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Sensitivity models for missing covariates in the analysis of survival data from multiple surveys

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Summary. Using individual patient data from five independent surveys, we evaluate regional variations in survival in cerebral palsy. The influence of four important variables measuring disability, which are only partially observed for many cases, are analysed. Results are compared between a naive complete case analysis; a full likelihood model in which the covariates are assumed to be missing at random and in which each of the binary predictor variables are modelled as independent Bernoulli random variables; a model in which the covariates are modelled by a conditional wise sequence, accommodating dependencies between the likelihoods of having various mixtures of disabilities; and a model in which the likelihood of a predictor variable being observed is allowed to depend on the value of the covariate itself (NMAR). Fully parametric survival regression models are used and analysis carried out in *BUGS*. Results suggest that proportions recorded as having severe visual or cognitive impairments are substantially lower than the actual proportions severely impaired. Associations between the likelihood of a particular covariate being recorded and the likelihood of a more severe disability imply that life expectancies for those who are very severely impaired may be up to 20% less than inferences based on complete case analyses.

Introduction

Cerebral palsy is a common cause of childhood disability, affecting both physical and mental abilities. Survival is known to be affected by severity of impairment, measured by such factors as severity of manual and ambulatory functions, severity of cognitive, visual and hearing abilities (Evans et al. (1990); Hutton et al. (2000); Hutton and Pharoah (2002); Hemming et al. (2006)). Estimates of life expectancies in cerebral palsy are important for effective planning of resources and knowledge of levels of care which will be required in the future. Life expectancies also play an important role in medico-legal settlements. Indirect comparisons between different regions and countries, from published survival studies, appeared to suggest that there may be some variation between life expectancies for those with cerebral palsy (Hutton et al., 2000). For example, of those with a severe manual disability, 47% were reported to survive to age 30 years for the Mersey Cerebral Palsy Register, compared to 60% for the North of England Collaborative Cerebral Palsy survey (Hutton et al., 2000).

The United Kingdom Cerebral Palsy collaboration (UKCP) brings together individual anonymised data from the five UK cerebral palsy registers into one database, harmonising definitions of potential predictor variables (Surman et al., 2006). Using this individual patient data, in those with observed information on all four categories of severe impairments, no regional variation was observed after adjusting for manual, ambulatory, cognitive and visual disabilities (Hemming et al., 2006). That analysis however, being a complete case analysis, was based on a subset (71%) of the initial dataset, and excluded one of the five

regions due to a particularly high proportion of missing data (63% for one covariate). It is a real concern that those who are very severely impaired may die before assessments of severity have been carried out, and they may therefore not be representative of the sample as a whole, thereby not meeting the underlying assumption of a complete case analysis, that of the data being missing completely at random. It is our aim here to consider how sensitive questions of regional variation in survival and estimates of life expectancies are to various patterns and assumptions of the missing covariate data.

Inferences for life expectancies in cerebral palsy from multiple data sources

In the UK there are five cerebral palsy registers, each covering a separate (although non exhaustive), region of the UK, comprising the Mersey region, the North of England, the Oxford region, Northern Ireland and Scotland. Each of these five registers have independently carried out retrospective surveys or have prospectively collected information on those born with cerebral palsy to mothers resident within defined geographical regions over various time periods. Furthermore, each register also flags all cases with the national births and deaths registers (the National Health Service Central Register for England, the Northern Ireland Central Services Agency or the General Register Office for Scotland). All individuals having been identified as having cerebral palsy from each of these five regions are represented using anonymised data on the collaborative UKCP database. The database therefore comprises a well defined cohort of people with cerebral palsy, for virtually all of whom vital status information is known.

Survival predictions as a function of severity of impairment are dependent on knowledge of the severity of the impairment. The severity of impairments is usually evaluated between the ages of two and five years. For children who die before an evaluation is made, information on severity of impairment may be missing. This covariate data can also be missing for those children who are still alive, perhaps because the child moved out of the region or was lost to follow-up (for covariate information and not death information, which are distinct processes) for some other reason. The proportions of cases with missing information on severity vary between the five regions, with some having high levels of recorded data, whilst others having high proportions of missing data.

Missing covariate data in survival analysis

Following standard notation, we use capital letters to denote random variables, lower case letters to denote realisations of random variables and distinguishing vectors using bold type face for realisations of random variables and for parameters. Let $Y = (Y_1, \dots, Y_P)$, with realisation \mathbf{y} , be a P dimensional vector representing complete data on P variables. Let R_p ($p = 1, \dots, P$) be an indicator for whether variable Y_p is observed, so that $R = (R_1, \dots, R_P)$ is a vector of observed data indicators, with realisation \mathbf{r} and with density function $f_R(\mathbf{r}; \boldsymbol{\psi})$. Without loss of generality, let Y , with density function $f_Y(\mathbf{y}; \boldsymbol{\theta})$, be partitioned into an observed and missing part, $Y = (Y_o, Y_m)$ where Y_o is the observed component of Y and Y_m the missing component of Y .

Following Rubin's terminology (Rubin, 2002), the data are said to be Missing Completely At Random (MCAR) if:

$$f_{R|Y_o, Y_m}(\mathbf{r}|\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\psi}) = f_R(\mathbf{r}; \boldsymbol{\psi}) \quad (1)$$

that is the missing data mechanism does not depend on either the observed (Y_o) or missing data (Y_m). A Complete Case Analysis (CCA), that is deleting or ignoring, any observations with incomplete or missing data, will only result in unbiased estimates where the data are MCAR. Even when data are MCAR estimates may be inefficient if large numbers of observations are excluded. Where the data are not MCAR, a CCA leads both to reduced efficiency and possible biased estimates of covariate effects.

Data are said to be Missing At Random (MAR) if:

$$f_{R|Y_o, Y_m}(\mathbf{r}|\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\psi}) = f_{R|Y_o}(\mathbf{r}|\mathbf{y}_o; \boldsymbol{\psi}) \quad (2)$$

that is, the data are missing in a way that is MCAR after conditioning on other observed data. MAR is therefore a less restrictive assumption than to MCAR.

Under likelihood based inferences, the observed data are (Y_o, R) and the likelihood of interest for this observed data is $L(\theta|Y_o, R)$, such that:

$$\begin{aligned} L(\theta|Y_o, R) &\propto f_{Y_o, R}(\mathbf{y}_o, \mathbf{r}; \boldsymbol{\theta}, \boldsymbol{\psi}) \\ &= \int_{Y_m} f_{(Y_o, Y_m, R)}(\mathbf{y}_o, \mathbf{y}_m, \mathbf{r}; \boldsymbol{\theta}, \boldsymbol{\psi}) dY_m \\ &= \int_{Y_m} f_{R|Y_o, Y_m}(\mathbf{r}|\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\psi}) f_{Y_o, Y_m}(\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\theta}) dY_m. \end{aligned} \quad (3)$$

This likelihood is intractable unless either a full specification of the missing data mechanism, $f_{R|Y_o, Y_m}(\mathbf{r}|\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\psi})$, is given, or if $f_{R|Y_o, Y_m}(\mathbf{r}|\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\psi}) = f_{R|Y_o}(\mathbf{r}|\mathbf{y}_o; \boldsymbol{\psi})$. Under this simplification the likelihood reduces to:

$$\begin{aligned} L(\theta|Y_o, R) &\propto \int_{Y_m} f_{R|Y_o, Y_m}(\mathbf{r}|\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\psi}) f_{Y_o, Y_m}(\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\theta}) dY_m \\ &= f_{R|Y_o}(\mathbf{r}|\mathbf{y}_o; \boldsymbol{\psi}) \int_{Y_m} f_{Y_o, Y_m}(\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\theta}) dY_m \\ &\propto \int_{Y_m} f_{Y_o, Y_m}(\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\theta}) dY_m \\ &= f_{Y_o}(\mathbf{y}_o; \boldsymbol{\theta}) \\ &\propto L(\boldsymbol{\theta}|Y_o). \end{aligned} \quad (4)$$

This simplification holds when the data are MAR, and θ and $\boldsymbol{\psi}$ are distinct, or for Bayesian inferences are a priori independent. The missing data mechanism is then said to be “ignorable”, since inferences can be made from the likelihood of the observed data, $L(\theta|Y_o)$.

In survival analysis, let $Y = (T, \Delta, Z)$, where T represents survival times, Δ censoring indicators and a Z vector of covariates. It is assumed throughout that the censoring mechanism does not depend on the missing failure time, a realistic assumption in the application considered. This assumption, in addition to the assumption that the probability of censoring does not depend on the missing value of the covariate, leads to ignorability of the censoring mechanism (Baker, 1994). We consider relaxing this assumption in part, by allowing the probability of a covariate being missing to depend on the censoring status, but retain the assumption of the censoring mechanism not depending on the missing failure time throughout. The covariates, $Z = (Z_1, \dots, Z_K)$, are only partially observed. We use the subscript $k = 1, \dots, K$, as opposed to p above to distinguish between a set of partially observed covariates (Z), different from partially observed data (Y). The complete

case analysis method is the routine and in many cases only survival method available in statistical packages. Likelihood based inferences for MAR covariates require the model is parameterised in terms of the conditional density function for T , $f_{T|Z}(\mathbf{t}|\mathbf{z}; \boldsymbol{\theta}_{T|Z})$, and the marginal density for Z , $f_Z(\mathbf{z}|\boldsymbol{\theta}_Z)$ where $\boldsymbol{\theta} = (\boldsymbol{\theta}_T, \boldsymbol{\theta}_Z)$ represents the parameters for the survival distribution and covariate distribution functions respectively (Lipsitz and Ibrahim, 1996a). Writing $Z = (Z_o, Z_m)$, so that $Y_o = (T, \delta, Z_o)$, for the case in which Z is MAR, likelihood based inferences reduce to:

$$\begin{aligned} L(\boldsymbol{\theta}|Y_o) &\propto f_{Y_o}(\mathbf{y}_o; \boldsymbol{\theta}) \\ &= f_{T,\delta,Z_o}(\mathbf{t}, \delta, \mathbf{z}_o; \boldsymbol{\theta}) \\ &= f_{T,\delta|Z_o}(\mathbf{t}, \delta|\mathbf{z}_o; \boldsymbol{\theta}_T) f_{Z_o}(\mathbf{z}_o; \boldsymbol{\theta}_Z) \end{aligned} \tag{5}$$

(6)

that is, specification of both $f_{T|Z}(t|\mathbf{z}, \boldsymbol{\theta}_T)$, of main interest, and $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$, often of lesser interest, are required.

For the Cox proportional hazards survival model methods have been developed to accommodate missing at random categorical or continuous covariate data (Herring and Ibrahim (2001); Herring et al. (2002); Chen et al. (2002)), as have methods for parametric survival models (Lipsitz and Ibrahim (1996a); Schluchter and Jackson (1989); Lipsitz and Ibrahim (1996b); Meng and Schenker (1999); Cho and Schenker (1999)). Of these, two have consisted of Bayesian approaches: a piecewise constant proportional hazards cure rate model (Chen et al., 2002); and a log-F accelerated failure time model for missing at random categorical and continuous covariates (Cho and Schenker, 1999).

Where the MAR assumption does not hold, the data are said to be Not Missing At Random (NMAR) and the missing data mechanism said to be “non-ignorable”. In this situation the mechanism for the missing data, that is $f_{R|Y_o, Y_m}(\mathbf{r}|\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\psi})$, must also be specified and included in likelihood inferences (equation 3). Since (Y_o, Y_m, R) is not observed in full, this may lead to issues of identifiability for $f_{R|Y_o, Y_m}(\mathbf{r}|\mathbf{y}_o, \mathbf{y}_m; \boldsymbol{\psi})$ (Baker and Laird, 1988). Within the remit of generalised linear models, non-ignorable missing data mechanisms have been modeled (Ibrahim et al., 1999; Stubbendick and Ibrahim, 2003), and Bayesian approaches have used informative priors to identify model parameters with NMAR covariates (Huang et al., 2005). In survival analysis, within a proportional hazards cure rate model, non-ignorable models have been proposed using maximum likelihood based techniques (Herring and Ibrahim, 2002).

Existing methods for dealing with missing covariate data in survival analysis, both Bayesian and frequentist, are often not easy useable by others and so have limited practical applicability due to computational programming required. Furthermore, to our knowledge there have been no Bayesian developments of survival regression models for non-ignorable missing data mechanisms. The accelerated failure time models are a wide class of log-linear models which provide an alternative to the Cox model when assumptions of proportionality are not met, and being fully parametric they can also be more powerful. The accelerated failure time models have also been shown to fit the UKCP data well (Kwong and Hutton, 2003) and may be more robust in cases of model misspecification (Hutton and Monaghan, 2002).

Here we develop a log-normal failure time model, which accommodates binary missing at random categorical covariates. We focus on the log-normal model and on categorical covariates as this allows us to provide a practical solution within BUGS (Spiegelhalter et al., 1999) and which is therefore accessible in practical applications. We further consider

extensions to covariate data not missing at random, and consider how robust estimates are to various non-ignorable missing data mechanisms supported by the data. Although specified in terms of impairment variables and regional effects, the model is more generally a log-normal survival model for multiple and partially observed binary covariates with extra variation due to center, region, or other familial variable (or some other fully observed categorical covariate). Extensions included allow for interactions between the covariates and the inclusion of fully observed continuous covariates.

This paper is organised as follows. In the next section the fully parametric Bayesian survival model is outlined and models for the covariate data proposed. Likelihood based methods using Markov Chain Monte Carlo (MCMC) simulations are used to draw parameter inferences under the less restrictive assumption of MAR. A parametric distribution is specified for the missing data mechanism, that is the likelihood of a covariate being observed, with dependencies on censoring status, survival time, and the covariate itself. Using weakly informative priors, we obtain posterior parameter estimates and consider how sensitive model inferences are to possible non-missing at random missing data mechanisms. In the final section, the model is applied to the UKCP data and inferences drawn. We compare inferences from a naive CCA to the better fitting models under a MAR analysis, and consider how robust parameter inferences are under possible NMAR mechanisms. *The BUGS code for this model is available from the first author on request.*

Proposed model

Survival distribution Let T have density function $f_{T|Z}(t|\mathbf{z}; \boldsymbol{\theta}_T)$ conditional on binary covariates $\mathbf{z} = (z_1, \dots, z_k)$. Full data, for individuals $i = 1, \dots, n_j$, out of $j = 1, \dots, J$ regions, are observed for event times t_{ij} and censoring indicators δ_{ij} . The binary covariate data on $\mathbf{z}_{ij} = (z_{1ij}, \dots, z_{Kij})$ are only partially observed. Let $y_{ij} = \ln t_{ij}$. Let $S_{T|Z}(t|\mathbf{z}; \boldsymbol{\theta}_T)$ and $h_{T|Z}(t|\mathbf{z}; \boldsymbol{\theta}_T)$ represent the corresponding survival and hazard functions respectively.

We initially assume that there is an underlying effect of severity of impairment on survival across all five regions. We allow the survival to differ between the regions, in addition to that which might be explained by random variation. We therefore propose a model which allows for extra regional variation:

$$\ln t_{ij} = \alpha_0 + \boldsymbol{\alpha}\mathbf{z}_{ij} + b_{(s)j} + \sigma\eta \quad \text{where } \eta \sim N[0, 1] \quad (7)$$

and where $b_{(s)j}$ is a fixed effect due to region. We use the notation $b_{(s)j}$ (for $j = 1, \dots, J-1$) to refer to a regional effect (j) on survival (s) - as opposed to regional effects on proportions impaired (to be introduced later). We focus on the log-normal survival model and consider generalisability to other distributions in the discussion. The resulting model is the log-normal survival model, with α_0 the intercept parameter, $\boldsymbol{\alpha}$ representing the set of covariate effects, $\mathbf{b}_{(s)} = (b_{(s)1}, \dots, b_{(s)J-1})$ a set of regional effects on survival and σ the scale parameter, with $\boldsymbol{\theta}_T = (\alpha_0, \boldsymbol{\alpha}, \sigma, \mathbf{b}_{(s)})$.

Regional variation in the survival of both the severely impaired and non severely impaired is incorporated by including an interaction between regions and the binary covariates:

$$\ln t_{ij} = \alpha_0 + \boldsymbol{\alpha}\mathbf{z}_{ij} + \mathbf{c}_j\mathbf{z}_{ij} + b_{(s)j} + \sigma\eta \quad (8)$$

where $\mathbf{c}_j = (c_{1j}, \dots, c_{Kj})$ represents an interaction between covariate k and region j . Hence $\boldsymbol{\theta}_T = (\alpha_0, \boldsymbol{\alpha}, \sigma, \mathbf{b}_{(s)}, \mathbf{c})$ where $\mathbf{c} = (\mathbf{c}_1, \dots, \mathbf{c}_{J-1})$.

Incorporating additional fully observed covariates ($\bar{\mathbf{z}}$) is straightforward:

$$\ln t_{ij} = \alpha_0 + \boldsymbol{\alpha}\mathbf{z}_{ij} + \mathbf{d}\bar{\mathbf{z}} + b_{(s)j} + \sigma\eta \quad (9)$$

where \mathbf{d} represents the effect of a fully observed vector of continuous covariates ($\bar{\mathbf{z}}$), and $\boldsymbol{\theta}_T = (\alpha_0, \boldsymbol{\alpha}, \sigma, \mathbf{b}_{(s)}, \mathbf{d})$.

Covariate distribution Let $\mathbf{z} = (z_1, \dots, z_K)$, the indicators of severity of impairment for K binary covariates (where 1 represents a severe impairment) have probability distribution $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$.

Since inferences for the covariates are not a prime interest of the analysis, one approach is to base inferences on a simple Bernoulli distribution of the form:

$$z_k \sim \text{Bern}(p_k) \quad (10)$$

where p_k represents the probability of a severe impairment for each of the K covariates and where each of the variables z_k are independently distributed binary variables:

$$f_Z(\mathbf{z}; \boldsymbol{\theta}_Z) = \prod_{k=1}^K f_{Z_k}(z_k; p_k) \quad (11)$$

where $\boldsymbol{\theta}_Z = (p_1, \dots, p_K)$. This implies that the probability of having a particular severe impairment is independent of any other severe impairments. This is not necessarily a valid assumption, but one which is investigated since it greatly reduces the computational time, as there are only K nuisance parameters.

An alternative is to specify the joint distribution for $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$ by a series of conditional distributions (Lipsitz and Ibrahim (1996b); Ibrahim et al. (1999)):

$$f_Z(\mathbf{z}; \boldsymbol{\theta}_Z) = \prod_{k=K}^1 f_{Z_k|Z_1:Z_{k-1}}(z_k|z_1 : z_{k-1}; \boldsymbol{\beta}_k) \quad (12)$$

where $\boldsymbol{\theta}_Z = (\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_K)$. In this way, correlations are induced between likelihoods of each severe impairment, in contrast to the independent specification of probabilities in equation 11. For example, in the case of four covariates:

$$f_Z(\mathbf{z}; \boldsymbol{\theta}_Z) = f_{Z_4|Z_1:Z_3}(z_4|z_1 : z_3; \boldsymbol{\beta}_4) f_{Z_3|Z_1:Z_2}(z_3|z_1 : z_2; \boldsymbol{\beta}_3) f_{Z_2|Z_1}(z_2|z_1; \boldsymbol{\beta}_2) f_{Z_1}(z_1; \boldsymbol{\beta}_1)$$

Each conditional distribution may be represented by a logistic relationship. That is, the conditional probability of having a particular severe impairment, given a subset of other impairments, is modeled as a linear combination of this subset on the logit scale:

$$\text{logit } \tilde{p}_k = \beta_{k0} + \beta_{k1}z_1 + \dots + \beta_{kk-1}z_{k-1} \quad (14)$$

where \tilde{p}_k denotes a conditional probability (of variable z_k being 1 (severe)) as opposed to the marginal probability of p_k , and where $\boldsymbol{\beta}_k = (\beta_{k0}, \dots, \beta_{kk-1})$. The number of nuisance parameters to be estimated under such a parameterisation is $\sum_{k=1}^K k = (K+1)K/2$. Inferences for GLMs and PH models have been shown to be robust to the order of the specification of this conditional wise model (Ibrahim et al. (1999); Stubbendick and Ibrahim (2003)).

To allow the conditional proportion (\tilde{p}_k) of those severely impaired to be similar between the regions, with some variability due to extra variation between the regions, a fixed regional effect can be incorporated:

$$\text{logit } \tilde{p}_k = \beta_{k0} + \beta_{k1}z_1 + \dots + \beta_{kk-1}z_{k-1} + b_{(\bar{p})kj} \quad (15)$$

where $b_{(\bar{p})kj}$ represents a fixed regional effect in (conditional) proportions (\tilde{p}_k) severely impaired. Under this conditional wise specification with regional variations in proportions severely impaired, $\boldsymbol{\theta}_z = (\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_K, \mathbf{b}_{(\bar{p})1}, \dots, \mathbf{b}_{(\bar{p})K})$, where $\mathbf{b}_{(\bar{p})k} = (b_{(\bar{p})k1}, \dots, b_{(\bar{p})k,J-1})$.

Sensitivity analysis (NMAR extension) The covariate data are not fully observed, and this is represented using a missing data indicator, r_{kij} , such that:

$$\begin{aligned} r_{kij} &= 1 \text{ if covariate } k \text{ is observed for individual } i \text{ in region } j \\ &= 0 \text{ otherwise.} \end{aligned} \quad (16)$$

Where the covariate data are not missing at random, likelihood based inferences require full specification of the missing data mechanism $f_R(\mathbf{r}; \boldsymbol{\psi})$, which is here modeled as a logistic function:

$$\text{logit } p_{(r)kij} = \gamma_{k0} + \gamma_{k1}z_{ijk} \quad (17)$$

where $p_{(r)kij}$ represents the probability of covariate k being observed for individual i from region j . Here the probability that a covariate is observed depends only on the covariate value itself, with $\boldsymbol{\psi} = (\boldsymbol{\psi}_1, \dots, \boldsymbol{\psi}_K)$ and where $\boldsymbol{\psi}_k = (\gamma_{k0}, \gamma_{k1})$.

The probability of observing a particular covariate may depend on event time and, or censoring status. Where the missing data mechanism is allowed to depend on the censoring status, the independent censoring assumption is replaced by independent censoring conditional on R (Rathouz, 2007).

$$\text{logit } p_{(r)kij} = \gamma_{k0} + \gamma_{k1}z_{ijk} + \gamma_{k2}t_{ij} + \gamma_{k3}\delta_{ij}. \quad (18)$$

So that $f_{R_k}(r_k; \boldsymbol{\psi}_k)$ follows a logistic distribution with parameter $\boldsymbol{\psi}_k = (\gamma_{k0}, \gamma_{k1}, \gamma_{k2}, \gamma_{k3})$ and $\boldsymbol{\psi} = (\boldsymbol{\psi}_1, \dots, \boldsymbol{\psi}_K)$. Regional effects on the missing data mechanism are not considered as this would require a large number of additional parameters.

Likelihood Inferences

All inferences are carried out in BUGs using Markov Chain Monte Carlo methods, and models compared using DIC (Spiegelhalter et al., 2002). For completeness the likelihood is presented.

Inferences ignoring the missing data mechanism lead to $L(\boldsymbol{\theta}|\mathbf{t}, \boldsymbol{\delta}, \mathbf{z}, \mathbf{r}) \propto L(\boldsymbol{\theta}|\mathbf{t}, \boldsymbol{\delta}, \mathbf{z})$, where $\mathbf{t} = (t_1, \dots, t_n)$, $\boldsymbol{\delta} = (\delta_1, \dots, \delta_n)$, $\mathbf{r} = (\mathbf{r}_1, \dots, \mathbf{r}_n)$ and the parameter of interest is $\boldsymbol{\theta}_T$ and nuisance parameter $\boldsymbol{\theta}_Z$. The full likelihood is based on the factorisation of the joint distribution for $f_{T,\delta,Z}(t, \delta, \mathbf{z}; \boldsymbol{\theta}) = f_{T,\delta|Z}(t, \delta|\mathbf{z}; \boldsymbol{\theta}_T)f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$ so that:

$$L(\boldsymbol{\theta}|\mathbf{t}, \boldsymbol{\delta}, \mathbf{z}) \propto \prod_{j=1}^N \prod_{i=1}^{n_j} l_{ij}. \quad (19)$$

Where \mathbf{z}_{ij} is fully observed:

$$l_{ij} = S_{T|Z}(t_{ij}|\mathbf{z}_{ij}; \boldsymbol{\theta}_T) h_{T|Z}(t_{ij}|\mathbf{z}_{ij}; \theta_T)^{\delta_{ij}} f_Z(\mathbf{z}_{ij}; \boldsymbol{\theta}_Z). \quad (20)$$

Where \mathbf{z}_{ij} is only partially observed, let $G = 2^K$ represent the number of possible combinations of covariates (from now on called groups), with associated covariate \mathbf{z}_g ($g = 1, \dots, G$), being one realisation of the G possible combinations of \mathbf{z} . Let W_{gij} be an indicator for whether the partial observed covariate information on individual i from region j is compatible with the realisation \mathbf{z}_g , then:

$$l_{ij} = \sum_{g=1}^G W_{gij} S_{T|\mathbf{Z}}(t_{ij}|\mathbf{z}_g; \boldsymbol{\theta}_T) h_{T|\mathbf{Z}}(t_{ij}|\mathbf{z}_g; \theta_T)^{\delta_{ij}} f_Z(\mathbf{z}_g; \boldsymbol{\theta}_Z). \quad (21)$$

For non-ignorable missing data mechanisms:

$$L(\boldsymbol{\theta}, \boldsymbol{\psi} | \mathbf{t}, \boldsymbol{\delta}, \mathbf{Z}, \mathbf{r}) = \prod_{j=1}^N \prod_{i=1}^{n_j} l_{ij}, \quad (22)$$

where for \mathbf{z}_{ij} only partially observed:

$$l_{ij} = \sum_{g=1}^G W_{gij} S_{T|\mathbf{Z}}(t_{ij}|\mathbf{z}_g; \boldsymbol{\theta}_T) h_{T|\mathbf{Z}}(t_{ij}|\mathbf{z}_g; \theta_T)^{\delta_{ij}} f_Z(\mathbf{z}_g; \boldsymbol{\theta}_Z) f_{R|T,Z}(\mathbf{r}_{ij} | t_{ij}, \delta_{ij}, \mathbf{z}_g; \boldsymbol{\psi}) \quad (23)$$

As the data (T, δ, Z, R) are not fully observed (since Z is only partially observed), it may not be possible to identify the parameters $\boldsymbol{\psi}$ in $f_R(r; \boldsymbol{\psi})$. No algebraic check of identifiability has been established, but in other similar models, using weakly informative priors, the parameter $\boldsymbol{\psi}$ has been identifiable (Huang et al., 2005). Non-identifiability in practical applications generally manifests as divergence or slow mixing of chains. Furthermore, even for an identifiable model, there is clearly no means of evaluating distributional assumptions for the missing data mechanism. Following others we therefore propose to use posterior inferences for $(\boldsymbol{\theta}_T, \boldsymbol{\theta}_Z)$ based on weakly informative priors for $\boldsymbol{\psi}$, in the role of a sensitivity analysis, to investigate how robust and sensitive inferences are to departures from the assumption of MAR.

Application

The data set consists of 3946 cases with early impairment cerebral palsy, born during the 1980s and 1990s, flagged with the relevant births and deaths register and followed-up until March 2004. All survival times are recorded to the nearest week. We condition throughout on survival to age two years (to allow for delays in initial notifications). Vital status information is complete for the entire cohort. Severity of any associated disability is measured by four known important binary predictor variables ($z=1$ for severe; and $z=0$ for not severe): z_1 : severe ambulatory impairment (unable to walk); z_2 : severe manual impairment (unable to self feed); z_3 : severe cognitive impairment ($\text{IQ} < 50$); z_4 : and severe visual impairment ($< 6/60$ in the better eye). All four binary predictor variables are only partially observed. A further fully observed continuous covariate, a standardised birthweight for gestational age (z -score) is also considered. Standard birthweights for gestational age values are taken from the Scottish population (Hemming et al., 2006). Large negative

z-score values (around -3) indicate that the child was born with a very low birthweight for gestation, and values around zero suggest an average birthweight. Birthweight and gestational age are routinely recorded neonatal variables and are fully observed for this data set and so z-scores are derivable for the entire data set. This variable is of particular interest in current epidemiological research in cerebral palsy (Jarvis et al. (2003); Hemming et al. (2007)).

Proportions of cases with a particular severe impairment vary quite considerably between the regions (Table 1). Scotland stands out as the most noticeable extreme. Proportions of cases for which information on severity of impairment is missing, varies both between the regions and between the variables (Table 1). For manual and ambulatory severity indicators, the proportions of cases for which the severity indicator is missing, is low (less than 10%). For the other associate impairments, proportions of missing data are higher, for both Northern Ireland and Oxford, and very high for Scotland (63% with missing IQ information). The vast majority of cases have covariate information on at least one variable, with many having information on two or three covariates: 2980 (76%) have full covariate information on four covariates; 661 have data on three covariates; 226 have data on two covariates; 26 have data on one covariate only; 53 cases have no information on any of the four covariates.

Some association between severity of disability and missing information may be due to the child having died before the assessment could be made or through lack of follow-up (for covariate data rather than death information). The Kaplan-Meier estimates of the survival by region and severity or missingness of impairment variables indicate there are many likely causes of missing covariate information (Figure 1). For example, for both manual and ambulatory variables for Mersey, missing data show a strong association with early deaths, although the absolute number of cases with missing information on these two variables for the Mersey region is small. In the North of England, Oxford and Scotland those with missing covariate information on ambulatory and manual variables constitute a mixture of early deaths and late censored observations, contrasting with Northern Ireland in which almost all those with missing information on these two variables are late censored observations. For those with missing cognitive impairment, again Mersey constitutes mainly deaths, and the North of England a mixture of early deaths and late censored observations, for Northern Ireland and Oxford, the missing covariate observations consist of mainly censored survival times. In Scotland, where the proportion of cases with missing information on severity of cognitive impairment is high, a significant proportion of those with missing cognitive data have died. For those with missing visual impairment, a large number of early deaths give a survival pattern which is not too dissimilar to those who are severely impaired for all five regions.

Implementation

We compare inferences from a complete case analysis where it is assumed the covariate data are MCAR, to inferences obtained under the less restrictive assumption of MAR and where to enable likelihood based inferences, not only $f_{T|Z}(t, \mathbf{z}; \boldsymbol{\theta}_T)$ is modeled (for which we consider models 7-9), but so too is $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$ (for which we consider models 10-11 and 12-15). We further consider how robust parameter estimates are to non-missing at random missing data mechanisms (models 17-18).

The parameters partition into those which are robust to prior specifications and those which are less so. For those which are robust we provide results based on a single set of independent non-informative priors (although a range of priors was used to obtain conclusions

that this set of parameters are robust to prior specification). These parameters consist of the regression coefficients ($\alpha_0, \boldsymbol{\alpha}, \mathbf{c}, \mathbf{d}$), for which we used diffuse independent normal priors $N(0, 0.001)$ - (parameterised by center and precision parameters); the scale parameter, σ a Gamma prior $G(1, 0.001)$ (approaching a uniform prior over the range (0,100) on the precision scale); for the covariates (Bernoulli specification), non-informative Beta priors $B(1, 1)$ for p_k and for the conditional wise specification, independent normal priors $N(0, 0.001)$ for β_{kk} .

Inferences might be less robust to priors for the parameters of the missing data mechanism $f_R(r; \boldsymbol{\psi})$. For these parameters we considered various weakly informative priors: $N(0, 0.1)$, a fairly tight prior centered around zero; $N(0, 0.01)$ a more uniform prior again centered at zero; $N(0, 0.001)$ towards a vague prior centered at zero.

All inferences were carried out in BUGS (Spiegelhalter et al., 1999) with convergence checked using CODA (Best et al., 1997). All inferences were compared over 100,000 iterations after an initial burn in of 10,000 iterations. A diffuse range of starting values were explored. For missing binary covariate data, zero and one values were randomly imputed to generate a set of initial covariate values. Model fits were compared using deviances with complexity penalised by twice the number of parameters, along the lines of DIC. The order of the conditional specification for $f_Z(\mathbf{z})$ was chosen by the order providing the lowest DIC value, although the main parameters of interest were not sensitive to choice of order. The order used was z_1 =ambulatory impairment; z_2 =manual impairment; z_3 =cognitive impairment; z_4 =visual impairment. No interactions were found to be significant and are not presented.

Sample run times for this fairly large data set ($n=3946$) were for the CCA 107 seconds; for the MAR analysis with independent Bernoulli model for $f_Z(z|\boldsymbol{\theta}_z)$ 206 seconds; for the MAR analysis with conditional model for $f_Z(z|\boldsymbol{\theta}_z)$ 1076 seconds; and for the NMAR model 21708 seconds.

Results

Inferences from a CCA

As expected, in a CCA the distribution assigned to $f_Z(\mathbf{z}; \boldsymbol{\theta}_z)$ has no impact on any survival analysis inferences (as is demonstrated in Table 2). Inferences for $f_{T|Z}(t, \mathbf{z}; \boldsymbol{\theta}_T)$ allowing dependencies on the four binary predictor variables and with extra variation due to region (modeled as in equation 7), suggests that out of the four predictive covariates, a severe ambulatory impairment reduces the median life time by the most (Table 2). Furthermore, a regional variation in survival is significant, with Scotland seemingly having a much more favorable outlook compared to the other four regions (coefficient 26(11.98)). Clearly since Scotland has such high proportions of missing sensory data, such a conclusion is questionable under anything but a completely naive CCA.

Inferences from a MAR analysis

Using a MAR likelihood based approach it is necessary to model $f_Z(\mathbf{z}; \boldsymbol{\theta}_z)$. Although attractive, a simple independent parameterisation for $f_Z(\mathbf{z}; \boldsymbol{\theta}_z)$ through independent Bernoulli random variables (equations 10-11), results in considerable loss of information on the relationship between the four impairment variables compared to the model in which likelihoods of severe impairments are modeled as functions of other severe impairments (equations 12 and 14). For models in which a regional variation in survival is included, the deviance is 20580 (14 parameters) for the Bernoulli specification compared to 15790 (20 parameters) for the conditional wise specification. Using the conditional wise specification for $f_Z(\mathbf{z}; \boldsymbol{\theta}_z)$

also results in smaller standard errors on parameter estimates compared to the model in which the Bernoulli specification is used (Table 2).

Using an independent Bernoulli model for $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$, a MAR analysis finds only a marginally significant difference between the five regions in survival: deviance of 20590 (no region effect - 10 parameters) vs 20580 (with region effect - 14 parameters) (Table 2). Contrasting this, under the conditional specification for $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$, a significant difference between Scotland and the other four regions persists under the MAR fit, with Scotland again standing out as having the most favorable outlook (deviance of 15820 (no region effect - 16 parameters) vs 15790 (with region effect 20 parameters)). Standard errors on fixed effect region parameters are smaller under the MAR fit compared to the CCA (this holds for both the conditional and Bernoulli models).

For models in which a regional effect on survival is included, under the Bernoulli specification for $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$, the effect of a severe ambulatory impairment decreases from $-0.86(0.19)$ in a CCA to $-1.08(0.28)$ in a MAR analysis (Table 2). This suggests a much greater reduction in life expectancies for those with a severe ambulatory impairment than previously thought (i.e. from inferences based on a CCA). However, using a conditional wise specification for $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$, the effect of a severe ambulatory impairment is $-0.87(0.18)$, very similar to that of the CCA, and with a much smaller standard error compared to the Bernoulli model for $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$. Using the conditional wise specification for $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$, the reduction in life expectancy associated with the three severe impairments, ambulation, manual and cognitive disabilities are fairly similar, as opposed to the CCA where a severe ambulatory impairment appears to have the greatest impact on a reduction in life expectancy.

Parameter estimates for the parameters of less interest (those of $f_{Z_k|Z_1:Z_{k-1}}(z_k|z_1 : z_{k-1}; \boldsymbol{\beta}_k)$) are given in Table 3, and an interpretation of these parameter estimates in Table 4. There is a strong correlation between a severe ambulatory impairment and the other three impairment variables: someone who has a severe ambulatory impairment has a very high likelihood of having a severe manual, visual or cognitive impairment (Table 4), with probabilities ranging from 0.81 to 0.98. A severe cognitive impairment is also highly correlated with a severe visual impairment, and having both a cognitive and visual impairment is highly correlated with a severe manual impairment.

There is some regional variation in proportions severely impaired (Table 1). In a further attempt to improve model fit under MAR inferences, we consider a fixed regional effect on the proportion severely impaired, using a conditional specification for $f_Z(\mathbf{z}; \boldsymbol{\theta}_Z)$ (equation 15). This model gives a lower deviance value compared to models in which no regional variation between proportions severely impaired are included (deviances 15760 (24 parameters) vs 15790 (20 parameters)). Posterior parameters indicate that there is noticeable regional variation in proportions of cases with each of the four severe impairments (Table 5). For instance, in Scotland, 37% are estimated to have a severe ambulatory impairment, compared to the 31% average over regions; only 6% in the North East are estimated to have a severe visual impairment (compared to the average of 12%, with as many as 18% in Scotland). Cognitive information has particularly high proportions of missing data in Scotland, with just 3% of those with known data on this variable having a severe impairment. MAR inferences estimates 15% of cases in Scotland as having a severe intellectual impairment, lower than the average (26%), but a vast improvement on 3%. Main parameter estimates for $f_{T|Z}(t|\mathbf{z}; \boldsymbol{\theta}_T)$ do not differ greatly to those of previous MAR fits, except that the coefficient for extra variation in Scotland decreases from 0.51 (0.15) to 0.34 (0.16).

We further consider including as an additional explanatory variable, a child's birthweight

measured in terms of a deviation from expected birthweight for a given gestational age, called “z-scores”. This is a fully observed continuous covariate. Both linear and quadratic effects are investigated as this variable is thought to have an inverse J relationship with the incidence of cerebral palsy and so might be expected to affect survival outcome in a similar way. Neither models provided a significant improvement in model fit (Table 5). This is in contrast to the CCA (previously published (Hemming et al., 2005) and not shown here) where it was found to have a significant influence.

NMAR sensitivity analysis

We consider allowing the missing data mechanism to be non-ignorable, parameterised by a logistic function with dependencies on the event time, censoring status and covariate (equations 17 and 18). We consider only the better fitting model for the covariate distribution, and all results presented on based on the full conditional wise specification (equation 14). Main parameter estimates were not sensitive to choice of prior and results presented are based on the prior $N(0, 0.01)$.

NMAR and MAR models are compared (Table 6) by fitting full NMAR models in which dependencies on time, censoring and covariate are included; with NMAR models in which only an intercept term is included in the distribution for the missing data mechanism (and so therefore reduces to a MAR model). The NMAR models provide a better fit, in terms of smaller deviances, than the NMAR models in which only the intercept term is included. This suggests the covariate data may be NMAR. As with MAR fits, a model in which region is included as a survival regression model appears to give a better fit. However, the extent to which the model is improved by including an effect due to region, both in terms of deviance and parameter estimates, is greater under the NMAR models. Under the better fitting NMAR model with variation due to region included, posterior estimates for α suggest that the coefficients for the effects of both manual and ambulatory functions are similar (around -0.86); and coefficients for both cognitive and visual disabilities are also similar (around -0.7). Scotland again appears to have significantly better survival outcomes compared to the other four regions, and variation between the other four regions is similar to that of the MAR models.

Parameter estimates for the missing data mechanism are presented in Table 7. The coefficients for the effect of failure time on the missing data mechanisms for motor functions are positive, suggesting that as age increases there is an increase in the likelihood of observing the covariate. In contrast, the coefficients for the effect of the failure time on the likelihood of observing the covariates for cognitive and visual disabilities are negative, suggesting that as the failure time increases there is a decrease in the likelihood of observing the covariate values. For both motor impairments and cognitive disabilities, the negative estimates of the effect of censoring status on the likelihood of observing covariate values imply an increased likelihood of observing covariates for deaths compared to censored observations. The corresponding negative coefficient for visual disability, suggests an increased likelihood of observing the visual covariate value for those who are censored compared to those who have died, although this effect is not significant. The effects of the covariates themselves on the likelihood of observing the covariate, all being negative, suggest a reduced likelihood of observation for those who are severely impaired compared to those who are not severely impaired.

Clinical conclusions

A naive CCA leads to conclusions of a regional variation in survival outcome, with estimated proportions severely impaired (26%, 20%, 24%, 9%), with order of magnitude on survival influence: ambulation, intelligence, manual dexterity, and vision; and order of influence of region on survival: the North of England (worst), Mersey, Oxford, Northern Ireland, Scotland (best). Scotland having the best survival outcome, is however dubious under such a complete case analysis as Scotland has such high proportions of missing data.

A full and in depth analysis of this data involves fitting a more complex model to the likelihood of a severe impairment, in which correlations between the four variables are induced (through a conditional wise specification), and this results in a better fitting model. Under such a MAR model fit, proportions severely impaired are higher (31%, 23%, 28%, 12%), the effect of the covariates on survival similar between models in which an effect of region is included and one which is not, and interestingly, Scotland again comes out as having the best survival outcome. All three of ambulation, manual dexterity and intelligence, are estimated to have similar affects on median life expectancies. This model leads to greater precision in main parameter inferences compared to the CCA, a reflection of the larger sample size and how the extra uncertainty introduced by the partial covariate information has been reduced by modeling correlations between each of the four impairments. A model in which the proportions of cases with each of the severe impairments are allowed to vary between the regions produces a slightly better fit. Under such a model the proportion of cases with a severe intellectual impairment in Scotland is estimated to be around 15%, which is much lower than the average over the regions, but substantially higher than the raw data for Scotland leads us to believe at just 3%. A sensitivity analysis, allowing for the covariate data to be missing in a non-random way, suggests a further increase in the proportions severely impaired, especially those with severe visual or cognitive impairments ((32%, 24%, 33%, 16%)).

One of the primary questions of interest in the analysis of this data, is whether there exists a regional variation in survival. Such regional variations may possibly be due to such factors, as variations in neonatal care, variations in racial, ethnic or socio-economic mixes of the background populations of the regions, which may indirectly affect rates of cerebral palsy, rates of the severely impaired and survival outcomes. A CCA, conditioning on severity of impairment, suggests no regional variation between four (Mersey, North of England, Northern Ireland, Scotland) of the regions, but suggests an improvement in survival in Scotland. Our in depth MAR analysis, again suggests no regional variation between these four regions, but finds that survival in Scotland is significantly improved. This apparent favorable outlook in Scotland may reflect a true increase in life expectancy or may be an artifact of the data, since even an analysis with complete covariate data may be compromised by differing ascertainment proportions between the regions. Scotland has higher proportions severely impaired for all impairment variables except for cognitive impairment, which could suggest an under-ascertainment of the less severely impaired cases. Further analysis have shown estimated prevalence rates of cerebral palsy in Scotland to be significantly lower than those of the other four regions, a further indication of reduced ascertainment (personal communication, Jane Hutton).

The other focus of this analysis is that of life expectancies for individuals with particular sets of impairments, used both in legal settlements and planning of resources. The CCA, estimates the median life expectancy for someone with four severe impairment variables, born in the Mersey region, is 18.7 years ($exp(6.88)/52$); compared to the MAR analysis, using the

conditional wise specification for the covariates, as 15.5 years ($\exp(6.69)/52$); interestingly under the NMAR analysis this median life expectancy is similar ($\exp(6.70)/52$), a reflection of those with missing covariate information being both deaths (providing a reduction in life expectancies) and censored observation (providing an increase in life expectancies). A CCA leads to the conclusion that ambulation provides the most predictive information for a reduction in life expectancy. Results here however suggest that the four severe impairment variables have similar predictive powers, a conclusion that has some clinical feasibility due to all four variables acting as surrogate markers for a degree of cerebral damage which is difficult to quantify.

The estimated proportion of cases with a severe visual impairment is high (around 16%) compared to less than 10% from the CCA. However, this pattern of informed missingness detected by the NMAR model, and the MAR model in which correlations between each of the impairments were included, would seem to be consistent with clinical expectations. Although in the UKCP no distinction is given to various types of missingness, in the Mersey region information is recorded on those with a possible visual impairment but one which is difficult to test (either because the child is too young or too severely impaired to test). For the Mersey region, 8% have “unknown” severity of visual impairment and an additional 12% fall into the group with a possible impairment, thereby suggesting that many more than the 8% have a severe visual impairment (Hutton and Pharoah, 2002). The NMAR analysis for the visual covariate, suggests a stronger association between the covariate being missing and the individual having died, than for the other three covariates.

Discussion

Missing covariate data often do not meet the assumption of being MCAR. The lesser restrictive assumption of MAR is attractive, although when using likelihood based inferences requires additional assumptions for variables often of secondary interest, that of the covariate distributions in survival analysis. For a conceptually simple problem of relating survival outcome to four binary predictor variables with extra variation due to region or center and allowing for interactions and a fully observed continuous covariate, we have shown that careful consideration to the joint distribution for the covariates is a minimal requirement in meeting the MAR assumption. In our application the simple independent Bernoulli model for these four binary variables resulted in a poorer and misleading fit, than did a conditional wise specification for this joint density. This addresses in part the common concern of using independent prior distributions.

We have presented a Bayesian model which allows sensitivity of parameter estimates to be evaluated, either to the MCAR or MAR assumptions. Almost all medical survival applications, being complete case analyses, assume that the data are MCAR. One of the advantages of the model presented here, is that being written in BUGS, it can potentially be used by others. Its short falling may be that being a fully parametric survival model, it is limited to use for data which meet the parametric assumptions. However, even the semi-parametric Cox proportional hazards model makes an underlying assumption of hazards being proportional, which of course will not always be feasible. We have focused on the log-normal survival model, belonging to the class of accelerated failure time (AFT) models. The BUGS code is adaptable to the Weibull model (also belonging to the class of AFT models). For other models within the AFT class, it is not immediately clear that they too would be programmable within BUGS, but could be implemented using Gibbs sampling

with an adaptive rejection sample. Our focus has been on binary covariates, which were of clinical interest in this example, though we have illustrated an extension to include a fully observed continuous covariate. Extensions to missing at random continuous covariates are a natural next extension, with incorporation into the conditional wise specifications, as too are informative Bayesian priors.

We recommend that applied statisticians consider not only MAR models, but also NMAR models informed by the substantive scientific knowledge. Such knowledge can inform the structure of the model and prior distributions. Both marginal and conditional interpretations of parameters should be considered, as well as the meaning of sensitivity parameters. Although some might regard sensitivity parameters as nuisance parameters, it is sensible to consider whether the implied mechanisms are consistent with clinical understanding and knowledge of data collection.

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Table 1. Cerebral palsy cases by region, severity of impairment and survival status

	ME			NE			NI			OX			SC		
	A	D	T	A	D	T	A	D	T	A	D	T	A	D	T
Cases	671	89	760	688	54	742	683	67	850	889	63	952	593	49	642
LLF severe	125	82	207	161	36	197	166	62	228	256	54	310	192	34	226
LLF not severe	546	6	552	510	4	514	584	3	587	614	3	617	390	3	393
LLF missing	0	1	1	17	14	31	33	2	35	19	6	25	11	12	23
ULF severe	89	77	166	145	35	180	109	55	164	144	49	193	122	34	156
ULF not severe	582	11	593	528	7	535	622	5	627	729	7	736	457	3	460
ULF missing	0	1	1	15	12	27	52	7	59	16	7	23	14	12	26
IQ severe	147	81	228	131	37	168	171	58	229	159	49	208	8	0	8
IQ not severe	522	5	527	541	8	549	536	5	541	600	6	606	228	1	229
IQ missing	2	3	5	16	9	25	76	4	80	130	8	138	357	48	405
Vision severe	31	43	74	26	12	38	51	29	80	74	30	104	50	18	68
Vision not severe	597	20	617	636	27	663	680	26	706	758	23	781	308	8	316
Vision missing	43	26	69	26	15	41	52	12	64	57	10	67	235	23	258

Key

LLF: Lower Limb Function (ambulation); ULF: Upper Limb Function (manual); IQ: Intelligence quotient

A: Alive; D: Dead; T: Total

ME: Mersey; NE: North of England; NI: Northern Ireland; OX: Oxford; SC: Scotland

Table 2. Model inferences for CCA and MAR models: posterior estimates and posterior standard deviations

	$f_Z(\cdot)$ modeled by independent Bernoulli variables				$f_Z(\cdot)$ modeled by a conditional wise specification				
	No region effect		Region effect		No region effect		Region effect		
	CCA	MAR	CCA	MAR	CCA	MAR	CCA	MAR	
$f_Z(\cdot)$									
p_1	0.26 (0.01)	0.31 (0.01)	0.26 (0.01)	0.31 (0.01)	0.26	0.31	0.26	0.31	
p_2	0.20 (0.01)	0.23 (0.01)	0.20 (0.01)	0.23 (0.01)	0.20	0.23	0.20	0.23	
p_3	0.24 (0.01)	0.26 (0.01)	0.24 (0.01)	0.26 (0.01)	0.24	0.24	0.24	0.26	
p_4	0.09 (0.01)	0.11 (0.01)	0.09 (0.01)	0.11 (0.01)	0.09	0.09	0.09	0.12	
$f_{T Z}(\cdot)$									
α_0	9.85 (0.21)	9.97 (0.20)	9.80 (0.24)	9.93 (0.21)	9.86 (0.23)	9.94 (0.19)	9.77 (0.24)	9.81 (0.21)	
α_1	-0.86 (0.19)	-0.97 (0.28)	-0.88 (0.20)	-1.08 (0.28)	-0.87 (0.20)	-0.83 (0.18)	-0.87 (0.20)	-0.87 (0.18)	
α_2	-0.72 (0.18)	-0.76 (0.25)	-0.70 (0.18)	-0.66 (0.25)	-0.72 (0.18)	-0.85 (0.16)	-0.70 (0.18)	-0.81 (0.17)	
α_3	-0.81 (0.15)	-0.95 (0.15)	-0.81 (0.16)	-0.96 (0.16)	-0.81 (0.16)	-0.85 (0.15)	-0.81 (0.16)	-0.84 (0.16)	
α_4	-0.58 (0.21)	-0.45 (0.12)	-0.60 (0.13)	-0.49 (0.13)	-0.58 (0.13)	-0.54 (0.12)	-0.60 (0.13)	-0.60 (0.12)	
$\frac{1}{\sigma^2}$	0.72 (0.08)	0.61 (0.06)	0.71 (0.09)	0.61 (0.06)	0.72(0.09)	0.66 (0.06)	0.72 (0.09)	0.67 (0.06)	
$b_{(s)1}$			26.00 (11.98)	0.10 (0.16)			21.6 (9.93)	0.51 (0.15)	
$b_{(s)2}$			-0.09 (0.16)	-0.12 (0.15)			-0.08 (0.15)	-0.09 (0.14)	
$b_{(s)3}$			0.14 (0.15)	0.19 (0.14)			0.14 (0.15)	0.19 (0.13)	
$b_{(s)4}$			0.18 (0.15)	0.24 (0.14)			0.19 (0.15)	0.25 (0.13)	
Deviance	15110 (4.4)	20590 (17.9)	15100 (18.48)	20580 (33.75)	11580.00 (21.32)	15820 (45.05)	11580 (21.47)	15790 (22.23)	
No.Parameters	10	10	14	14	16	16	20	20	

Missing data in survival surveys

Table 2 Key

For the independent Bernoulli model, $f_Z(\cdot)$ is modeled as in equation 10-11. For the conditional wise specification, $f_Z(\cdot)$ is modeled as in equation 12-15. p_k for $k = 1, \dots, 4$ represent marginal probabilities for the four severe impairment variables in the order Lower Limb Function (ambulation); Upper Limb Function (manual); Intelligence Quotient; and Vision.

$f_{T|Z}(\cdot)$ is modeled as in equation 7, with α_0 is the intercept parameter; σ the scale parameter; and $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_4)$, where α_k for $k = 1, \dots, 4$ are the covariate effect parameters for the four severe impairment variables (order as above); For the survival model with additional variation due to region, $b_{(s)j}$ for $j = 1, \dots, 4$ represent fixed region effects (order: Scotland, the North of England, Northern Ireland, Oxford, and Mersey as the baseline).

Table 3. Conditional wise specification for $f_Z(\cdot)$: posterior estimates and posterior standard deviations

	No region effect		Region effect	
	CCA	MAR	CCA	MAR
β_{10}	-2.57 (0.08)	-2.33 (0.07)	-2.57 (0.08)	-2.33 (0.07)
β_{11}	4.00 (0.19)	3.93 (0.17)	3.99 (0.19)	3.92 (0.17)
β_{12}	1.45 (0.15)	1.41 (0.15)	1.45 (0.15)	1.41 (0.15)
β_{13}	1.26 (0.28)	1.22 (0.24)	1.27 (0.28)	1.22 (0.24)
β_{20}	-2.86 (0.09)	-2.78 (0.09)	-2.87 (0.09)	-2.79 (0.09)
β_{21}	2.98 (0.09)	3.03 (0.12)	2.99 (0.13)	3.04 (0.12)
β_{22}	1.80 (0.20)	1.68 (0.17)	1.81 (0.20)	1.67 (0.17)
β_{30}	-1.53 (0.05)	-1.43 (0.05)	-1.53 (0.05)	-1.43 (0.05)
β_{31}	3.83 (0.22)	3.72 (0.24)	3.83 (0.22)	3.73 (0.20)
β_{30}	-2.31 (0.06)	-2.03 (0.05)	-2.31 (0.06)	-2.03 (0.05)

The conditional wise specification for $f_Z(\cdot)$ is given in equation 14. Note: other parameter estimates for these four models are presented in Table 2 (columns 5 to 8). An interpretation of these parameter estimates is given in Table 4.

Table 4. Influence of conditional wise specification for $f_Z(z)$

		No region effect						Region Effect				
		z_1, \dots, z_{k-1}			CCA		MAR		CCA		MAR	
	z_k	ULF	IQ	Vision	$p(\cdot \cdot)$	$p(\cdot)$	$p(\cdot \cdot)$	$p(\cdot)$	$p(\cdot \cdot)$	$p(\cdot)$	$p(\cdot \cdot)$	$p(\cdot)$
LLF	1	1	1	1	0.98	0.26	0.99	0.28	0.98	0.28	0.99	0.31
	1	1	1	0	0.95		0.95		0.95		0.95	
	1	1	0	1	0.94		0.94		0.94		0.94	
	1	0	1	1	0.54		0.57		0.54		0.57	
	1	1	0	0	0.81		0.83		0.81		0.83	
	1	0	1	0	0.25		0.28		0.25		0.29	
	1	0	0	1	0.21		0.25		0.21		0.25	
	1	0	0	0	0.07		0.09		0.07		0.09	
ULF	1		1	1	0.87	0.20	0.87	0.31	0.87	0.20	0.87	0.23
	1		1	0	0.53		0.56		0.53		0.56	
	1		0	1	0.26		0.25		0.26		0.25	
	1		0	0	0.05		0.06		0.05		0.06	
IQ	1			1	0.91	0.24	0.91	0.23	0.91	0.24	0.91	0.28
	1			0	0.18		0.19		0.18		0.19	
Vision	1				0.09	0.09	0.12	0.12	0.09	0.09	0.12	0.12

Table 4 Key

LLF: Lower Limb Function (ambulation); ULF: Upper Limb Function (manual); IQ: Intelligence quotient

$p(\cdot|\cdot)$: conditional probability of $p(z_k|z_{k+1} \dots z_K)$

$p(\cdot)$: marginal probability of $p(z_k)$

The conditional wise specification for $f_Z(z|\beta)$ is given in equation 14.

Note: other parameter estimates for these four models are presented in Table 2 (columns 5 to 8) and Table 3.

Table 5. Influence of regional variations in proportions severely impaired (Model A) and deviations from expected birthweight (Model B): posterior estimates and posterior standard deviations

	Model A	Model B	
$f_Z(\cdot)$			
p_1	(0.37,0.29,0.28,0.33,0.27)	0.31	0.31
p_2	(0.26,0.26,0.21,0.21,0.22)	0.23	0.23
p_3	(0.15,0.24,0.29,0.26,0.31)	0.28	0.28
p_4	(0.18,0.06,0.10,0.12,0.13)	0.12	0.12
$f_{T Z}(\cdot)$			
α_0	9.80 (0.21)	9.80 (0.21)	9.78 (0.23)
α_1	-0.89 (0.18)	-0.88 (0.29)	-0.88 (0.19)
α_2	-0.83 (0.17)	-0.81 (0.17)	-0.81 (0.17)
α_3	-0.80 (0.15)	-0.85 (0.16)	-0.86 (0.16)
α_4	-0.58 (0.12)	-0.60 (0.12)	-0.60 (0.13)
$\frac{1}{\sigma^2}$	0.68 (0.07)		0.67 (0.07)
$b_{(s)1}$	0.34 (0.16)	0.52 (0.15)	0.52 (0.15)
$b_{(s)2}$	-0.11 (0.14)	-0.09 (0.14)	-0.09 (0.14)
$b_{(s)3}$	0.18 (0.13)	0.21 (0.13)	0.20 (0.14)
$b_{(s)4}$	0.24 (0.13)	0.25 (0.13)	0.26 (0.14)
z-score		-0.08 (0.04)	-0.08 (0.04)
z-score ²			0.02 (0.02)
Deviance	15670 (51.42)	15790 (52.28)	15790 (53.40)
No.Parameters	24	21	22

Key

Using the conditional wise specification, $f_Z(\cdot)$ is modeled as in equation 9 and where p_k for $k = 1, \dots, 4$ represent marginal probabilities (by row) for the four severe impairment variables in the order Lower Limb Function (ambulation); Upper Limb Function (manual); Intelligence Quotient; and Vision. In each row, individual region estimates are presented for Model A (order: SC,NE,NI,OX,ME). $b_{(p)j}$ for $j = 1, \dots, 4$ represent fixed regional effects on proportions severely impaired (order: Scotland, the North of England, Northern Ireland, Oxford, and Mersey as the baseline.)

$f_{T|Z}(\cdot)$ is modeled as in equation 4, with α_0 is the intercept parameter; σ the scale parameter; and $\alpha = (\alpha_1, \dots, \alpha_4)$, where α_k for $k = 1, \dots, 4$ are the covariate effect parameters for the four severe impairment variables (order as above); For the survival model with additional variation due to region, $b_{(s)j}$ for $j = 1, \dots, 4$ represent fixed region effects on survival (order: Scotland, the North of England, Northern Ireland, Oxford, and Mersey as the baseline).

Table 6. Sensitivity analysis: comparing different NMAR models (main parameters) : posterior estimates and standard deviations

$f_Z(\cdot)$	No region effect			Region effect		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
p_1	0.31	0.32	0.32	0.31	0.32	0.32
p_2	0.23	0.25	0.24	0.23	0.25	0.24
p_3	0.28	0.33	0.33	0.28	0.33	0.33
p_4	0.12	0.15	0.16	0.12	0.15	0.16
$f_{T Z}(\cdot)$						
α_0	9.95 (0.21)	10.06 (0.22)	10.07 (0.21)	9.82 (0.22)	9.83 (0.22)	9.86 (0.23)
α_1	-0.84 (0.19)	-0.84 (0.17)	-0.85 (0.17)	-0.88 (0.19)	-0.84 (0.18)	-0.86 (0.18)
α_2	-0.85 (0.16)	-0.94 (0.16)	-0.96 (0.15)	-0.81 (0.17)	-0.86 (0.15)	-0.87 (0.16)
α_3	-0.84 (0.16)	-0.69 (0.15)	-0.65 (0.15)	-0.84 (0.16)	-0.75 (0.15)	-0.73 (0.15)
α_4	-0.54 (0.12)	-0.62 (0.11)	-0.62 (0.11)	-0.60 (0.13)	-0.69 (0.11)	-0.70 (0.11)
$\frac{1}{\sigma^2}$	0.66 (0.06)	0.65 (0.06)	0.65 (0.06)	0.67 (0.06)	0.68 (0.06)	0.68 (0.07)
$b_{(s)1}$			–	0.51 (0.15)	0.71 (0.15)	0.73 (0.15)
$b_{(s)2}$			–	-0.10 (0.14)	-0.03 (0.14)	-0.03 (0.14)
$b_{(s)3}$			–	0.18 (0.13)	0.22 (0.13)	0.22 (0.13)
$b_{(s)4}$			–	0.25 (0.14)	0.26 (0.13)	0.26 (0.14)
Deviance	24580 (51)	23470 (77)	23240 (105)	24560 (53)	23420 (78)	23160 (113)
No.Parameters	20	24	32	24	28	36

Key

$f_Z(\cdot)$ is modeled as in equation 12-15 using the conditional wise specification. p_k for $k = 1, \dots, 4$ represent marginal probabilities for the four severe impairment variables in the order Lower Limb Function (ambulation); Upper Limb Function (manual); Intelligence Quotient; and Vision.

$f_{T|Z}(\cdot)$ is modeled as in equation 7, with α_0 is the intercept parameter; σ the scale parameter; and $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_4)$, where α_k for $k = 1, \dots, 4$ are the covariate effect parameters for the four severe impairment variables (order as above); For the survival model with additional variation due to region, $b_{(s)j}$ for $j = 1, \dots, 4$ represent fixed region effects (order: Scotland, the North of England, Northern Ireland, Oxford, and Mersey as the baseline).

$f_R(\mathbf{r}; \boldsymbol{\psi})$ is modelled by equations 17 and 18

Notes:

Model 1: Intercept only (MAR); Model 2: Intercept, covariate; Model 3: Intercept, covariate, time, censoring status.

Estimates for NMAR parameters are given in Table 7.

Table 7. Sensitivity analysis: comparing different NMAR models (NMAR parameters): posterior estimates and standard deviations

$f_R(\cdot)$	No region effect			Region effect		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
γ_{10}	7.77 (1.32)	5.21 (1.40)	3.51 (0.10)	7.69 (1.36)	5.25 (1.39)	
γ_{13}	-5.43 (1.33)	-4.87 (1.38)	-	-5.35 (1.37)	-4.91 (1.38)	
γ_{11}		0.15 (0.02)	-		0.15 (0.03)	
γ_{12}		0.60 (0.28)	-		0.60 (0.28)	
γ_{20}	5.16 (0.36)	3.03 (0.44)	3.33 (0.09)	-3.19 (0.40)	3.01 (0.44)	
γ_{23}	-3.17 (0.39)	-2.92 (0.38)	-	-2.51 (0.13)	-2.96 (0.38)	
γ_{21}		0.14 (0.02)	-		0.14 (0.02)	
γ_{22}		0.73 (0.26)	-		0.74 (0.26)	
γ_{30}	2.48 (0.13)	2.88 (0.27)	1.62 (0.04)	2.51 (0.13)	2.96 (0.31)	
γ_{33}	-1.84 (0.18)	-2.40 (0.28)	-	-1.89 (0.19)	-2.51 (0.33)	
γ_{31}		-0.01 (0.01)	-		-0.01 (0.01)	
γ_{32}		0.74 (0.18)	-		0.76 (0.18)	
γ_{40}	2.47 (0.08)	3.26 (0.22)	1.93 (0.08)	2.45 (0.09)	3.30 (0.22)	
γ_{43}	-2.03 (0.16)	-2.22 (0.21)	-	-2.07 (0.16)	-2.30 (0.20)	
γ_{41}		-0.04 (0.01)	-		-0.05 (0.01)	
γ_{42}		-0.12 (0.21)	-		-0.09 (0.21)	

Key

$f_R(\cdot)$ is modeled as in equation 17.

γ_1 refers to the distribution of the missing data mechanism for the covariate lower limb function;

γ_2 refers to the distribution of the missing data mechanism for the covariate upper limb function;

γ_3 refers to the distribution of the missing data mechanism for the covariate cognitive function;

γ_4 refers to the distribution of the missing data mechanism for the covariate visual function;

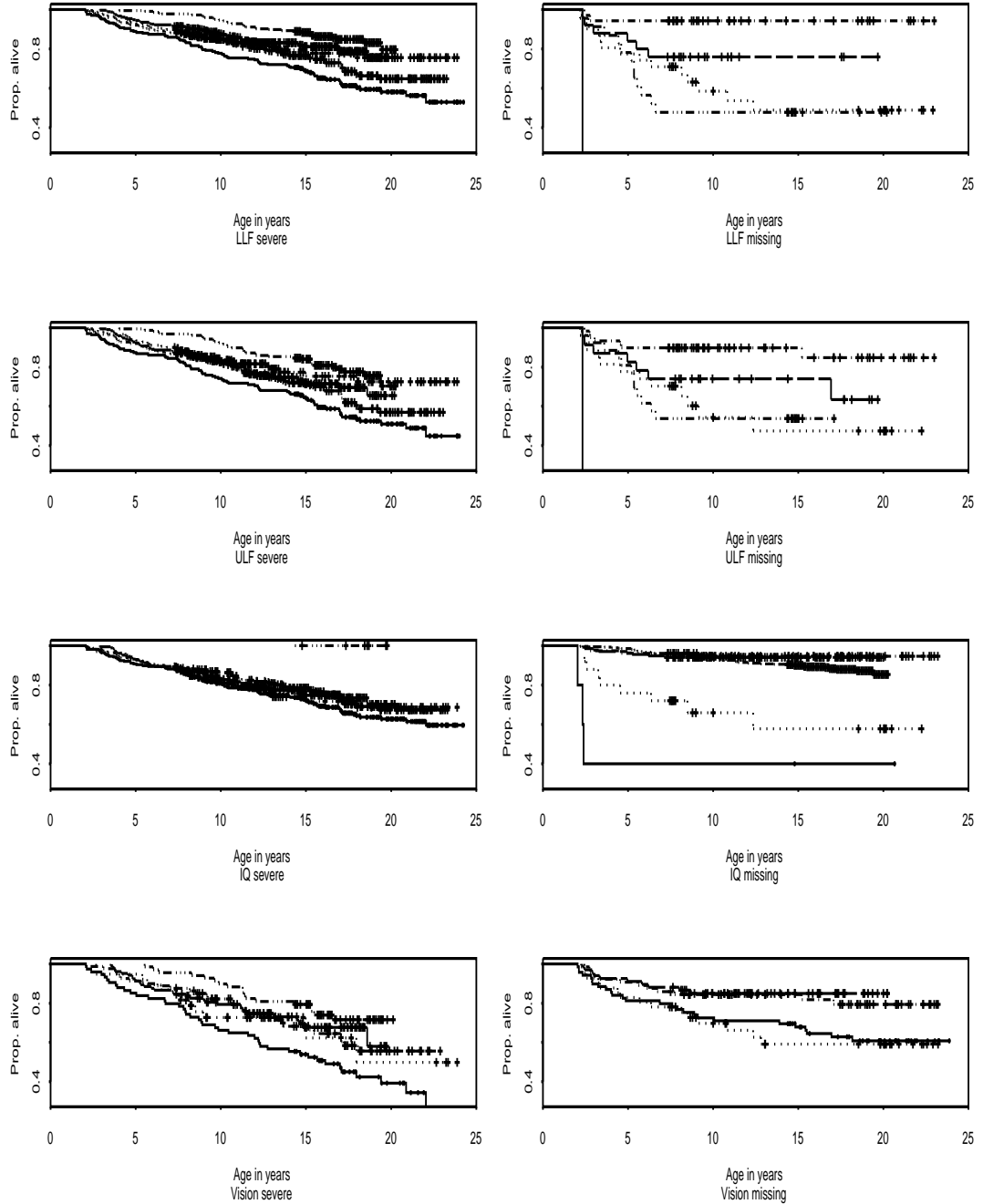
The parameters $\gamma_{k0}, \gamma_{k1}, \gamma_{k2}, \gamma_{k3}$ for variable k (1...4) (above) represents an intercept value, an effect due to the covariate, and effect due to the event time, and an effect due to the censoring status.

Notes:

Model 1: Intercept only (MAR); Model 2: Intercept, covariate; Model 3: Intercept, covariate, time, censoring status.

Estimates for main parameters are given in Table 6.

Figure 1. Kaplan-Meier estimates of survival for the severe and unknown impairments



Key

LLF: Lower Limb Function (ambulation); ULF: Upper Limb Function (manual); IQ: Intelligence quotient

Mersey: - ; North of England: ··· ; Northern Ireland: - · - ; Oxford: - - - ; Scotland: - - - -