

A hybrid model for optimal decisions within personal finance and retirement

Bell, Agnieszka Karolina Konicz

Publication date:
2013

Document Version
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

Citation (APA):
Konicz, A. K. (2013). A hybrid model for optimal decisions within personal finance and retirement [Sound/Visual production (digital)]. Recent Research Results in Operations Research, Lyngby, Denmark, 06/09/2012

DTU Library

Technical Information Center of Denmark

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

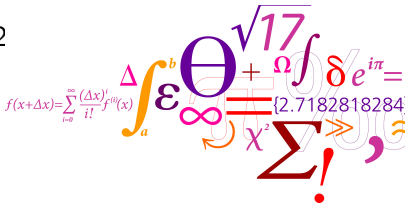
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

A hybrid model for optimal decisions within personal finance and retirement

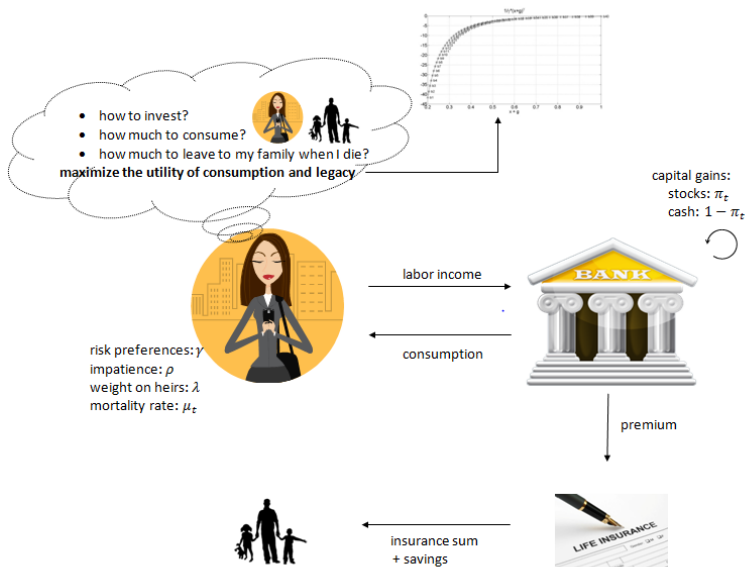
Agnieszka Karolina Konicz

Technical University of Denmark

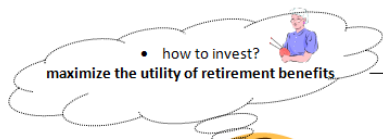
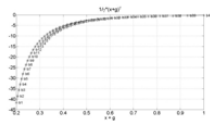
OR Seminar
Dec. 20, 2012



case (A) - optimal investment, consumption and life insurance, [Richard, 1975]



case (B) - optimal investment with optimal annuities



risk preferences: γ
impatience: ρ
mortality rate: μ_t

before retirement:
premiums

after retirement:
benefits



capital gains:
stocks: π_t
cash: $1 - \pi_t$

• how to define the benefits
given the pension saver's preferences?

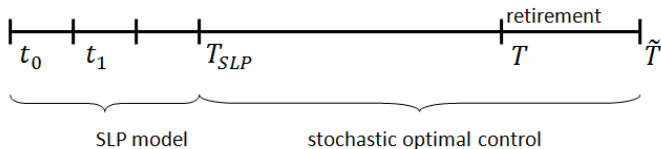
stochastic optimal control - explicit solutions

- ✓ ideal framework - produce an optimal policy that is easy to understand and implement
- ✗ explicit solution may not exist
- ✗ can't deal with details

stochastic (linear) programming (SLP)

- ✓ general purpose decision model with an objective function that can take a wide variety of forms
- ✓ can address realistic considerations, such as transaction costs
- ✓ can deal with details
- ✗ problem size grows quickly as a function of number of periods and scenarios
- ✗ challenge to select a representative set of scenarios for the model

- first years decisions - multi-stage stochastic linear programming (**SLP**)
- decisions for the long steady period - **stochastic optimal control** (dynamic programming)



Wealth dynamics

$$dX_t = (r + \pi_t(\alpha - r))X_t dt + \pi_t \sigma X_t dW_t + I_t dt - c_t dt - \mu_t^* I_t dt, \\ X_0 = x_0.$$

Maximize expected utility of consumption and bequest

$$V(t, x) = \sup_{\pi, c, I \in \mathcal{Q}[t, \tilde{T}]} E_{t,x} \left[\int_t^{\tilde{T}} e^{-\int_t^s \mu_\tau d\tau} \left(u(s, c) + \mu_s U(s, X_s + I_s) \right) ds \right],$$

with the utility functions:

$$u(c, t) = \frac{1}{\gamma} w^{1-\gamma}(t) c^\gamma = \frac{1}{\gamma} e^{-\rho t} c^\gamma, \quad U(x) = \frac{1}{\gamma} v^{1-\gamma}(t) x^\gamma = \frac{1}{\gamma} \lambda^{-\gamma} e^{-\rho t} x^\gamma,$$

$1 - \gamma$ - risk aversion, ρ - impatience factor, λ - weight on bequest, μ_t - mortality rate, μ_t^* - pricing mortality rate.

Solution (optimal value function)

$$V(t, x) = \frac{1}{\gamma} f^{1-\gamma}(t) (x + g(t))^\gamma,$$

$$f(t) = \int_t^{\tilde{T}} e^{-\frac{1}{1-\gamma} \int_t^s (\mu_\tau - \gamma(\mu_\tau^* + \varphi)) d\tau} \left[w(s) + \left(\frac{\mu_s}{(\mu_s^*)^\gamma} \right)^{1/(1-\gamma)} v(s) \right] ds,$$

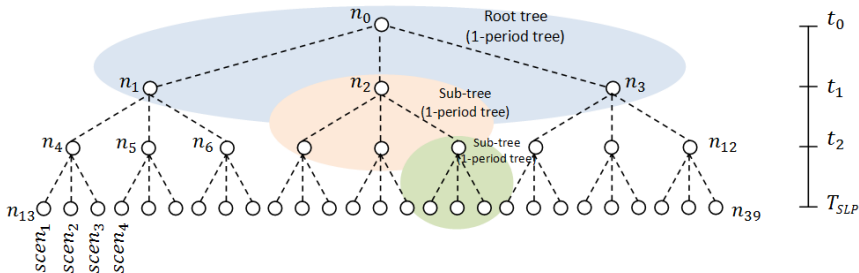
$$g(t) = \int_t^{\tilde{T}} e^{-\int_t^s (r + \mu_\tau^*) d\tau} l(s) ds.$$

The optimal controls

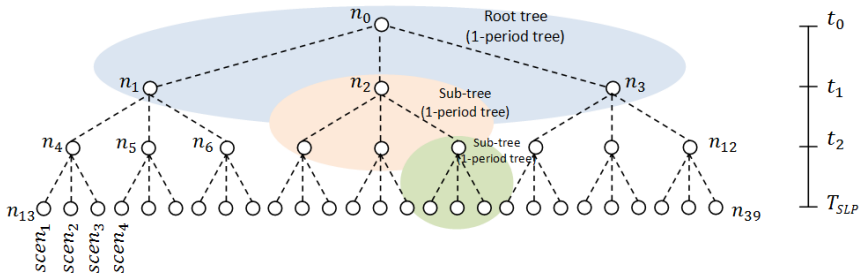
$$\pi_t^* = \frac{\alpha - r}{\sigma^2(1-\gamma)} \frac{X_t + g(t)}{X_t}, \quad c_t^* = \frac{w(t)}{f(t)} (X_t + g(t)),$$

$$l_t^* = \left(\frac{\mu_t}{\mu_t^*} \right)^{1/(1-\gamma)} \frac{v(t)}{f(t)} (X_t + g(t)) - X_t.$$

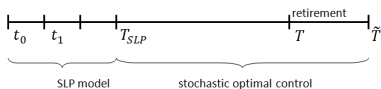
SLP model - objective



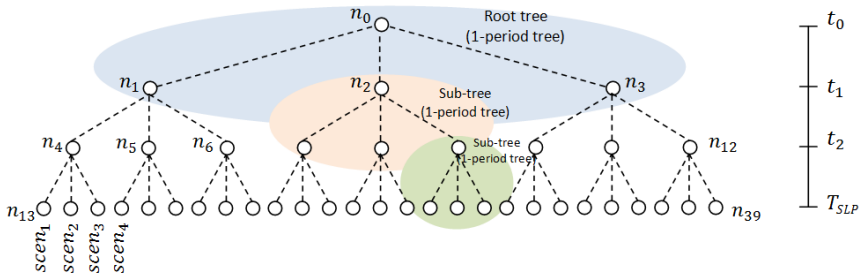
SLP model - objective



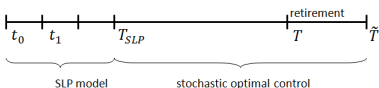
$$\sum_{s=t_0}^{T_{SLP}-1} \sum_{n_{\nu(s)}} Pr_{n_{\nu(s)}} \cdot e^{-\int_{t_0}^s \mu_{\tau} d\tau} \left[u(s, \tilde{C}_{n_{\nu(s)}}) + \mu_s U(s, \sum_{i=1}^N \tilde{X}_{n_{\nu(s)}}^i + \tilde{I}_{n_{\nu(s)}}) \right] + \sum_{n_{\nu(T_{SLP})}} Pr_{n_{\nu(T_{SLP})}} \cdot e^{-\int_{t_0}^{T_{SLP}} \mu_{\tau} d\tau} \cdot V \left(T_{SLP}, \sum_{i=1}^N \tilde{X}_{n_{\nu(T_{SLP})}}^i \right) \rightarrow \max$$



SLP model - objective



$$\sum_{s=t_0}^{T_{SLP}-1} \sum_{n_{\nu(s)}} Pr_{n_{\nu(s)}} \cdot e^{-\int_{t_0}^s \mu_{\tau} d\tau} \left[u(s, \tilde{C}_{n_{\nu(s)}}) + \mu_s U(s, \sum_{i=1}^N \tilde{X}_{n_{\nu(s)}}^i + \tilde{I}_{n_{\nu(s)}}) \right] + \sum_{n_{\nu(T_{SLP})}} Pr_{n_{\nu(T_{SLP})}} \cdot e^{-\int_{t_0}^{T_{SLP}} \mu_{\tau} d\tau} \cdot V \left(T_{SLP}, \sum_{i=1}^N \tilde{X}_{n_{\nu(T_{SLP})}}^i \right) \rightarrow \max$$



- **Obs!** linearize the objective

budget equation, $t = t_0, \dots, T_{SLP} - 1$ and $\nu(t) = 1, \dots, K_t$,

$$\sum_{i=1}^N \tilde{P}_{n\nu(t)}^i + \tilde{C}_{n\nu(t)} + \mu_t^* \tilde{I}_{n\nu(t)} = x_0 \mathbb{1}_{\{t=t_0\}} + \sum_{i=1}^N \tilde{S}_{n\nu(t)}^i + I_t, \quad (1)$$

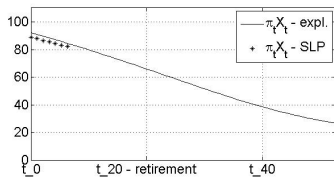
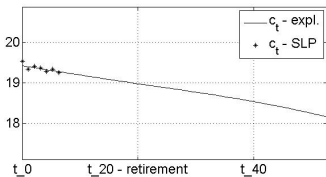
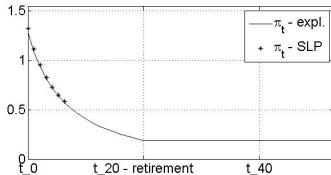
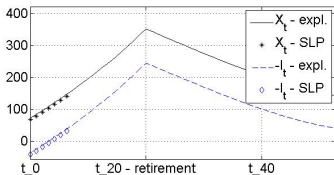
asset inventory balance, $t = t_0, \dots, T_{SLP}$, $\nu(t) = 1, \dots, K_t$, $i = 1, \dots, N$:

$$\tilde{X}_{n\nu(t)}^i = (1 + R_{n\nu(t)}^i) \tilde{X}_{n\nu(t-1)}^i \mathbb{1}_{\{t > t_0\}} + \tilde{P}_{n\nu(t)}^i \mathbb{1}_{\{t < T_{SLP}\}} - \tilde{S}_{n\nu(t)}^i \mathbb{1}_{\{t < T_{SLP}\}}, \quad (2)$$

non-negativity, $t = t_0, \dots, T_{SLP} - 1$, $\nu(t) = 1, \dots, K_t$:

$$\tilde{C}_{n\nu(t)} > 0, \quad \sum_{i=1}^N \tilde{X}_{n\nu(t)}^i + \tilde{I}_{n\nu(t)} > 0, \quad \tilde{P}_{n\nu(t)}^i \geq 0, \quad \tilde{S}_{n\nu(t)}^i \geq 0, \quad (3)$$

Results I - case (A)



- payments: $l_t = 27.000$ EUR, $x_0 = 60.000$ EUR,
- market: $N = 2$, $r = 0.02$, $\alpha = 0.04$, $\sigma = 0.2$,
- utility function: $\gamma = -3$, $\rho = 0.04$, $\lambda = 10$,
- $age_0 = 45$, $age_T = 65$,
- life uncertainty: $\theta = 0.0$, $\beta = 4.59364$,
 $\delta = 0.05032$,

- scenario tree: $T_{SLP} = 8$, $bf = 3$, number of trees = 10,
- linearization: $m = 40$,
- $bp_1^c = 0.7E[c_{T_{SLP}}^*]$, $bp_m^c = 2E[c_{T_{SLP}}^*]$,
- $bp_1^{x+g} = 0.5E[X_{T_{SLP}}^* + g_{T_{SLP}}]$, $bp_m^{x+g} = 2E[X_{T_{SLP}}^* + g_{T_{SLP}}]$,
- $bp_1^{x+ins} = 0.5E[X_{T_{SLP}}^* + I_{T_{SLP}}^*]$, $bp_m^{x+ins} = 1.5E[X_{T_{SLP}}^* + I_{T_{SLP}}^*]$.

budget equation with transaction costs, $t = t_0, \dots, T_{SLP} - 1$, $\nu(t) = 1, \dots, K_t$

$$\sum_{i=1}^N \tilde{P}_{n_{\nu(t)}}^i (1 + q^i) + \tilde{C}_{n_{\nu(t)}} + \mu_t^* \tilde{I}_{n_{\nu(t)}} = x_0 \mathbb{1}_{\{t=t_0\}} + \sum_{i=1}^N \tilde{S}_{n_{\nu(t)}}^i (1 - q^i) + I_t, \quad (1')$$

asset inventory balance with taxes on capital gains, $t = t_0, \dots, T_{SLP}$, $\nu(t) = 1, \dots, K_t$,

$i = 1, \dots, N$:

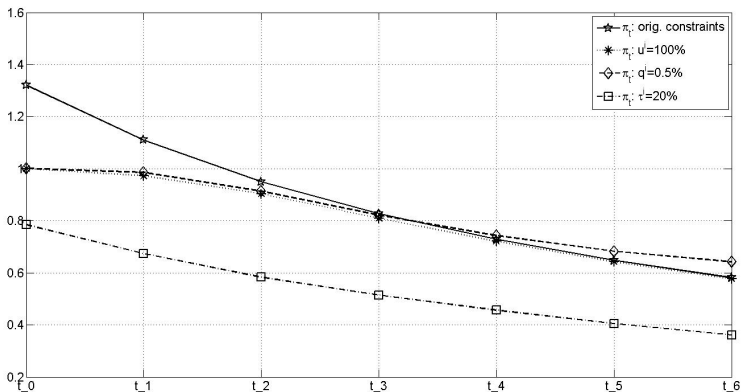
$$\tilde{X}_{n_{\nu(t)}}^i = (1 + \text{net} R_{n_{\nu(t)}}^i) \tilde{X}_{n_{\nu(t-1)}}^i \mathbb{1}_{\{t > t_0\}} + \tilde{P}_{n_{\nu(t)}}^i \mathbb{1}_{\{t < T_{SLP}\}} - \tilde{S}_{n_{\nu(t)}}^i \mathbb{1}_{\{t < T_{SLP}\}}, \quad (2')$$

limits on portfolio composition, $t = t_0, \dots, T_{SLP}$, $\nu(t) = 1, \dots, K_t$:

$$\tilde{X}_{n_{\nu(t)}}^i \geq d_i \sum_{i=1}^N \tilde{X}_{n_{\nu(t)}}^i, \quad \tilde{X}_{n_{\nu(t)}}^i \leq u_i \sum_{i=1}^N \tilde{X}_{n_{\nu(t)}}^i. \quad (4)$$

Results II - case (A) with modifications

Optimal investment: a) original constraints, b) limit on portfolio composition, $u = 100\%$, c) transaction costs, $q = 0.5\%$, d) taxes on capital gains, $\tau = 20\%$.





Bruhn, K. and Steffensen, M. (2011).

Household Consumption, Investment and Life Insurance.

Insurance: Mathematics and Economics, 48(3):315–325.



Geyer, A., Hanke, M., and Weissensteiner, A. (2009).

Life-cycle Asset Allocation and Consumption Using Stochastic Linear Programming.

The Journal of Computational Finance, 12(4):29–50.



Høyland, K. and Wallace, S. W. (2001).

Generating Scenario Trees for Multistage Decision Problems.

Management Science, 48(11):1512–1516.



Klaasen, P. (2002).

Comment on: "Generating Scenario Trees for Multistage Decision Problems".

Management Science, 47(2):295–307.



Richard, S. F. (1975).

Optimal Consumption, Portfolio and Life Insurance Rules for an Uncertain Lived Individual in a Continuous Time Model.

Journal of Financial Economics, 2(2):187–203.