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Seventeen years of global SSH anomalies analyzed by a maximum information based extension to EOF analysis

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Introduction

Empirical Orthogonal Function (EOF) analysis is traditionally used for analyzing sea level data as it provides a compact description of the spatial and temporal variability in terms of orthogonal functions. The motivation for extending the Empirical Orthogonal Function (EOF) analysis to an information maximization based approach, is to find descriptive subspaces where a large amount of information is present.

17 years of global Sea Surface Height (SSH) anomalies are analyzed using the Infomax algorithm as the application of this method can identify latent oceanographic features in the data set, with no other constraints than independence.

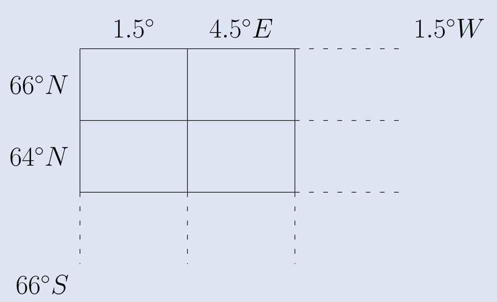
Expected to be prominent in the analyses is the El Niño Southern Oscillation (ENSO), which is a combined oceanographic and atmospheric phenomenon. The occurence of El Niño is periodic with an inexact cycle time of 2-7 years and has enormous socio-economic consequences all over the planet.

Data

The data set are 17 years of global Sea Surface Height (SSH) anomalies from TOPEX/Poseidon, Jason-1 and Jason-2 missions.

Period: January 1993 $\rightarrow$ December 2009 (204 months).

Arranged in a grid of size 67×120 with resolution of $2^\circ \times 3^\circ$.



The $P = 204$ months of SSH values are represented in the data matrix $\mathbf{X}$ with the months column wise and $N = 8040$ observations row wise.

$$\mathbf{X} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,P} \\ \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,P} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_P \end{bmatrix} . \quad (1)$$

Prior to these analyses, the data have been preprocessed to extract seasonal signals, tide, Earth tide and impacts from high/low pressure weather systems.

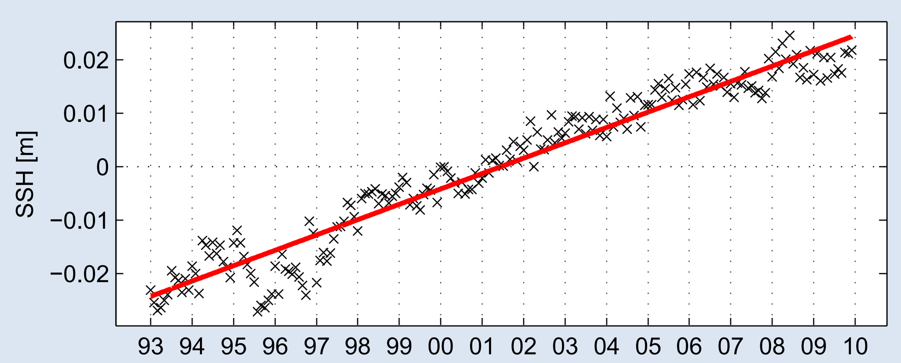


Figure 1: Mean values and first order fit. Mean increase per year approximately 2.9 mm

EOF analysis - a summary

Empirical Orthogonal Function (EOF) analysis is a variant of PCA, where the assumption of zero spatial mean is replaced by an assumption of zero temporal mean.

To find the principal directions, the empirical dispersion matrix is calculated as

$$\hat{\Sigma} = \frac{1}{N-1}\mathbf{X}^T\mathbf{X} \quad (2)$$

which is recognized as T-mode analysis [3]. The EOFs are calculated by finding the the eigenvectors of $\hat{\Sigma}$ that satisfy

$$\hat{\Sigma}\mathbf{u}_j = \lambda_j \mathbf{u}_j \quad (3)$$

followed by a projection of the original data onto these

$$\text{EOF}_j = \mathbf{X}\mathbf{u}_j \quad \forall j \in [1, P] \, . \quad (4)$$

The EOFs are sorted decreasingly according to contained variance.

Results The five most significant EOFs and loadings are displayed in Figure 2. All have a large peak with relation to El Niño of 1997-1998. Not identical - shifted approx. one month.

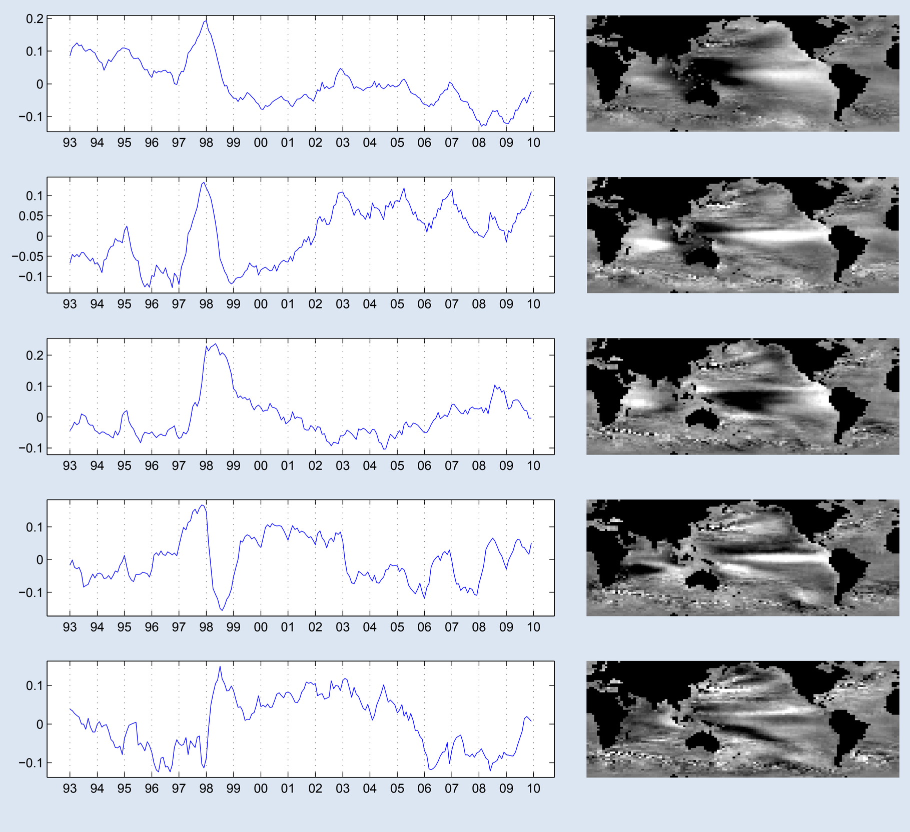


Figure 2: First five EOFs and loadings. All dominated by El Niño 1997-1998 in different phases.

Further analysis

Highlighted points south of Greenland in IC4 and mean SSH anomalies for these in Figure 4.

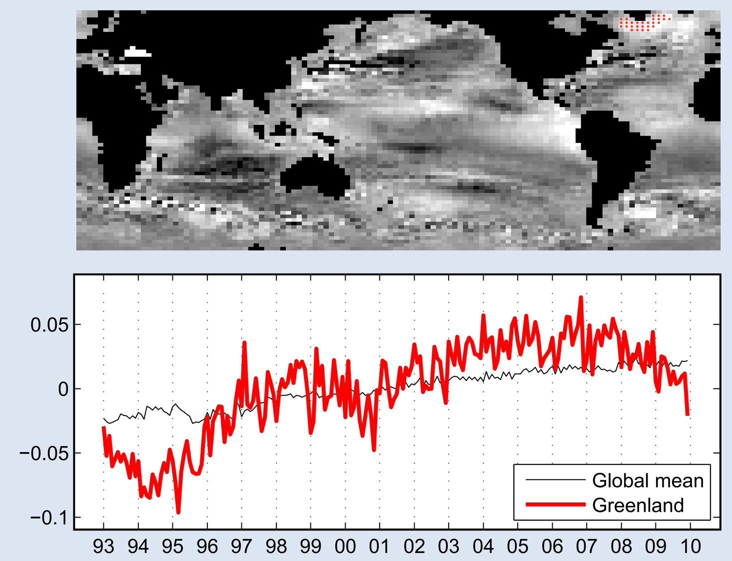
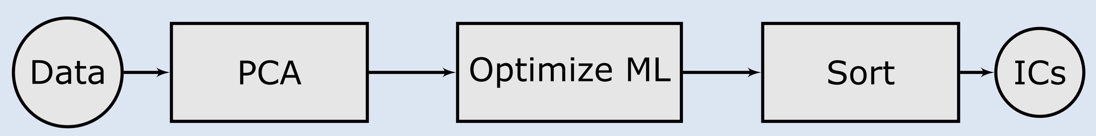


Figure 4: SSH anomalies in area highlighted in IC4

A stronger trend than the global average through majority of period.

Trend started turning in 2006-2007?

Maximizing information (Infomax)



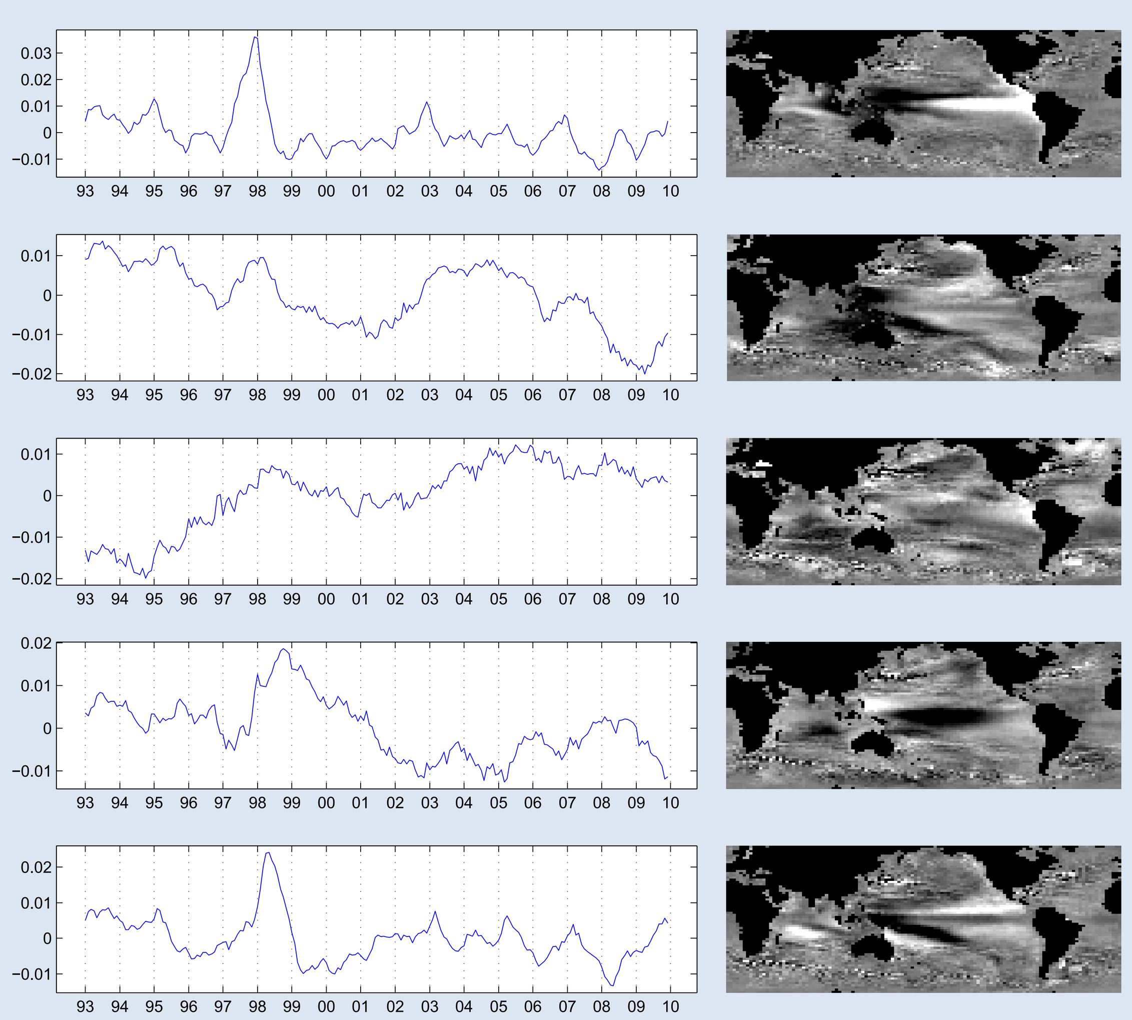


Figure 3: First five ICs and corresponding loadings

The Infomax method [1] is an ICA type method. Seeks to uncover a number of latent sources. The sources $\mathbf{s}_i$ have been put through a mixing matrix $\mathbf{G}$ and intercepted as signals in $\mathbf{X}$

$$\mathbf{X}^T = \mathbf{G}\mathbf{S} \quad (5)$$

Maximum information = nongaussianity [2]. There is no orthogonality constraint.

Maximum likelihood based approach to maximize nongaussianity, using e.g. steepest ascent.

Nonlinear mapping $\phi(\mathbf{w}^T\mathbf{x}) = -\tanh(\mathbf{w}^T\mathbf{x}) \Rightarrow$ implicit assumption that the sources are leptokurtic (supergaussian).

Results Temporal and spatial patterns of IC1-5 in Figure 3 show:

1. A large pool of water hits South America from the west. Large peak at El Niño of 1997-1998, and peaks at other known El Niño years.
2. Captures periods following El Niños - part of the build-up phase to the next.
3. The southern Pacific is left colder than usual (La Niña).
4. Strong linear trend in the temporal pattern. The spatial representation highlights - among others - an area south of Greenland.
5. Captures a connection between El Niños and the SSH anomalies in the Indian Ocean.

Conclusions

Main results

1. Purer separation of the ENSO signal in the first component. Pure separation of signals of interest simplifies further analysis and hypothesis generation or confirmation.
2. Detection of a sea level rise south of Greenland through the majority of the time period. The information extracted from this data set, however, suggests that the trend started turning in 2006-2007, similarly to 1999-2001.

Infomax pros and cons

+ No demand for mutual orthogonality.
+ Flexibility by allowing for different nonlinear mappings and thereby different assumptions about the source priors.
– Assumption of equal number of sources and intercepted signals.
– Iterative optimization.

Overall, when analyzing these 17 years of SSH anomalies, Infomax has shown superior to EOF analysis in extracting information of interest.

References

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