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Published in:
Scripta Materialia

Link to article, DOI:
10.1016/j.scriptamat.2012.11.032

Publication date:
2013

Citation (APA):
Hansen, J. Z., & Brøndsted, P. (2013). Determination of the minimum size of a statistical representative volume element from a fibre-reinforced composite based on point pattern statistics. Scripta Materialia, 68, 503-505. DOI: 10.1016/j.scriptamat.2012.11.032

Determination of the minimum size of a statistical representative volume element from a fibre reinforced composite based on point pattern statistics

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Abstract

In a previous study, Trias et al. [1] determined the minimum size of a statistical representative volume element (SRVE) of a unidirectional fibre reinforced composite primarily based on numerical analyses of the stress/strain field. In continuation to this, the present study determines the minimum size of a SRVE based on a statistical analysis on the spatial statistics of the fibre packing patterns found in genuine laminates, and those generated numerically using a microstructure generator.

Keywords: 2. 2D quantitative analysis, 2. Scanning electron microscopy (SEM), 3. Fibre reinforced composites, 3. Glass fibre, 5. Monte Carlo simulation

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In order to numerically investigate mechanical properties and damage evolution of fibre reinforced composites, statistical representative volume elements (SRVE's) are widely used, see e.g. [1, 2, 3] for a general presentation. The size of the SRVE is fundamental since it must be large enough to catch the correct stress/strain fields and spatial statistics of the fibre packing pattern. At the same time, the SRVE must be as small as possible to minimize the computational effort in a finite element context. For transverse cross-sections of unidirectional fibre reinforced composites, limited information is available regarding the minimum size of the SRVE; however, Trias et al. [1] have determined the minimum SRVE size based on different mechanical analyses (e.g. stress/strain field, effective properties, and energy) of generated microstructures made of a carbon/epoxy system. In order to replicate the evaluation parameters considered with reasonable reliability, the minimum side length obtained is $L = 50r$ for a square shaped SRVE with uniform fibre radius r. They did not consider any comparisons to actual microstructures found in genuine composite laminates, and as continuation of the work of Trias et al., the present study determines the minimum size of a SRVE with non-uniform fibre radii distribution considering only the spatial point pattern statistics. For varying size of the square shaped SRVE, a comparison is made between the fibre packing patterns found in genuine composite laminates, and that produced numerically using a microstructure generator. The fibre packing pattern of 3 different unidirectional composite laminates with varying fibre volume fraction (FVF) is analysed using digital image analysis. The laminates (glass fibres/polyester matrix) are manufactured using the VARTM process, and micrographs of the microstructure are captured from transverse cross-sections using a SEM. The fibre architecture is mapped by the circular Hough transformation, which yields the fibre location and the individual fibre radii [4]. The obtained fibre radii statistics (mean, standard deviation, and skewness) from the analysed laminates are: $r_{mean} = 8.59\mu$m, $r_{std} = 0.74\mu$m, and $r_{skew} = 0.35\mu$m. These fibre radii statistics are used as input to a numerical microstructure generator to replicate the fibre architecture. The microstructure generator is inspired by the work of Melro et al. [2] and is based on a hard-core generator with a sophisticated stirring criteria to ensure the fibre packing, especially for fibre volume fractions above 60%. The fibre stirring criteria is a combination of a compaction step of the closest fibre neighbours and the shortest path thorugh all fibres. Fibre allocation for fibre volume fractions above 60% is ensured using a sub-domain allocation based on the area of the largest Delaunay triangles. By comparison between genuine laminates and generated microstructures, it is found that the numerical microstructure generator is capable of producing statistically similar fibre architectures with respect to the point pattern, and thus it is suitable for numerical generation of SRVE's [5]. These two tools are used in the present to study the minimum size of the SRVE that can be used to replicate the spatial statistics of the composite microstructure. 5 different sizes of the square shaped SRVE ($L \times L$, varying from $L = 300 - 700\mu$m) are simulated, and selected microstructural parameters are used to evaluate the consistency. For each size of the SRVE, different FVF's are generated matching those found in the laminates. 5 different

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microsturctures are generated for each FVF to identify individual variations. The microstructural characterisation parameters are: the number of neighbours per fibre, the nearest neighbour distance, the number of contact points per fibre, and the local FVF (fibre area in relation to the area of the associated Voronoi cell), see e.g. [5]. These parameters are described by the mean value obtained from the different analyses. The spatial point pattern is evaluated for a single FVF using a second order statistical analysis based on the Ripley K-function, $K(h)$, and the pair distribution function, $g(h)$ [5, 6, 7, 8]. The second order statistics are presented as the deviation from complete spatial randomness (CSR) in terms of the expressions $L(h) = \sqrt{K(h)/\pi} - h$ and $g(h) - 1$, which for the case of CSR yields $L(h) = g(h) - 1 = 0$ [5, 6, 8]. Fibre centres are used as input for the determination of the second order statistics. The fibre volume fractions obtained in the laminates are [60.4 65.4 70.2]%, and the simulated microstructures are chosen to be with FVF = [60 65 70]%. The number of neighbours per fibre, the nearest neighbour distance, the number of contact points per fibre, and the local FVF are presented in Fig. 1 for the different sizes of the SRVE. For each FVF presented, the respective property is normalised with the associated value from the corresponding laminate. This means that an identical replication of the microstructure is obtained for the properties equal to unity. From Fig. 1 it is observed that the number of neighbours per fibre, the nearest neighbour distance, the number of contact points per fibre, and the local FVF are all unaffected by the size of the SRVE; nonetheless, there is a larger individual scatter for smaller sizes of the SRVE (larger magnitude of the errorbar). Fig 2 shows the deviation between the requested and obtained FVF from the microstructure generator. It is noticed from Fig. 2 that a smaller SRVE size and lower requested FVF imply a larger deviation in the obtained FVF in the simulations. Furthermore, there seems to be an asymtotic behaviour for $L \geq 600\mu$m. Therefore, in order to obtain the least deviation on the required FVF, and to reduce individual scattering, the side length should be minimum $L = 600\mu$m. The second order statistics are presented in Fig. 3 for a selected FVF, namely FVF = 65%. It is observed from Fig. 3 that the functions show characteristic peaks in multiples of the mean fibre diameter plus the communal fibre-to-fibre distance. This means an increased probability of clustering for these distances. For increasing values of h, the packing pattern tends towards CSR with $L(h) = g(h) - 1 = 0$. It is noted from Fig. 3 that the packing pattern of the laminate follows a CSR pattern if $h \gtrsim 60\mu\text{m} \simeq 7r_{mean}$. This means that the size of the SRVE must be larger than $7r_{mean}$ in order to replicate the CSR pattern. This observation is in accordance with Zangenberg & Brøndsted, [5]. For the smaller side lengths of the SRVE, the second order statistics cannot be replicated due to the low amount of fibres, which is reflected in the deviating patterns, especially distinct for $L < 600\mu$m, see also magnification in Fig. 3(a). For $L < 600\mu$m the functions seem to drift away from the laminate data, and by inspection and comparison to the genuine laminate, it is found that the deviation is reduced for $L \geq 600\mu$m. Therefore, $L \geq 600\mu$m appears to be a suitable minimum side length for the SRVE. If the fibre radii distribution is assumed to follow a normal behaviour, then the 99+% of the distribution is covered within the range of $r_{mean} \pm 3r_{std}$. Since the larger radii imply a more difficult fibre allocation in the simulations, an equivalent radius, r_{eq}, accounting for the non-uniformity of the radii distribution, is proposed as: $r_{eq} = r_{mean} + 3r_{std}$. Trias et al. [1] found that a normalised side length equal to $\delta = L/r = 50$ for a square shaped SRVE with uniform fibre radius, r, would be sufficient to replicate the correct stress/strain field. Using this notation with the equivalent radius and $L = 600\mu$m, it is found that $\delta = L/r_{eq} = 56$ is sufficient for the spatial point pattern statistics to be satisfied. Following the conclusion from Trias et al., it is noticed that for this value of δ the stress and strain fields will also be satisfied. Therefore, the spatial point pattern statistic of the fibre microstructure can be replicated for approximately the same size of the SRVE. In summary, the minimum size of a SRVE of a transverse cross-section from a unidirectional fibre reinforced composite with non-uniform fibre radii distribution is investigated using point pattern statistics. Numerical microstructures are generated for different sizes of the volume element, and the resulting fibre packing pattern is compared to that found in genuine composite laminates. For side lengths of the square shaped SRVE $L > 600\mu$m all considered evaluation parameters are met. Introducing an equivalent fibre radius $r_{eq} = r_{mean} + 3r_{std}$, the point pattern statistics are replicated for $\delta = L/r_{eq} = 56$. This value corresponds to the studies by Trias et al. [1] considering numerically generated SRVE's and the mechanical performance.

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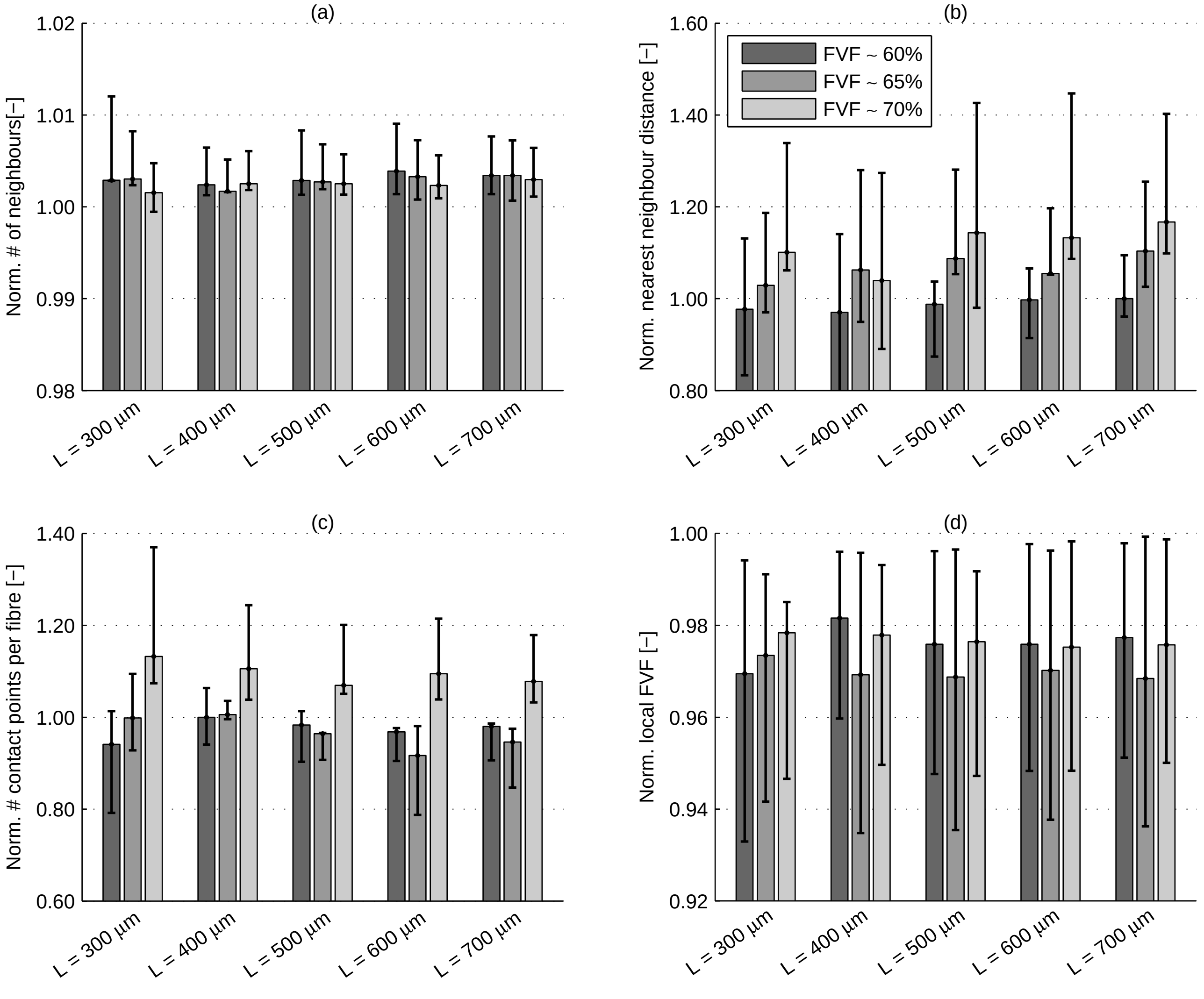

Figure 1: Differences between the fibre packing pattern for a numerically generated SRVE and composite laminates. The SRVE is square shaped with side length L. Results are normalised with the value obtained from the respective composite laminate with the same FVF, and the error bar reflects the minimum/maximum value obtained in the simulation. (a) Number of neighbours. (b) Nearest neighbour distance. (c) Number of contact points per fibre. (d) Local FVF.

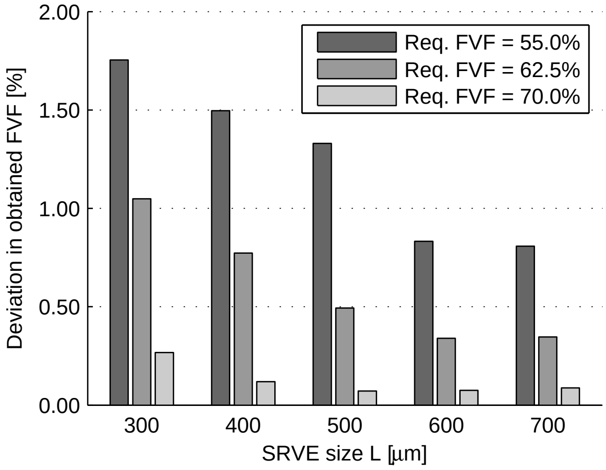

Figure 2: Deviation in the requested and obtained fibre volume fraction from the numerical microstructure generator.

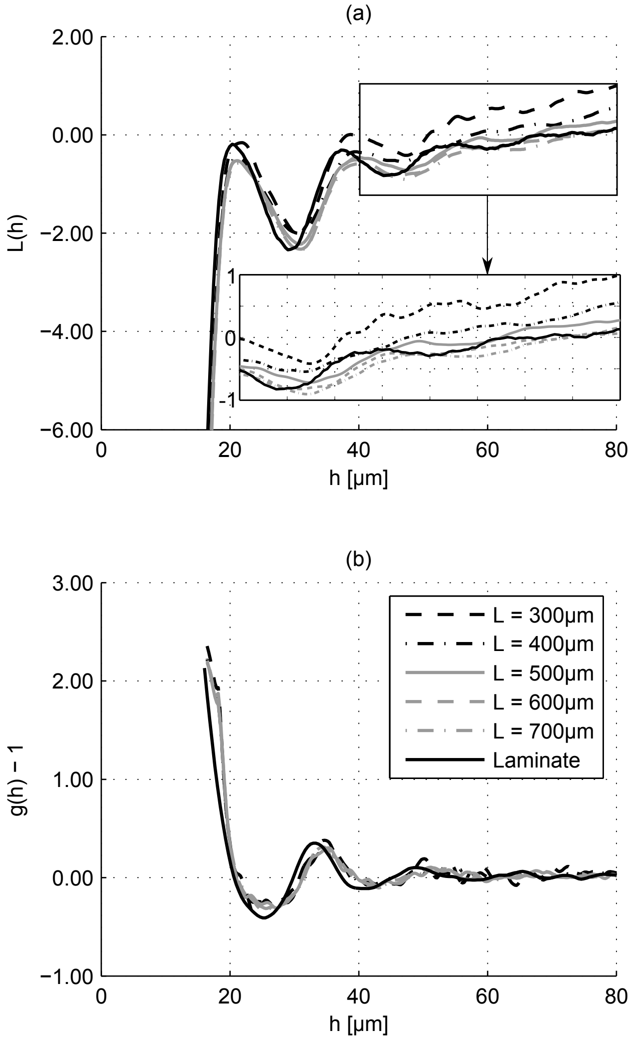

Figure 3: Second order statistics of composite architecture from genuine laminates and simulated microstructures. Results are shown for FVF = 65%. The SRVE is square shaped with side length *L*. (a) The *L*-function. (b) Pair distribution function.