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Can we meaningfully speak of changes in price under the regime of changes in techniques?

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Abstract
The paper discusses the role of the Standard Commodity as a numeraire. We argue that the heart of Sraffa's motivation for introducing the Standard Commodity was the dependence of relative prices on technical conditions in the basic sector. We show, by constructing a large class of worked examples that the direction of price movements under technical change will be dependent on the numeraire. If the choice of numeraire is arbitrary then so are the relative movement directions of prices under technical change.

Keywords: Sraffa, Basic System, Standard Commodity, Numeraire

Introduction

Hahn (1982) in his famous critique of Sraffa's *production of Commodities by Means of Commodities* claims, "A numeraire is a numeraire. The price of the numeraire can be set equal to one. Sraffa has chosen Standard net product as numeraire and there's an end to it." (358).[1] Unfortunately Sraffians have more or less agreed with Hahn on this point, as Kurz and Salvadori (1993) write, "the Standard commodity is a useful, although not a necessary, tool of analysis." (p. 117). This poses a prima facie problem for an interpretation of Sraffa's book. Even a casual reading of the book leaves a deep impression on the reader of the aesthetic design of the book, and the author's extreme concern for brevity--it appears that the author wanted to make sure that not even one word in the whole book could be declared superfluous. The book is hardly Ninety pages long in large type settings. Out of the Ninety pages Sraffa devotes two full chapters (Chs. 4 and 5) on the Standard commodity and the uniqueness of the Standard system under

[1] Had Hahn, however, considered the case of joint production, which is the most general case for Sraffa, he would have noticed that he could not derive an inverse profit-wage frontier with any arbitrary numeraire.

single product industries and one full chapter (ch. 8) on the Standard commodity with joint products, totalling Twenty-Five pages in all. The question is: Why Sraffa, of all people, decided to devote more than Twenty-Five percent of his life's most important work on something so trivial, if it happens to be so trivial, as the question of a numeraire? There is obviously something a miss here, as we again witness in the debate on Sraffa's reswitching proposition in the last chapter of the book. We find that in this chapter Sraffa was highly concerned with the theoretical problem posed by the uniqueness of the Standard commodity for his famous reswitching proposition,[2] which led him to develop the proposition via a tortuous method of devising a particular and highly artificial system where two methods are supposed to be producing identical basic goods, and thus have the same Standard commodity, but are distinct in their use as non-basics; the subsequent literature on the reswitching proposition, however, has shown no concern with the problem of numeraire in this case either.

One reason for this attitude has been a common misunderstanding that Sraffa's Standard commodity was mainly concerned with solving Ricardo's problem (Sraffa 1951). As Hahn writes, "Reader of Sraffa may be surprised at this since they know that he is much concerned with an '*invariant standard of value*'. This has something to do with Ricardian theory but to a modern theorist it is almost incomprehensible." (p. 358). And again, Kurz and Salvadori (1993) write, "Sraffa relates the Standard commodity to

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[2] "If the product is a basic one, the problem is complicated by the circumstance that each of the two alternative methods of producing it implies a distinct economic system, with a distinct Maximum rate of profit. As a result we seem to lack a common ground on which the comparison between the two methods can be carried out: since, according as one or the other method is used, we are in one or the other economic system, and to any given rate of profits there will correspond in each system, a different wage, even though in the same standard, and a different set of relative prices; as a consequence a comparison of the prices by the two methods becomes meaningless since its result appears to depend on which commodity is chosen as standard of prices." (Sraffa 1960, 82).

Ricardo's search for an 'invariable measure of value',…" (p. 117). On this the verdict seems to be unanimous. As early as 1970, Maurice Dobb wrote, “some explanation may be sought by some about the so-called Standard Commodity (or set of commodities) and the reason for the prominence given to it in the Sraffa-system. I will be as brief as possible about this. This is devised to deal with Ricardo’s problem of finding an ‘invariable standard’, or measure of value, that will be invariant to changes in the profit-wage ratio: in other words a measure of commodity-magnitudes and of relations between them that is independent of distribution and of relative prices.” (359-60). All this, notwithstanding Sraffa's (1960) explicit claim that "It should perhaps be stated that it was only when the Standard system and the distinction between basics and non-basics had emerged in the course of the present investigation that the above interpretation of Ricardo's theory [corn-profit model] suggested itself as a natural consequence." (p. 93). Thus the Standard system was an integral and logical part of the *production of Commodities*-- an interesting interpretation of Ricardo was simply a by-product of it.

The problem with these interpretations is that they create a prima-facie difficulty in interpreting Sraffa’s book. If the Standard commodity was devised to solve Ricardo’s problem, then what was the point in proving that the Standard system and the Standard commodity were unique to any given system of production? The uniqueness property of the Standard commodity has no bearing on Ricardo’s problem of ‘invariable measure of value’. Interestingly, we find scantly anything on the question of uniqueness of the Standard commodity in the vast literature on Sraffa’s book. Sraffa uses the uniqueness property of the Standard commodity to first show that his analysis of a given system of production can be done without having to measure wages in terms of this highly awkward theoretical devise and he later goes on to highlight the fact that the uniqueness property makes it clear that no logical grounds exist for comparing prices once the technique of the basic good system is even marginally perturbed (see f.n.2). We think that this goes to the heart of Sraffa's critique of neoclassical theory.

As is well known, the neoclassical demand and supply theory uses any arbitrary commodity or a combination of commodities as numeraire. The stability of the general equilibrium of a system depends upon the supply responses to hypothetical changes in prices measured by the chosen numeraire. These supply responses are generally accompanied by changes in techniques of production. Not only in the cases of increasing or decreasing returns, but also in the case of constant returns an increase or decrease in the quantity supplied of a commodity will be accompanied by changes in the techniques of production, as long as it is not assumed that all sectors in the economy have equal 'capital/labor' ratio.[3] This is because general equilibrium ensures full employment of all the resources. Thus if a relatively 'capital intensive' sector increases its quantity supplied it raises the relative price of capital vis-à-vis labor. Thus all the sectors readjust to relatively more 'labor intensive' techniques. Thus every point on a supply function represents a distinct Sraffian system. Yet the prices throughout a supply function are measured by the arbitrarily chosen numeraire. Sraffa's proof that there is no logical ground for comparing prices in two separate systems implies that the stability or instability of a general equilibrium is an artificial product of an arbitrary choice of the numeraire. In other words, comparing prices of a commodity x in the two systems may show a rise in the price of x when measured in terms of a numeraire commodity m_1;

[3] In this case Labor Theory of Value sails through too.

however, may show a fall in its price when measured in terms of a numeraire commodity m_2 . Below we present a simulation exercise that shows exactly this.

Experiment on reversing of prices after technical change

This describes a simple experiment to see if after technical change the direction of change in prices is independent of the numeraire used. The pricing model used is that of Sraffa in his book *Production of commodities by means of commodities*. Suppose that we have:

1. a system of production with n products
2. two possible numeraires m_1 and m_2
3. an initial technology matrix t_a and a slightly perturbed technology matrix t_b
4. then the Sraffian profit rate equalising rule gives us a tensor of prices

$\mathbf{P}_{1,a}$	$\mathbf{P}_{1,b}$
$\mathbf{P}_{2,a}$	$\mathbf{P}_{2,b}$

where the $\mathbf{p}_{i,j}$ are price vectors.

It is clear that in such a configuration we would expect that in going from $\mathbf{p}_{i,a} \to \mathbf{p}_{i,b}$ we would see some prices rising and some falling. Our hypothesis is that there will exist some prices that are rising in the transition $\mathbf{p}_{1,a} \to \mathbf{p}_{1,b}$, however will be falling in the transition $\mathbf{p}_{2,a} \to \mathbf{p}_{2,b}$. That is to say that a change in the numeraire will result in a change in the direction of price movements under technical change.

Summary Results

We constructed a large sample of random technical input output matrices and real wages by the method described later. Our method of construction ensured that each such combination of I/O matrix and real wage was economically feasible. We then chose the first two commodities of each random input output matrix and used them as two alternative numeraires. Using each of these numeraires we calculated Sraffian price vectors.

We then introduced a technical change to the matrix and recalculated the prices using each numeraire. We used two different sorts of technical change – neutral changes, which did not alter the net product, and output enhancing changes. Over a large number of runs we calculated:

1. The fraction of i/o tables that had at least one commodity show price direction reversal under technical change, for (a) neutral technical change or (b) output enhancing technical change.
2. The fraction of all commodities over all runs that showed price direction reversal under technical change, for (a) neutral technical change or (b) output enhancing technical change.

The results are summarized below, with *n* being the number of commodities:

n	Tables tested	1(a)	1(b)	2(a)	2(b)
5	500	0.236	0.51	0.047	0.134
20	500	0.894	0.888	0.262	0.280
80	500	0.982	0.978	0.341	0.312
200	40	0.975	1.0	0.373	0.344

It appears that as the number of commodities grows, the fraction of price vectors that exhibit at least one instance of price reversal under change of technology and numeraire tends to unity. The probability that any given commodity will show price reversal under change of numeraire appears to tend to about 1/3. Whether this limit is a property of the degree of sparseness of the I/O tables could be the subject of further investigation. However it is clear from these results that the problem of price direction reversal under change of numeraire is likely to affect a significant fraction of commodities.

Experimental procedure

We now describe our procedure for the construction of the experimental I/O tables. The input output matrices and real wage vectors that we used were subject to the following constraints:

1. For all commodities the net physical output of the economy will be positive.
2. Only part of the net physical output will be absorbed by the real wage ensuring the possibility of positive profits.

Our models will be defined in terms of a Use matrix U, whose element u_{ij} represents the amount of the *j*th commodity used in industry *i*. It is assumed that these quantities are in some sort of natural units. Conversely the Generate matrix G, represents the outputs of the industries such that g_{ij} is the output of commodity *j* by the *i*th industry.

- P is the price vector before technical change, Q the price vector after technical change.
- *w* is the real wage represented as a vector of physical units of each commodity.
- ρ is the rate of profit, which is assumed to be equal in all industries.
- λ specifies the labor usage of the economy.
- The variable *m* indicates which commodity is currently used as the numeraire.

The matrices U and G are set up by an algorithm to form a consistent pair of production matrices created in such a way that the economy has a net positive product of all commodities. The construction is parameterized to control how sparse the U and G matrices will be. That is to say, we can control the number of zero elements in the matrices. The I/O table construction procedure simultaneously constructs the labor input vector and initializes the rate of profit, which we chose to be 50%.

The approach taken to constructing the matrices is to start out with a simple pair of matrices containing a single commodity for $g_{1,1} > u_{1,1}$ and then carrying out a series of operations that grow the matrices whilst preserving the net input/output ratio of the system.

Essentially we start of with a simple ’corn economy’ whose expansion ratio is well defined, we then divide the corn into two categories of commodity whilst preserving the same overall expansion ratio, and recursively apply the process. We start out such that 1 seed planted yields 2 at harvest. Of the net product half a seed goes in wages and half a seed in profit. Thus the initial profit rate is 0.5 and the wage is 0.5.

The aim is to generate an I/O table that approximates the structure of real I/O tables. These can be presented with successively greater degrees of disaggregation-- thus at one level one might have a sector called timber products. On disaggregating this might divide into plywood, sawn timber, and fiberboard products. The three sub-sectors will show substantial similarity in their input structure, one would certainly expect them to be more similar in terms of cost structure than any of them were to for example non-ferrous metal production. This genetic similarity of sibling industries is to be emulated by the procedure of successively splitting industries, represented by rows in the U and G matrices, into two daughter industries that are similar to but not identical to each other. The two basic operations are to split the matrices along the columns to increase the number of products, and to split them along the rows to increase the number of industries. The process of constructing the I/O matrix is designed to work by decomposition, such that at each iterative step we have a new standard commodity with the same expansion ratio in terms of itself : i.e, a 2 to 1 expansion. If we can demonstrate that within the basic sector one can encounter price direction reversal after technical change under change of numeraire, then the same will obviously apply to the whole economy, since each economy contains a basic sector as a subpart.

Documentation describing the algorithms used in the experiment and the source of the computer program itself are available as the files joint pdf and joint pas from the website <http://www.dcs.gla.ac.uk/~wpc/reports/index.html>.

A simple worked example

An example of what we end up with is shown in Table 1.

```
G
     0.61684     0.00000     0.00000     0.00000     0.00000
     0.00000     0.03318     0.00000     0.00000     0.00000
     0.00000     0.00000     0.55197     0.00000     0.00000
     0.00000     0.00000     0.00000     0.41343     0.00000
     0.00000     0.00000     0.00000     0.00000     0.38458

U
     0.01304     0.00519     0.05109     0.11997     0.19107
     0.00000     0.00000     0.02063     0.00000     0.00000
     0.14247     0.00000     0.14848     0.00000     0.00000
     0.05249     0.00000     0.00535     0.00000     0.00000
     0.10042     0.01140     0.05044     0.08674     0.00122


   λ
     0.12921     0.00537     0.11651     0.01417     0.73474
w = real wage
     0.15421     0.00829     0.13799     0.10336     0.09615
```

Table 1: An example of the U and G matrices after removing joint production.

If we now select commodities 1 and 2 as two possible numeraires the profit equalizing price vectors are:

(1)

1.00000	0.82186	0.80477	0.22116	1.50925
1.21675	1.00000	0.97920	0.26909	1.83638

with a profit rate of 50%. We can now consider some types of technical change:

1. A neutral technical change, i.e., one that does not alter the overall productivity of labor in the economy, so that the economy produces the same bundle of goods before and after the change.
2. A productivity enhancing technical change, in which the net output of the economy in physical terms rises.

Neutral change

We can get a neutral technical change by swapping two rows of the commodity usage matrix, with the new usage matrix shown in Table 2,

U

0.01304	0.00519	0.05109	0.11997	0.19107
0.00000	0.00000	0.02063	0.00000	0.00000
0.05249	0.00000	0.00535	0.00000	0.00000
0.14247	0.00000	0.14848	0.00000	0.00000
0.10042	0.01140	0.05044	0.08674	0.00122

Table 2: The usage matrix after neutral technical change

in which we have swapped rows 3 and 4 of the matrix. Clearly the total usage of each commodity remains unchanged, as does the gross output, but the prices would be expected to alter. This is in fact the case, the resulting prices are:

(2)

before technical change

1.00000	0.82186	0.80477	0.22116	1.50925
1.21675	1.00000	0.97920	0.26909	1.83638

after technical change

1.00000	0.27711	0.22892	0.65370	1.42767
3.60868	1.00000	0.82609	2.35898	5.15202

change direction

down	down	down	up	**down**
up	down	down	up	**up**

Observe that when using commodity 1 as the numeraire the price of commodity 5 has fallen from 1.50925 to 1.42767, but using commodity 2 as numeraire, it has risen from 1.83638 to 5.15202. Thus the direction of price change is dependent on the numeraire chosen. For the other non-numeraire commodities the direction of change is independent of the numeraire.

Productivity enhancing change

We can improve the productivity of the economy by increasing the output of one of the diagonal elements in the G matrix. If we are to isolate the effect of technical change from the effect of class distribution of income, we must ensure that the increased net output is divided between labour and capital in the same ratio as existing national income. The resulting matrices are shown in Table 3

G

0.61684	0.00000	0.00000	0.00000	0.00000
0.00000	0.03318	0.00000	0.00000	0.00000
0.00000	0.00000	1.05197	0.00000	0.00000
0.00000	0.00000	0.00000	0.41343	0.00000
0.00000	0.00000	0.00000	0.00000	0.38458

U

0.01304	0.00519	0.05109	0.11997	0.19107
0.00000	0.00000	0.02063	0.00000	0.00000
0.14247	0.00000	0.14848	0.00000	0.00000
0.05249	0.00000	0.00535	0.00000	0.00000
0.10042	0.01140	0.05044	0.08674	0.00122

λ

0.12921	0.00537	0.11651	0.01417	0.73474
real wage				
0.15421	0.00829	0.38799	0.10336	0.09615

Table 3: Matrices with augmented productivity in industry 3, and compensating change in the real wage.

If we again solve for prices and get a third pair of price vectors:

(3)

before technical change				
1.00000	0.82186	0.80477	0.22116	1.50925
1.21675	1.00000	0.97920	0.26909	1.83638
after technical change				
1.00000	0.42632	0.34986	0.22845	1.48689
2.34565	1.00000	0.82066	0.53586	3.48772
change direction				
down	down	down	up	**down**
up	down	down	up	**up**

Again examining the prices of industry 5, we observe that using commodity 1 as the numeraire the price has fallen compared with (1) but using commodity 2 as the numeraire, the price has risen when compared with price system (1).

Conclusion

Of course, now it is well accepted among the high brow neoclassical economists that the stability of the general equilibrium is dependent on the choice of numeraire: "The reason why uniqueness, for example, does not depend on the choice of *numeraire* while stability may, is that stability depends on the adjustment process. Strictly speaking, a change of numeraire is simply a change of adjustment process: it is quite natural that the economy may be stable under one adjustment process but not under another" (Allingham, 1989, 203).

Above we have shown that this insight was already there in Sraffa's treatment of the numeraire. This also points to what Sraffa meant by "prelude to a critique of economic theory." Sraffa's proposition regarding the uniqueness of the Standard commodity points to the limitation of economics as science. All scientific theories are predictive in nature but in economics its predictions become meaningless once those predictions entail technical change in its *basic* system, since it loses all grounds for measuring the changes in its variables. This has a close relation with the later Wittgenstein's (1953) philosophy of language, which was, as acknowledged by Wittgenstein himself, highly influenced by Sraffa. Wittgenstein had argued that the meaning of a word is not attached to the thing it refers to but rather depends on the context, or what he called the "language game", of its use. One produces nonsense by attempting to drag a word from one language game to another; for example, words such as God or Soul make perfect sense in a religious discourse but produce nonsense in a scientific discourse. This is because there are no grounds for translating meaning from one language game to another. For Sraffa value of a commodity in economics occupies the same space as meaning of a word in language. The value of a commodity is well defined for a given *basic* good system but value of the same commodity in two different *basic* good systems cannot be compared, as there exists no grounds for measuring them in two systems.

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