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# Scheduling and Location Issues in Transforming Service Fleet Vehicles to Electric Vehicles

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## **ABSTRACT**

There is much reason to believe that fleets of service vehicles of many organizations will transform their vehicles that utilize alternative fuels which are more sustainable. The electric vehicle is a good candidate for this transformation, especially which “refuels” by exchanging its spent batteries with charged ones. This paper discusses the issues that must be addressed if a transit service were to use electric vehicles, principally the issues related to the limited driving range of each electric vehicle’s set of charged batteries and the possible detouring for battery exchanges. In particular, the paper addresses the optimization and analysis of infrastructure design alternatives dealing with (1) the number of battery-exchange stations, (2) their locations, (3) the recharging capacity and inventory management of batteries at each facility, and (4) routing and scheduling of the fleet. Several optimization models are developed and some preliminary results are given in the paper.

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## Introduction and Rationale

The environmental, geopolitical, and financial implications of the world's dependence on gasoline-powered vehicles are well known and documented, and much has been done to lessen our dependence on gasoline. One thrust on this issue has been the investigation of alternative-fueled vehicles, such as electric vehicles, hydrogen-gas vehicles and bio-fuel based vehicles which seem to be viable environmentally friendly contenders, and perhaps could replace gasoline-powered vehicles [Ogden et al 1999], . This paper discusses logistical issues that need to be considered when transforming a service fleet to electric vehicles (EV). These vehicles have an electric motor rather than a gasoline engine, and a battery to store the energy required for to move the vehicle. From the success of hybrid vehicles in fleets, and among personal ownerships of cars, there is a reason to believe that there will be future opportunities to utilize EVs [Axsen and Kurani 2009, Bapna et al. 2002]. The “*range anxiety*” issue (where a driver feels that the vehicle will run out of fuel leaving the driver stranded) is among the most concerns in replacing current gasoline vehicles with EVs [Bakker 2011].

With current technology, EVs require “refueling” more often than gasoline vehicles, that is, they need a charge or exchange their batteries before exceeding a range, say  $c$ , that an EV can travel on a charge. EVs could recharge its batteries at located *recharging stations*, or swap its batteries at *battery exchange facilities* where charge-spent batteries can be replaced by charged ones.

Because the spent battery requires an extended period of time to recharge, this method has the implicit assumption that vehicle will be used only for driving short distances, such as for daily commuting. EV companies are trying to overcome this limited range requirement with fast charging stations; locations where a vehicle can be charged in only a few minutes to near full capacity using high-power chargers; the vehicles still take more time to recharge than a standard gasoline vehicle would take to refuel [Botsford and Szczepanek 2009]. The battery-exchange “refueling” infrastructure design is to locate quick battery exchange (BE) facilities. These stations will remove a pallet of batteries that are nearly depleted from a vehicle and replace the battery pallet with one that has already been charged [Senart et al. 2010, Shemer 2012]. This method of refueling has the advantage that it is reasonably quick. Battery exchange stations have been tried out by taxi vehicles in Tokyo in 2010 [Schultz 2010]. In fact, the country of Denmark is investigating the possibility of having sufficient battery exchange locations so that the country relies on no, or very few, gasoline-powered vehicles.

In the case for transforming a fleet of service vehicles to electric vehicles, it means that an initial infrastructure needs to be well designed so that “refueling” stations are located at suitable points, which consider all the service routes the vehicles have to cover, be they delivery-pickup tours, or passenger boarding and alighting trip. In the public transit scenario, a bus would need to refuel after servicing a fewer number of bus routes than when the buses are gasoline powered. In fact, in this case, there is more reason to believe that battery exchange facilities are more appropriate than battery charging since the latter would require the buses to be idle for long periods of time which lowers the productivity of buses that are normally quite expensive.

Battery-exchange stops will need to be planned in the bus itineraries and these battery exchange stops may require detouring. Furthermore, the locations of BE depots will significantly affect the needed detouring and hence the productivity of transit operations. Optimally locating BE facilities is a critical concern when switching to electric buses.

Another issue we need to address is the capacity issue and attendant inventory control design. The underlying question here is whether there is appropriate inventory at each location so that drivers do not have to be sent elsewhere or have to wait too long for a fully-charged battery to be available.

In general, a good infrastructure design calls for (1) assessing the requirements for the infrastructure, (2) modeling and analysis of the design alternatives, (3) optimization of design parameters such as the number of stations, their locations and the capacity at each location, (4) provision of real-time availability information to drivers and (5) operational protocol and procedures. Factor (1) focuses on the demand for refueling, (4) focuses on the technologies for informing each driver within the fleet of available fuel/charging capacities, and (5) on how the system is envisioned to operate. This paper focuses on factors (2) and (3) with respect to routing, scheduling, location and capacity sizing decisions for a service fleet.

### **EV Routing issues**

Taking a trip, especially one through lowly populated areas, requires the driver to plan when the vehicle will need to be refueled. Given the abundance of gasoline stations for standard vehicle, drivers usually consider refueling only when their fuel tank is low. The search for a good refueling point can be further aided by navigation systems and smart phone apps, such as Google Maps, that provide motorists the location of gasoline stations in the vicinity. In the case of most current electric vehicles, planning refueling is more important than for gasoline vehicles, since there would be few battery recharging or exchange facilities, at least initially. Hence, one needs to develop models which look for the shortest routes from origins to destinations that include detouring when necessary. Objectives for these models could be to (a) minimize the total detouring distances and (b) minimize the total number of battery-exchange stops. Such shortest route models relate to the constrained shortest path problem [see, e.g. Handler and Zang 1980, Beasley and Christofides 1989, Desrochers and Soumis 1988, Xiao et al. 2005]. Detouring plays a major role in the problems discussed in this paper.

Given a start point and an end point in a network with several battery exchange (BE) facilities, we are interested in finding the shortest route such that an electric vehicle (EV) with range  $C$  can successfully navigate it. In the case that the shortest path between the starting and ending points is less than  $C$ , then this problem can be trivially solved by any standard shortest route algorithm such as Dijkstra's [see e.g., Ahuja et al. 1993]. In the case that the shortest path between the two points is greater than  $C$ , we need to consider where the vehicle can swap its batteries. We also may only be interested in paths that require at most  $P$  battery swaps, in which case the problem can be considered as a modification of the classic constrained shortest path problem.

Let  $G = (V, E)$  be a directed network with node set  $V$  and arc set  $E$ . Let vertices  $s, t \in V$  represent the starting and ending points of a trip by the EV. Let  $d_{ij}$  denote the length each arc  $(i, j) \in E$  and let  $R \subseteq V$  be the given set of BE locations, which are assumed to be at nodes without loss of generality. Let  $p$  be the maximum number of times we are allowed to stop for a battery exchange on the trip. We define the *EV shortest route problem* (EV-SRP) as the problem of finding the shortest path in  $G$  starting at  $s$  and ending at  $t$  such that any walk (or sub-path) without a battery-exchange stop contained in the path starting and ending at nodes in  $\{s, t\} \cup R$  has the length at most  $c$ . This path may contain at most  $p$  elements of  $R$ , since a vehicle would never visit the same BE twice.

We can formulate EV-SRP as a linear integer program. In our solution, the EV will likely service several trips between different refueling stations. Let these walks between  $\{s, t\} \cup R$  be called *super-trips*. For simplicity define the set  $F = \{s, t\} \cup R$ . The vehicle will make at most  $p + 1$  of these super-trips, since the vehicle can only stop at  $p$  refueling stations. We define the decision variable  $x_{ijk}$  for  $(i, j) \in \tilde{E} = E \cup \{(t, t)\}$  and  $k \in \{1, \dots, p + 1\}$  as whether or not the EV will use arc  $(i, j)$  during its  $k^{\text{th}}$  super-trip. Since a vehicle may need less than  $p + 1$  super-trips, we add an additional set of dummy decision variables  $x_{ttk} \in \{0, 1\}$  for  $k \in \{1, \dots, p + 1\}$  (even though  $(t, t) \notin E$ ). These variables will allow the bus to have its final super trips be loops at the end node. The integer program is now the following:

$$\text{Minimize } \sum_{k=1}^{p+1} \sum_{(i,j) \in \tilde{E}} d_{ij} x_{ijk}$$

$$\sum_{j \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0 \quad \forall i \in V \setminus (\{s, t\} \cup R), \quad k = 1, \dots, p + 1 \quad (1)$$

$$\sum_{j \in V} x_{sj1} = 1 \quad (2)$$

$$\sum_{j \in V} x_{jt(p+1)} = 1 \quad (3)$$

$$\sum_{i \in F} \sum_{j \in V} x_{ijk} = 1 \quad k = 1, \dots, p + 1 \quad (4)$$

$$\sum_{i \in F} \sum_{j \in V} x_{jik} = 1 \quad k = 1, \dots, p + 1 \quad (5)$$

$$\sum_{(i,j) \in \tilde{E}} d_{ij} x_{ijk} \leq c \quad k = 1, \dots, p + 1 \quad (6)$$

$$\sum_{k \in \{1, \dots, |R| + 1\}} x_{00k} \geq |R| + 1 - p \quad k = 1, \dots, |R| + 1 \quad (7)$$

Constraint (1) ensures that conservation laws hold. Constraints (2) and (3) ensure that the vehicle starts and ends at the proper destination. Constraint (4) and (5) ensure that every super-trip starts and ends at either BE stations or at either  $s$  or  $t$ . Constraint (6) ensures that the vehicle can indeed travel without running out of battery power.

If we are allowed to make an unlimited number of stops (i.e.,  $p = \infty$ ) and our only concern is to minimize total distance, then the problem can be easily solved in polynomial time. First, find the shortest path between each pair of elements in  $\{s, t\} \cup R$ . Then, create a new graph  $G' = (V', E')$  with node set  $V' = \{s, t\} \cup R$ . Define  $E' = \{(i, j) : i, j \in V', \text{ the shortest path between } i \text{ and } j \text{ in } G \text{ is length at most } c\}$ . Define a length function  $d'$  where  $d'$  maps arc  $(i, j)$  to the length of the shortest path between  $i$  and  $j$  in  $G$ . This new graph can be generated in polynomial time. Now, running Dijkstra's algorithm on graph  $G'$  will get us the shortest path between  $s$  and  $t$ , which will be the solution to this case of EV-SRP.

The more interesting and more difficult cases of EV-SRP are when refueling stops are penalized with a "stop cost" representing, for example, the price of each battery exchange. Then we need to consider both the distance cost and the stop cost to find the optimal route. When stops are very costly then the problem becomes that of minimizing the number of battery exchange stops to reach the destination: a problem that we are currently studying. Another problem is the multi-objective one, where stop costs are traded off with total distance costs. Related constrained shortest path problems (but without the range constraint) are the problem of finding the route with the least number of arcs [Dumitrescu and Boland 2001, 2003] and finding the best route that trades off total distance with number of arcs [Beasley 1989] have been shown to be NP-hard.

## Route scheduling and bus itinerary planning

The problems involved in scheduling a fleet of vehicles to service routes at specific times, for example scheduling a fleet of buses in a city, are broadly referred to as *vehicle scheduling problems* which have been well studied and documented [e.g., Golden 1988, Dror 2000, Freling 1995, Laporte 2009]. Changing the fleet to one that uses electric vehicles increases the complexity of the problem because of the limited range of the electric charge on the batteries. This range limit will require the buses to refuel several times during their use throughout the day, sometimes having to detour significantly from the original route just to swap the batteries. Thus, despite the fact that EVs require less energy to operate than gasoline-powered ones, inefficiency is added by requiring frequent battery swaps. An explicit requirement of refueling, sometimes several times during a tour, adds not only the refueling requirements for the bus but also a detouring component to itinerary planning that is not present in the standard VSP problems. Electric buses which have battery pallets that can be swapped have been tested at the Shanghai EXPO [Zhu 2012]. Good scheduling of the EV fleet of buses to the routes and refueling stations can lower the amount of energy the buses use. This topic has been studied for fleets of delivery vehicles, where there is not a time-table requirement on the scheduling [Davis and Migliozi, 2012].

Given a set of bus routes with given start and end times that need to be serviced, and a set of BE facilities for the new EV buses, we need to find the best way to assign buses to routes. The limited range of each charged battery pallet, as well as the small number of BE stations, means that it is important to plan exactly where and when the buses will have their batteries exchanged.

The classic *Single Depot Vehicle Scheduling Problem* (VSP), in the context of public transit, may be defined as follows: given a depot at location  $d$  and  $n$  trips to be serviced, with  $j^{\text{th}}$  trip starting at location  $s_j$  and ending at locations  $e_j$  with corresponding starting and ending times  $st_j$  and  $et_j$  for  $j = 1, \dots, n$ , and given traveling times and costs for all pairs  $(e_j, s_j)$ ,  $(e_j, d)$ , and  $(d, s_j)$ , find the minimum cost assignment of buses to trips, such that every trip is served by exactly one bus. Here each trip could be considered as a job with a given start and end time and a start node and end node on a graph. Later we will represent each trip as a single node on a graph, and arcs between two nodes will represent if two trips can be serviced by the same bus. This problem has several polynomial time approaches to solve it [see, e.g., Li 2007, Laporte 2009]. However, upon adding the constraint that each bus can only travel a certain distance before needing to visit a BE station the problem becomes NP-hard. The addition of the limited range constraint changes the problem into what we shall refer to as *Electric Vehicle Scheduling Problem* (EV-VSP), which is what we plan to address.

Formally the single depot EV-VSP is the following: we are given a depot  $d$  at location  $d$ ,  $k$  BE stations at locations  $FS = \{fs_1, fs_2, \dots, fs_k\}$ , with EV buses needing to service  $n$  trips, where  $j^{\text{th}}$  trip is from location  $s_j$  to location  $e_j$  for  $j \in N$ ,  $N = \{1, \dots, n\}$ , having for start and end times  $st_j$  and  $et_j$ , and electric charge requirements  $f_j$ . Each bus has an electric charge capacity  $w$  that gives it a limited range. For each dead-heading trip  $\{(a, b): a, b \in FS \cup d \cup \{s_j: j = 1, \dots, n\} \cup \{e_j: j = 1, \dots, n\}, a \neq b\}$ , we have a travel time  $t_{ab}$ , cost  $c_{ab}$  and energy consumption  $f_{ab}$ . The EV-VSP problem is to find a feasible minimum cost assignment of buses to trips, and the BE stations between trips being serviced, such that each trip is serviced by exactly one bus, each bus route starts and ends at the depot, and any route a bus takes between two refuel stops, or the depot and a refuel stop, it uses at most  $w$  charge. For meaningful application, but with no loss of generality, we assume that at most one BE station will be visited between two trips, a BE station may be visited between the depot and the first trip and/or after the last trip before returning to the depot. Each bus starts at the depot with a full charge; if a depot also has charging facilities a BE station can be placed in the same location as the depot, although this is not assumed by default. The EV-VSP is a generalization of VSP since the single depot vehicle scheduling problem is a special case of where  $w = \infty$  and  $k = 0$ . The EV-VSP has many similarities to the capacitated arc routing problem [see e.g., Golden and Wong 1981, Wøhlk 2008].

Without loss of generality, assume that the trips are ordered by their start times (so  $st_i \leq st_j$  for  $1 \leq i \leq j \leq n$ ). We can create a directed graph  $G = (V, A)$ , cost and fuel requirement functions  $c, f: A \rightarrow \mathbf{R}^+$ , and a constant value  $w$  to represent the problem. This representation of the problem

fully captures the time compatibility component in the connections of  $G$ . We say that trips  $i, j$  are *compatible* if  $et_i + t_{ij} \leq st_j$ . Two trips  $i, j$  are compatible with BE station  $l$  if  $et_i + t_{il} + t_{ij} \leq st_j$ . We write the relationship for compatibility of  $i$  and  $j$  as  $comp(i, j)$  and  $i$  and  $j$  are compatible with BE station  $l$  as  $comp_f(i, j, l)$ . Define the set  $H$  as:

$$H = \{h_{ab}^l : a, b \in \{1, \dots, n\}, a < b, l \in \{1, \dots, k\}\} \cup \{h_{ad}^l : a \in \{1, \dots, n\}, l \in \{1, \dots, k\}\} \cup \{h_{da}^l : a \in \{1, \dots, n\}, l \in \{1, \dots, k\}\}$$

Set  $H$  represents the possible BE station visits that could occur between trips. So  $h_{ab}^l \in H$  represents a bus stop for battery exchange at BE station  $l$  after servicing trip  $a$  but before servicing trip  $b$ . We now define node set  $V = N \cup H \cup \{d\}$ . The arcs of  $G$  as well as definitions of  $c$  and  $f$  are given in Table 1. Arc set  $A_1$  represents a bus taking a dead-heading trip between two trips. The fuel requirement for the origin trip is added to the fuel cost of the dead-heading trip so that we do not need to associate fuel requirements with vertices. Arc sets  $A_2, A_3$ , and  $A_4$  represent dead-heading trips between the depot

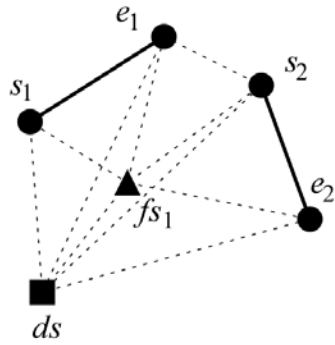
Set	Definition	Fuel	Cost
$A_1$	$\{(a, b) : a, b \in N, comp(a, b)\}$	$f_a + f_{ab}$	$c_{ab}$
$A_2$	$\{(a, d) : a \in N\}$	$f_a + f_{ad}$	$c_{ad}$
$A_3$	$\{(a, d) : a \in H\}$	$f_{ad}$	$c_{ad}$
$A_4$	$\{(d, a) : a \in N \cup H\}$	$f_{da}$	$c_{da}$
$A_5$	$\{(a, h_{ab}^l) : a \in N, b \in N, l \in H, comp_f(a, b, l)\}$	$f_a + f_{al}$	$c_{al}$
$A_6$	$\{(h_{ab}^l, b) : a \in N, b \in N, l \in H, comp_f(a, b, l)\}$	$f_{lb}$	$c_{lb}$
$A_7$	$\{(a, h_{ad}^l) : a \in N, l \in H\}$	$f_a + f_{al}$	$c_{al}$
$A_8$	$\{(h_{ad}^l, d) : a \in N, l \in H\}$	$f_{ld}$	$c_{ld}$
$A_9$	$\{(d, h_{da}^l) : a \in N, l \in H\}$	$f_{dl}$	$c_{dl}$
$A_{10}$	$\{(h_{da}^l, a) : a \in N, l \in H\}$	$f_{la}$	$c_{la}$

Table 1: The arcs in graph  $G = (V, A)$  where  $A = \cup_{i=1}^{10} A_i$

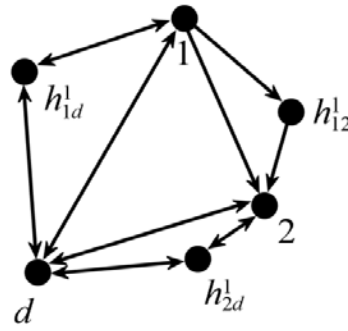
and trips or refuel stations. Arc sets  $A_5$  and  $A_6$  represent dead-heading trips between trips and fuel stations. Arc sets  $A_7$  and  $A_8$  represent traveling from a trip, refueling, then returning to the depot. Arc sets  $A_9$  and  $A_{10}$  represent traveling from the depot to a refueling station then from a refueling station to a trip. An example  $G$  is given in Figures 1 and 2.

Given  $G, c, f$ , and  $w$ , The EV-VSP is now the following: find a minimum cost set of cycles  $C$  in  $G$  that visit each node in  $N \subset V$  exactly once, where each cycle includes vertex  $d$  and no induced walks of  $C$  between starting and ending at vertices in the set  $\{d\} \cup H$  and containing no intermediate vertices of  $\{d\} \cup H$  have a fuel requirement greater than  $w$ . Not only can EV-VSP be shown to be NP-hard, but the proposers have recently shown that even finding a solution that is guaranteed to be within 150% of optimal is NP-hard.





**Figure 1:** An instance of the EV-VSP



**Figure 2:** The corresponding graph G

### Location of BE Facilities

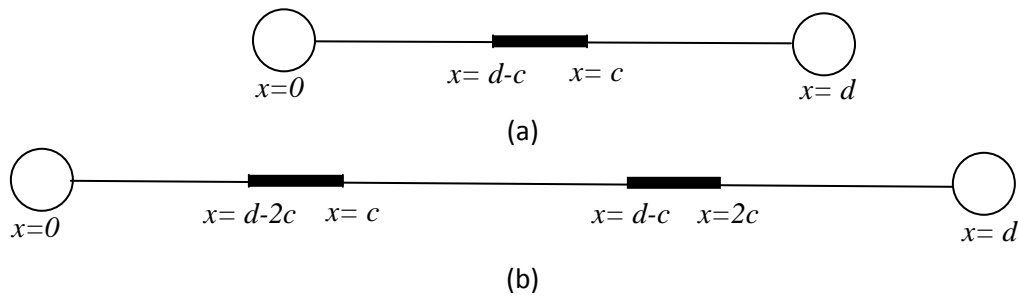
We can define two prototypical location problems that need consideration of refueling detours: (1) Location of BE exchange stations for minimizing the total detouring for given origin-destination demands, and (2) location of BE facilities for a fleet of service electric vehicles, for example, a fleet of EV buses for public transit. We will assume the underlying optimal routing between network points as described above.

The problem of optimally locating such refueling stations (battery recharging, battery exchanging and, other alternative refueling options can all addressed similarly) has been investigated by Kuby and co-researchers [e.g., Kuby 2005, Kuby and Lim 2006, Kuby and Lim 2006, Lim and Kuby, 2010, Capar et al. 2012] and others [Wang and Lin 2009, and Wang and Wang, 2010]. They used modifications of flow capturing or flow inspection models [see, e.g., Hodgson 1990, Berman et al., 1992, Mirchandani et al., 1995] to cover as much origin-destination flows as possible with a given number ( $p$ ) of stations. They compared their proposed model with standard  $p$ -median and  $p$ -center models [see, e.g., Mirchandani and Francis, 1990] which were used as proxies for maximizing proximity to refueling stations and coverage by refueling stations, respectively. In particular, the proposed models were compared empirically for specific scenarios (e.g., for the State of Florida) in order to choose one location model over another [Kuby et al. 2009]. They also extended their model to the capacitated case [Upchurch et al. 2009]. However, these models do not take into consideration the likely possibility of vehicles making detours to refuel; therefore, the direct consideration of locating facilities to minimize detouring distances and or minimizing detouring stops have not been addressed in their *Flow-Refueling Location* model.

Consider the first problem of locating BE facilities for a given set of O-D demand. The research will first consider the case when the transportation network is a tree network. The location models and algorithms developed for trees can then be extended to general networks. We begin by considering the simple special case where we have an EV planning to travel between two points  $O$  and  $D$  along a road.

Let  $c$  be the maximum distance the vehicle can travel before needing to refuel and let  $d$  be the length of the road, where  $d > c$ . We are interested in finding the optimal locations for BE stations so that the electric vehicle can travel between the two points. We can describe points along the route by their distance from starting point  $0$ .

If  $c < d < 2c$ , then we only need a single BE station. That refueling station can fall anywhere along the interval  $[d - c, c]$  (see Fig 3-a). If  $2c < d < 3c$  we need to place two BE stations along the route, where the first one falls in the interval  $[d - 2c, c]$ , and the second in the interval  $[d - c, 2c]$  (see Fig 3-b).

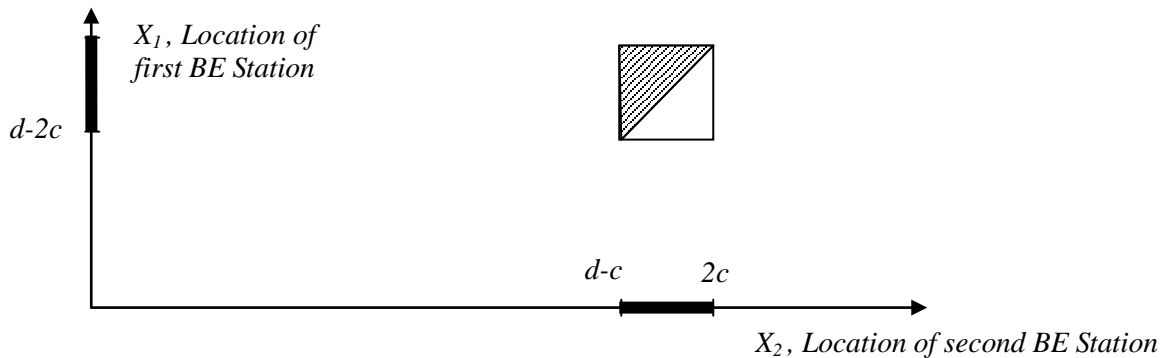


**Figure 3:** Illustration of localization sets for BE facilities on a single route.

The optimal refueling station placements are not independent of each other since the distance between the two stations must be at most  $c$  units. If you consider the refueling station locations as a point in space  $[0, D]^2$ , the feasible solutions form a convex polyhedral (see Fig 4). Depending on what we try to optimize, the optimal solution may fall at an extreme point of the convex polyhedral or in an interior point. For example, if the EV driver prefers to maximize the lowest charge level the EV vehicle will have

$$\left( \frac{d}{3}, \frac{2d}{3} \right)$$

with only two stops, then interior point  $\left( \frac{d}{3}, \frac{2d}{3} \right)$  will be optimal. If the driver is indifferent to level of charge as long as she reaches  $D$ , then all points in the polyhedral are equally optimal; in particular the extreme points are optimal, which are computationally easy to identify. We may refer to these extreme



points as *breakpoints* on the line. We are interested in modeling different *anxiety functions* and characterize the facility localization set within the polyhedral.

*c*

**Figure 4:** Localization polyhedral for two facility case.

Generalizing to a path of any length, suppose the minimum number of BE facilities required is  $p$ , using the above arguments. Then the feasible space can be shown to be a polyhedral in  $p$ -space. Define location variables as  $x_1, \dots, x_p$  for some  $p \in \mathbf{Z}^+$ , where  $x_i \in [0, D]$  for  $i = 1, \dots, p$ . Then the location model can be formalized as follows

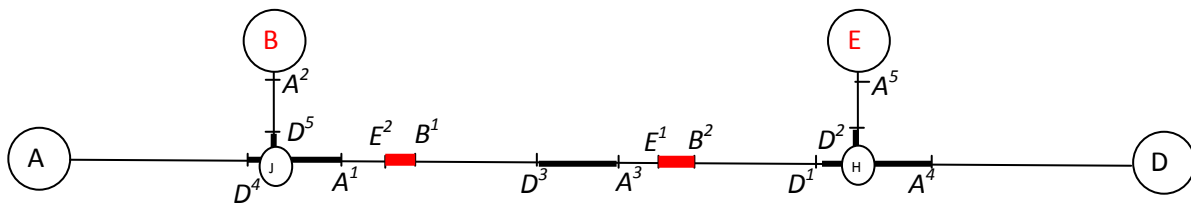
$$\text{minimize } Z(x_1, x_2, \dots, x_p)$$

$$\text{s.t. } x_i + x_{i+1} \leq c \quad \forall i = 1, \dots, p-1$$

$$x_i < x_{i+1} \quad \forall i = 1, \dots, p-1$$

where  $Z(x_1, x_2, \dots, x_p)$  is the objective function representing the anxiety function of the driver. If  $Z$  is a linear or concave function, then a solution will lie at an extreme point of the polyhedral. If  $Z$  is convex and differentiable, then the solution may lie on an interior point.

Now consider the situation where we have  $n$  points  $v_1, \dots, v_n$ , and a collection of OD flows connecting these points. This will create a tree network where the leaves of the tree are  $v_1, \dots, v_n$ . A problem of interest is to find the minimum number of refueling stations and their locations such that the total detouring cost for the OD flows is minimized. Each vehicle may travel between any pair  $[(v_i, v_j)]$  for  $i, j \in \{1, \dots, n\}$ , for some cost function  $Z$ . We may then narrow down the search space with appropriately defined segments between break points. (See example in Figure 5). The search for optimal placement of the EV stations, along with the study of the complexity of the problem and the possible solution algorithms, would be both intellectual and relevant research directions. This could then lead to the more general location problem on general networks, either from the modeling sights gained from the tree case or the algorithmic methods from solving for tree case.



**Figure 5:** Localization segments for a tree network

The second location model identified above deals with locating BE stations for a fleet of EVs. Assume we are given a set of bus trips and we are interested in serving them with EV buses, like in the EV-VSP

defined in section 3.2. Now however, we add variables representing BE station locations. Instead of  $FS = \{fs_1, fs_2, \dots, fs_k\}$  being fixed, they are variables that we would like to adjust to minimize the overall cost to the system. We are given a cost function  $C_f: FS \rightarrow \mathbf{R}$  which assigns a cost to placing a BE station at a location. The travel times and fuel costs between trip start and end locations and refuel stations are no longer fixed; they are also functions of the BE locations. A simplifying assumption would be to have these functions depend only on the distance between the dead-heading trips, however depending on the real-world considerations that may not be realistic. We will refer to this problem as the *Battery Exchange Station Location Problem for Transit* (BESLPT). Since the EV-VSP is NP-hard, it follows that BESLPT is also NP-hard, because it contains EV-VSP as a subproblem. Still, can study special cases and develop exact and/or approximate algorithms for them, as well as for general cases.

### **Facility sizing and inventory management issues**

When a vehicle arrives at a BE station, it requests a fully charged battery pallet (an output of the station) to replace the nearly depleted batteries it currently holds. The request could either be satisfied by a fully charged battery pallet from the facility's storage, or by a pallet that is just completing its charging. If the request is indeed satisfied, the vehicle in turn deposits a fully or partially spent pallet. If there are idle battery pallet chargers (BPC) at the station, the spent battery pallet is placed on a BPC and its recharging begins, otherwise it is kept in a queue until a BPC is available. If instead there is no fully charged battery available at the facility, then the vehicle could leave and go to a different facility (i.e., it balks). Alternatively, it could wait for a battery to fully charge, which may take some time. The bus could even take, if necessary, a replacement battery that is only partially charged and use that partially charged battery to travel to another battery-exchange facility on its route.

The size and attendant cost of the facility depends on both the number of BPCs it holds and number of battery pallets the facility keeps on hand. The availability of charged battery pallets at any given time depends on the size of the facility, the inventory of pallets, and demand for charged pallets the facility is experiencing. The facility incurs an indirect cost from the unavailability of charged pallets when an EV arrives for an exchange because the driver will not have to pay for a battery swap, and there may be a loss of goodwill from the unserved customer. Models to evaluate total direct and indirect costs for possible decisions on facility sizing and inventory holding would be very important in designing the BE infrastructure.

At first glance, it may appear that one may be able to model arrivals of EVs with spent charge at a facility as a non-homogeneous Poisson process with the service times for exchanging batteries as a deterministic. Hence, with a charging system with  $p$  BPCs, the system may be approximately treated as an  $M/D/p$  queuing system, for which there are several analytical results for periods when the arrival rate is homogenous [e.g., Tijms 1994, Franx 2001]. However, consider the following additional consideration. If the request for a recharged battery pallet is satisfied, the vehicle, in turn, deposits a fully- or a partially-spent pallet. If there is an idle BPC, the spent pallet is assigned to it. Otherwise, it is kept in a queue for an available BPC. If there is no fully charged battery available, then the vehicle could leave and go

elsewhere (i.e., it balks), or it could wait for a fully charged battery and experience some discomfort. The existence of two interacting queues, EVs waiting for battery pallet swaps and spent pallets waiting for a BPC. To the authors' knowledge, analytical models for such a system are not available.

The authors have built a preliminary simulation model of a BE facility using the ARENA [Kelton et al. 2001] software. In fact the model included some additional realistic considerations such as that the driver will wait up to a specified tolerance period (say 10 minutes) if a charged battery will soon be available. Each simulation run was characterized by (1) the arrival rate of EVs, (2) the number of BPC in the facility, and (3) the inventory level of batteries. The simulation showed, as expected, smaller percentage of customers leave before being served if (1) the arrival rate decreases, (2) number of BPCs are increased, (3) number of battery-pallets in inventory are increased.

Finally, there is the issue of evaluating a **network of facilities**, given by their locations and sizes. This is a much more complex problem than the conventional capacitated facility location problems [e.g., Jacobsen 1990, Melkote and Daskin 2001]; here the interactions between facilities are much more complex due to the limited ranges of EVs and the detouring issue to reach BE stations.

Furthermore, if vehicles can accept partially charged battery pallets when there are other BE facilities within range on their routes, then the stations can have multiple available products at any time, defined by the "*charged levels*" of the pallets. Instead of facilities having only fully charged and spent batteries, the facilities will have a range of available battery charge levels. Then an EV could pick up a partially charged pallet with enough charge to reach the next BE station on its planned route. This issue further couples the location and sizing decisions. These considerations cause the sizing and inventory management decisions for a network of facilities to become much more complex, both from a modeling and computational point of views.

The authors are developing a more complex simulation model that includes a network of BE facilities, where a driver may swap his spent pallet with a partially charged pallet when he can reach another BE facility for a swap along his route to the destination. Development of analytical models that provide similar results could then be used in an optimization framework that both locates and sizes BE facilities to minimize overall costs that includes fixed costs of facilities, costs to drivers for detouring, service-related costs (e.g., waiting time for a battery exchange), and energy and environmental society costs.

## **Conclusions**

This paper argued that there are good reasons to believe that fleets of service vehicles of many organizations will transform their vehicles to electric vehicles (EV), where each service vehicle would "refuel" by exchanging its spent batteries with charged ones. The limited driving range of an electric vehicle's set of charged batteries adds complication to operating the fleets since refueling stops, and potential detouring just for refueling, must be explicitly taken into account in routing and scheduling of each EV when designing the fleets daily task schedule. In the context of public transportation, the "tasks" each bus performs are the servicing of several bus routes.

This paper addressed the optimization and analysis of infrastructure design alternatives dealing with (1) the number of battery-exchange stations, (2) their locations, (3) the recharging capacity and inventory management of batteries at each facility, and (4) the routing and scheduling of the EV fleet. With regard to routing of vehicles from point to point, the *EV shortest route problem* (EV-SRP) was defined and modeled as an optimization problem. When the number of stops is unconstrained, the EV shortest route can be found in polynomial time. However, the more interesting and more difficult cases of EV-SRP are when refueling stops are penalized with a “stop cost” representing, for example, the price of each battery exchange. Then we need to consider both the distance cost and the stop cost to find the optimal route. When stops are very costly then the problem becomes that of minimizing the number of battery exchange stops to reach the destination and this may not be polynomially solvable, a problem that we are currently studying.

The problems of scheduling a fleet of EV vehicles to service routes at specific times, referred here as *Electric Vehicle Scheduling Problem (EV-VSP)*, was defined and formulated. By preprocessing the underlying network to include potential battery-exchange stops, we developed a vehicle routing problem that can be shown to be NP hard. Even the problem to obtain a solution that is 150% of the optimal is NP-hard. Heuristics to solve the problem are now been studied.

Although there is some reported work on the location of battery-exchange (BE) facilities, it is surprising that these models do not explicitly consider detouring to exchange batteries. Rather all these papers focus on locating facilities along the origin-destination routes either to find minimum number to cover all the routes or to locate a given number to maximize the traffic volume covered. The paper presented some preliminary models for locating BE facilities on trees where detouring for battery exchanges are allowed.

Finally the paper briefly addressed the combined problem of sizing a facility in terms of number of battery-pallet chargers (BPC) it has and of inventory management of battery pallets at the facility. Preliminary simulation models showed that smaller percentage of customers leave before being served if numbers of BPCs is increased, or the number of battery-pallets in inventory is increased. The authors are currently developing analytical models that may be used in configuring a network of facilities in terms of sizes and locations.

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