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INVERSION ASSUMING WEAK SCATTERING

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Abstract: *The study of weak scattering from inhomogeneous media or interface roughness has long been of interest in sonar applications. In an acoustic backscattering model of a stationary field of volume inhomogeneities, a stochastic description of the field is more useful than a deterministic description due to the complex nature of the field. A method based on linear inversion is employed to infer information about the statistical properties of the scattering field from the obtained cross-spectral matrix. A synthetic example based on an active high-frequency sonar demonstrates that the proposed method provides a quantitative description of a weak scattering field in terms of its second-order statistics.*

Keywords: *Weak scattering, least-squares inversion, high-frequency active sonar, covariance matrix*

1. INTRODUCTION

The challenge of a deep-water oil leak is that a significant quantity of oil remains in the water column and possibly changes properties. It is of interest to determine the physical properties of the new forms of oil in order to monitor the degradation process. The weak scattering approach is applied to model monostatic backscattering from submerged oil, which is modelled as a fluid medium with spatial heterogeneity. A high-frequency active sonar is selected to collect the backscattered returns, which can both overcome the optical opacity of the water and resolve small scale structure of the new forms of oil. The parameters of the spatial covariance of the contaminated region can be inferred by relating the statistical properties of the scattered field to the statistical properties of the scattering medium.

2. FORWARD PROBLEM

The scattered sound pressure, p_s, observed at a remote position $\mathbf{r}_0$ due to scattering from spatial fluctuations of the compressibility, ε_κ, and density, ε_ρ, of the medium within a scattering region R is given by the integral equation [1],

$$p_s(\mathbf{r}_0) = \int_R (k^2 \varepsilon_\kappa(\mathbf{r}) p(\mathbf{r}) - div(\varepsilon_\rho(\mathbf{r}) \nabla p(\mathbf{r}))) g(\mathbf{r}_0 \mid \mathbf{r}) d\mathbf{r}\,, \tag{1}$$

where k is the wavenumber, p is the wave insonifying the scatterer located at $\mathbf{r}$ and $g(\mathbf{r}_0 \mid \mathbf{r}) = \dfrac{1}{4\pi |\mathbf{r}_0 - \mathbf{r}|} e^{-ik|\mathbf{r}_0 - \mathbf{r}|}$ is the free-space Green's function relating the field observed at $\mathbf{r}_0$ due to a point source at $\mathbf{r}$. The harmonic dependence $e^{i\omega t}$ is implied and neglected for simplicity and the mean sound speed is assumed constant throughout the medium. The compressibility and density fluctuations are normalized to their mean values, $\varepsilon_\kappa(\mathbf{r}) = \delta\kappa(\mathbf{r}) / \langle \kappa \rangle$, $\varepsilon_\rho(\mathbf{r}) = \delta\rho(\mathbf{r}) / \langle \rho \rangle$, thus are dimensionless quantities.

For far field radiation, the Fraunhofer approximation for the range term is valid, $|\mathbf{r}_0 - \mathbf{r}| \approx r - \hat{\mathbf{r}} \cdot \mathbf{r}_0$, where $r = |\mathbf{r}|$ and $\hat{\mathbf{r}} = \mathbf{r}/r$, and the Green's function takes the simpler form [2],

$$g(\mathbf{r}_0 \mid \mathbf{r}) = \frac{1}{4\pi r} e^{-ik(r - \hat{\mathbf{r}} \cdot \mathbf{r}_0)}. \tag{2}$$

The incident wave which insonifies the region R emanates from a monopole located at the origin of the coordinate system out of the scattering region R,

$$p_i(\mathbf{r}) = A \frac{e^{-ikr}}{r}\,, \tag{3}$$

where A is the pressure amplitude at a distance 1 m from the source and r denotes the range of the insonified point.

Assuming weak scattering, the Born approximation applies, $p \approx p_i$. Thus, inserting Eqs. (2) and (3) in Eq. (1) the pressure scattered from inhomogeneities in the acoustic parameters of the medium is,

$$p_s(\mathbf{r}_0) \approx \frac{k^2 A}{4\pi} \int_R (\varepsilon_\kappa(\mathbf{r}) - \varepsilon_\rho(\mathbf{r})) \frac{e^{-ik(2r - \hat{\mathbf{r}}\cdot\mathbf{r}_0)}}{r^2} d\mathbf{r}. \tag{4}$$

Owing to the Born approximation, Eq. (4) relates linearly the backscattered pressure and the fluctuations in the acoustic parameters thus can be discretized and rearranged in a matrix-vector formulation,

$$\mathbf{d} = \mathbf{G}\mathbf{m} + \mathbf{n} \tag{5}$$

where $\mathbf{d}$ is the vector comprising the acquired data (the scattered returns possibly contaminated with additive noise $\mathbf{n}$), $\mathbf{G}$ is the linear forward operator and $\mathbf{m}$ is the vector of model parameters, namely the compressibility fluctuations. The density fluctuations are neglected henceforth since they are proportional to the compressibility fluctuations and are less significant in fluid media [3].

3. INVERSE PROBLEM

Assuming that the random field of model parameters is stationary, the model covariance matrix, $\mathbf{C}_m$, has a Toeplitz structure determined by the covariance function. Due to the fact that model parameters, which are more than a correlation length apart, are uncorrelated the dimensions of the problem can be significantly reduced [4] when the interest is in inferring the model covariance function and not the model parameters *per se*.

The forward linear problem yields,

$$\mathbf{C}_d = \mathbf{G}\mathbf{C}_m\mathbf{G}^H + \mathbf{C}_n, \tag{6}$$

where $\mathbf{C}_d = \left\langle \mathbf{d}\mathbf{d}^H \right\rangle$ is the data covariance matrix, $\mathbf{C}_m = \left\langle \mathbf{m}\mathbf{m}^T \right\rangle$ is the model covariance matrix and $\mathbf{C}_n = \left\langle \mathbf{n}\mathbf{n}^H \right\rangle$ is the noise covariance matrix, ^{T} denotes transpose, ^{H} denotes conjugate transpose of a vector or matrix and $\langle\ \rangle$ is the ensemble average. The noise is assumed uncorrelated with the model parameters.

Inversion of Eq. (6) with the least-squares approach yields,

$$\hat{\mathbf{C}}_m = \mathbf{G}^+\mathbf{C}_d\left(\mathbf{G}^+\right)^H \tag{7}$$

where $^+$ denotes generalized inverse.

4. SIMULATION RESULTS

A monostatic configuration is assumed. The receiver is a uniform linear array (ULA) centered at the origin of the coordinate system such that the sensors locations are $\mathbf{x} = \left[\mathbf{q} - (n_m - 1)/2\right]$, $\mathbf{q} = \left[1, 2, \ldots, n_m\right]$, n_m is the number of sensors with interelement spacing d_m. Time varying gain is applied to compensate for spreading loss and absorption due to propagation in the medium. Autofocusing is used to relate the focusing distance to the arrival time. Thus, the forward matrix has a depth dependent structure,

$$\mathbf{G}(r_i)_{n_m \times n_\theta} \propto e^{-ik2r_i} \begin{bmatrix} e^{ikx_1 \sin(\theta_1)} & e^{ikx_1 \sin(\theta_2)} & \cdots & e^{ikx_1 \sin(\theta_{n\theta})} \\ e^{ikx_2 \sin(\theta_1)} & e^{ikx_2 \sin(\theta_2)} & \cdots & e^{ikx_2 \sin(\theta_{n\theta})} \\ \vdots & \vdots & \ddots & \vdots \\ e^{ikx_{n_m} \sin(\theta_1)} & e^{ikx_{n_m} \sin(\theta_2)} & \cdots & e^{ikx_{n_m} \sin(\theta_{n\theta})} \end{bmatrix} \tag{8}$$

The total $\mathbf{G}_{N \times M}$ matrix, where $N = n_m n_r$ and $M = n_\theta n_r$ (n_r is the number of focusing ranges and n_θ is the number of broadside angles to the scatterers positions), is a block matrix which is constructed by the direct sum of $\mathbf{G}(r_i)$ for $i = 1, 2, \ldots, n_r$. Its eigenvalues are the combined eigenvalues of the $\mathbf{G}(r_i)$ matrices.

$$\mathbf{G}_{N \times M} = \bigoplus_{i=1}^{n_r} \mathbf{G}(r_i)_{n_m \times n_\theta} \tag{9}$$

In the overdetermined case ($n_m > n_\theta$) and assuming equidistant spacing in $\sin(\theta)$, such that $\sin(\theta)_i - \sin(\theta)_j = d_{\sin}(i - j)$, it is easily deduced that asymptotically the matrix $\left[\mathbf{G}^H \mathbf{G}\right]_{ij} \to \sin c\left(\frac{n_m d_m}{\lambda} d_{\sin}(i - j)\right)$ is a Toeplitz sinc matrix. The eigenvalues of a Toeplitz matrix are connected to the Fourier transform of the series [5]. And by choosing $d_{\sin} \geq \lambda / (n_m d_m)$ the matrix $\mathbf{G}^H \mathbf{G}$ is full rank, thus invertible [6]. The higher the frequency and/or the longer the receiving array, the finer the resolution that can be achieved.

Naturally, the field of model parameters exhibits stationarity in the Cartesian coordinate system. However confining the insonified area within an opening angle [-15°, 15°] the curvature is negligible and $d_{\sin} \approx d_x / r$.

A synthetic example is implemented to demonstrate the method. A high-frequency active sonar is considered [7]. The receiver is a ULA with $n_m = 256$, $d_m = 1.6$ mm. The field is insonified by a narrowband 200 kHz source. The duration of the pulse is 120 μs corresponding to a range resolution of 0.1 m (c=1500 m/s). The transmitter is assumed to have a narrow directivity pattern in the along-track plane, thus only the 2D across-track

plane is modelled. The data covariance matrix is calculated by an ensemble average from 500 pings and additive Gaussian noise is assumed $n \sim CN(0,0.01)$.

Figure 1 shows the measurement setup and a realization of the 2D field of compressibility fluctuations confined to the area considered for the inference. The field is described by an anisotropic spherical covariance function with characteristic length (the lag where the covariance function has decayed by at least 95%) 2 m in the x-direction and 0.5 m in the z-direction [8]. Figure 2 shows the actual and reconstructed covariance function with respect to the lag distance in x-direction and z-direction respectively. The characteristic lengths are denoted by vertical dotted lines and the variance corresponds to the values at zero lag distance.

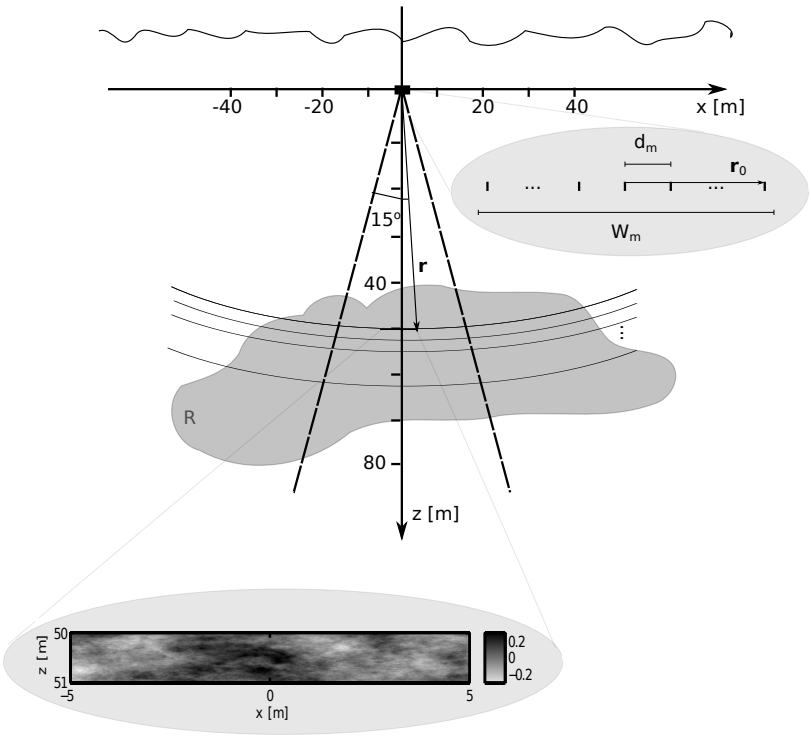

Fig.1: Measurement setup.

5. CONCLUSION

For stationary scattering fields the method of covariance inference basically allows significant reduction of the dimensions of the problem. Generally, in a medium where there is flow as in the water column, the scattering field will not be static so a deterministic description has less to offer. Localization of the contaminated region can be provided by beamforming and identification by inference of the covariance characteristics of the model covariance.

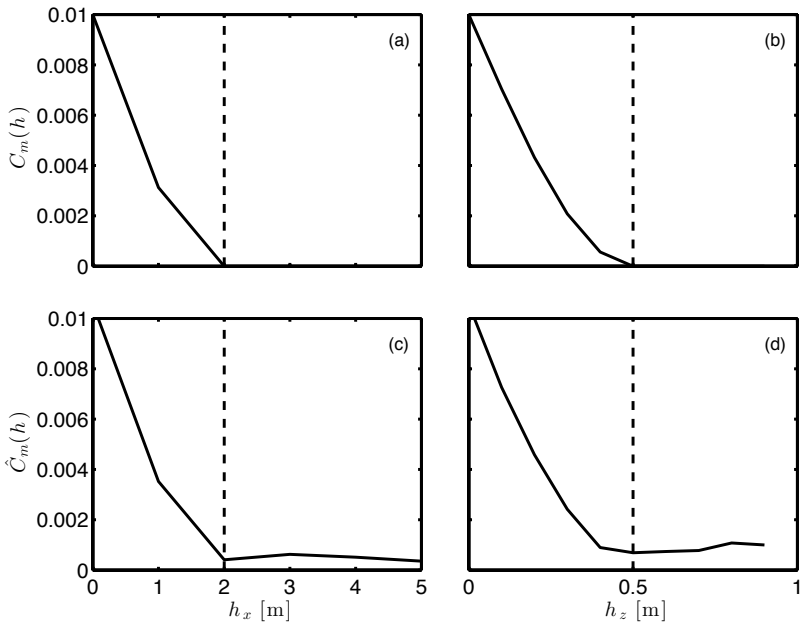

Fig.2: True, C_m, and reconstructed, $\hat{C}_m$, covariance function as a function of lag-distance in x and z-direction respectively. The characteristic lengths are denoted by dashed lines in each case.

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