Korean J. Chem. Eng., **21**(2), 329-337 (2004)

Nonparametric Nonlinear Model Predictive Control

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(*Received 18 August 2003 • accepted 5 December 2003*)

Abstract−Model Predictive Control (MPC) has recently found wide acceptance in industrial applications, but its potential has been much impeded by linear models due to the lack of a similarly accepted nonlinear modeling or data-based technique. Aimed at solving this problem, the paper addresses three issues: (i) extending second-order Volterra nonlinear MPC (NMPC) to higher-order for improved prediction and control; (ii) formulating NMPC directly with plant data without needing for parametric modeling, which has hindered the progress of NMPC; and (iii) incorporating an error estimator directly in the formulation and hence eliminating the need for a nonlinear state observer. Following analysis of NMPC objectives and existing solutions, nonparametric NMPC is derived in discrete-time using multi-dimensional convolution between plant data and Volterra kernel measurements. This approach is validated against the benchmark van de Vusse nonlinear process control problem and is applied to an industrial polymerization process by using Volterra kernels of up to the third order. Results show that the nonparametric approach is very efficient and effective and considerably outperforms existing methods, while retaining the original data-based spirit and characteristics of linear MPC.

Key words: Model Predictive Control, Process Control, Nonlinear Modeling, Volterra Kernels, M-Sequence

INTRODUCTION

Model Predictive Control (MPC) represents a class of control schemes where the control signal generation involves the on-line use of a parametric or a nonparametric model of the plant. Major design techniques of MPC include Model Algorithm Control, Dynamic Matrix Control, Internal Model Control and Generalized Predictive Control, etc. [Garcia et al., 1989]. The underlying strategy of MPC is, at any given time, to solve on-line a receding open-loop optimal control problem over a finite time horizon, where only the first control of the resulting control sequence is actually implemented on the plant. MPC algorithms are very intuitive and easy to understand, and practical constraints can often be included in the on-line open-loop algorithm [Mayne et al., 2000]. MPC has received worldwide attention because it is straightforward to implement in industrial applications, particularly in chemical processes, where the dynamics is relatively slow and can hence accommodate on-line optimization easily [Garcia et al., 1989].

However, much of the work has been confined to a linear control strategy, based on a linear model in predicting future values of the plant response [Doyle III et al., 1995; Henson, 1998]. Since severe nonlinearity often exists in an industrial process that can hardly be ignored in practice, higher control performance can only be achieved through using a nonlinear model [Pearson, 2002], including polynomial ARMA model [Hernansez and Arkun, 1993], bilinear model [Yeo and Williams, 1987], combined ARMA-Hammerstein model [Fruzzetti et al., 1997], extended Kalman filter [Ahn et al., 1999] and neural networks [Saintdonat et al., 1991]. In a bid to achieve improved performance, however, these approaches sacrifice the simplicity, accuracy and characteristics arising from process I/O data [Henson, 1998]. This limits the range of industrial applications, as these models are often difficult and inaccurate to obtain in practice [Pearson, 2002; Tan and Li, 2002].

This has stimulated work on formulating MPC for use with a nonparametric Wiener model [Norquay et al., 1998; Jeong et al., 2001] and (its more practical version) Volterra model [Doyle III et al., 1995; Genceli and Nikolaou, 1995; Maner et al., 1996] and on Volterra modeling [Kashiwagi and Sun, 1995; Kashiwagi and Rong, 2002; Pearson, 2002]. A potential advantage of using a nonparametric model is that it can yield nonlinear MPC (NMPC) directly from process I/O data [Kashiwagi et al., 1998]. However, methods developed elsewhere require a first-principles model as so to derive a second-order Volterra model analytically from the bi-linearized fundamental model [Doyle III et al., 1995; Genceli and Nikolaou, 1995; Maner et al., 1996]. Hence, the original limitation on the range of applications still remains. Further, these NMPC methods are so far limited to one nonlinear kernel only, *i.e.*, up to the second-order Volterra kernels may be obtained and utilized. Hence, the fuller potential of NMPC remains yet to be realized.

Recently, progress on Volterra modeling with a high degree of accuracy has been made at Kashiwagi Laboratory, Kumamoto University, using Volterra kernels of up to the third order which can now be measured easily by perturbing the plant with a pseudorandom M-sequence signal that provides enough excitation and yet is acceptable in an industrial situation [Kashiwagi and Sun, 1995; Kashiwagi, 1996]. This progress permits Volterra NMPC schemes to be extended to the third order. In place of a Kalman estimator for the linear case, Volterra measurements offer the potential to realize NMPC the same way as linear MPC and hence the full capability of the

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‡This paper is dedicated to Professor Hyun-Ku Rhee on the occasion of his retirement from Seoul National University.

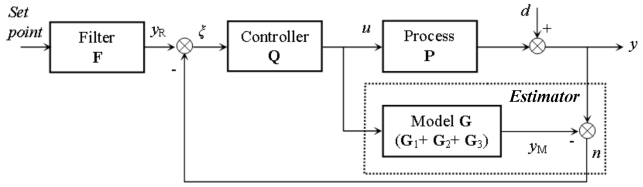

Fig. 1. Model predictive control framework.

Volterra methodology, as reported in this paper.

In the next section, objectives and solutions of NMPC are first analyzed, followed by time-domain formulation using multi-dimensional convolution with Volterra kernels. For this, M-sequence based high-order Volterra identification techniques are detailed in Section 3. Case studies are reported in Section 4 and conclusions are drawn in Section 5.

NONPARAMETRIC NMPC

1. Design Objective and Exact Solution

Refer to Fig. 1 for notations in a general framework of model-based control. The pre-filter **F** outside the loop is for robust considerations in model-following and is often a unity-gain first-order low-pass with a relatively small time-constant or a critically damped second-order filter with a relatively high natural frequency. The process **P** is modeled by **G**. The controller **Q** generates a control sequence u through observed plant output y and output predicted by **G**. The estimator block will be interpreted in Section 2.2.

Suppose that **P** has a fading memory, as indeed found in many industrial processes. Then its output can be represented by the Volterra series as a temporal extension of the Taylor series expansion [Boyd and Chua, 1985], as given by

$$y(t)=\sum_{i=1}^{\infty}\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty}p_i(\tau_1,\tau_2,\cdots\tau_i)u(t-\tau_1)\cdots u(t-\tau_1)d\tau_1 \cdots d\tau_i+d(t) \quad (1)$$

where p_i is the *i*-th order Volterra kernel, an *i*-dimensional impulse response of the nonlinear process. This equation can be compactly rewritten as [Doyle III et al., 1995]

$$y(t)=\sum_{i=1}^{\infty}y^{(i)}(t)+d(t) \quad (2)$$

$$=\sum_{i=1}^{\infty}p_i*u(t)+d(t) \quad (3)$$

where $y^{(i)}(t)$ is the degree-*i* Volterra contribution to the overall output and * denotes *i*-dimensional convolution.

Similar to any other control schemes, the design objective, J: $R^K \rightarrow R^+$, is to find a **Q** such that

$$J=\min_u|e(t)| \quad (4)$$

subject to constraints imposed upon by saturations of actuators and their change rates:

$$u_{Min}\le u(t)\le u_{Max} \quad (5)$$

$$\Delta u_{Min}\le \Delta u(t)\le \Delta u_{Max} \quad (6)$$

where

$$u(t)=q*\xi(t), \quad \xi(t)=y_R-n(t), \quad n(t)=y(t)-y_M(t) \quad (7)$$

$$e(t)=y_R(t)-y(t) \quad (8)$$

Here q is the impulse response of the controller **Q** and K is the degree of freedom in $\mathbf{u}\in R^K$ under optimization and $y_R(t)$ is the desired output. Denoting the finite number of Volterra kernels of **G** that may be identified from **P** by g_i, i=1, 2, …V,

$$y_M(t)=\sum_{i=1}^{V}g_i*u(t)=g*q*\xi(t) \quad (9)$$

From this and Eq. (8),

$$e(t)=(\xi(t)-n(t))-(y_M(t)+n(t))=\xi(t)-g*q*\xi(t) \quad (10)$$

The objective of Eq. (4) is therefore strictly met if

$$(g*q)*\xi(t)=\xi(t), \qquad \forall\ \xi(t) \quad (11)$$

i.e., if an exact inverse controller $\mathbf{Q}=\mathbf{G}^{-1}$ is found [Doyle III et al., 1995].

However, obtaining a strictly zero J of Eq. (4) or strictly satisfying Eq. (11) for an exact inverse would be impossible in practice, as otherwise the controller 'gain' would, under constraints, need to be infinite for all frequency and time [Li et al., 2002]. This is also because an exact inverse implies that the MPC will reduce to open-loop control in effect, which in turn cannot guarantee Eq. (11) or Eq. (4). Such a realization can lead to steady-state offsets if a disturbance d(t) or estimation error n(t) exists [Genceli and Nikolaou, 1995; Henson, 1998].

To resolve this problem, Doyle III et al. [1995] have decomposed **G** into degree-1 and degree-2 Volterra components, G_1 and G_2 (Fig. 1), and derived a 'generalized inverse' based on the 'left inverse' of the degree-1 component. Their Volterra NMPC framework developed is in a second-order analytic domain, as it needs an analytical Volterra model and this can only be identified up to the second order via the bilinear Carlemann 'linearization' applied to a first-principles model. This means that their NMPC controller offers only one nonlinear kernel and also loses the ease of realization present in the data-based linear MPC.

Further, this treatment still does not solve the offset and robustness problem. Unfortunately, a well-developed theory for nonlinear state observers to combat this is unavailable [Henson, 1998]. While this issue remains unaddressed in the main derivations, Doyle III et al. [1995] in their case studies have intuitively augmented the controller with the pre-filter **F**, by moving it inside the loop to prevent an exact inverse. Clearly, this adds complexity to controller synthesis and revolts the rigor of their prior theoretical derivations. We shall show in the following section that this arrangement is unnecessary and the offset problem will be solved neatly by using a nonparametric formulation.

2. Nonparametric Formulation

To relieve application engineers from needing to obtain, and linearize, a first-principle based nonlinear model, NMPC is to be formulated here by using the nonparametric model given by Eq. (10). Requiring only a modest computational power in the realization and not needing an on-line Runge-Kutta solver (as does the analytical method by Maner et al. [1996]), this should be more suitable for computer implementation and for retaining the discrete-time characteristics of linear MPC. Further, a third or higher-order Volt-

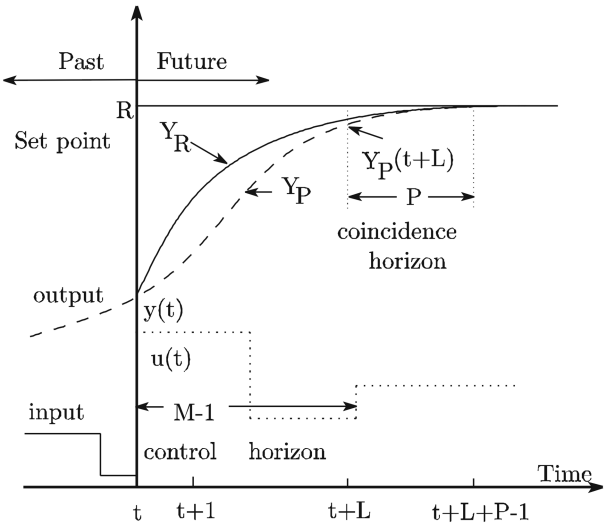

Fig. 2. General principle of discrete-time MPC.

erra controller formulated in this way does not add more structural complications to a second-order one.

Discrete-time formulation of MPC can be illustrated as shown in Fig. 2 [Kashiwagi et al., 1998]. It requires that an 'open-loop' optimization problem be solved on-line only over a finite prediction horizon P for a finite control horizon M. Within these receding horizons, the objective of Eq. (4) becomes

$$J_d = \min_{u(t+1|t),\ldots,u(t+M|t)} \left| e(t+j|t) \right|_{j=L}^{L+P-1} \tag{12}$$

where L≥1 is the minimum prediction step desired and e(t+j|t) $\forall$ j∈ {L, L+1, …, L+P−1} is estimated from the model and output information available at time t. Here the metric norm is evaluated within the finite discrete set {L, L+1, …, L+P−1} and may be either L_1, L_2, L_∞, mixed, weighted or any other norm provided J_d can be optimized on-line.

The open-loop offset problem encountered in an analytical inverse can be handled in discrete-time by designing an error estimator, which will give the controller an implicit integral action [Henson, 1998]. As we shall see in the sequel, this is naturally and elegantly realized without needing full-state feedback, given an I/O based nonparametric, as opposed to an analytic, model. The simplest error estimator is a zero-order one, *i.e.*, the discrepancy between the actual and modeled output at time t is used throughout the prediction horizon [Henson, 1998]. Genceli and Nikolaou [1995] have shown that, using such an error estimator, the closed-loop system with a Volterra controller is asymptotically stable with zero offsets if the uncertainty and its rate of change are bounded and the end-condition $y_M(\infty)=R$ is met. Here we extend this result to the generic MPC framework of Fig. 1.

Theorem: The linear MPC framework of Fig. 1 is directly applicable to nonlinear MPC in the time-domain with a Volterra predictor

$$y_P(t+j|t)=y(t)+[y_M(t+j)-y_M(t)], \quad \forall j\in\{L, L+1, \ldots, L+P-1\} \tag{13}$$

and objective

$$J_d = \min_{u(t+1|t),\ldots,u(t+M|t)} \left| \xi(t+j) - y_M(t+j) \right|_{j=L}^{L+P-1} \tag{14}$$

The resultant nonlinear controller is asymptotically stable with zero offsets from set-point R, if the uncertainty and its rate of change are bounded and the end-condition $y_M(\infty)=R$ is met.

Proof: Eq. (13) is equivalent to

$$y_P(t+j|t)-y_M(t+j)=y(t)-y_M(t), \quad \forall j\in\{L, L+1, \ldots, L+P-1\} \tag{15}$$

This implies that the modeling error $y(t)-y_M(t)$ estimated from the nonparametric Volterra model is held constant throughout the prediction horizon by a 'zero-order estimator'. Denote this amount of unmeasured state variables by n(t) and error-correct the set-point to $y_R(t+j)-n(t)=\xi(t+j)$. This agrees with Fig. 1 for the linear case. Therefore, the normed quantity in Eq. (14) evaluates to

$$y_R(t+j)-[y(t)-y_M(t)]-y_M(t+j)=y_R(t+j)-y(t+j|t) \tag{16}$$

and is hence the same as that in Eq. (12). <qed>.

Note that here $y_M(t)$ can be estimated $\forall$ t≥0 from convolution between u(t) and the Volterra kernels, given the assumption that plant **P** has a fading memory and hence the Volterra model converges [Boyd and Chua, 1985]. While Genceli and Nikolaou have shown that prediction using a second-order Volterra model will meet the sufficient condition for robust tracking with zero offsets [Genceli and Nikolaou, 1995; Henson, 1998], a third-order one will further reduce the estimation error (as we shall see in Section 3). Hence the theorem proves following their derivations [Genceli and Nikolaou, 1995; Henson, 1998].

The discrete-time Volterra model allows a rigorous retention of the original characteristics and spirit of linear MPC. The theorem signifies that the control sequence can be optimized such that $y_M(t)$ tracks $\xi(t)$ for an implicit inverse within the finite horizons, instead of mathematically formulating y(t) to track $y_R(t)$ $\forall$ t. To further reduce the estimation error and improve robustness, the 'hard-command' of a set-point change can be replaced by a 'soft-command' trajectory $y_R(t)$ for the process to follow, provided $y_R(\infty)=R$ [Li et al., 2002].

Without loss of generality, consider a first-order low-pass pre-filter with unity-gain

$$F(s)=\frac{1}{1+\tau s} \tag{17}$$

The reference trajectory in the continuous domain is given by

$$y_R(t)=R(1-e^{-t/\tau}) \tag{18}$$

It is not difficult to derive its discrete-time version in difference equation

$$y_R(t+1)=\alpha y_R(t)+(1-\alpha)R \tag{19}$$

where, with a given sampling interval T,

$$\alpha=1-\frac{T}{\tau} \tag{20}$$

Iterating the first-order equation yields

$$y_R(t+j)=\alpha^j y_R(t)+(1-\alpha^j)R \tag{21}$$

for calculating the reference trajectory j steps ahead [Kashiwagi et al., 1998].

Since the control signal will go through a D/A converter, the resolution of u will be finite. Hence we can search for a discrete value

of Δu for each element in $\mathbf{u} \in R^K$ [Kashiwagi et al., 1998]. A simple optimization algorithm that accommodates constraints easily is an *a posteriori* hill-climbing algorithm, *i.e.*, hill-climbing guided by trail-and-error [Li et al., 2002]. Compared with conventional nonlinear programming, an *a posteriori* search takes a longer time, but is much more straightforward to implement for any objective metric under any constraints [Li et al., 2002; Tan and Li, 2002].

Now the only task left is to obtain Volterra kernels in Eq. (10) for i=1, 2 and 3, by using a pseudorandom sequence that provides the plant with enough excitation and yet are acceptable in an industrial situation [Kashiwagi, 1996; Pearson, 2002].

THIRD-ORDER VOLTERRA MODEL

Consider the identification of the nonlinear process of Eq. (1). In order to identify Volterra kernels $g_i(\tau_1, \tau_2, \ldots)$, an M-sequence [Kashiwagi, 1996] is used to excite the nonlinear system with acceptable amplitude. The resultant cross-correlation function $\phi_{uy}(\tau)$ between the input u(t) and the output y(t) can be written as [Kashiwagi, 1996]

$$\begin{aligned}\phi_{uy}(\tau) &= \overline{u(t-\tau)y(t)} \\ &= \sum_{i=1}^{\infty} \int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} p_i(\tau_1, \tau_2, \ldots \tau_i) \\ &\quad \overline{u(t-\tau)u(t-\tau_1)\ldots u(t-\tau_i)}\, d\tau_1 \ldots d\tau_i \end{aligned} \tag{22}$$

where $\overline{\ }$ denotes time average. Usually the moment of u(t) is difficult to obtain, but with an M-sequence, the *n*-th moment of u(t) yields easily. Here, the (i+1)th moment of the input M-sequence u(t) is given by [Kashiwagi, 1996; Kashiwagi and Rong, 2002]

$$\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)\ldots u(t-\tau_i)} = \begin{cases} 1 & (\text{for certain } \tau) \\ -1/N & (\text{otherwise}) \end{cases} \tag{23}$$

where N is the period of the M-sequence. For an M-sequence with the degree greater than 16, 1/N is in the order below 10^{-5}. Hence Eq. (23) can be approximated as a set of impulses which appear at certain τ's.

Let us consider measuring the *i*-th Volterra kernel. Then for integers $k_{i1}^{(j)} < k_{i2}^{(j)} < \ldots, k_{i,i-1}^{(j)}$, there exists a unique $k_{ii}^{(j)}$ (mod N) such that [Kashiwagi and Sun, 1995]

$$u(t)u(t+k_{i1}^{(j)})\ldots u(t+k_{i,i-1}^{(j)}) = u(t+k_{ii}^{(j)}) \tag{24}$$

where j is the number of the group $(k_{i1}, k_{i2}, \ldots, K_{i,i-1})$ for which Eq. (24) holds. This property is called the *Shift and Add Property* of the M-sequence [Kashiwagi, 1996]. Assume that the total number of those groups is m_i (that is, j=1, 2, …, m_i). Then Eq. (23) becomes unity when

$$\tau_1 = \tau - k_{i1}^{(j)}, \tau_2 = \tau - k_{i2}^{(j)}, \ldots, \tau_i = \tau - k_{ii}^{(j)} \tag{25}$$

Therefore Eq. (22) is approximated by

$$\phi_{uy}(\tau) = \sum_{i=1}^{\infty}\sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \ldots, \tau - k_{ii}^{(j)}) \tag{26}$$

Since $g_i(\tau_1, \tau_2, \ldots, \tau_i)$ is zero when any of τ_i is smaller than zero, each $g_i(\tau - k_{i1}^{(j)}, \ldots, \tau - k_{ii}^{(j)})$ in Eq. (26) appears in the cross-correlation function $\phi_{uy}(\tau)$ when $\tau > k_{ii}^{(j)}$.

In order to obtain Volterra kernels from Eq. (26), $k_{ii}^{(j)}$ must appear sufficiently apart from one another. For this to be realized, we should select suitable M-sequences that set the cross-sections of the Volterra kernels sufficiently apart from one another. Some appropriate M-sequences are given in Kashiwagi [Kashiwagi and Sun, 1995; Kashiwagi, 1996].

When measuring Volterra kernels up to the third order, the cross-correlation function $\phi_{uy}(\tau)$ becomes

$$\begin{aligned}\phi_{uy}(\tau) &= \Delta t g_1(\tau) + F(\tau) + 2(\Delta t)^2 \sum_{j=1}^{m_2} g_2(\tau - k_{21}^{(j)}, \tau - k_{22}^{(j)}) \\ &\quad + 6(\Delta t)^3 \sum_{j=1}^{m_3} g_3(\tau - k_{31}^{(j)}, \tau - k_{32}^{(j)}, \tau - k_{33}^{(j)}) \end{aligned} \tag{27}$$

where

$$F(\tau) = (\Delta t)^3 g_3(\tau, \tau, \tau) + 3(\Delta t)^3 \sum_{q=1}^{m_1} g_3(\tau, q, q) \tag{28}$$

and Δt is the time increment or sampling period. To generalize, we have,

$$\phi_{uy}(\tau) = \Delta t g_1(\tau) + F(\tau) + \sum_{i=2}^{\infty} i!(\Delta t)^i \sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \ldots, \tau - k_{ii}^{(j)}) \tag{29}$$

Here $F(\tau)$ is a function of τ and is the sum of the odd order Volterra kernels when some of its arguments are equal. Since $F(\tau)$ appears together with $g_1(\tau)$ in an overlapped manner, $F(\tau)$ must be calculated from the odd order Volterra kernels and be subtracted from the measured $g_1(\tau)$ in order to obtain an accurate $g_1(\tau)$. Following this, a Volterra model can be identified for use with Eq. (13) [Kashiwagi et al., 1998].

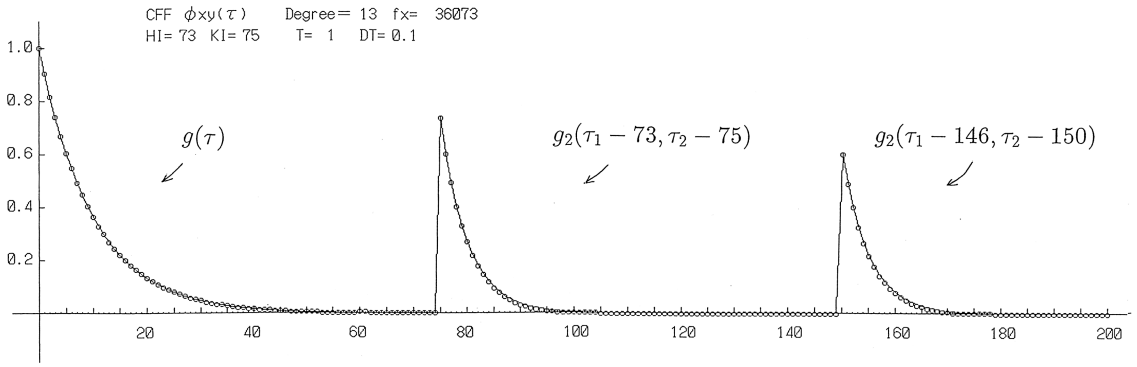

Fig. 3. An example of crosscorrelation function $\phi_{uy}(\tau)$.

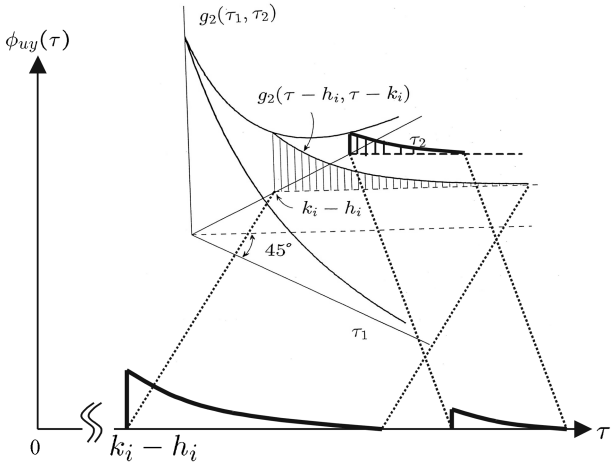

Fig. 4. Reconstruction of second Volterra kernel.

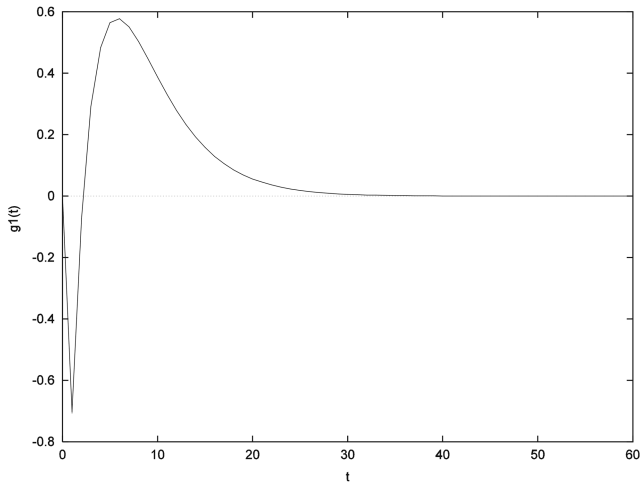

Fig. 5. The 1st-order Volterra kernel of the van de Vusse reactor.

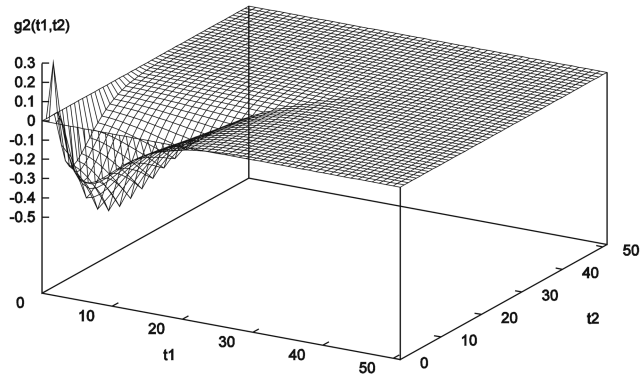

Fig. 6. The 2nd-order Volterra kernel of the reactor.

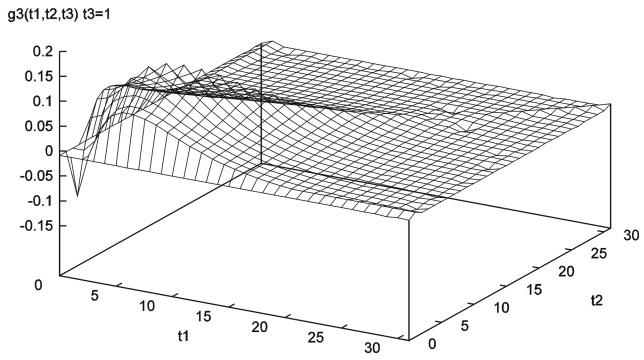

Fig. 7. The 3rd-order Volterra kernel (t_3=1) of the reactor.

To illustrate this method, an example is shown below. When we use M-sequence of 13 degree with the generating characteristic polynomial f(x)=36073 in octal notation, we can get, by computer search, those $k_{ir}^{(j)}$ in Eq. (24) as

$$k_{21}^{(1)}=73, k_{22}^{(1)}=75, k_{21}^{(2)}=146, k_{22}^{(2)}=150, \ldots$$

That is,

$$\phi_{uy}(\tau)=\Delta t g_1(\tau)+2(\Delta t)^2\{g_2(\tau-73\Delta t, \tau-75\Delta t) + g_2(\tau-146\Delta t, \tau-150\Delta t)+\ldots\}+\ldots$$

where $F(\tau)$ and third order kernel are omitted for simplicity. Fig. 3 shows an example of $\phi_{uy}(\tau)$ actually calculated for some nonlinear system, where two cross-sections of second Volterra kernel can be seen. Since M-sequence is selected in order for those kernels to be separated from each other, we can cut those kernel slices from $\phi_{uy}(\tau)$, and reconstruct the second order kernel as shown in Fig. 4. When third order kernel exists, we reconstruct $g_3(\tau_1, \tau_2, \tau_3)$ first, calculate $F(\tau)$ in Eq. (18) and subtract $F(\tau)$ from the part of crosscorrelation function in the neighborhood of origin to obtain $g_1(\tau)$.

CASE STUDIES

1. Benchmark van de Vusse Reactor Problem

The van de Vusse reactor is studied in the second example of Doyle III et al. [1995]. This is an isothermal continuous mixed tank reactor. Control of two density components is carried out by adjusting the amount of input flow. The process is known to be highly nonlinear and is used in nonlinear process control as a benchmark problem.

Denote the two component concentrations as x_1 and x_2, and the amount of input flowing as u. Under the same condition as that in Doyle III et al. [1995], the process is modeled by

$$\begin{cases} \dfrac{dx_1}{dt} = -50x_1 - 10x_1^2 + (10 - x_1)u \\ \dfrac{dx_2}{dt} = 50x_1 - 100x_2 - x_2 u \\ y = x_2 \end{cases} \quad (30)$$

Using the method presented in Section 3 to identify the Volterra kernels, the results obtained are shown in Figs. 5-7. The M-sequence used here has an amplitude of 0.025 and a characteristic polynomial f(x)=260577. With the measured Volterra kernels g_1, g_2, and g_3, responses of the process can be estimated. In Fig. 8, these are validated against the actual output from Eq. (30) with respect to two step inputs. When a first and a second-order Volterra models are used, the results are almost the same as those in Doyle III et al. [1995] (Figs. 9 and 10). However, it is clear that a third-order model significantly outperforms both.

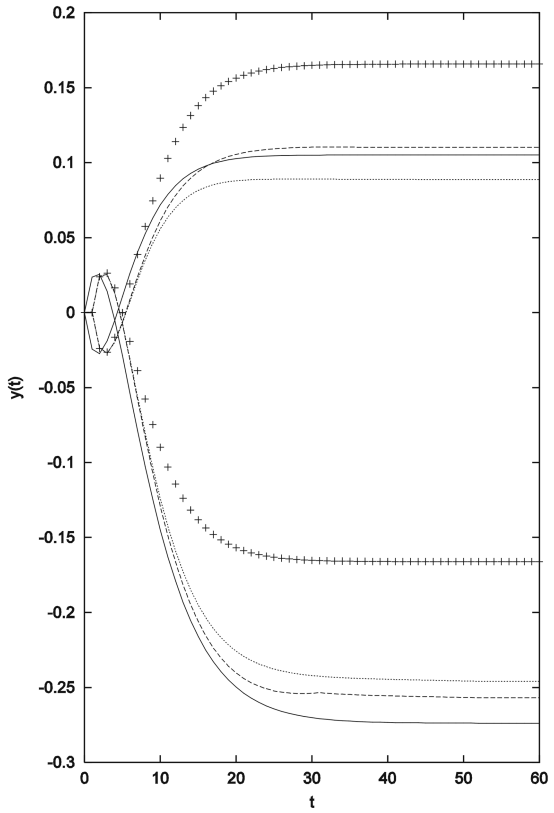

Fig. 8. Actual and modeled step responses of the van de Vusse reactor (solid: actual output; +: linear model; dotted: up to the 2nd-order Volterra kernels; dashed: up to the 3rd-order kernels).

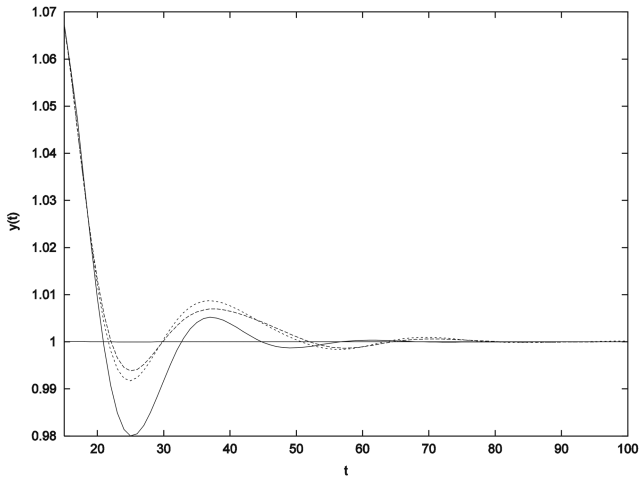

Fig. 9. NMPC performance comparison on the van de Vusse reactor (solid: linear model; dotted: 2nd-order Volterra model; dashed: 3rd-order model).

Using the measured Volterra kernels g_1, g_2, and g_3, nonparametric NMPC is realized for α=0.5 and M=P=10. Here, for comparison on the same ground, a quadratic objective (L_2 norm) is used, whilst any objective (including non-differentiable ones) can be used with an *a-posteriori* optimizer [Li et al., 2002; Tan and Li, 2002]. The result of the NMPC is shown in Fig. 9 for a set-point change

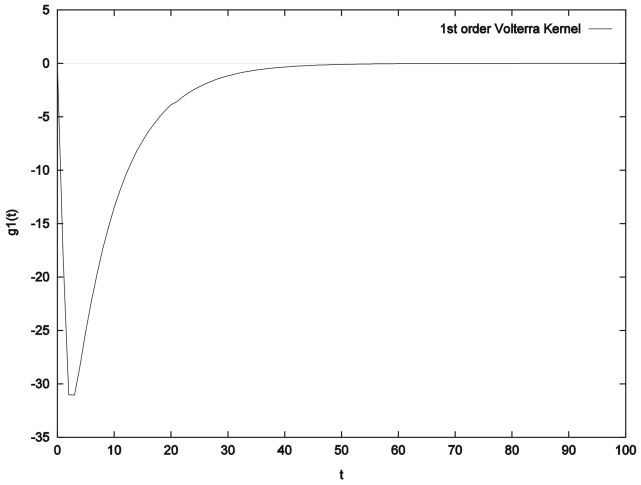

Fig. 10. Obtained first-order Volterra kernel of the Mitsubishi polymerization process.

from 1.12 to 1.0. We see that, while all nonparametric NMPC controllers offer zero-offsets, the third-order one offers superior performance. It is not only improved over the second-order controller, but this is also achieved without requiring an analytical model, which is needed for a parametric method [Doyle III et al., 1995].

2. Application to a Polymerization Process

Following the benchmark tests, the developed higher performing and easier to implement NMPC method is applied to a process problem of Mitsubishi Chemical Corp. Their chemical reactor is described by the differential equation

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{1}{Tp_1}(-x_1 + Kp_1 u_1) \\ \dfrac{dx_2}{dt} = \dfrac{1}{Tp_2}(Kp_2 x_1 x_2 - x_2 + Kp_3 u_2) \\ y = x_2 \end{cases} \tag{31}$$

with initial conditions

$$\begin{cases} x_1 = 0.02\ \text{kgh}^{-1}, \quad x_2 = 5.0\ \text{kgcm}^{-2} \\ u_1 = 0.05\ \text{kgh}^{-1}, \quad u_2 = 3195\ \text{kgh}^{-1} \end{cases} \tag{32}$$

where x_1 is the consumption velocity of catalyst, x_2 is gas density, u_1 is the supply quantity of catalyst, u_2 is the supply quantity of polyethylene, and Tp_1, Tp_2, Kp_1, Kp_2 and Kp_3 are constants. Here the control input to optimize is u_1 and the output to control is x_2, required to follow a step change in reference to R=10 kg cm^{-2} from 5.

For this, an M-sequence, denoted by Δu, with amplitude ±0.025 and characteristic polynomial f(x)=260577 in octal notation is applied to the reactor, with a sampling period of 0.3 h. Taking cross-correlations between Δu and Δy, Volterra kernels are measured. These are shown in Figs. 10-12. In Figs. 13-15, comparison is made between the actual output and the Volterra estimates responding to a sinusoidal input. We see again that the third-order model offers the best estimation which should be sufficient enough to preclude the need for a further higher-order model.

Then, using the measured Volterra kernels, NMPC is realized with the same settings in the van de Vusse case. The search for the con-

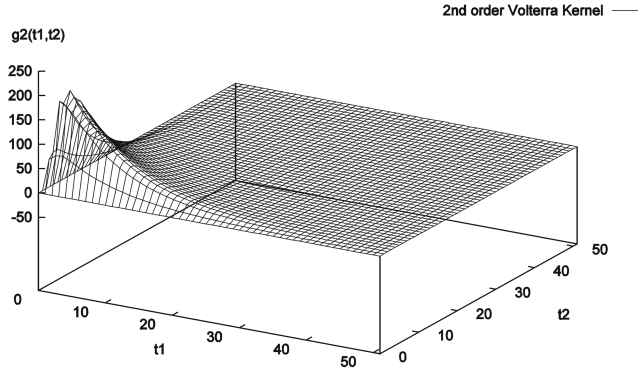

Fig. 11. Obtained second-order Volterra kernel of the process.

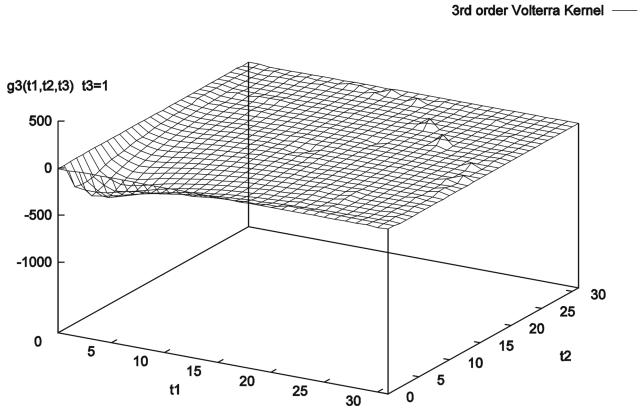

Fig. 12. Obtained third-order Volterra kernel of the process.

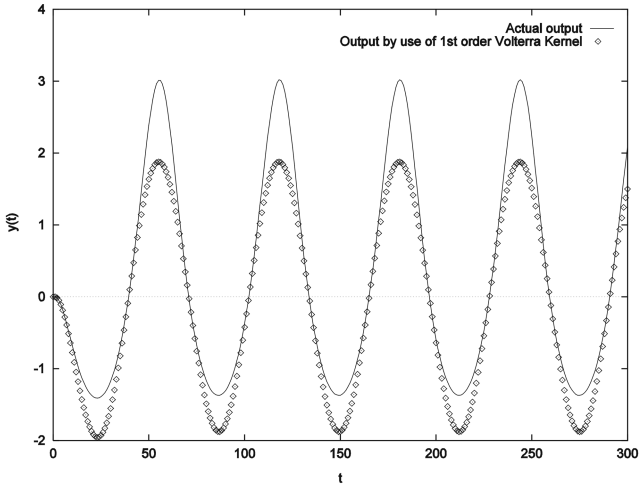

Fig. 13. Comparison between the actual output and the output estimated from a first-order Volterra model of the Mitsubishi polymerization process.

trol sequence is carried out within the range of ±0.05 with a 0.001 increment. Figs. 16 and 17 present the controlled performance and control signals. From the results, we see that the nonparametric NMPC formulated from the third-order Volterra model offers the best closed-loop performance.

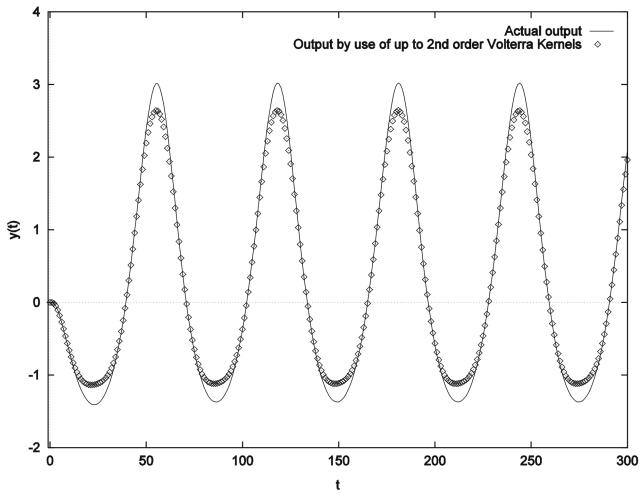

Fig. 14. Comparison between the actual output and the output estimated from a second-order model.

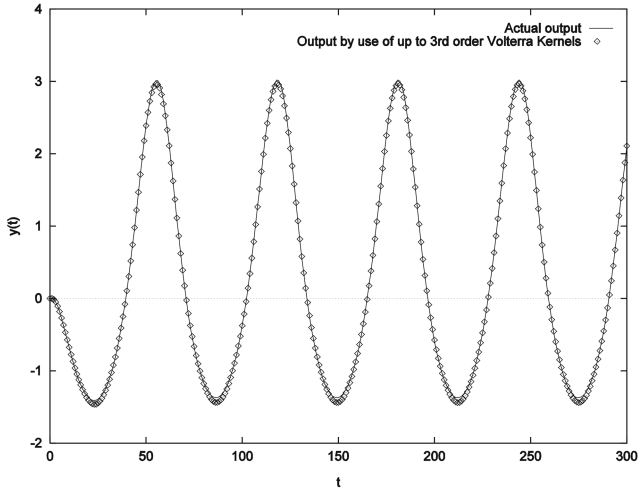

Fig. 15. Comparison between the actual output and the output estimated from a third-order model.

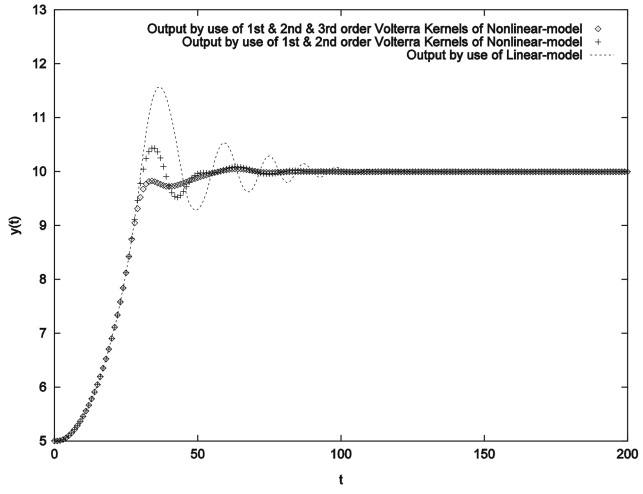

Fig. 16. Performance of the NMPC controllers formulated using the first, second and third-order Volterra models for the Mitsubishi polymerization process.

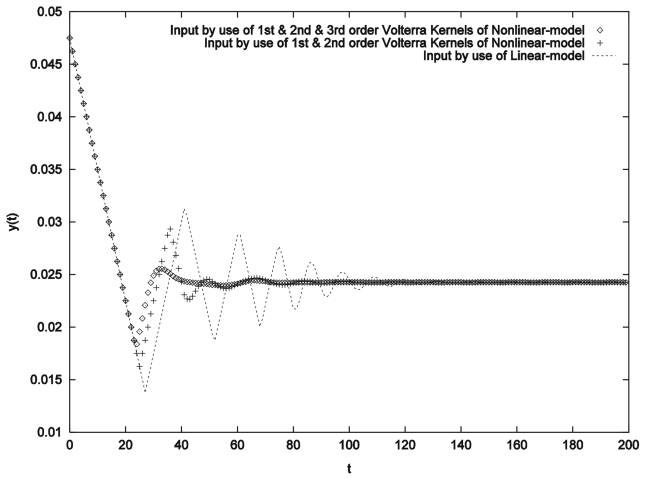

Fig. 17. Nonparametric control inputs optimized for models with three different levels of nonlinearity.

CONCLUSION

In this paper, methods for formulating and realizing nonparametric Volterra NMPC have been developed. They allow retention of the original characteristics of linear MPC and relieve practicing engineers from the tedious task of obtaining an often less accurate and less computerized fundamental model. This simple and yet rigorous approach to NMPC incorporates an error estimator automatically in the realization with zero offsets, needing no explicit nonlinear observer. The nonparametric NMPC has been validated against the benchmark van de Vusse nonlinear process control problem and then applied to the control of an industrial polymerization process by using Volterra kernels of up to the third order. Results show that it is very efficient and effective and considerably outperforms existing methods. Further work includes the development of powerful nonlinear search algorithms for use with on-line implementation under multiple constraints.

ACKNOWLEDGMENTS

This paper was partly presented at the 3rd Korea-Japan Joint Seminar on Chemical Processes held in October 2002 at Seoul National University, Korea. The authors would like to express their gratitude to Prof. Hyun-Ku Rhee of Seoul National University for organizing this seminar and for giving the authors valuable comments on this paper. Also the authors would like to thank Mitsubishi Chemical Corp., Japan, for providing the authors with the data of polymerization process.

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