure 14 shows both PF and BE axial force versus shear angle data following normalisation. With regard to the BE data, as predicted in Figure 9, the BE sample with $\lambda = 2$ produces an axial force slightly lower than the samples with $\lambda = 2.5$ and 3 and is therefore most affected by the normalisation process. Normalisation produces a slight improvement in the reliability of the BE data which can be seen when comparing Figures 13 and 14.

Theoretically, the normalised BE and PF data should match. However, Figure 14 shows the PF data is significantly higher than the BE data, even at low shear angles and also contains much greater variability. This behaviour is the opposite of that observed for viscous textile composites where BE force versus shear angle curves tend to be higher than equivalent PF curves, see Figure 8 in ref. [39]. The explanation for the behaviour of viscous textile composites was that the axial force increased with sample extension due mainly to intra-ply slip while the specimen approached its locking angle, creating an apparent increase in axial force when plotted against shear angle. It is not clear why the opposite behaviour is found for dry fabrics in this investigation. One suggestion is that intra-ply slip in dry fabrics is a lower energy process than for viscous textile composites and therefore does not produce the same large increase in axial force with increasing extension. Nevertheless, this should not result in a lower BE force versus shear angle curve than that produced by a PF test unless much of the stored energy due to trellis shear is able to relax after fabric locking during a BE test; relaxation facilitated by intra-ply slip mechanisms occurring at large shear angles. The latter possibility could be explored in subsequent investigations. Another more likely possibility is that the adverse effect of the PF boundary conditions is more prominent for dry fabrics than viscous textile composites since dry fabrics can be fastened more securely within the PF clamps. If this is the case then assuming that PF test results in which test samples buckle at low shear angles are discarded, the lowest of the PF curves is the most accurate of all the PF data. With regard to which is the more reliable data; the lowest PF test curve or the BE data, the argument presented above coupled with the extremely good repeatability of the BE test results suggests the latter, at least until a shear angle of about 30 degrees. Beyond 30 degrees the BE test induces intra-ply slip (see Figures 3 and 12), a mode of deformation not particularly relevant to the forming process and therefore less indicative of the material's behaviour under forming conditions.

5 CONCLUSIONS

An energy normalisation method has been devised to allow direct comparison between PF and BE test data for rate-independent compressible fabrics. The method takes into account the non-homogeneous deformation kinematics that occur throughout the BE test specimen. A large amount of variability in results was found to be inevitable when conducting PF tests. Variation was also found in BE data, though a method of correcting the data, based on results of visual analysis, was described. Agreement between BE data conducted using different sample dimension ratios was found to be very good following correction of the data and improved further following application of the energy normalisation method. Results of the energy normalisation method indicate that the shape of the axial force versus shear angle curve, produced during a BE test, is important in determining the amount by which the data is modified following normalisation; linear-shaped curves undergo a greater correction than non-linear curves. This suggests that stitched fabrics will usually be more affected by the normalisation than woven fabrics.

Comparison between PF and BE data following normalisation was poor. This poor comparison, coupled with the observation that the PF axial force versus shear angle data lay above that of the BE tests indicate that the boundary conditions of the PF test may severely affect axial force versus shear angle results, more so than for viscous textile composites. Further work should be performed to investigate this possibility.

Perhaps the most significant conclusion of this work has been the validation of the theoretical argument using experimental data. For woven fabric, the normalisation procedure produces results which are not significantly different to results normalised by L_5. As a result it can be stated that a reasonable method of producing a close approximation to properly normalised data for woven fabrics is simply to divide the axial force by L_5 and also to use greater specimen ratios for the tests. This approximate procedure might be adopted if high accuracy in the results is not required. Furthermore, the same energy normalisation method can now be applied with confidence to biaxial testing methods [13, 23]. The latter can potentially overcome the limitations of the PF and BE tests, namely, fabrics under significant in-plane tension can be tested (not possible in uniaxial BE tests) and boundary conditions are not so problematic (as with the PF test). The resulting data could be normalised reliably using energy arguments analogous to those presented here for the uniaxial BE test.

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Appendix A: Mechanical Behaviour of an Elastic Four-Bar Linkage Connected by Elastic Hinges

The following argument illustrates how the proportion of energy stored in reinforcement fibres decreases with increasing fibre stiffness relative to trellis shear stiffness. Figure A1(a) shows a four bar elastic linkage, freely jointed (pinned) at A and D but with elastic hinges at B and C. The springs represent the stiff reinforcement fibres while the hinges represent the trellis shear stiffness of an analogous biaxially reinforced composite material. Figure A1(b) shows one quarter of the system which is in equilibrium. The linkage has been extended by applying a force in the positive *y*-direction (opposed by an extensive force on the opposite corner of the linkage in the negative *y*-direction). The equilibrium configuration for the system (i.e. the relative amount of elastic strain in the springs versus the elastic rotation of the hinges at which equilibrium is reached) depends on the elastic modulus of both the springs and hinges. One way of determining the equilibrium configuration is by using the principal of virtual work.

Figure A1 about here

In Figure 1A(b) the system is moved from its equilibrium position through a virtual displacement in the *x* direction. The force at point C due to the spring is,

$$\mathbf{F}_s = -Ke\cos\chi\,\hat{\mathbf{i}} + ke\sin\chi\,\hat{\mathbf{j}} \qquad \text{(A1)}$$

where the spring elongation $e = L - L_o$ and L_o is the un-extended length of the spring, K is the spring constant (the force required to stretch the spring a unit length), $\hat{\text{i}}$ and $\hat{\text{j}}$ are unit vectors in the x and y directions. Also, the moment due to the hinges

$$\mathbf{M} = C\theta\hat{\mathbf{k}} \tag{A2}$$

where C is the elastic constant of the spring (the moment required to turn the hinge an angular unit), $\hat{\mathbf{k}}$ is a unit vector in the z-direction (i.e. out of the plane of the page). Using the principal of virtual work and referring to Figure A1(b)

$$\delta W = \mathbf{F}_s.\boldsymbol{\delta}r - \mathbf{M}.\boldsymbol{\delta}\phi = 0 \tag{A3}$$

where use of the δ (or $\boldsymbol{\delta}$ for a vector quantity) indicates a virtual increment. From Figure A1

$$\mathbf{r} = L\sin\phi\,\hat{\mathbf{i}} \tag{A4}$$

and

$$\boldsymbol{\delta}\mathbf{r} = \hat{\mathbf{i}}\delta r \tag{A5}$$

thus

$$d\phi = \frac{dr}{L\cos\phi} \tag{A6}$$

Combining Eq's (A1-A3) and (A5) the following is obtained

$$Ke\sin\phi = \frac{C\theta}{L\cos\phi} \tag{A7}$$

From Figure A1(b)

$$\phi = \frac{\pi}{4} - \frac{\theta}{2} \tag{A8}$$

and by combining with the identity: $\sin\phi\cos\phi = \frac{1}{2}\sin 2\phi$ and substituting *e*, Eq (A7) gives

$$L^2 - LL_o - \frac{2C\theta}{k\cos\theta} = 0 \qquad \text{(A9)}$$

Thus given *K*, *C*, L_o and θ it is possible to find L by solving Eq (A9) and therefore find the strain in the springs, *e*. Figure A2 shows the tensile strain versus θ for various ratios of *k/C* (the spring constants of the linear springs and hinges). As expected the tensile strain of the connecting bars increases with shear angle and decreases with the magnitude of the *k/C* ratio.

Figure A2 about here

Using the tensile strain, the energy stored in the linear elastic springs and hinges can be calculated as a function of shear angle. The ratio between the two can then be found as

$$\frac{E_s}{E_h} = \frac{Ke^2}{C\theta^2} \qquad \text{(A10)}$$

Using Eq (A10) the ratio of energies stored in the springs and the hinges versus shear angle is plotted in Figure A3 for various *k/C* ratios. It is clear that for a higher *k/C* ratio, a lower amount of energy is stored in the springs. While this analysis is clearly a simplification of actual engineering fabric behaviour, the same basic principles hold true.

Appendix B: Determining the Relationship between c_A and c_B

Consider a constant displacement rate test. At any given instant c_A and c_B can each be related to two factors; the shear resistance and the rate of strain in the two respective regions of the fabric, i.e.

$c_A(\theta) \propto F_A(\theta)\dot{\theta}_A(\theta)$ and $c_B(\theta) \propto F_B(\theta)\dot{\theta}_B(\theta)$

One method of determining the relationship between c_A and c_B might be to compare the ratio between the two functions at any given time. This produces the equation:

$$\frac{c_A(\theta)}{c_B(\theta)} = \frac{F_A(\theta)\dot{\theta}_A(\theta)}{F_B(\theta)\dot{\theta}_B(\theta)} \tag{B1}$$

and using Eq (18) find

$$\frac{c_A(\theta)}{c_B(\theta)} = \frac{2F_A(\theta)}{F_B(\theta)} \tag{B2}$$

However, the ratio on the right hand side of Eq (B2) depends on the material behaviour which is unknown (see Figure B1).

Figure B1

Thus, comparing c_A and c_B at any given time does not provide a useful relationship between c_A and c_B and so some other method must be found. An alternative is to compare c_A and c_B when the fabric in each region is at the same state of strain, i.e. at two different times during the same constant displacement rate test. According to Eq (17) the shear angle in region A is always twice that in region B. Thus, the shearing resistance in Region A when the shear angle in Region A is $\theta/2$ is the same as that in Region B when the shear angle in Region A is θ. Thus:

$c_A(\theta/2) \propto F_A(\theta/2)\dot{\theta}_A(\theta/2)$ or $c_A(\theta/2) \propto F_B(\theta)\dot{\theta}_A(\theta/2)$ and $c_B(\theta) \propto F_B(\theta)\dot{\theta}_B(\theta)$

and

$$\frac{c_A(\theta/2)}{c_B(\theta)} = \frac{F_B(\theta)\dot{\theta}_A(\theta/2)}{F_B(\theta)\dot{\theta}_B(\theta)} = \frac{\dot{\theta}_A(\theta/2)}{\dot{\theta}_B(\theta)} \tag{B3}$$

For a constant displacement rate, $\dot{d}$ and for any given sample size, L, Eq (6) can be used to find the angular velocity in Regions A and B, i.e. in Region B

$$\dot{\theta}_B(\theta) = \frac{\dot{\theta}_A(\theta)}{2} = \frac{\dot{d}}{2L\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}$$

and in region A:

$$\dot{\theta}_A\left(\frac{\theta}{2}\right) = \frac{\dot{d}}{L\sin\left(\frac{\pi}{4} - \frac{\theta}{4}\right)}$$

Thus the right hand side of Eq (B3) is easily found and gives Eq (29).