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# Gas-Plasma compressional wave coupling by momentum transfer

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Abstract. Pressure disturbances in a gas-plasma mixed fluid will result in a hybrid response, with magnetosonic plasma waves coupled to acoustic waves in the neutral gas. In the analytical and numerical treatment presented here, we demonstrate the evolution of the total fluid medium response under a variety of conditions, with the gas-plasma linkage achieved by additional coupling terms in the momentum equations of each species. The significance of this treatment lies in the consideration of density perturbations in such fluids: there is no 'pure' mode response, only a collective one in which elements of the characteristics of each component are present. For example, an initially isotropic gas sound wave can trigger an anisotropic magnetic response in the plasma, with the character of each being blended in the global evolution. Hence sound waves don't remain wholly isotropic, and magnetic responses are less constrained by pure magnetoplasma dynamics.

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# 1. Introduction

Realistic plasmas generally consist of a mixture of ionized and neutral components, with the latter either being the non-ionized source of plasma or a buffer gas. There is a huge variety of plasma contexts in which the neutral gas has a significant effect on the plasma evolution. In some of these cases, the influence arises because the plasma is the ionized component of the neutral, so that there is a continual exchange between species: Miyagawa et al. (2006) discusses gas and plasma flow inside a closed plasma processing system, in which the neutral pressure has a significant effect on the plasma surface treatment. Fruchtman *et al.* (2005) take this to a greater extreme by examining the case where the ionization is sufficiently strong to deplete the neutral density significantly, leading to dramatic ionization effects. Such considerations of charge-exchange, ionization and recombination are also addressed by Helander et al, (1994), but mainly from the perspective of the evolution of plasma transport properties, rather than the whole ensemble (neutral-neutral collisions are not treated). Apart from plasma processing applications and plasma-surface considerations in the tokamak divertor context, plasma-neutral physics plays a leading role in astrophysical and space plasmas, in which a wide variety of parameter values can be experienced (in fact the analysis of Fruchtman *et al.* was partly motivated by the wide-ranging degree of ionization and neutral depletion that can be encountered in space plasmas). The interaction between the solar wind and the neutral interstellar medium is the topic of the article by Pogorelov & Zank (2005) in which the geometry of the interaction region at the heliopause boundary is significantly affected by the relative abundance of neutrals.

This article considers a different problem from those cited in the preceding paragraph in that here we wish to consider the global pressure response to small amplitude disturbances without triggering ionization. Sigalotti et al. (2004) addresses part of this by examining the coupling between MHD magnetosonic waves and plasma sound waves as a direct result of the presence of neutral component, showing that the phase speeds of such waves are significantly altered from the fully ionized case. Our analysis will agree with this, but encompasses the neutral response simultaneously: feedback from the plasma influences the neutral, and vice versa. The closest in concept to the analysis presented here is the work by Koshevaya et al. (2001), in which atmospheric acoustic modes trigger plasma waves in the ionosphere by collisions between neutral gas molecules and ions. However, we believe that coupled solutions for waves in different media have not been examined in detail, and we present the results of our analysis of the interaction between the velocity field of a magnetofluid wave and the sound wave of an inert, neutral gas. The two fluids are interpenetrating and exchange momentum, but only the plasma responds directly to magnetic influences. In this way, the velocity field of each species induces in the other a compressional response.

# 2. The model equations

The interaction between the two interpenetrating gases is modelled in terms of a frictional drag term in the momentum equation for each species, in such a way that when the one-fluid momentum equation for the entire medium is constructed, the interspecies frictional terms cancel exactly.

The following subsections detail the modelling of the plasma, and then the neutral gas, in isolation, and finally the interaction term and its consequences. Throughout this article, variables with subscript 0 are equilibrium quantities.

Consider a one-fluid magnetized plasma mixed with a simple single component neutral gas, each at the same temperature. The plasma and the neutral are assumed not to interact chemically or by charge-exchange, and we will not consider ionization processes here. We will take the single-fluid ideal MHD equations to govern the plasma, and the usual hydrodynamical equations for the neutral gas. Hence, for an independent magnetofluid, we will use the following plasma equations:

$$\dot{\rho} = -\nabla \cdot (\rho \boldsymbol{w}) \tag{1}$$

$$\rho \dot{\boldsymbol{w}} = -\rho (\boldsymbol{w} \cdot \nabla) \boldsymbol{w} - \nabla p + \boldsymbol{J} \times \boldsymbol{B}$$
<sup>(2)</sup>

$$p\rho^{-\gamma_a} = \text{constant}$$
 (3)

$$\dot{\boldsymbol{B}} = \nabla \times (\boldsymbol{w} \times \boldsymbol{B}) \tag{4}$$

in which  $\rho$ ,  $\boldsymbol{w}$ , p,  $\boldsymbol{J}$  and  $\boldsymbol{B}$  are the plasma density, velocity field, pressure, current density and flux density respectively;  $\gamma_a = 5/3$  is the adiabatic constant. Time derivatives are denoted by the usual dot notation.

The hydrodynamical equations for the neutral gas evolution in isolation are simpler:

$$\dot{\hat{\rho}} = -\nabla \cdot (\hat{\rho} \boldsymbol{v}) \tag{5}$$

$$\hat{\rho}\dot{\boldsymbol{v}} = -\hat{\rho}(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} - \nabla\hat{p}$$
(6)

where  $\hat{\rho}$ ,  $\boldsymbol{v}$  and  $\hat{p}$  are the neutral gas density, velocity field and pressure, respectively. The neutral gas will also obey an adiabatic equation of state.

For small amplitude, low frequency waves confined to the x, z-plane perturbing a homogeneous equilibrium, and with the uniform equilibrium magnetic field  $B_0$  aligned in the z-direction, the following equations quantify the overall fluid behaviour:

$$\dot{\rho} = -\rho_0(\partial_x w_x + \partial_z w_z) \tag{7a}$$

$$\rho_0 \dot{w}_x = -\sigma^2 \partial_x \rho + (B_0/\mu_0)(\partial_z B_x - \partial_x B_z) + K_x \tag{7b}$$

$$\rho_0 \dot{w}_z = -\sigma^2 \partial_z \rho + K_z \tag{7c}$$

$$\dot{B}_x = B_0 \partial_z w_x \tag{7d}$$

$$\dot{B}_z = -B_0 \partial_x w_x \tag{7e}$$

$$\hat{\rho} = -\hat{\rho}_0(\partial_x v_x + \partial_z v_z) \tag{7f}$$

$$\hat{\rho}_0 \dot{v}_x = -c^2 \partial_x \hat{\rho} - K_x \tag{7g}$$

$$\hat{\rho}_0 \dot{v}_z = -c^2 \partial_z \hat{\rho} - K_z \tag{7h}$$

where  $\boldsymbol{w} = \hat{\mathbf{x}}w_x + \hat{\mathbf{z}}w_z$  is the plasma velocity field,  $\boldsymbol{v} = \hat{\mathbf{x}}v_x + \hat{\mathbf{z}}v_z$  is the neutral gas velocity field,  $\rho$  and  $\hat{\rho}$  are the plasma and neutral gas densities respectively,  $\boldsymbol{B} = \hat{\mathbf{x}}B_x + \hat{\mathbf{z}}(B_0 + B_z)$ is the magnetic field, and  $\sigma$ , c denote the equilibrium sound speed in the plasma and neutral gas, respectively. We have also used the notation  $\partial_x$  to denote the partial derivative  $\partial/\partial x$ , and so on. The momentum coupling terms  $K_x$  and  $K_z$  are introduced to the plasma and neutral gas equations in such a way that the total fluid (that is, plasma plus neutral gas) momentum is conserved. We will assume a simple form for the vector momentum coupling,  $\boldsymbol{K}$ , based on the Boltzmann collision operator (see Jancel and Kahan, 1963), namely that

$$K_{x,z} = \pm \gamma |w_{x,z} - v_{x,z}|,\tag{8}$$

where  $\gamma$  is taken as a constant of proportionality; that is, the momentum coupling takes the form of a drag or friction term that operates when the two fluids have a relative velocity.

Since we are assuming that the two fluids are at the same equilibrium temperature, we will not require explicit coupling terms in the respective energy equations: the redistribution of energy between the plasma and neutral gas for small fluctuations will be accommodated by the resulting pressure variations in each fluid driven by the momentum drag.

#### 2.1. coupled dispersion relations

Fourier transforming the equations to examine the response to a plane wave mode of the form  $\exp i(\mathbf{k} \cdot \mathbf{r} - \omega t)$  yields the following relations for each fluid density perturbation:

$$\rho \left[ \omega^4 - k^2 (\alpha^2 + \sigma^2) \omega^2 + k_z^2 k^2 \sigma^2 \alpha^2 \right] = i \omega^2 \left( k_x K_x + \delta k_z K_z \right) \tag{9}$$

$$\hat{\rho}\left(\omega^2 - k^2 c^2\right) = -i\left(k_x K_x + k_z K_z\right) \tag{10}$$

in which  $\alpha = [B_0^2/(\mu_0\rho_0)]^{1/2}$  is the Alfvén speed and  $\delta = 1 - k^2 \alpha^2/\omega^2$ . Notice that in the absence of coupling  $(K_x = K_z = 0)$  the familiar dispersion relations for the Fast and Slow Magnetosonic modes (see Boyd and Sanderson, 2001), and the usual gas acoustic mode, drop out from the equations. Note that the dispersion relation that gives the Fast and Slow Magnetosonic modes yields the compressional Alfvén and plasma sound waves when the propagation direction is aligned parallel to the equilibrium magnetic field.

It is clear that the friction term will be non-zero unless the waves are identical and in phase in each fluid, an unlikely outcome given that the plasma has the additional degree of freedom provided by the magnetic field, to which the neutral gas does not respond directly. Hence growth and decay of plasma and gas waves will be inevitable, as momentum transfer influences the evolution of disturbances in each gas.

#### 2.2. damping and growth for matched fluids

In this section we will assume that the wave characteristics of the gas are closely matched with at least one of the plasma modes. In general, this will not be true, but this illustration does offer a little insight into the possible phenomena. This means that it is useful to retain the original dispersion relations, and see how they are perturbed by the interaction.

Consider the non-dimensional wavenumbers  $n = k\alpha/\omega$  and  $\hat{n} = kc/\omega$ . Assume that the growth or decay of waves can be characterized by a small imaginary component  $n_i$ of n (and  $\hat{n}_i$  of  $\hat{n}$ ), such that  $n = n_r + in_i$  is the complex non-dimensional wavenumber (similarly,  $\hat{n} = \hat{n}_r + i\hat{n}_i$ ). Substituting these forms into the coupled dispersion relations, assuming that the correction to the real part is negligible, yields:

$$1 - n_r^2 (1 + r^2) + n_r^4 r^2 \cos^2 \theta \approx 0$$
(11)

$$n_i \approx \frac{\Lambda}{2n_r \left(2n_r^2 r^2 \cos^2\theta - (1+r^2)\right)} \approx \frac{\Lambda}{2n_r \left(1+r^2 - 2/n_r^2\right)}$$
(12)

$$\begin{array}{ccc}
1 - \hat{n}_r^2 \approx 0 & (13) \\
\vdots & \Xi & (13)
\end{array}$$

$$\hat{n}_i \approx \frac{-}{2\hat{n}_r} \tag{14}$$

Note that we have taken  $k_z/k = \cos \theta$ ,  $r = \sigma/\alpha$  and written the interaction terms in the form

$$\Lambda = \left(k_x K_x + \delta k_z K_z\right) / (\rho \omega^2) \tag{15}$$

$$\Xi = \left(k_x K_x + k_z K_z\right) / (\hat{\rho}\omega^2) \tag{16}$$

In breaking the expressions into real and imaginary parts, we have assumed that any real contributions from  $\Lambda$ ,  $\Xi$  are negligible; although this is consistent with (8) with respect to the momentum transfer terms themselves, we must account carefully for the complex parts of  $\omega$  and k, since we are assuming that n is complex. However taking only the lowest order correction to the real and imaginary parts affords a limited insight.

Given the assumption that the gas and plasma modes are similar, we can consider propagation parallel to, and perpendicular to, the magnetic field direction.

For parallel propagation  $(k_x = 0)$ , note that the plasma dispersion relation factors exactly, with a sonic mode and the compressional Alfvén mode, the latter being a transverse wave in that plasma motion is orthogonal to the wave-vector. Hence for a simultaneous plane wave solution in both plasma and neutral gas, we must consider only the sonic wave in each, so that the dispersion relations and damping terms are:

$$n_r^2 \approx 1/r^2 \tag{17}$$

$$n_i \approx -\frac{k_z K_z}{2r \rho \omega^2} \tag{18}$$

$$\hat{n}_r^2 \approx 1 \tag{19}$$

$$\hat{n}_i \approx \frac{k_z K_z}{2\hat{\rho}\omega^2} \tag{20}$$

If the waves are perfectly synchronised then  $K_z = 0$  and damping vanishes. This can only happen if the phase speeds are the same in both the plasma and the neutral gas, and so we are assuming that  $\sigma = c$  here. Phase mismatching will lead to damping of the response in one medium at the expense of growth in the other, and vice-versa. The The contrasting case is perpendicular propagation, in which the plasma only has the Fast Magnetosonic mode as a solution. The corresponding dispersion relations and damping terms are:

$$n_r^2 \approx 1/(1+r^2) \tag{21}$$

$$n_i \approx -\frac{\kappa_x \kappa_x}{2\rho \omega_2 \sqrt{1+r^2}} \tag{22}$$

$$\hat{n}_r \approx 1 \tag{23}$$

$$\hat{n_i} \approx \frac{\kappa_x \kappa_x}{2\hat{\rho}\omega^2} \tag{24}$$

Once more, if the phase velocities are to be the same in both media, then the plasma damping term is the same factor  $\alpha/c$  different from the corresponding gas term as we saw in the parallel case. However there is a very important point to note here: the phase speeds in the plasma are different in the two directions, and therefore parallel and perpendicular propagating waves cannot simultaneously be undamped in the same gas-plasma mixture.

This means that in general, there may be angular directions with respect to the background magnetic field in which the damping (or momentum transfer) is least, and so the overall response to a localised disturbance which is initially symmetric in one medium will not necessarily retain its symmetry once the other medium is entrained.

In general, the two fluids will not even be closely matched, so that a deeper insight requires a full numerical solution of the equations.

#### 3. Numerical Simulation

The linearised model equations were recast using the following dimensionless dependent variables:

p	=	$w_x/\alpha$	normalised plasma velocity in x-direction
q	=	$w_z/\alpha$	normalised plasma velocity in z-direction
g	=	$v_x/\alpha$	normalised neutral velocity in x-direction
h	=	$v_z/\alpha$	normalised neutral velocity in z-direction
m	=	$b_x/B_0$	normalised magnetic field perturbation in x-direction
n	=	$b_z/B_0$	normalised magnetic field perturbation in z-direction
d	=	$\hat{ ho}/\hat{ ho}_0$	normalised neutral density
e	=	$ ho/ ho_0$	normalised plasma density

and the time and space co-ordinates were normalised according to

$$t = \tau/T$$

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$$\begin{aligned} x &= \xi/L \\ z &= \zeta/L \\ L/T &= \alpha \end{aligned} \tag{25}$$

where L and T are the characteristic length and time in the problem. The full dimensionless set of equations is then

$$d_{\tau} = -g_{\xi} - h_{\zeta} \tag{26a}$$

$$e_{\tau} = -p_{\xi} - q_{\zeta} \tag{26b}$$

$$p_{\tau} = -r^2 e_{\xi} + m_{\zeta} - n_{\xi} + \Gamma(g - p)$$
(26c)

$$q_{\tau} = -r^2 e_{\zeta} + \Gamma(h-q) \tag{26d}$$

$$g_{\tau} = -s^2 d_{\xi} - \Gamma(g - p_{\tau})/\kappa \tag{26e}$$

$$h_{\tau} = -s^2 d_{\zeta} - \Gamma(h - q)/\kappa \tag{26f}$$

$$m_{\tau} = p_{\zeta} \tag{26g}$$

$$n_{\tau} = -p_{\xi} \tag{26h}$$

in which subscript  $\tau$  denotes  $\partial/\partial \tau$ , and so on. Four key parameters appear:

$$r = \sigma/\alpha \tag{27}$$

$$s = c/\alpha \tag{28}$$

$$\Gamma = \gamma T / \rho_0 \tag{29}$$

$$\kappa = \hat{\rho}_0 / \rho_0 \tag{30}$$

These are the normalised plasma and gas sound speeds, the normalised momentum transfer coefficient and the mass density ratio, respectively. We can determine  $\kappa$  from the ratio of sound speeds:

$$s/r = c/\sigma \tag{31}$$

$$= (\rho_0/\hat{\rho}_0)^{1/3} \tag{32}$$

if we assume an adiabatic equation of state, and assume that  $\sigma^2 = (dp/d\rho)_0$  for the plasma, and similarly for the gas. This yields

$$\kappa = (s/r)^3. \tag{33}$$

The entire set of equations is solved numerically using a Lax-Wendroff hyperbolic solver (for example see Mitchell & Griffiths, 1980). In all simulations we use perfectly transmitting boundary conditions, so that there are no reflections.

As a simple guide, Figure 1 shows the behaviour of the uncoupled neutral gas to a central density perturbation in that medium. The region in which the perturbation is driven is excised from the plot, for clarity (as is the case in all subsequent plots). The density disturbance propagates radially outwards, as expected, with computational parameters set here to be r = s = 1, although the plasma has no influence in this particular case.

Clearly there are many different permutations of the parameters and variable sets that can be presented. We have chosen to restrict attention to 5 general cases:

- (i) all speeds are equal, and hence so are the gas and plasma densities
- (ii) r < 1, s = 1 so that the plasma sound speed (and density) is smaller than the Alfvén speed, the latter being equal to the gas sound speed
- (iii) r > 1, s = 1 so that the plasma sound speed (and density) is greater than the Alfvén speed, the latter being equal to the gas sound speed
- (iv) r = 1, s < 1, which is the case where the gas sound speed is less than the Alfvén speed, the latter being equal to the plasma sound speed
- (v) r = 1, s > 1 so that the gas sound speed is larger than the plasma sound speed and Alfvén speed, the latter two being equal.

In all the numerical examples shown here, the disturbance is initiated by a gaussian perturbation of the neutral gas density, of maximum non-dimensional amplitude of 10%, driven harmonically in time. The momentum coupling term  $\Gamma$  is also set to 10% in all coupled cases. Snapshot images (after two cycles of the driving perturbation) are shown of the non-dimensional plasma and gas density perturbations (*e* and *d* respectively), and the non-dimensional parallel and perpendicular magnetic field perturbations (*m* and *n* respectively), for the five general cases.

- (i) In Figure 2 all the critical speeds are identical, and the gas density disturbance is only slightly anisotropic; the plasma density perturbation is elongated on the vertical axis, showing a phase speed consistent with fast-mode, and has a four-fold symmetry showing maxima where the total magnetic pressure is at a minimum. The (approximately) isotropic flow of the neutral gas has similarly entrained the plasma, and given that the Alfvén and plasma sound speeds are identical to the gas sound speed, the compressional Alfvén and gas sound modes are equally excited; this is revealed by the relatively similar response in x and z magnetic field components, though note that the latter is also excited in the fast mode, and so has a larger response.
- (ii) Figure 3 presents the case where the plasma sound speed is greater than the Alfvén and gas sound speeds, the latter two being identical. This is clearly manifested in the density plots, where the elongation of the plasma density in the vertical direction reflects the more dominant fast mode, and has distorted the gas density wavefronts. The weaker magnetic pressure exerts less of an influence than in case (i), leaving the plasma density response less structured than before. Notice that the plasma gas density dominates over the neutral gas density, causing significant distortion in the latter even though the neutral gas is the initial driver.
- (iii) Figure 4 reveals the behaviour when the plasma sound speed is less than both the Alfvén speed and the neutral gas speed, the latter two being identical. In this case the plasma density is significantly affected by the x-component of the magnetic field perturbation, reflecting the fact that the compressional Alfvén mode is more readily coupled to the gas behaviour than the plasma acoustic mode. Note also that the gas density is significantly greater than the plasma density, making the

gas the primary driver in the momentum coupling. Hence we expect to see the least distortion in the neutral density, and the greatest structure in the plasma density as the latter is driven at phase speeds that only match the neutral gas density wave at particular angles to the background magnetic field.

- (iv) Figure 5 shows the situation where s = 0.6, r = 1.0, meaning that the gas sound speed is now the smallest speed, and hence the gas density is small compared to the plasma density. The effect on the gas density wave is significant: even though the neutral gas is excited as an isotropic gaussian, the acoustic wave takes on the structure dictated by the magnetised plasma. There is also significant suppression of the x-component of magnetic field, principally because of the disparity in speeds leading to inefficient excitation of the compressional Alfvén mode parallel to the background field.
- (v) In Figure 6 we see the neutral gas dominating, and forcing a symmetric response from the plasma. Notice also that the disturbance in all variables has travelled further in the same time.

## 4. Discussion

We have shown here a treatment of compressional waves in a gas-magnetoplasma mix when feedback is incorporated via momentum coupling. No ionization processes have been taken into account, so the collisional exchanges are all assumed below the impact ionization activation threshold. This would be a good approximation for a plasma in a background gas of a different chemical species, for example. It could also be valid for small perturbations of a weakly-ionized gas, provided recombination and ionization events were negligible.

The analysis reveals that the combined response is a hybrid one: fluid disturbances can generate magnetic perturbations, and vice-versa. Where the characteristic speeds are similar, the nature of the disturbance in each medium is not dominated by the specific modes of that medium, but is instead altered by the presence of the other species. This complements other research in the literature in which either the plasma or the neutral gas is dominant, and so broadly retains its characteristic behaviour, albeit slightly modified by the minority species.

It is important to note that resonant coupling will be directional, since the phase and group velocities of the plasma waves depend strongly on the propagation direction with respect to the magnetic field. Consequently the properties of compressional disturbances may have significant variation depending on orientation to the local magnetic field, making diagnostics of density variations problematic if only one perspective is afforded.

Given that gas-plasma mixtures are the norm, rather than the exception, it is worthwhile commenting briefly on possible implications of the ever-present cross-species coupling.

Cosmological gas-plasma mixtures may be particularly sensitive to momentum



Figure 1. The plot shows the uncoupled neutral gas density for the case r = s = 1, when subjected to a time-dependent Gaussian perturbation at the centre of the plotting region. The equilibrium magnetic field lies in the horizontal direction, pointing left to right, although since the plasma is uncoupled from the behaviour, there is no disturbance other than in the neutral gas, and no magnetic influence. This plot helps set the scene for subsequent cases where the plasma and the neutral gas are linked via momentum transfer.

transfer, given the anisotropy of the effect and the limitations of the simplest line-ofsight observations; careful modelling of the coupling process may prove critical for the interpretations of the distribution of matter density and magnetic field characteristics.

Dust in plasmas is also a common phenomenon; the behaviour of suspended particles in neutral gas, and separately in plasmas, has been a growing research area recently. A unified treatment in which the total pressure field is considered may offer greater insight to existing concepts, and might also motivate new applications. For example, acoustic scavenging of dust in air (Magill *et al.* 1989, Riera *et al.* 2006) relies on the entrainment of suspended dust particles in the hydrodynamical forces associated with compressional waves. Such techniques have been usefully exploited in liquids, but gas-based scavenging suffers from greater acoustic field attenuation. Plasmas are rather efficient at growing and precipitating dust particles, both in technological applications (Boufendi & Bouchoule 2002, Ganguly *et al.*, 2002, Diver & Clarke 1996, Stark *et al.*, 2006) and in tokamaks (Krashennenikov *et al.*, 2004). Astrophysical dust



Figure 2. The panel shows the plasma density (top left), the gas density (bottom left) and the x- and z-components of the magnetic field (top and bottom right, respectively) for the case r = s = 1. The equilibrium magnetic field lies in the horizontal direction, pointing left to right.

in ionized material also plays a significant role, for example in comets (Bemporad *et al.* 2005) and in stellar envelopes (Dwek 2004). There is therefore the possibility that gas-magnetoplasma acoustic agglomeration of suspended particles may benefit from the enhanced pressure characteristics of the combined medium, increasing the efficacy of the technique over the neutral gas case, albeit introducing an element of anisotropy. We intend to report the results of investigations into such applications in subsequent publications.



Figure 3. The panel shows the plasma density (top left), the gas density (bottom left) and the x- and z-components of the magnetic field (top and bottom right, respectively) for the case r = 1.4, s = 1. The equilibrium magnetic field lies in the horizontal direction, pointing left to right.

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Figure 4. The panel shows the plasma density (top left), the gas density (bottom left) and the x- and z-components of the magnetic field (top and bottom right, respectively) for the case r = 0.6, s = 1. The equilibrium magnetic field lies in the horizontal direction, pointing left to right.

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Figure 5. The panel shows the plasma density (top left), the gas density (bottom left) and the x- and z-components of the magnetic field (top and bottom right, respectively) for the case r = 1.0, s = 0.6. The equilibrium magnetic field lies in the horizontal direction, pointing left to right.

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Figure 6. The panel shows the plasma density (top left), the gas density (bottom left) and the x- and z-components of the magnetic field (top and bottom right, respectively) for the case r = 1.0, s = 1.4. The equilibrium magnetic field lies in the horizontal direction, pointing left to right.

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