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Article Title: Dimensional Response Analysis of Multistory Regular Steel MRF Subjected to Pulselike Earthquake Ground Motions Year of publication: 2011

Link to published article: http://dx.doi.org/10.1061/(ASCE)ST.1943-541X.0000193

Publisher statement: None

Dimensional response analysis of multi-storey regular steel MRF

subjected to pulse-like earthquake ground motions

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Abstract

 An alternative and efficient procedure to estimate the maximum inelastic roof displacement and the maximum inelastic interstorey drift ratio along the height of regular multi-storey steel MRF subjected to pulse-like ground motions is proposed. The method and the normalized response quantities emerge from formal dimensional analysis which makes use of the distinct time scale and length scale that characterize the most energetic component of the ground shaking. Such time and length scales emerge naturally from the distinguishable pulses which dominate a wide class of strong earthquake records and can be formally extracted with validated mathematical models published in literature. The proposed method is liberated from the maximum displacement of the elastic single-degree-of-freedom structure since the self similar master curve which results from dimensional analysis involves solely the shear strength and yield roof displacement of the inelastic multi-degree-of-freedom system in association with the duration and acceleration amplitude of the dominant pulse. The estimated inelastic response quantities are in superior agreement with the results from nonlinear time history analysis than any inelastic response estimation published previously.

KEY WORDS: Dimensional Analysis, Drift, Near-fault, Pulse, Steel MRF, Seismic design, Self-Similarity , Earthquake engineering

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INTRODUCTION

The increasing number of recordings in the near-source area has provided strong evidence that their ground velocity and acceleration time-histories may exhibit coherent pulses, capable of imposing high drift demands in building structures (Bertero et al. 1978, Hall et al 1995).

 Research on inelastic seismic response in the near-source has mainly focused on singledegree-of-freedom (SDOF) systems. The early works of Veletsos and Newmark (1960), Veletsos et al. (1965) and subsequent studies by Chopra and Chintanapakdee (2003, 2004) confirmed that the equal-displacement rule is valid under near-fault (pulse-type) ground motions. Mavroeidis *et al.* (2004) concluded that the Newmark-Hall (1969) design equations are applicable to near-fault ground motions, provided that the period axis of the inelastic spectrum is normalized with the duration, T_p , of the predominant pulse of the ground motion. The unique advantages of normalizing the response with a time scale and a length scale of the excitation was first proposed by Makris and co-workers (2004a, 2004b, 2006) who showed using dimensional analysis (Barenblatt 1996, Langhaar 1951) that the inelastic response curves assume similar shapes for different values of the normalized yield displacement and concluded using the concept of self similarity that a single inelastic response curve can offer the maximum inelastic displacement of the structure given the pulse period and amplitude of the ground shaking. Recently, Mylonakis and Voyagaki (2006) developed closed form solutions for elastic-perfectly plastic SDOF systems subjected to simple waveforms and confirmed that the use of the strength reduction factor, *R*, complicates the results since parameter *R* is inherently rooted in the elastic response.

 A handful of studies have investigated the response of multi-degree-of-freedom (MDOF) systems to near-fault ground motions. Initially Bertero et al (1978) and subsequently Hall *et al.* (1995) concluded that the demands imposed on structures located in the near-source area

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could far exceed the capacity of flexible high-rise buildings. Alavi and Krawinkler (2004) used the ratio of the base shear coefficient over the amplitude of the pulse of the ground motion for expressing the strength of the structure and demonstrated that structures with fundamental periods longer than the period of the pulse of the ground motion respond very differently from structures with a shorter period. Recently, Kalkan and Kunnath (2006) showed that motions with forward directivity excite higher modes, while motions with flingstep displacement tend to accentuate first-mode behavior. All the aforementioned studies concluded that the current near-fault seismic design practice (ATC 1996), i.e., the constant amplification of the design response spectrum, is facing challenges that remain to be addressed.

 Current design guidelines (ATC 1996, FEMA273 1997, FEMA356 2000, FEMA440 2004, Eurocode 8 2004) for estimating maximum deformations of buildings adopt the equivalent SDOF systems by using the results of a pushover analysis of the corresponding MDOF system. The maximum inelastic displacement of the SDOF system is calculated either with the displacement coefficient method (FEMA273 1997) or the equivalent linearization (ATC 1996) method. The translation of the maximum SDOF displacement to the maximum roof displacement, $u_{r, \text{max}}$, of the MDOF is then achieved by using appropriate conversion factors which are based, either on statistical analysis of a large number of nonlinear time history analyses (FEMA273 1997), or on the concept of the constant deformed shape of the structure during the seismic excitation (EC8 2004). The study by Chopra and Goel (2002) showed that the above-mentioned first-mode approach may yield poor estimates on the maximum interstorey drifts along the height of the building, and therefore a multi-mode inelastic static procedure is needed to better estimate the inelastic interstorey drifts of building structures.

 Current seismic codes, such as the EC8 (2004), calculate the maximum interstorey drifts by relying entirely on the equal-displacement rule; while assuming that the maximum

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interstorey drift profile remains constant during the seismic excitation. According to the results presented in (Alavi and Krawinkler 2004) these assumptions depart from reality in the case of buildings subjected to pulse-like ground motions. Studies from Miranda (1999) and Miranda and Reyes (2002) estimate the maximum interstorey drift ratio (*IDR*_{max}: difference in successive floor displacements normalized with the storey height) along the height of the frame via correlation studies with the maximum roof drift, while, recent work of Akkar *et al*. (2005) presents correlation studies between $u_{r,\text{max}}$ and IDR_{max} in the near-source but with emphasis on elastic and not on inelastic buildings.

 In this paper, the response of SDOF systems with period of vibration in the range of interest with respect to the fundamental period of vibration of steel MRF is first examined. Based on formal dimensional analysis, a self-similar (master) curve that offers the peak inelastic SDOF displacement normalized to the energetic length scale of the predominant pulse of the earthquake ground motion (a measure of the persistence of the excitation to generate inelastic response) is derived and yields favorable estimates when compared with the estimates offered by the inelastic deformation ratio method available in the literature.

 The premise that the maximum inelastic roof displacement of a multi-storey steel MRF can be estimated from the maximum inelastic displacement of an equivalent SDOF system is next evaluated. It is shown that the combination of the error due to the SDOF representation of the real MDOF structure together with the error due to the approximate equation used to predict the peak response of the SDOF system may lead to appreciable overestimated values of the peak inelastic roof displacement.

 The aforementioned overestimated values of the peak inelastic roof displacement motivated the exploration of an alternative and more efficient way for estimating the peak values of global and local inelastic deformation demands in regular multi-storey steel moment-resisting frames (MRF) under pulse-like ground motions. More specifically, the

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paper proposes simple formulae which offer a) on the basis of dimensional analysis, the peak inelastic roof displacement, $u_{r, \text{max}}$, in association with the yield roof displacement, $u_{r, v}$, the base shear strength*, V*y, the total mass of the real MDOF frame together with the amplitude and duration of distinguishable acceleration pulses of the excitation and b) the relation between $u_{r,\text{max}}$ and *IDR*_{max}, associated with the number of stories, n_s , and the beam-to-column stiffness ratio, ρ , of the frame.

KINEMATIC CHARACTERISTICS OF PULSE-TYPE GROUND MOTIONS USED IN THIS STUDY

The relative simple form, yet destructive potential of near source ground motions has motivated the development of various closed form expressions which approximate their kinematic characteristics. The early work of Veletsos *et al*. (1965) was followed by the papers of Hall *et al.* (1995), Heaton *et al.* (1995), Makris (1997), Makris and Chang (2000), Alavi and Krawinkler (2004) and more recently by the paper of Mavroeidis and Papageorgiou (2003). Physically realizable pulses can adequately describe the impulsive character of near-fault ground motions both qualitatively and quantitatively by usually adopting two input parameters, which are either the acceleration amplitude, a_p , and duration, T_p , or the velocity amplitude, v_p , and duration, T_p (Makris 1997). The more sophisticated model of Mavroeidis and Papageorgiou (2003) is described by the following analytical function of the ground velocity

$$
v(t) = A \cdot \frac{1}{2} \cdot \left[1 + \cos\left(\frac{2 \cdot \pi \cdot f_{\text{p}}}{\gamma} \cdot (t - t_0)\right) \right] \cdot \cos\left[2 \cdot \pi \cdot f_{\text{p}} \cdot (t - t_0) + v\right],
$$

\n
$$
t_0 - \frac{\gamma}{2 \cdot f_{\text{p}}} \le t \le t_0 + \frac{\gamma}{2 \cdot f_{\text{p}}}
$$
 (1)

where *A* and f_p (=1/ T_p) are the amplitude and frequency of the pulse, *v* is the phase between the half-cycles of the pulse, *γ* is a parameter which controls the number of zero-crossings of the signal and *t*^o is a parameter that controls the time at which the amplitude of the signal occurs. Recently, Vassiliou and Makris (2009, 2010) have developed a mathematically formal, objective and easily reproducible procedure to estimate the parameters of the Mavroeidis and Papageorgiou (2003) using wavelet analysis. The pulse duration of this model was found to be strongly correlated with the moment magnitude, M_w , of the event,

$$
\log T_{\rm p} = -2.9 + 0.5 \cdot M_{\rm w} \tag{2}
$$

Alternative equations to Equation (2) are known to the literature (Okamoto 1984).

 Figure 1 plots 17 pulse-like ground motions used in the nonlinear time-history analyses of this study. The moment magnitude, M_w , closest distance to the causative fault, *D*, peak ground acceleration, *PGA*, peak ground velocity, *PGV* and peak ground displacement, *PGD*, of the 17 ground motions are presented in Table 1, together with the pulse period T_p , the velocity amplitude v_p and the acceleration amplitude a_p (=2 $\pi \cdot v_p/T_p$) used in the model of Mavroeidis and Papageorgiou (2003) which produces the mathematical approximations plotted with heavy lines in Figure 1.

ESTIMATION OF THE PEAK INELASTIC SDOF DISPLACEMNT: A COMPARISON BETWEEN INELASTIC DEFORMATION RATIO AND DIMENSIONAL RESPONSE ANALYSIS

The comparison is based on an ensemble of 3400 inelastic responses resulted from the 17 pulse-like ground motions of Table 1 that excited 200 elastic perfectly-plastic SDOF systems with pre-yielding periods of vibration in the range of values 0.5 to 3 sec (50 equally spaced values) and yield strengths which correspond to four values (2, 4, 6 and 8) of the strength reduction factor, *R*. The study focuses on systems with periods of vibration between 0.5 and 3 sec since this is the range of interest with respect to the fundamental period of vibration of code-dictated steel MRF.

The inelastic deformation ratio, C_R , defined as the ratio of maximum displacements of inelastic and corresponding (of the same period) linear systems, can be obtained from published *R-μ-Τ* relations (Vidic *et al*. 1994, Miranda and Bertero 1994) or by directly using the results of statistical analysis (Ruiz-Garcia and Miranda 2003, Chopra and Chintanapakdee 2004). The recommendations of FEMA440 (2004) adopt the relation of Ruiz-Garcia and Miranda (2003) for the inelastic deformation ratio, i.e.

$$
C_R = 1 + \frac{R-1}{a \cdot T^2} \tag{3}
$$

where *a* takes the values 130, 90 and 60 for NEHRP site classes B, C and D, respectively. According to FEMA440 (2004), Eq.(3) may not be applicable for near-fault ground motions. Chopra and Chintanapakdee (2004) proposed inelastic deformation ratios which were found to be generally applicable to a wide range of conditions, except for soft-soil sites, and even for a large ensemble of near-fault motions. The aforementioned inelastic deformation ratio for the elastic-perfectly-plastic SDOF system is described by the following equation

$$
C_{\rm R} = 1 + \left[\left(\frac{61}{R^{2.4}} + 1.5 \right) \cdot \left(\frac{T}{T_{\rm c}} \right)^{2.4} \right]^{-1} \tag{4}
$$

where T_c is a corner period of the elastic response spectrum; calculated by employing the iterative algorithm of Riddell and Newmark (1979). Figure 2 presents a graphical comparison of the exact (computed) inelastic deformation ratios with those obtained with the aid of Equation (3) and Equation (4). Equation (4) fits better the response databank than Equation (3) since it takes into account the frequency content of the ground motion by employing the ratio T/T_c . It is though evident that as the strength reduction factor increases, both Equation (3) and Equation (4) offer unconservative estimates.

 In view of this challenge this paper adopts the dimensional response analysis technique (Makris and co-workers 2004a, 2004b, 2006), and proposes the following design master

curve for estimating the maximum dimensionless inelastic displacement $\Pi_1 = u_{\text{inel}} \omega_p^2 / a_p$ of elastic-perfectly-plastic SDOF systems

$$
\Pi_1 = (p + q \cdot \Pi'_3) \cdot \Pi'_2 \tag{5}
$$

where *p*, *q, r* and *s* are constants to be determined on the basis of regression analysis on the data of an available response databank and Π_1 , Π_2 and Π_3 are the dimensionless variables:

$$
\Pi_1 = \frac{u_{\text{inel}} \cdot \omega_p^2}{a_p} \tag{6}
$$

$$
\Pi_2 = \frac{F_{\rm y}}{m \cdot a_{\rm p}}\tag{7}
$$

$$
\Pi_3 = \frac{u_y \cdot \omega_p^2}{a_p} \tag{8}
$$

where $\omega_p = 2\pi/T_p$ and $a_p = \omega_p \nu_p$ are the cyclic frequency and the amplitude of the distinct predominant acceleration pulse of the near-fault pulse-like earthquake ground motion.

 The Levenberg-Marquardt algorithm (MATLAB 1997) was adopted for nonlinear regression analysis of the response databank (3400 points) presented herein, leading to the following explicit form of Equation (5):

$$
\Pi_1 = (-0.92 + 2.61 \cdot \Pi_3^{0.14}) \cdot \Pi_2^{-0.13}
$$
\n(9)

 Figure 3 portrays schematically the approximation of the whole response databank with the proposed Equation (9). This figure bears out the interesting mild dependence of the inelastic displacement to the normalized strength and also illustrates that most of the SDOF systems of interest have yield strengths associated with Π_2 values lower than 1.0. While the proposed curve originates from a best fit, it systematically overestimates the displacements at larger values of Π_2 . The early work of Makris and Psychogios (2006) presented the response analysis of SDOF systems which idealized three frames well known in the literature with corresponding normalized strengths within the range $0.0 \leq \Pi_2 \leq 4.0$. Given the smaller number

of data points, concluded to a variation of Equation (9), $\Pi_1 = (-0.46 + 2.4 \Pi_3) \Pi_2^{-0.57}$, where the structure of the equation is the same (Equation (5)) yet the parameters *p*, *q*, *r* and *s* are different.

Figure 4 compares the statistical distributions of the ratio $u_{\text{inel,app}}/u_{\text{inel,exact}}$ offered by the inelastic deformation ratio method (Equation (4)) and the dimensional response analysis technique (Equation (9)). As the strength reduction factor increases, the self-similar (master) curve from the dimensional analysis provides better estimates than the inelastic deformation ratio method since the distributions are sharper and narrower. Only for the lowest value of the strength reduction factor $(R=2)$, the inelastic deformation ratio method offers superior results to the dimensional analysis method. Both the ratio C_R and the dimensional master curve underestimate the exact maximum inelastic displacement as the strength of the system decreases. The most important observation for both the inelastic deformation ratio method and the dimensional analysis method is that the estimate of maximum inelastic displacement of the associated SDOF systems due to individual ground motions may be alarmingly small (say equal to 20% of the true displacement) or exceedingly large (say 400% of the true displacement).

SDOF-SYSTEM ESTIMATE OF THE PEAK INELASTIC ROOF DISPLACEMENT OF STEEL MRF

According to the displacement modification method presented in FEMA440 (2004), the maximum inelastic roof displacement of a building structure may be estimated on the basis of a pushover analysis in the form of a plot of base shear, *V*, versus roof displacement, *u*^r . By assuming the normalization of modes with $\Phi_{ri}=1$ (element of eigenvector at roof level=1), the yield strength, F_y , the yield displacement, u_y , and the mass, m , of the equivalent SDOF system are readily available. Then, the period, *T*, and the strength reduction factor, *R*, are obtained and thus, the maximum inelastic displacement of the equivalent SDOF system, *u*inel, can be easily derived from the inelastic deformation ratio (Equation (4)). To this end, the maximum inelastic roof displacement of the building, $u_{r,\text{max}}$, may be obtained as

$$
u_{r,\text{max}} = \Gamma \cdot u_{\text{inel}} \tag{10}
$$

 The inelastic response databank of a large collection of steel MRF (that is described later in the paper) is used to evaluate the accuracy of the above SDOF-system estimate of the peak inelastic roof displacement. Figure 5 (left) shows the statistical distribution of the ratio $u_{r,\text{max,app}}/u_{r,\text{max,exact}}$ but for values of the *R* factor of the equivalent SDOF system larger than 2. $u_{r,\text{max,app}}$ is obtained with Equation (10) together with Equation (4), while $u_{r,\text{max,exact}}$ is the exact value from nonlinear dynamic analysis. A significant overestimation (median value equal to 1.5) of the exact peak roof displacement is observed. Chopra et al. (2003) investigated the premise of the SDOF-estimate of the peak roof displacement by determining the responses of both the MDOF and the corresponding equivalent SDOF system rigorously by nonlinear dynamic analyses and concluded that the first-mode SDOF system overestimates the roof displacement as the ductility demand increases. The same conclusion was also derived by Tjhin *et al*. (2005). Figure 5 (right) shows the statistical distribution of the ratio $u_{r,\text{max,app}}/u_{r,\text{max,exact}}$ obtained with Equation (10) together with Equation (9) derived from dimensional analysis but for values of the *R* factor of the equivalent SDOF system larger than 2. The median value of this ratio is equal to 1.25, while the coefficient of variation is equal to 0.4. Figure 5 (left and right) reveal that for individual ground motions, the combination of the error due to the SDOF representation of the real MDOF structure (Equation (10)) together with the error due to the approximate equation used to predict the peak response of the SDOF system (Equation (4) or Equation (9)) may lead to exceedingly large values (say 400% larger than the true ones) of the peak inelastic roof displacement.

 In view of these challenges this paper proceeds with application of the dimensional analysis method on the response of MDOF structures without any reduction of the problem to the SDOF system.

REGULAR PLANE STEEL MOMENT RESISTING FRAMES, SEISMIC ANALYSES AND RESPONSE DATABANK

Design and structural characteristics

The study is based on 2-dimensional frames with storey heights and bay widths equal to 3 m and 5 m, respectively. It should be pointed out that a bay width from 4 to 6 m is the usual case in European practice but quite low compared to that of the American practice. The frames have the following geometrical characteristics: number of stories, n_s , with values 3, 6, 9, 12, 15 and 20 and number of bays, n_b , with values only 3 and 6.

 The frames are designed in accordance with the structural Eurocodes EC3 (1993) and EC8 (2004) by using the software SAP2000 (2005). The yield stress of the material is set equal to 235 MPa, while gravity load on the beams is assumed equal to 27.5 kN/m (dead and live loads of the floors). The expected earthquake ground motion is defined by the design spectrum of the EC8 (2004) with peak ground acceleration, *PGA*, equal to 0.4g and soil class B. The design process of the frames resulted in optimum cross-sections of the columns which satisfy both the requirements for strength/stiffness (EC3 2003) and the capacity design rule (EC8 2004). For each of the frames, the column cross-sections were subsequently increased two times in order to obtain three different values of the beam-to-column stiffness ratio, *ρ*, defined as (Akkar et al. 2005)

$$
\rho = \frac{\sum (I/l)_b}{\sum (I/l)_c} \tag{11}
$$

where *I* and *l* are the moment of inertia and length of the steel member (column *c* or beam *b*), respectively. The parameter *ρ* varies along the height of the frames and therefore, its nominal value was calculated for the storey closest to the mid-height of each of the frames.

 The design of the frames led to a flexible family of frames, while, a stiff family of frames were directly obtained by keeping the strength and stiffness constant while reducing the mass. Moreover, in order to cover even conservative design cases, for both the stiff and flexible frames three values of the yield strength of the material, i.e., S235 (considered in the design procedure described in the above paragraphs), S275 and S355, were considered. The aforementioned process led to 6 (number of stories) * 2 (number of bays) * 3 (beam-tocolumn stiffness ratio) $*$ 2 (fundamental period of vibration) $*$ 3 (strength of material) = 216 frames.

Data of the frames, including values for n_s , n_b , ρ , beam and column cross-sections and fundamental periods of vibration (flexible and stiff), are presented in Table 2. In that table, expressions of the form, e.g., $260-360(1-4) + 240-330(5-6)$ mean that the first four stories have columns with HEB260 cross-sections (Androic *et al.* 2000) and beams with IPE360 cross-sections, whereas the next two higher stories have columns with HEB240 crosssections and beams with IPE330 cross-sections.

Modelling for nonlinear static and dynamic analysis

The software DRAIN-2DX (Prakash *et al*. 1993) was used for performing nonlinear static or dynamic analyses. The analytical models of the frames were centreline representations in which inelastic behaviour was modelled by means of bilinear (hysteretic) point plastic hinges with 3% hardening (Gupta and Krawinkler 1999). Therefore, the modelling is more representative of steel frames with an overall response that is not significantly influenced by the deformations of panel zones and connections. In addition, diaphragm action was assumed at every floor due to the presence of the slab, P-delta effects were also taken into account,

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while Rayleigh damping corresponding to 3% of critical damping at the first two modes was adopted.

Structural characteristics based on nonlinear static (pushover) analysis

 For each of the 216 frames described herein, a first-mode inelastic static (pushover) analysis has been performed. The base shear coefficient, V_v/W (V_v : base shear yield strength, *W*: seismically effective weight), and yield roof displacement, $u_{r,v}$, of the frames were calculated on the basis of a bilinear idealization of the pushover curve (FEMA440 2004) and are presented in Table 3. Figure 6 plots the base shear coefficient versus the number of stories of the frames and shows that as the number of stories increases, the base shear coefficient decreases.

Seismic analyses and response databank

 The family of the frames described in this Section was subjected to the ensemble of the 17 pulse-like ground motions of Table 1. The results of the 216 (frames) $*$ 17 (accelerograms) = 3672 nonlinear time history analyses were post-processed in order to create a response databank with the response quantities of interest, i.e., the maximum roof displacement and the maximum interstorey drift ratio along the height of the frame.

 The response of a frame to a particular ground motion may be elastic or inelastic. Since this study focuses on the inelastic seismic response of steel MRFs, the results associated with the elastic response of the frames were deleted from the response databank. Of the 3672 analyses, 443 were found to be elastic and are mainly offered by the 3-storey stiff frames since these frames have large values of the base shear coefficient (Figure 6).

THE PROPOSED PROCEDURE FOR ESTIMATING INELASTIC DRIFT DEMANDS IN

STEEL MRF

Estimation of the maximum inelastic roof displacement

By analyzing the MDOF inelastic response databank described in the previous Section in association with the use of dimensional analysis introduced earlier, it is possible to derive a single design master curve which directly involves the mechanical properties (base shear yield strength V_y and yield roof displacement $u_{r,y}$ of the actual MDOF structure. The dimensionless parameters are defined as

$$
\Pi_1 = \frac{u_{r,\text{max}} \cdot \omega_p^2}{a_p} \tag{12}
$$

$$
\Pi_2 = \frac{V_y}{m \cdot a_p} \tag{13}
$$

$$
\Pi_3 = \frac{u_{\text{r},\text{y}} \cdot \omega_{\text{p}}^2}{a_{\text{p}}} \tag{14}
$$

where *m* is the mass of the frame.

 Figure 7 plots the computed peak inelastic roof displacements from 3229 nonlinear time history analyses in terms of the dimensionless Π products given by Equations (12), (13) and (14). Figure 7 reveals remarkable order where a relative narrow band of data exhibits mild decrease as dimensionless strength $\Pi_2=V_y/m a_p$ increases. Most importantly the dimensionless graph of Figure 7 uncovers that near the low value of the dimensionless strength, $\Pi_2=V_y/ma_p=0.3$, the dimensionless roof displacement exhibits a well-defined concentration of lower values (valley) and subsequently exhibits a well-defined concentration of peak values when the dimensionless strength reaches the vale of $\Pi_2=V_y/m a_p=0.7$. Assuming an acceleration amplitude for a strong earthquake, $a_p=0.5g$, the concentration at low values of the roof displacement happens at

$$
\frac{V_{y}}{W} = \frac{V_{y}}{m \cdot g} = \frac{V_{y}}{m \cdot \alpha_{p}} \cdot \frac{\alpha_{p}}{g} = 0.3 \cdot 0.5 = 0.15
$$
 (15)

The discussion offered in the previous paragraph along with Equation (15) indicate why buildings with relative low strength $V_v/W=0.15$ perform well (low displacement demands) even when excited by strong ground motions. Even more important is the result that by doubling the strength $\Pi_2=V_v/m a_p=0.6$ (\Rightarrow *V_v*/W=0.30 with $a_p=0.5g$) the inelastic displacement demand may increase up to 70%. This counterintuitive result has been known to several researchers (Priestley et al. 2001 among others), nevertheless the dimensional analysis method adopted in this paper and Figure 8 demonstrates it in a decisive manner.

 After having established this well-defined concentration of low values of roof displacements at $\Pi_2=0.3$, the overall trend of the peak inelastic roof displacement is approximated again with Equation (5) (the dimensionless terms defined by Equations (12), (13) and (14)) which for the nonlinear response of the 216 MDOF frames, nonlinear regression analysis produced the following approximation

$$
\Pi_1 = (-3.1 + 4.7 \cdot \Pi_3^{0.17}) \cdot \Pi_2^{-0.24} \tag{16}
$$

Equation (16) offers a ratio $u_{r,\text{max,app}}/u_{r,\text{max,exact}}$ with median value equal to 0.92 and coefficient of variation equal to 0.19 (Figure 8); a significantly better estimation than the estimations obtained in Figure 5 of the paper with the aid of the equivalent SDOF system. Since Equation (16) approximates the overall trend of the peak inelastic roof displacements, captures neither the concentration of low displacements at $\Pi_2=0.3$ nor the concentration of high displacements at $\Pi_2=0.7$.

Figure 7 also shows that the Π_2 values obtained by using the base shear strength and the mass of the real MDOF steel frames are substantially different than those corresponding to the SDOF system and shown previously in Figure 3. This partially explains the different coefficients appearing in Equation (9) and Equation (16).

Estimation of the maximum interstorey drift ratio along the height of the frame

An accepted way for estimating the maximum interstorey drift ratio along the height of the frame (*IDR*_{max}) is via correlation studies with the maximum roof drift $u_{r,\text{max}}/H$ (Karavasilis *et al.* 2007)*.* By analysing the response databank described in this paper, the ratio *β=* (*u*r,max/*Η*)*/IDR*max was found to be strongly dependent on the number of stories. A dependence

on the parameters *ρ* was also identified and thus, nonlinear regression analysis produced the following approximation

$$
\beta = 1.0 - 0.18 \cdot (n_s - 1)^{0.45} \cdot \rho^{0.17}
$$
 (17)

The above-mentioned relation is simple and satisfies the physical constraint $\beta=1$ for $n_s=1$. With the maximum roof displacement known $(u_{r,\text{max,exact}})$, Equation (17) offers a ratio *IDR*max,app/*IDR*max,exact with a median value equal to 1.0 and coefficient of variation equal to 0.11 (Figure 9 left). The dependence of the ration *β* on the level of inelastic deformation expressed through the ductility factor $(u_{r,\text{max}}/ u_{r,y})$ was also examined; yet poor correlation was identified (correlation coefficient lower than 0.15). This means that the effect of the number of stories on the ratio between the peak roof displacement and the peak interstorey drift ratio is significantly larger than the effect of the drift concentrations as the structure moves further in the inelastic range of the response.

While the described statistics for predicting the *IDR*_{max} for a known maximum roof displacement $(u_{r,max,exact})$ are extremely encouraging, it is of significant interest to calculate the error introduced in the prediction of the *IDR*max by combining the uncertainties of both Equations (17) and (16). For a given base shear strength and yield roof displacement, i.e., given the approximate maximum roof displacement $(u_{r,\text{max,app}})$, Equation (17) offers a ratio *IDR*_{max,app}/*IDR*_{max,exact} with a median value equal to 1.0 and coefficient of variation equal to 0.35 (Figure 9 right).

EXAMPLE ON THE APPLICATION OF THE PROPOSED METHOD

Assume that we are interested in estimating the inelastic deformation demands of a 6 storey regular steel MRF with storey height equal to 3 m, parameter ρ equal to 0.47, base shear strength equal to 0.24*W* and yield roof displacement equal to 0.125 m when subjected to the Pacoima dam recording $(v_p=1.15 \text{ m/sec}$ and $T_p=1.47 \text{ sec}$) from the 1971 San Fernando

earthquake, the Rinaldi recording (v_p =1.42 m/sec and T_p =1.25 sec) from the 1994 Northridge earthquake and the OTE recording $(v_p=0.45 \text{ m/sec}$ and $T_p=0.71 \text{ sec}$) offered by the 1995 Aigion earthquake. The acceleration amplitude of the pulse of the Pacoima dam recording is equal to $a_p = (2*3.14/1.47)*1.15 = 0.5g$, while the same calculation gives an acceleration amplitude $a_p=0.73g$ for the Rinaldi recording and $a_p=0.41g$ for the Aigion recording.

 The following calculations refer to the Pacoima dam recording, while for the Rinaldi and Aigion recordings, the final results (drift estimations) are only discussed.

Makris and Psychogios (2006)

By assuming an inverted triangular mode shape, the participation factor of the frame is Γ=1.38 and the effective modal mass coefficient is *a*=0.83 and therefore, the mechanical properties of the SDOF approximation of the frame are $F_v/W=0.24/0.83=0.29$ and $u_y=0.125/1.38=0.091$ m. With the above values the dimensionless parameter Π_2 (Equation (7)) is calculated equal to 0.58, while the dimensionless parameter Π_3 (Equation (8)) is calculated equal to 0.34. Substitution of these values into Equation (13) of Makris and Psychogios (2006) gives a value of the dimensionless parameter Π_1 equal to 1.93 and therefore, a value of the peak SDOF displacement equal to 0.52 m. The peak roof displacement is then obtained as $u_{\text{r,max}} = \Gamma^* 0.52 = 1.38^* 0.52 = 0.72 \text{ m}$. The work of Makris and Psychogios does not offer tools for estimating the maximum interstorey drift ratio.

Proposed Equation (9)

The proposed Equation (9) can be used instead of Equation (13) of Makris and Psychogios (2006). Substitution of $\Pi_2=0.58$ and $\Pi_3=0.34$ into Equation (9) gives a value of the dimensionless parameter Π_1 equal to 1.42 and therefore, a value of the peak SDOF displacement equal to 0.384 m. The peak roof displacement is then obtained as $u_{r,\text{max}}$ $\Gamma^*0.384 = 1.38 \cdot 0.384 = 0.53$ m.

Proposed procedure for estimating the peak inelastic roof displacement and the peak inelastic interstorey drif ratio along the height of the frame

The proposed procedure does not rely on the SDOF representation of the real MDOF steel frame. Equation (13) gives the dimensionless parameter Π_2 equal to 0.48, while Equation (14) gives the dimensionless parameter Π_3 equal to 0.46. Substitution of these values into Equation (16) gives a value of the dimensionless parameter Π_1 equal to 1.215 and therefore, a value of the peak roof displacement equal to 0.33 m.

Equation (17) provides a value of the parameter β equal to 0.7 and therefore, the maximum interstorey drift ratio along the height of the frame is equal to $0.33/(6*3*0.7) = 2.6\%$.

Inelastic deformation ratio

The period of the equivalent SDOF system is $T=2\pi (mu_y/F_y)^{0.5} = 2\pi (u_y/(0.29g))^{0.5} = 1.12$ sec and the corresponding cyclic frequency $\omega = 2\pi/T = 5.61$ rad/sec. The ordinate of the pseudoacceleration spectrum for a period equal to 1.12 sec is $S_a=14.5 \text{ m/sec}^2$ and therefore, the maximum displacement of the elastic SDOF system of the same period is equal to S_d =14.5/5.61²=0.461 m. The associated strength reduction factor can be easily obtained as $R=S_d/u_v=0.461/0.091=5.06$. With the strength reduction factor and the period of the inelastic SDOF system known, Equation (3) (Ruiz-Garcia and Miranda 2003) provide the inelastic deformation ratio C_R =1.05 (a value of C_R =1.0 denotes that the equal displacement rule is valid) and therefore, the peak inelastic displacement is $u_{\text{inel}} = C_R S_d = 0.484 \text{ m}$. The peak roof displacement is then obtained as $u_{r,\text{max}} = \Gamma^* 0.484 = 1.38^* 0.4844 = 0.67 \text{ m}$. Based on the same calculations, Equation (4) (Chopra and Chintanapakdee 2004) provide the inelastic deformation ratio C_R =1.022 which finally leads to a peak roof displacement $u_{r,max}$ =0.65 m.

Nonlinear dynamic analysis

Figure 10 (top) presents the peak floor displacement and interstorey drift profiles of the frame under the Pacoima dam, Rinaldi and Aigion recordings, and reveals that the distribution of inelastic deformation demands departs from the assumption of the first-mode dominated response, while different profiles are noted for the three ground motions. Figure 10 (bottom) compares the estimations of the peak roof displacement with the exact peak roof displacements from nonlinear dynamic analysis and reveals the clear advantage of the proposed procedure over all the other inelastic estimations.

 The maximum interstorey drift ratio under the Pacoima dam recording was calculated with the aid of Equation (16) equal to 2.6%. The same equation provides for the Rinaldi recording an *IDR*=3.0% and for the Aigion recording an *IDR*=1.12%. These values are close to the values obtained from nonlinear dynamic analyses (Figure 10 top-right).

CONCLUSIONS

In this paper, the response of SDOF systems with period of vibration in the range of interest with respect to the fundamental period of vibration of steel MRF was first examined. Based on formal dimensional analysis, a self-similar (master) curve that offers the peak SDOF displacement in association with the pulse and period of the earthquake ground motion was derived. A comparison of the peak SDOF displacement estimates offered by the proposed self-similar curve with the estimates offered by the inelastic deformation ratio method yields favorable results for dimensional analysis.

 The premise that the maximum inelastic roof displacement of a multi-storey steel MRF can be estimated from the maximum inelastic displacement of an equivalent SDOF system was next evaluated. It was shown that the combination of the error due to the SDOF representation of the real MDOF structure together with the error due to the approximate equation used to predict the peak response of the SDOF system may lead to appreciable overestimated values of the peak inelastic roof displacement.

 The aforementioned overestimated values of the peak inelastic roof displacement motivated the development of a new method for estimating peak global and local inelastic deformation demands in building structures under pulse-like earthquake ground motions. The proposed method results from dimensional analysis and involves solely the shear strength and yield roof displacement of the inelastic multi-degree-of-freedom system in association with the duration and acceleration amplitude of the dominant pulse of the excitation. The estimated inelastic response quantities are in superior agreement with the results from nonlinear time history analysis than any inelastic estimation published previously.

 Interpretation of the above conclusions needs to be made in the context of the structural models and ground motions considered in the paper.

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CAPTION OF FIGURES

Figure 1: Ground velocity time histories of 17 recorded pulse-like ground motions (light lines) together with the mathematical approximation of the predominant pulse (heavy lines) proposed by Mavroeidis and Papageorgiou (2004)

Figure 2: Comparison of the computed (light lines) and the approximate (heavy lines) inelastic deformation ratios of elastic-perfectly-plastic SDOF systems proposed by Ruiz-Garcia and Miranda (2003) (left) and Chopra and Chintanapakdee (2004) (right).

Figure 3: Dimensional maximum inelastic displacements (points) of SDOF systems $(\Pi_1 = u_{\text{inel}} \omega_p^2 / a_p)$ and the proposed master curve (solid line)

Figure 4: Distribution of the ratio $u_{\text{inel.} \text{ap}}/u_{\text{inel.} \text{exact}}$. $u_{\text{inel.} \text{ap}}$ is computed with the inelastic deformation ratio (Equation (4) (left) and the dimensional response analysis (Equation (9)) (right).

Figure 5: SDOF-system estimate of the peak inelastic roof displacement; distribution of the ratio $u_{r,ap}$, $u_{r,ap}$, $u_{r,ap}$ is computed with Equation (10) together with the inelastic deformation ratio (Equation (4)) (left) and with Equation (10) together with the dimensional response analysis (Equation (9)) (right). The values of the *R* factor of the equivalent SDOF system are *R*>2.

Figure 6: Base shear coefficient vs. number of stories of the frames considered in this study.

Figure 7: Dimensionless maximum inelastic roof displacements, $\Pi_1 = u_{r,\text{max}} \omega_p^2 / a_p$, of a large collection of MDOF frames when subjected to 17 strong ground motions together with the proposed single master curve. At the value of dimensionless strength $\Pi_2=V_v/m a_p=0.3$ the peak inelastic roof displacements exhibit a remarkable concentration at relative low values

Figure 8: Distribution of the ratio $u_{r,app}/u_{r,exact}$. $u_{r,app}$ is computed with the dimensional response analysis (Equation (16)).

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Figure 9: Distributions of the ratio $IDR_{\text{max,app}}/IDR_{\text{max}}$. $IDR_{\text{max,app}}$ is computed with the proposed relation (Equation (17)) by assuming the $u_{r, max}$ known (left) and the $u_{r, max}$ unknown and calculated with the dimensional response analysis (Equation (16)) (right).

Figure 10: Six storey steel MRF ($V_v/w = 0.24$ and $u_{r,v} = 0.125$ m) subjected to the 1971 Pacoima dam (CA), 1994 Rinaldi (CA) and 1996 Aigion (Greece) recordings: Top: Maximum displacement and interstorey drift profiles; Bottom: Comparison of the peak roof displacement estimates with the proposed equations and with others published in the past, together with the results from nonlinear dynamic analysis.

Table 1: Data pertinent to the pulse-like ground motions considered in this study

Table 2: Data pertinent to the design structural characteristics of the frames considered in this study

