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A branch-and-price algorithm for the long-term home care scheduling problem

M. Gamst and T. Sejr Jensen

Abstract In several countries, home care is provided for certain citizens living at home. The long-term home care scheduling problem is to generate work plans such that a high quality of service is maintained, the work hours of the employees are respected, and the overall cost is kept as low as possible. We propose a branch-and-price algorithm for the long-term home care scheduling problem. The pricing problem generates a one-day plan for an employee, and the master problem merges the plans with respect to regularity constraints. The method is capable of generating plans with up to 44 visits during one week.

1 Introduction

In several countries, home care is provided for certain citizens living at home. Home care offers cleaning, grocery shopping, help with personal hygiene and medicine, etc. The long-term home care scheduling problem is to generate work plans spanning a longer period of time, such that a high quality of service is maintained, the work hours of the employees are respected, and the overall cost is kept as low as possible.

Quality of service consists of the following. *Regularity*: all visits at a citizen should be conducted at the same time of the day and by the same (small group of) employee(s) in order for the citizen to feel safe. *Skill set requirements*: certain tasks can only be performed by a subset of the employees due to skill requirements.

All time windows are soft, i.e., preferred visit times and employee work hours can be violated at a cost. Such violations are denoted "busyness". The overall cost of a solution consists of a linear combination of travel time between visits, quality of service, and busyness.

The long-term home care scheduling problem is *NP*-hard and is typically approached in one of two ways in the literature: (1) plans for employees for a single day are generated. This corresponds to a modified VRPTW and is denoted the *daily*

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planning problem [1, 4, 5]. (2) Otherwise a Periodic VRPTW is solved, i.e., plans are made for several days, but regularity constraints are ignored [2, 6, 7].

We propose a branch-and-price (BP) algorithm for the full long-term home care scheduling problem. The pricing problem generates a one-day plan for an employee, and the master problem merges the plans with respect to regularity constraints. The method is capable of generating plans with up to 44 visits during one week. This truly illustrates the complexity of the problem.

2 Exact solution algorithm

The problem considers a given period of time consisting of $L \in \mathbb{N}$ days. Time is discretized into time steps, which together span L. The set of employees is denoted E. For each employee $j \in E$ is given a set of days H_j and time windows $[a_{jh}, b_{jh}], h \in H_j$, that specify the work hours of j.

The set of visits is denoted V, the set of citizens is C, and the set of visits at citizen c is $V_c \subseteq V$. Each visit $j \in V$ is repeated after p_j amount of time. The visit repetitions are scheduled independently of each other and are denoted *activities*. Let A be the set of all activities and $A_j \subseteq A$ the set of activities for visit $j \in V$. Two consecutive activities from visit j are denoted $(i,k) \in A_j$. An activity $i \in A$ has attached a prioritized list of employees to conduct the activity, denoted pr_i. The prioritized list represents the *skill set requirement*. The duration of $i \in A$ is d_i and the time window is $[a_i, b_i]$. The travel time between two activities $i, j \in A$ is $c_{ij} \geq 0$. Finally, busyness, i.e., the amount of time employee j is late for conducting visit i, is denoted $b_{ij} \geq 0$.

Dantzig-Wolfe decomposing the problem results in a pricing problem, which generates a daily schedule for a given employee on a given day, and a master problem, which merges the daily schedules into an overall feasible solution.

The daily schedule for an employee is denoted a *path* and contains an ordered list of activities with attached starting times. The overall solution consists of paths for appropriate employees on appropriate days, covering all activities. Let $s_i \in \mathbb{N}$ denote the starting time of activity $i \in A$. Recall that the objective function consists of travel time, quality of service, and busyness. We aggregate these into a single, weighted objective function. Let $\mathbf{w}$ be the non-negative weight vector. The objective function which is to be minimized consists of:

1. Travel time (TT) between activities.
2. Busyness (B), i.e., how late an employee j is for conducting an activity i with respect to travel times and time windows.
3. Employee priority (EP).
4. Employee regularity (ER), i.e., the number of different employees at a citizen.
5. Visit regularity (VR), i.e., if the time between two consecutive activities $(i,k) \in A_j$ differs from p_j.

Let p be a path and P the set of all generated paths. Let $x_p \in \{0,1\}$ denote whether or not path p is part of the solution, and let $u_{ik} \geq 0$ denote the difference in the starting times between two consecutive activities $(i,k) \in A_j$. Each path p has a number of constants attached: $\delta_{Bp}^{ij} \geq 0$ denotes the amount of busyness for employee j and activity i, $\delta_p^i \in \{0,1\}$ denotes if activity i is visited, $\delta_{sp}^i \geq 0$ denotes the start time at activity i, $\delta_p^{ij} \in \{0,1\}$ denotes if employee j visits activity i in the path, and $\delta_p^{jh} \in \{0,1\}$ denotes if the path is generated for employee j on day $h \in H_j$. The master problem is formulated as:

$$\min \sum_{p\in P}\sum_{i\in A}\Bigg(\sum_{k\in A} w^{TT} c_{ik}\delta_p^i\delta_p^k x_p + \sum_{j\in E}\big(w^B\ \delta_{Bp}^{ij}\ x_p + w^{EP}\ pr_i(j)\ \delta_p^{ij}x_p\big)\Bigg) + \sum_{c\in C}\sum_{j\in E} w^{ER}\ y_c^j + \sum_{j\in V}\sum_{(i,k)\in A_j} w^{VR}\ u_{ik} \tag{1}$$

$$\text{s. t.} \quad \sum_{p\in P}\delta_p^i x_p = 1 \qquad \forall i\in A \tag{2}$$

$$\sum_{p\in P}\delta_{sp}^i x_p + p_j - \sum_{p\in P}\delta_{sp}^k x_p \leq u_{ik} \qquad \forall j\in V,\ \forall (i,k)\in A_j \tag{3}$$

$$\sum_{p\in P}\delta_{sp}^k x_p - \Big(\sum_{p\in P}\delta_{sp}^i x_p + p_j\Big) \leq u_{ik} \qquad \forall j\in V,\ \forall (i,k)\in A_j \tag{4}$$

$$\sum_{p\in P}\delta_p^{ij} x_p \leq y_c^j \qquad \forall c\in C,\ \forall v\in V_c,\ \forall i\in A_v,\ \forall j\in E \tag{5}$$

$$\sum_{p\in P}\delta_p^{jh} x_p \leq 1 \qquad \forall j\in E,\ \forall h\in H_j \tag{6}$$

$$x_p\in\{0,1\} \qquad \forall p\in P \tag{7}$$

$$u_{ik}\geq 0 \qquad \forall j\in V,\ \forall (i,k)\in A_j \tag{8}$$

$$y_c^j\in\{0,1\} \qquad \forall c\in C, \forall j\in E \tag{9}$$

The objective function (1) minimizes a weighted sum of travel times, busyness, employee priorities, employee regularity and visit regularity. Constraints (2) ensure that every activity is visited. Constraints (3) and (4) measure visit regularity. Constraints (5) measure employee regularity. Constraints (6) ensure that at most one path per employee per day is part of a solution. The number of columns in the master problem is reduced by fixing daily visits to days: if a visit must be repeated every day, then the corresponding activities are fixed to day 1, 2, 3, etc., respectively. The pricing problem only allows such activities to be part of paths on appropriate days.

Let $\pi_i^{(2)} \in \mathbb{R}$ be the dual of constraints (2), $\pi_{ik}^{(3)} \leq 0$ the dual of (3), $\pi_{ik}^{(4)} \leq 0$ the dual of (4), $\pi_{icj}^{(5)} \leq 0$ the dual of (5), and $\pi_{jh}^{(6)} \leq 0$ the dual of (6). The pricing problem is solved for each employee $j \in E$ on each day $h \in H_j$. The reduced cost of visiting activity $i \in A_v, v \in V_c, c \in C$ is:

$$\bar{c}_{jh}^i = w^{EP}\ pr_i(j) - \pi_i^{(2)} - \pi_{icj}^{(5)} - \begin{cases} s_i(\pi_{ik}^{(3)} - \pi_{ik}^{(4)}) & \exists k\in A : (i,k)\in A_v \\ 0 & \text{otherwise} \end{cases}$$

Activity $i \in A$ is visited exactly once, hence the reduced cost for employee $j \in E$ on day $h \in H_j$ is defined as:

$$\bar{c}_{jh} = \sum_{i \in A} \left(\bar{c}^{i}_{jh} + w^B b_{ij} + \sum_{k \in A} w^{TT} \, c_{ik} \right) \leq \pi^{(6)}_{jh} \tag{10}$$

Recall that $b_{ij} \geq 0$ is the amount of busyness for employee j at activity i and that $c_{ik} \geq 0$ is the travel time between activities $i,k \in A$. Now, $\pi^{(6)}_{jh}$ is a constant, so if the pricing problem generates a path where $\bar{c}_{jh} \leq \pi^{(6)}_{jh}$, then the path has negative reduced cost and the corresponding column is added to the master problem. The pricing problem is recognized as a shortest path problem with time constraints and potentially negative edge weights. This is also denoted the Elementary Shortest Path Problem with Resource Constrained (ESPPRC), which is NP-hard. We solve the problem to optimality using the labeling algorithm in [3]. Initially, we try to solve the pricing problem heuristically using the labeling algorithm, where only a single label is stored at each activity.

Branching is necessary when the optimal solution in a branch node is fractional. The following strategy is finite and eventually ensures a feasible solution. Fractional solutions occur when:

An employee j visits a citizen c fractionally. Two branching children are generated with added cut: $\left(y^j_c = 0\right)$ resp. $\left(y^j_c = 1\right)$.

An activity i is visited by several employees j, j' or on several days h, h'. Two branching children are generated with rules: $\left(\sum_{p \in P} \delta^i_p \delta^{jh}_p x_p = 0\right)$ resp. $\left(\sum_{p \in P} \delta^i_p \delta^{j'h'}_p x_p = 0\right)$. The pricing problem ensures that employee j (resp. j') never visits activity i on day h (resp. h').

An employee j travels on edges $(ik), (ik')$ on a given day h a fractional number of times. Let i be the first activity, from which employee j travels on different edges or at different times. Let constant $\delta^{s_{ik}}_p$ denote whether or not path p travels from i to k at time s_{ik}. Two branching children are generated with the following rules: $\left(\sum_{p \in P} \delta^{jh}_p \delta^{s_{ik}}_p x_p = 0\right)$ resp. $\left(\sum_{p \in P} \delta^{jh}_p \delta^{s_{ik'}}_p x_p = 0\right)$. The pricing problem ensures that employee j never travels from i to k (resp. k') at time s_{ik} (resp. $s_{ik'}$) on day h.

The branching rules do not complicate the pricing problem, because they either consider different columns (y^j_c) or consist of rules, which can trivially be handled by forbidding appropriate extensions in the labeling algorithm.

3 Computational Results

The BP algorithm is implemented using the framework COIN Bcp [8] and tested on an Intel 2.13GHz Xeon CPU with 4 cores and 8 GB RAM. Note that test results stem from using a single core. CPLEX 12.1 is used as standard MIP solver.

The BP algorithm is tested on a number of real-life benchmark instances provided by Papirgrden, a home care center in Funen, Denmark. Time is discretized into either 5 or 10 minute time steps. The objective weights are as follows: $W^{TT} = 500$, $w^B = 750$, $w^{EP} = 0$, $w^{ER} = 750 \cdot 5/\tau$ and $w^{VR} = 50$, where τ denotes the size of a time step. We have tested two algorithms: (1) The exact BP algorithm as described and (2) a heuristic BP algorithm where columns are only generated heuristically.

Instance	Master Cols	Problem Rows	Tree Size	Gap	Sol. Value	Time Sec.	Master Cols	Problem Rows	Tree Size	Sol. Value	Tim Sec.
1-20-5	6327	199	1	0.00	27000.0	3.97	1794	199	1	38750.0	0.75
1-25-5	8671	250	4687	0.79	29750.0	1801.33*	608	250	1	44500.0	0.18
1-30-5	12883	295	2079	3.17	38000.0	1801.97*	2852	295	1	86000.0	1.45
1-40-5	28365	397	171	0.00	43000.0	812.94	903	397	1	67500.0	0.55
1-50-5	31480	493	27	110.91	87000.0	1803.35*	1235	493	1	97500.0	1.45
1-80-5	8737	787	1	333.89	259250.0	1906.50*	2954	787	1	259250.0	10.96
2-20-5	8917	346	5	0.00	27000.0	8.58	1917	346	1	31750.0	0.66
2-25-5	10089	432	2405	0.94	29750.0	1803.39*	1416	432	3	34750.0	0.27
2-30-5	16383	512	695	1.41	38400.0	1803.29*	5637	512	481	73500.0	117.37
2-80-5	8261	1354	1	297.49	237500.0	1908.73*	12677	1354	185	237500.0	388.09
1-20-10	4877	199	59	0.00	17250.0	6.37	1286	199	1	24750.0	0.29
1-25-10	5507	250	277	0.00	18125.0	27.36	324	250	1	29625.0	0.11
1-30-10	10934	295	2449	0.32	25300.0	1801.07*	1845	295	1	54250.0	0.71
1-40-10	14037	397	63	0.00	27750.0	90.16	733	397	1	48750.0	0.32
1-50-10	19907	493	3	0.00	36500.0	337.04	1002	493	1	67500.0	0.71
1-55-10	19912	544	3	0.00	60375.0	123.64	1409	544	1	86875.0	0.69
1-58-10	40434	571	111	73.60	64450.0	1803.82*	2270	571	1	106650.0	1.89
1-80-10	19711	787	1	224.63	161100.0	1847.78*	2255	787	1	161100.0	4.44
2-20-10	5913	346	39	0.00	17250.0	9.19	1770	346	1	20750.0	0.43
2-25-10	6594	432	81	0.00	18125.0	16.30	714	432	3	23125.0	0.15
2-30-10	10109	512	1245	0.20	25200.0	1802.52*	4585	512	751	48000.0	115.49
2-40-10	15937	684	77	0.00	27250.0	162.93	3267	684	7	37500.0	1.00
2-50-10	25457	850	21	0.00	35750.0	654.53	6914	850	191	61250.0	58.38
2-55-10	29331	936	49	133.70	80625.0	1841.08*	6090	936	67	80625.0	21.94
2-58-10	21593	984	31	160.61	96750.0	1826.54*	7242	984	143	96750.0	76.81
2-80-10	22136	1354	3	201.76	149750.0	2475.74*	8552	1354	221	149500.0	265.50

Table 1 Results for the exact BP (left hand side) and heuristic BP (right hand side) for instances named "$|E|-|A|-\tau$", τ =time step size. Tests marked with * exceeded the 30 minute time bound.

Test results are summarized in Table 1. As can be seen from the left hand size of the Table, only few instances with 5 minute time steps can be solved to optimality within half an hour. A coarser discretization helps, but the BP algorithm still suffers from a large time usage. The number of columns is large, which is caused partly by large time windows and partly by busyness, i. e., that time windows may be violated. The tree size grows very large for some instances not solved to optimality, hence branching also constitutes a bottleneck. As can be seen from the right hand side of the Table, the heuristic BP algorithm is generally faster. Few instances suffer from large tree sizes and many columns, but the far majority of instances are solved in seconds. Unfortunately, the objective values generally suffer from the heuristic approach.

Improving the exact BP approach would require methods for reducing the number of columns and for improving the bounds to prune larger parts of the tree. The authors attempted stabilizing the value of dual variables using an interior point method [9], but with no avail. Other stabilization methods could be investigated, as better values for the dual variables could reduce the number of generated columns. The authors also tried different primal and incumbent heuristics for improving the bounds with little luck. Future work could continue on such heuristics or on changing the branching strategy.

4 Conclusion

In this paper, we presented a BP algorithm for the long-term home care scheduling problem. The *NP*-hard pricing problem consisted of calculating a work plan on a given day for a given employee and the master problem merged the plans into an overall optimal solution. The BP algorithm was tested on a number of real-life instances and was capable of only solving smaller instances due to the large number of combinations of visits, visit times and employees. This truly illustrates the complexity of the problem.

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References

1. Begur S. V., Miller D. M., Weaver J. R.: An Integrated Spatial DSS for Scheduling and Routing Home-Health-Care Nurses. Interfaces **27**, 35–48 (1997)
2. Claassen G.D.H., Hendriks Th.H.B.: An application of Special Ordered Sets to a periodic milk collection problem. European Journal of Operational Research **180**, 754 – 769, (2007)
3. Dumitrescu I., Boland N.: Improved preprocessing, labelling and scaling algorithms for the weight-constrained shortest path problem. Networks **42**, 135–153 (2004)
4. Eveborn P., Flisberg P., Rönnqvist M.: Laps Care - an operational system for staff planning of home care. European Journal of Operational Research **171**, 962–976 (2006)
5. Hansen A. D., Rasmussen M. S., Justensen T., Larsen J.: The Home Care Crew Scheduling Problem. In 1st International Conference on Applied Operational Research volume **1** of Lecture Notes in Management Science, 1–9 (2008)
6. Hemmelmayr V., Doerner K., Hartl R., Savelsbergh M.: Delivery strategies for blood products supplies. OR Spectrum **31**, 707–725 (2008)
7. Jang W., Lim H. H., Crowe T. J., Raskin G., Perkins T. E.: The Missouri Lottery Optimizes Its Scheduling and Routing to Improve Efficiency and Balance. Interfaces **36**, 302–313 (2006)
8. Lougee-Heimer R.: The Common Optimization INterface for Operations Research. IBM Journal of Research and Development **47**, 57–66 (2003)
9. Rousseau L.-M., Gendreau M., Feillet D.: Interior point stabilization for column generation. Operations Research Letters **35**, 660–668 (2007)