Enhanced Vibrational Energy Harvesting Using Non-linear Stochastic Resonance

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Abstract

Stochastic resonance has seen wide application in the physical sciences as a tool to understand weak signal amplification by noise. However, this apparently counter-intuitive phenomenon does not appear to have been exploited as a tool to enhance vibrational energy harvesting. In this note we demonstrate that by adding periodic forcing to a vibrationally excited energy harvesting mechanism, the power available from the device is apparently enhanced over a mechanism without periodic forcing. In order to illustrate this novel effect, a conceptually simple, but plausible model of such a device is proposed to explore the use of stochastic resonance to enhance vibrational energy harvesting.

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1 Introduction

Stochastic resonance is an intriguing and counter intuitive phenomenon. First proposed by Benzi and co-workers in the early 1980s, it was devised as a mechanism to explain the means by which weak forcing of solar insolation can produce the dramatic swings in the Earth's climate seen in periodic ice ages [1]. Subsequently, a significant body of work has explored applications in a diverse range of fields such neurophysiology [2], quantum systems [3] and signal processing [4]. Experimental work has also demonstrated stochastic resonance in a range of physical systems such as an ac-driven Schmitt trigger [5], a bistable ring laser [6] and more recently MEMS-scale cantilevered beams [7]. On-going work is continuing to exploit stochastic resonance in climate science, human cognition and the development of nano-scale devices.

The underlying mechanism of stochastic resonance requires a bi-stable non-linear system which is excited by noise, such as a double well potential [1]. If the system is trapped in either potential well, the effect of noise is merely to excite the dynamics locally, with the probability of a transition between the potential wells determined by the so-called Kramers rate [8]. For a large potential barrier between the two potential wells this probability is clearly small. However, if the dynamics are now forced such that the height of the potential barrier oscillates, then the transition probability is also forced. If this forcing is matched to the mean time between transitions (inverse Kramers rate), then stochastic resonance can occur. In stochastic resonance the system is driven across the weakened potential barrier by noise with the result that a large amplitude response occurs. This has the counterintuitive effect that the addition of noise to a weak periodic signal can amplify the signal with a greatly enhanced signal-to-noise ratio. It is this amplification effect which has led to a diverse range of applications of stochastic resonance and which we will exploit to enhance vibrational energy harvesting.

Cartmell has shown that combined excitation and parametric forcing can modify energy flows in a deterministic oscillator [9]. In addition, although early work on stochastic resonance focused on bi-stable systems excited by white noise, more recent work has shown that similar effects are possible in systems where stochastic noise is replaced with high frequency excitation [10]. This analogous phenomenon occurs when the excitation frequency is well separated from the forcing frequency of the potential well. Since machine vibration is never truly stochastic, this provides a mechanism to link stochastic resonance to real mechanical devices, such as those used for vibrational energy harvesting.

Energy harvesting has emerged as an important new topic with the goal of fabricating devices that can generate electrical power by exploiting ambient vibrational energy [11] or thermal gradients [12]. These mechanisms are seen as a practical means of powering remote wireless sensors in automotive or aerospace applications, without the need for a battery or wiring harness. Typically, a cantilevered beam with a piezoelectric strip is used to transform vibrational energy into electrical energy through damping [13]. For small displacements of the beam, peak power generation in the mechanism will occur when the natural frequency of the beam is tuned to the peak of the vibration noise spectrum. Here, we propose to exploit the phenomenon of stochastic resonance to enhance the performance of such devices. In particular, if the cantilevered beam is instead clamped at both ends it forms a simple bi-stable mechanical system with a double potential well. If the beam is then forced (periodically compressed and relaxed) so as to modulate the height of the potential barrier while being excited by noise, stochastic resonance can occur. It is proposed that additional energy can then be extracted from the ambient vibration noise spectrum, leading to enhanced power generation over a conventional linear oscillator. We expect further exploitation of stochastic resonance in a range of mechanical devices.

2 Energy Harvesting Device

2.1 Free vibration of a clamped-clamped beam

In order to explore the application of stochastic resonance to vibrational energy harvesting, a conceptually simple mechanism will now be investigated. We will consider a beam under a modest compressive load which can buckle into one of two symmetric equilibrium states. The beam is supported by a base of negligible mass. The essential behaviour of the beam can be captured by representing it as a single lumped mass m with two linear springs of stiffness k and natural length l, as shown in figure 1. Dissipation in the beam will be modeled by a single linear damper c. Using this lumped mass model, the displacement of the mass will be defined by x from the datum A-A', (such that $x \leq \sqrt{l^2 - d^2}$), while the springs are separated by $2d$, such that $d < l$. It will initially be assumed assumed that the distance A-A' is fixed. Following the analysis of Roundy [14], if the base is excited by a displacement X, it can be shown that the dynamics of the problem are described by

$$m\ddot{x} + c\dot{x} + 2kx\left(1 - \frac{l}{\sqrt{x^2 + d^2}}\right) = -m\ddot{X} \tag{1}$$

where the new non-linear term in (1) represents the dynamics of the spring-mass system. This non-linear term can then be expanded by assuming $x/d \ll 1$. Although this restriction will not be used later, the resulting equation of motion captures the full non-linearity of the problem. It can then be shown that

$$m\ddot{x} + c\dot{x} - 2k\left(\frac{l}{d} - 1\right)x + \frac{kl}{d^3}x^3 + \ldots = -m\ddot{X} \tag{2}$$

A non-dimensional position coordinate $\xi = \sqrt{l/d^3}x$ and non-dimensional time $\tau = t/\sqrt{m/k}$ can be defined. The qualitative non-linear model for the beam is therefore defined by

$$\xi'' + \bar{c}\xi' - \mu\xi + \xi^3 = Q(t) \tag{3}$$

where ($'$) indicates differentiation with respect to τ. The free parameter $\mu = 2(l/d - 1)$ is used as a measure of the compressive load acting on the beam, while $\bar{c} = c/\sqrt{km}$ and $Q(t) = -(m/k)\sqrt{l/d^3}\ddot{X}$.

In order to proceed, we will firstly consider an undamped, unexcited system with $\bar{c} = 0$ and $Q = 0$. Clearly, if the beam is in tension ($l < d$) then $\mu < 0$ while if the beam is in compression ($l > d$) then $\mu > 0$ with the critical buckling load corresponding to $\mu = 0$. It can be seen that for $\mu < 0$, equation (3) admits a single real equilibrium solution ($\xi'' = 0$) at $\tilde{\xi}_0 = 0$ corresponding to an undeflected beam in tension. For $\mu > 0$, equation (3) admits 3 equilibria defined as $\tilde{\xi}_0 = 0$, $\tilde{\xi}_1 = +\sqrt{\mu}$ and $\tilde{\xi}_2 = -\sqrt{\mu}$, corresponding to a symmetric buckled configuration. A supercriticial bifurcation then occurs when μ changes sign [15].

The change to the qualitative behaviour of the system detailed above can be seen through the use of an effective potential for the problem $V(\xi)$ such that $\xi'' = -\partial V(\xi)/\partial\xi$. The potential can then be defined as

$$V(\xi) = -\frac{1}{2}\mu\xi^2 + \frac{1}{4}\xi^4 \tag{4}$$

The stability properties of the equilibria defined above can be determined from the turning points of $V(\xi)$, as can be seen in figure 2. For $\mu < 0$ the single equilbrium point at $\tilde{\xi}_0$ is stable with $\partial^2 V(\xi)/\partial\xi^2 > 0$, while for $\mu > 0$ it becomes unstable with $\partial^2 V(\xi)/\partial\xi^2 < 0$ and the equilibria at $\tilde{\xi}_1$ and $\tilde{\xi}_2$ are stable with $\partial^2 V(\xi)/\partial\xi^2 > 0$. It is clear that $\tilde{\xi}_0$ becomes unstable when the two new (stable) equilibria $\tilde{\xi}_1$ and $\tilde{\xi}_2$ appear at the supercritical bifurcation. It will be assumed that the beam is initially in a post-buckled state and is in one of the two symmetric equilibria $\tilde{\xi}_1$ or $\tilde{\xi}_2$ corresponding to one of the two available potential wells.

2.2 Forced vibration of a clamped-clamped beam

The simple model of the clamped-clamped beam will now be extended to include the excitation and linear damping terms discussed above. It will be assumed that the parameter μ can be forced at frequency ω and with amplitude η. This implies that the beam is compressed and relaxed in an oscillatory manner so that the distance A-A' is now time varying. Such forcing could be achieved with an electromechanical actuator at the support points A and A', as indicated in figure 1. This forcing will modulate the height of the potential barrier to allow stochastic resonance. It will also be assumed that the beam is excited by external noise $Q(t)$, the properties of which will be discussed later. The dynamics of the mechanism are now parametrically forced and are defined by

$$\xi'' + \bar{c}\xi' - \mu(1 - \eta\cos(\omega t))\xi + \xi^3 = Q(t) \tag{5}$$

It can be seen that there is now an external input of energy to the mechanism from the excitation $Q(t)$ which is then dissipated by the linear damping $\bar{c}\xi'$. Importantly, this flow of energy from excitation to the response of the beam is modulated by the parametric forcing of the beam at frequency ω [9]. The effective potential of the problem can now be defined as a time dependent, oscillatory function given by

$$V(\xi, t) = -\frac{1}{2}\mu(1 - \eta\cos(\omega t))\xi^2 + \frac{1}{4}\xi^4 \tag{6}$$

The forcing of the potential is shown in figure 3 over a half cycle. It can be seen that the height of the potential barrier between the two stable equilibria of the system is modulated. As will be seen, when properly tuned through the forcing frequency ω, this modulation will allow the excitation $Q(t)$ to drive the mechanism between the two potential wells in a stochastic resonance. The significantly enhanced response of the beam will then provide greater power to be dissipated by the damper and exploited for energy harvesting.

2.3 Enhanced vibrational energy harvesting

In order to assess the use of stochastic resonance for vibrational energy harvesting, the total power dissipated by the damper will now be investigated. In principle, this power is available for energy harvesting. The details of the damper are not considered, although it may represent an electromechanical device [11] or a piezoelectric strip [13] (recalling that the spring-mass system represents a continuous beam). We note that power is required to drive the oscillatory forcing of the beam at frequency ω. This will be subtracted from the total power available. We note that traditionally stochastic resonance adds noise to a periodic signal, whereas our system is stochastically excited and then periodically forced.

From equation (5) it can be seen that

$$\xi'\xi'' - \mu\xi\xi' + \xi^3\xi' = -\bar{c}\xi'^2 - \mu\eta\cos(\omega t)\xi\xi' + \xi' Q(t) \tag{7}$$

which can be written as

$$\frac{d}{d\tau}\left(\frac{1}{2}\xi'^2 - \frac{\mu}{2}\xi^2 + \frac{1}{4}\xi^4\right) = -\bar{c}\xi'^2 - \mu\eta\cos(\omega t)\xi\xi' + \xi' Q(t) \tag{8}$$

and is clearly a statement of conservation of power [11]. We interpret equation (8) as the balance between the instantaneous power input due to excitation $Q(t)$, balanced by the rate of change of the kinetic energy and potential energy of the mechanism and the linear dissipation. Therefore, identifying the total energy of the system as $E = \xi'^2/2 + V(\xi)$, where the effective potential energy is defined by equation (4), it can be seen that

$$E' = -\bar{c}\xi'^2 - \mu\eta\cos(\omega t)\xi\xi' + \xi' Q(t) \tag{9}$$

We now propose from equation (9) that the instantaneous power P available for energy harvesting is given by

$$P = \bar{c}\xi'^2 - \delta\mu\eta\cos(\omega t)\xi\xi' \tag{10}$$

The first term, $\bar{c}\xi'^2$, is the usual linear dissipation due to damping. This is assumed to be harvested by the damper attached to the beam. It is proposed that the second term, $\delta\mu\eta\cos(\omega t)\xi\xi'$, represents the rate at which work is done in forcing the beam at frequency ω. However, this term can be of either sign corresponding to energy input to compress the beam and energy release when the beam relaxes. In order to provide a conservative estimate of the net power generated, we ensure that the term only represents a sink of energy and so we define

$$\delta = \begin{cases} 1 \text{ if } \cos(\omega t)\xi\xi' \geq 0 \\ 0 \text{ if } \cos(\omega t)\xi\xi' < 0 \end{cases} \tag{11}$$

Therefore, the power available for energy harvesting is reduced due to the power required to force the beam at frequency ω.

In order to simulate stochastic resonance the excitation $Q(t)$ will now be defined. Rather than pure white noise, a number of harmonics are summed and a strong white noise component added. The harmonics are postulated to represent the noise emitted from a single cylinder engine rotating with angular velocity Ω and with crank length to con-rod length ratio 1/3 [16].

White noise is then added to this periodic signal to represent the un-modelled high frequency dynamics of the engine so that the excitation is defined as

$$Q(t) = \frac{1}{2}\rho(t) + \sin(\Omega t) + \frac{1}{3}\sin(2\Omega t) - 0.00926\sin(4\Omega t) + 0.0003\sin(6\Omega t) \quad (12)$$

where the white noise $\rho(t)$ has zero mean and unit variance and the coefficients represent an approximation to the periodic vibration spectrum [16].

We now consider the response of the mechanism with and without periodic forcing of the beam. We select $\mu = 1$ and $\eta = 0.7$, corresponding to the springs being compressed to approximately 65% of their natural length and then modulated such that the distance A-A' changes by approximately 15% with frequency ω. With zero forcing ($\eta = 0$), the mechanism is excited in a single potential well with small amplitude vibration, as shown in figure 4. However, with the addition of periodic forcing ($\eta \neq 0$) the mechanism fluctuates between the two potential wells in a state of stochastic resonance with large amplitude displacements, again shown in figure 4. In stochastic resonance the mechanism is highly excited such that the linear dissipation $\bar{c}\xi'^2$ is greatly enhanced. However, the additional term $\delta\mu\eta\cos(\omega t)\xi\xi'$ in equation (10) will reduce this improvement in power output due to the work done in forcing the beam. The net integrated energy output from both cases is shown in figure 4. It can be seen that the forced mechanism in stochastic resonance apparently delivers significantly more energy from the excitation $Q(t)$ than the unforced mechanism. The forcing frequency ω required for stochastic resonance can be estimated from the Kramer's rate, the probability of transition between the potential wells [8].

The response of the mechanism away from stochastic resonance is shown in figure 5. In this case the mechanism is unable to transition between the two potential wells. It can be seen that the integrated energy output is now greater for the unforced mechanism, due to the work done in forcing the beam. Finally, the power available from the forced and unforced mechanisms is shown in figure 6. It can be seen that the forced mechanism dissipates significantly more power than the unforced mechanism, but that some of this power is required to force the beam. However, the net power available and integrated energy output is greater at stochastic resonance. Having introduced the application of stochastic resonance to vibrational energy harvesting, it is clear that other mechanical systems could be shown to be capable of exhibiting this phenomena.

3 Conclusions

The concept of stochastic resonance has been investigated as an effective new means of enhancing vibrational energy harvesting. Using a simple conceptual model of an energy harvesting mechanism it has been shown that periodic forcing can apparently be used to increase the mechanical energy available for extraction through energy harvesting. While a device using stochastic resonance will be mechanically more complex than a conventional device, and will be less efficient than the ideal mechanism investigated here, it is believed that the apparent enhancement in energy harvesting may be significant in practice and will be pursued through further analytical and experimental investigation.

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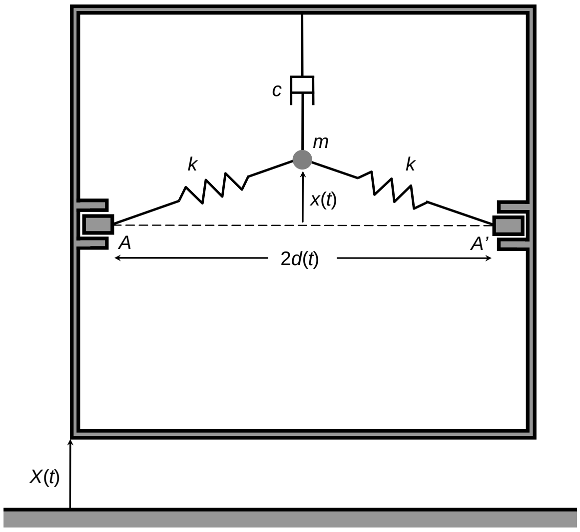

Fig. 1. 1 degree-of-freedom beam model comprising a single lumped mass m with spring constant k and displacement $x(t)$ from (unstable) equilibrium driving a damper with damping coefficient c. The device experiences base excitation with displacement $X(t)$ and the distance A-A' is modulated at frequency ω.

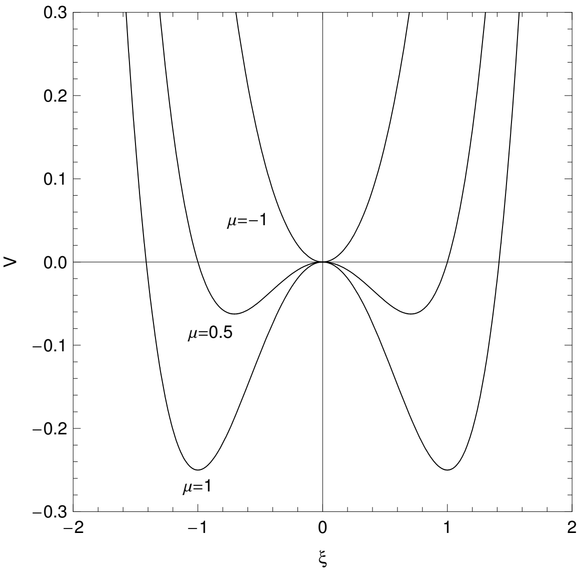

Fig. 2. Effective potential $V(\xi)$ for a 1 degree-of-freedom beam model. Single equilibrium $\widetilde{\xi}_0$ for $\mu < 0$ with two new equilibria $\widetilde{\xi}_1$ and $\widetilde{\xi}_2$ appearing after the supercriticial bifurcation at $\mu = 0$.

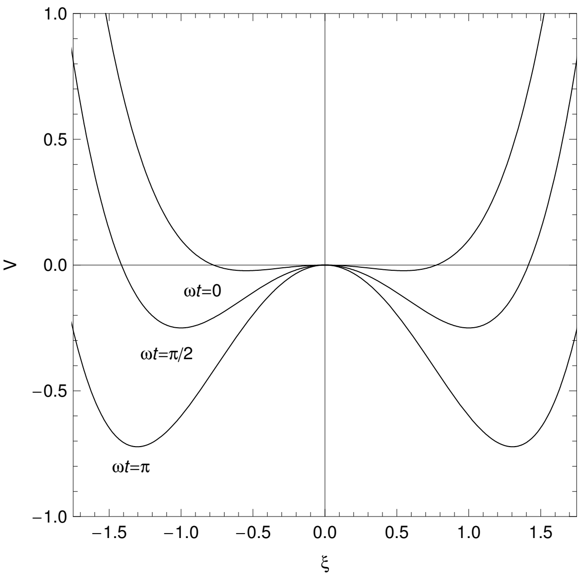

Fig. 3. Forcing of the double potential well with $\mu = 1$ and forcing amplitude $\eta = 0.7$

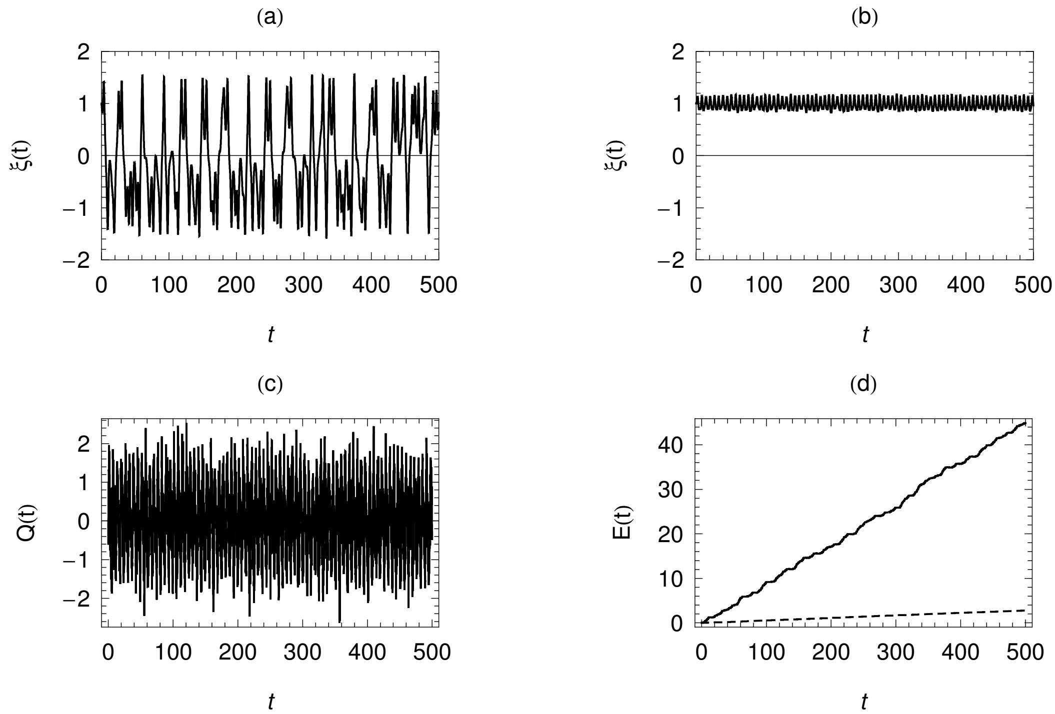

Fig. 4. Tuned system in stochastic resonance with $\omega = 1.2$ a: Response with forcing ($\eta = 0.7$) b: Response without forcing ($\eta = 0$) c: External noise $Q(t)$ with $\langle \rho(t) \rangle = 0$ d: Energy available from the mechanism with forcing (solid line) and without forcing (dashed line) with damping $\overline{c} = 0.5$.

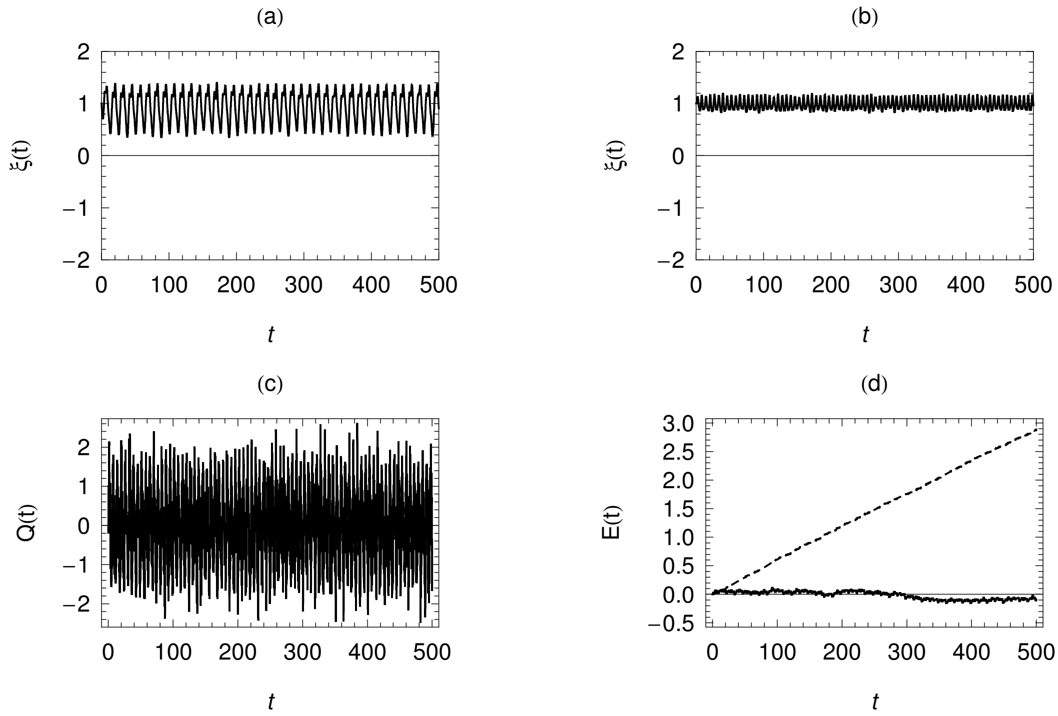

Fig. 5. Un-tuned system far from stochastic resonance with $\omega = 0.5$ a: Response with forcing ($\eta = 0.7$) b: Response without forcing ($\eta = 0.$) c: External noise $Q(t)$ and $\langle \rho(t) \rangle = 0$ d: Energy available from the mechanism with forcing (solid line) and without forcing (dashed line) with damping $\bar{c} = 0.5$.

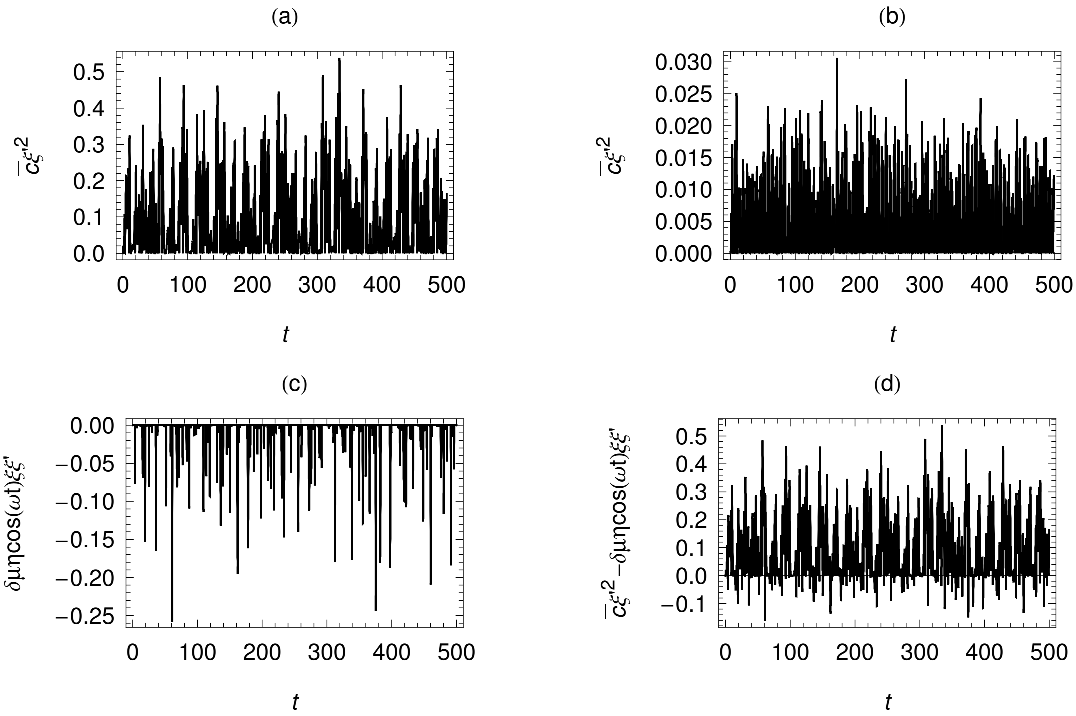

Fig. 6. Available power in stochastic resonance a: Total dissipated power from forced system b: Net available power from unforced system c: Power to modulate potential d: Net power available from forced system.