Computational Methods for Model Predictive Control

New Opportunities for Computational Scientists

Jørgensen, John Bagterp; Boiroux, Dimitri; Hovgaard, Tobias Gybel; Halvgaard, Rasmus Fogtmann; Skajaa, Anders; Gade-Nielsen, Nicolai Fog; Standardi, Laura; Sokoler, Leo Emil; Völcker, Carsten; Capolei, Andrea; Frison, Gianluca; Schmidt, Signe; Duun-Henriksen, Anne Katrine

Publication date:
2012

Document Version
Publisher's PDF, also known as Version of record

Computational Methods for Model Predictive Control

New Opportunities for Computational Scientists

John Bagterp Jørgensen
Dimitri Boiroux, Tobias Gybel Hovgaard, Rasmus Halvgaard,
Anders Skajaa, Nicolai Fog Gade-Nielsen, Laura Standardi,
Leo Emil Sokoler, Carsten Völcker, Andrea Capolei,
Gianluca Frison, Signe Schmidt, Anne-Katrine Dunn Henriksen
Technical University of Denmark

PARA 2012, Finlandia Hall, Helsinki, Finland
June 10-13, 2012

DTU Informatics
Department of Informatics and Mathematical Modeling

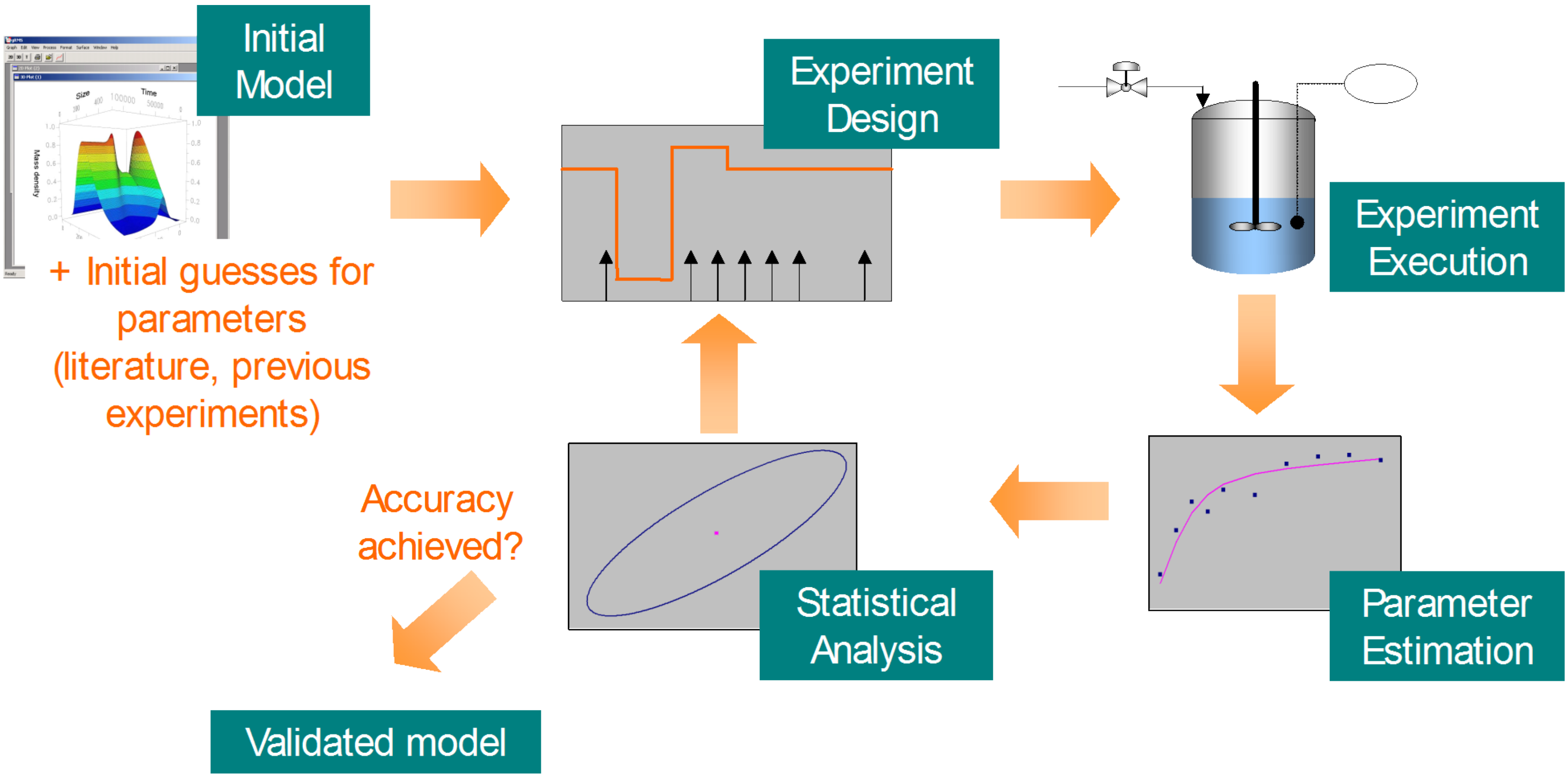
CERE
Center for Energy Resources Engineering

Introduction

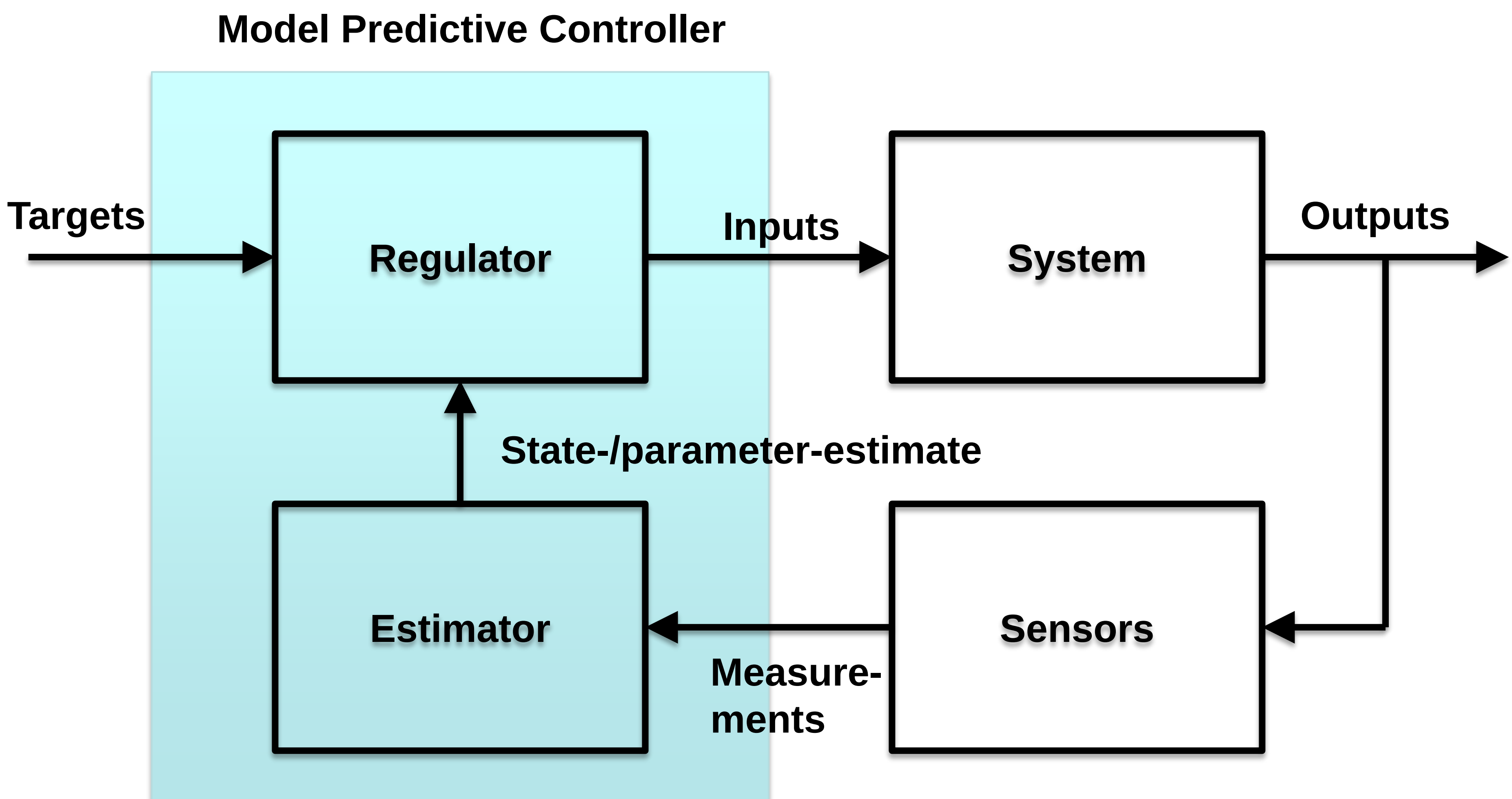
- Model Predictive Control
  - Optimal Experimental Design
  - Parameter Estimation
  - State Estimation
  - Regulation
- Constrained Optimization of Dynamical Systems
- Case Studies
  - The Artificial Pancreas
  - Closed-Loop Oil Reservoir Management
  - Smart Energy Systems

MODEL PREDICTIVE CONTROL

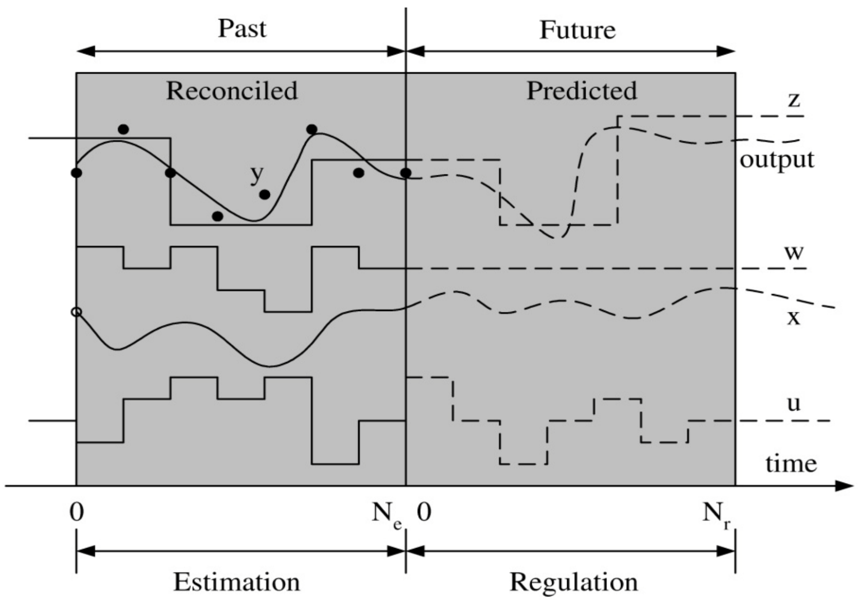
Systematic Model Building

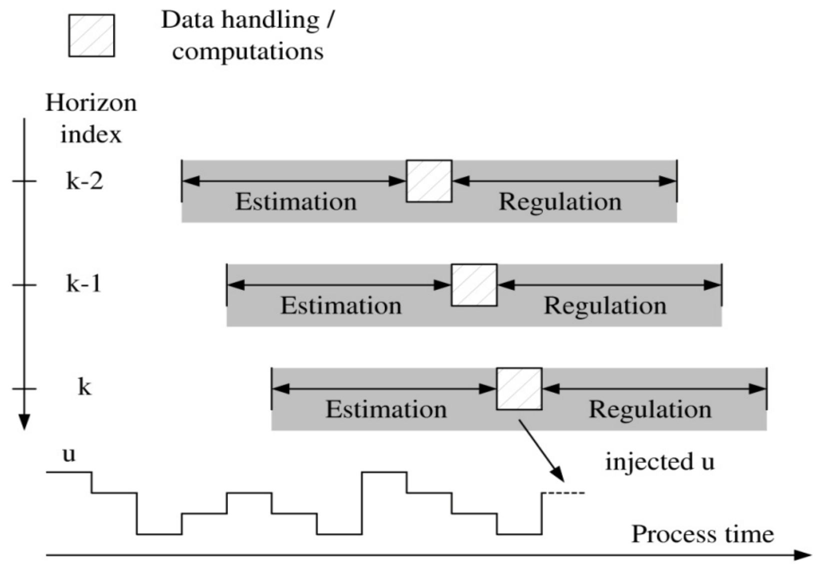


Model Predictive Controller



Model Predictive Control





MPC

=

Estimation + Regulation

Moving horizon implementation

Filtering and Prediction - EKF

$$d\boldsymbol{x}(t) = f(\boldsymbol{x}(t), u(t), \theta)dt + \sigma d\boldsymbol{\omega}(t)$$

$$\boldsymbol{y}(t_k) = g(\boldsymbol{x}(t_k), \theta) + \boldsymbol{v}(t_k)$$

Filtering

$$C_k = \frac{\partial g}{\partial x}(\hat{x}_{k|k-1}, \theta)$$

$$R_{k|k-1} = C_k P_{k|k-1} C_k' + R_k$$

$$K_k = P_{k|k-1} C_k' R_{k|k-1}^{-1}$$

$$e_k = y_k - g(\hat{x}_{k|k-1}, \theta)$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k e_k$$

$$P_{k|k} = P_{k|k-1} - K_k R_{k|k-1} K_k'$$

Prediction

$$\frac{d\hat{x}_k(t)}{dt} = f(\hat{x}_k(t), u(t), \theta) \qquad \hat{x}(t_k) = \hat{x}_{k|k}$$

$$\frac{dP_k(t)}{dt} = A(t)P_k(t) + P_k(t)A(t)' + \sigma\sigma' \qquad P_k(t_k) = P_{k|k}$$

$$A(t) = \frac{\partial f}{\partial x}(\hat{x}_k(t), \theta)$$

$$\hat{x}_{k+1|k} = \hat{x}_k(t_{k+1})$$

$$P_{k+1|k} = P_k(t_{k+1})$$

Optimal Control Problem

$$
\begin{aligned}
\min \quad & \phi = \int_{t_k}^{t_k+T} g(x(t), u(t))dt \\
s.t. \quad & x(t_k) = \hat{x}_{k|k} \\
& \dot{x}(t) = f(x(t), u(t)) && t \in [t_k,\, t_k + T] \\
& c(x(t), u(t)) \geq 0 && t \in [t_k,\, t_k + T]
\end{aligned}
$$

Integrator (Runge-Kutta Methods)

- DOPRI54 (non-stiff systems)
- ESDIRK12 / ESDIRK23 / ESDIRK34 (stiff systems)

Sensitivities

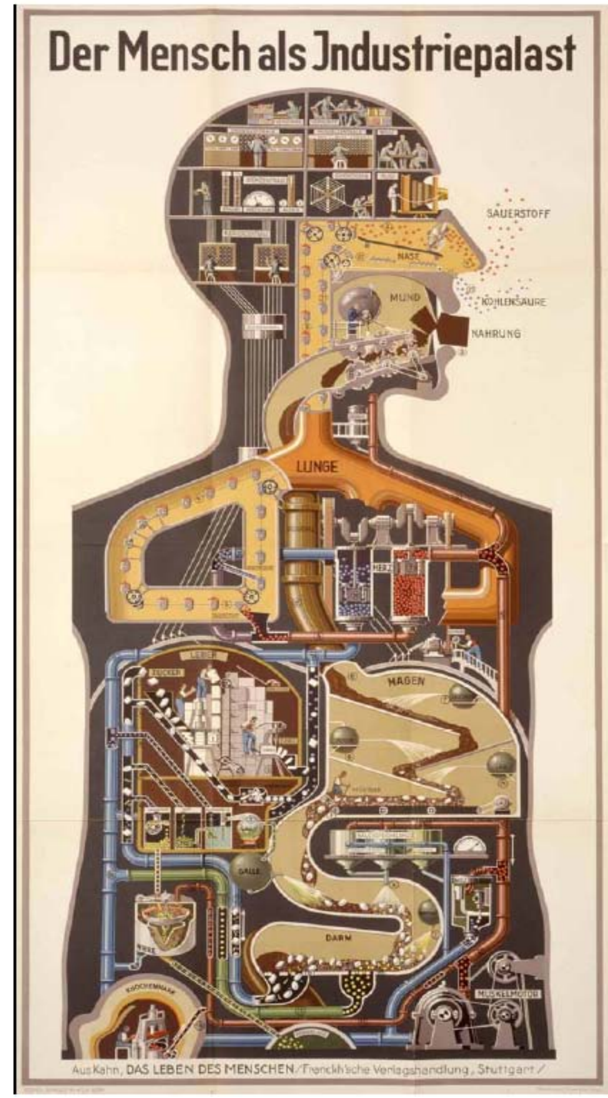
- Forward, Adjoint

Optimization

- SQP
- Single-shooting
- Multiple-shooting

Insulin Administration Systems
for
People with Diabetes

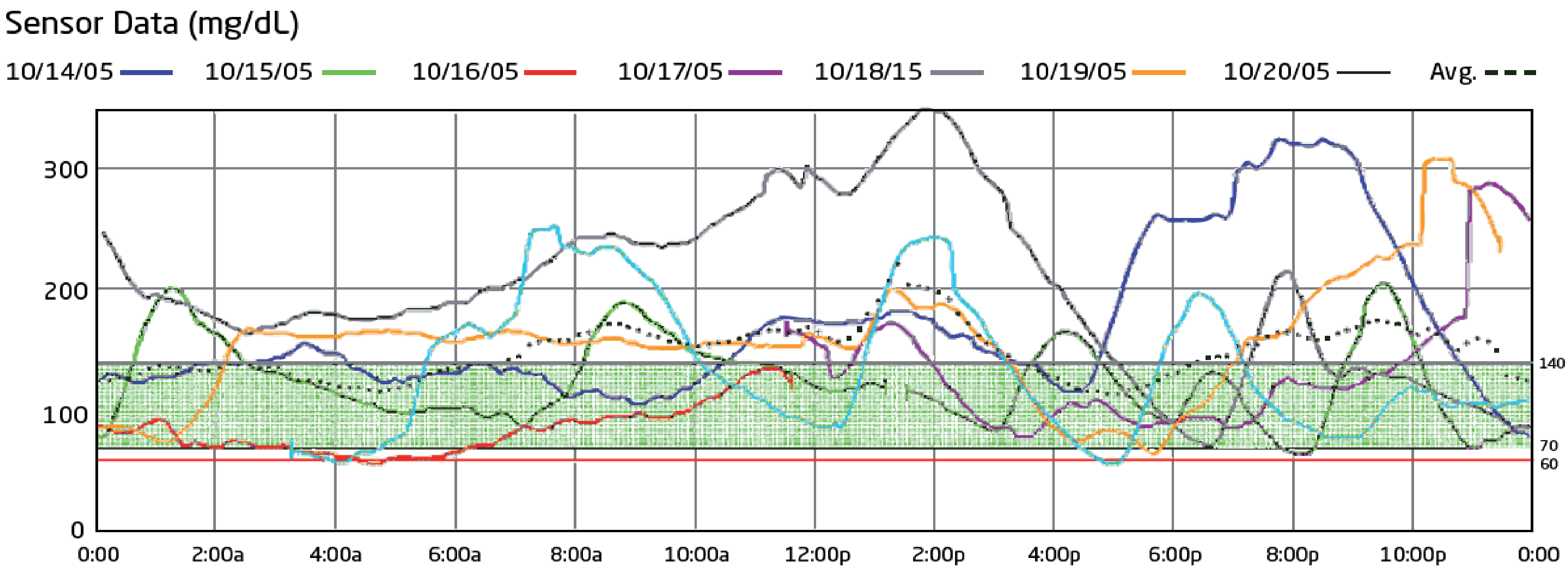
The Artificial Pancreas



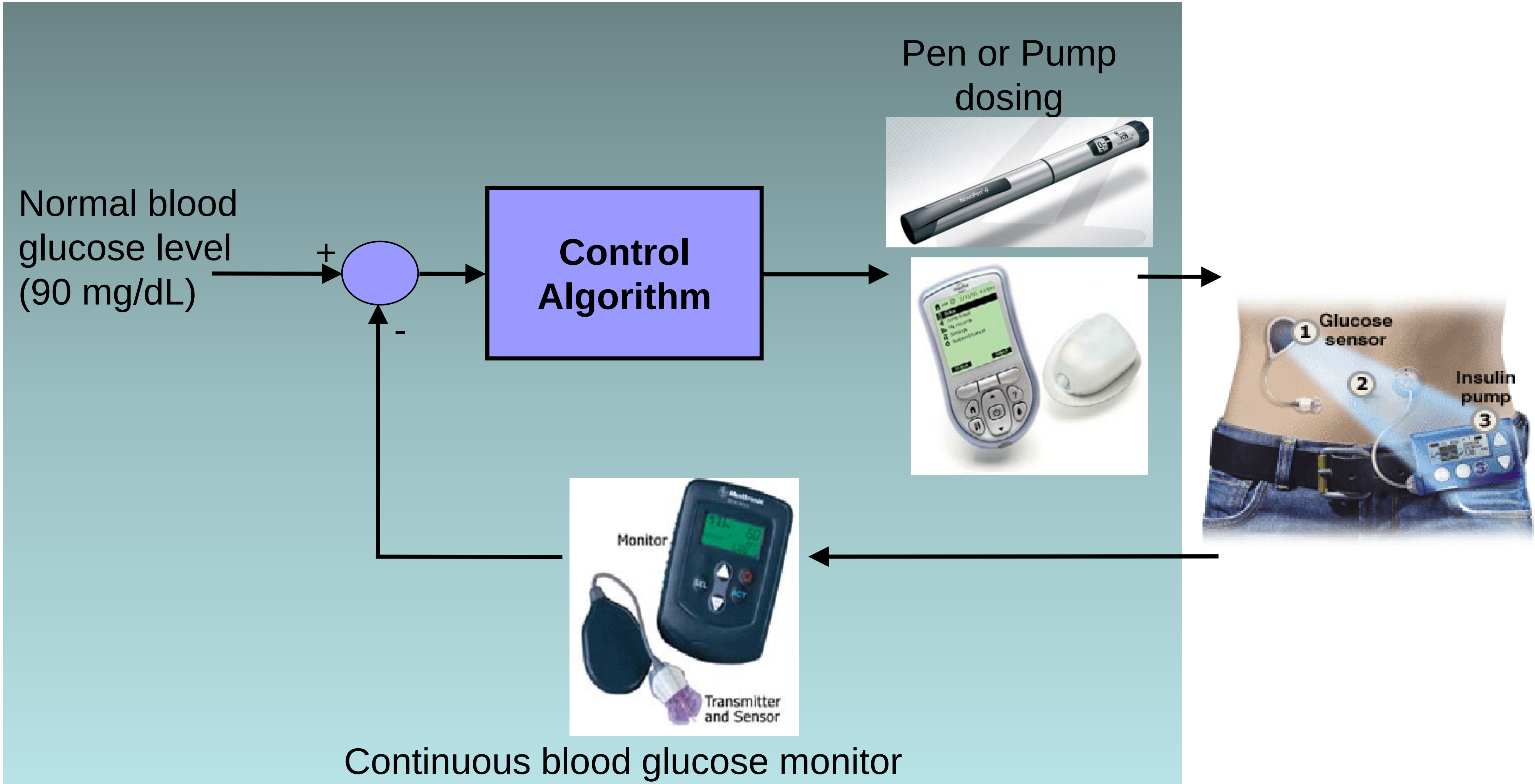
Blood glucose control

The blood glucose must stay within certain upper and lower bounds!

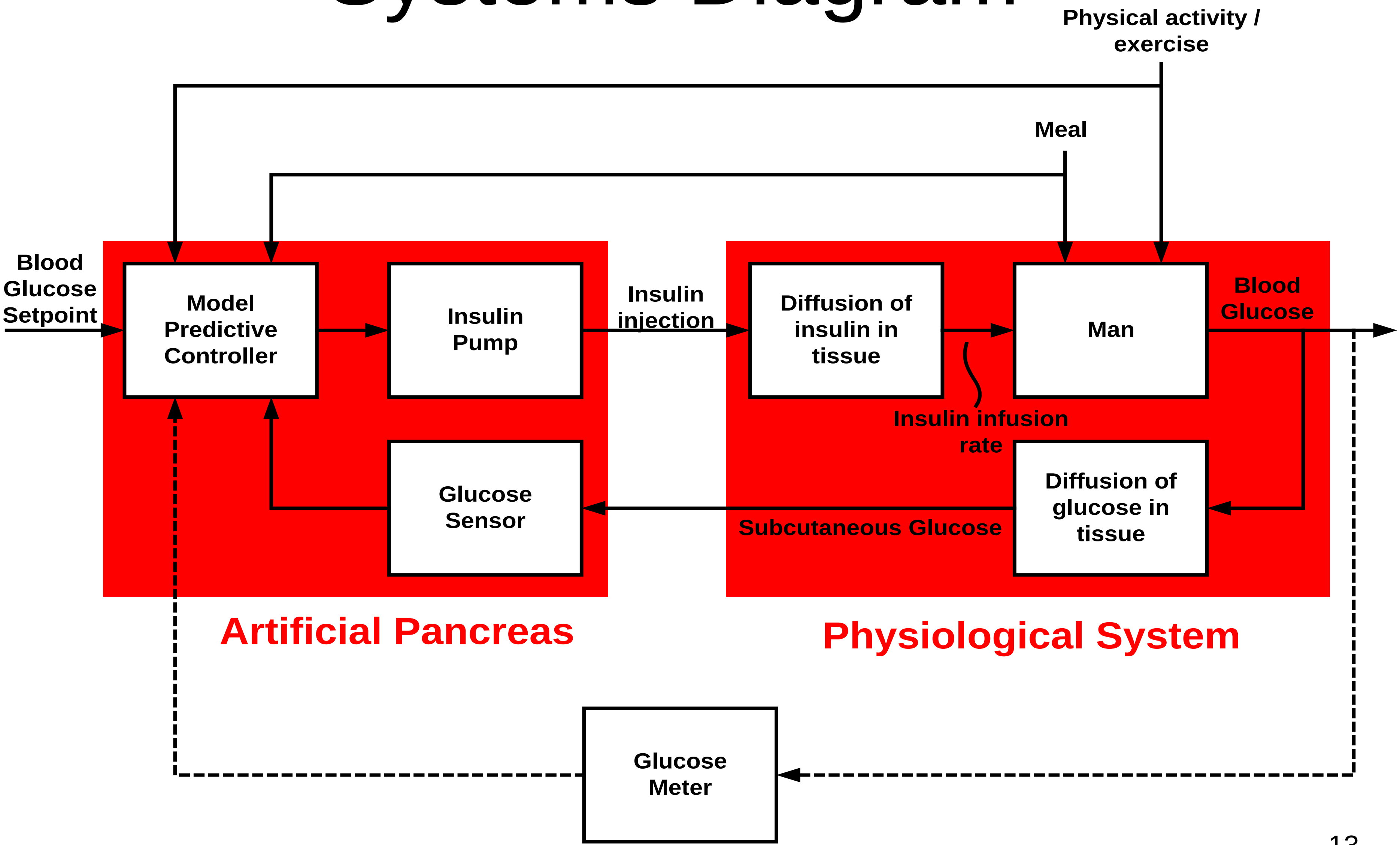
– Too low: coma (immediate effect)
– Too high: blindness and other long-term effects



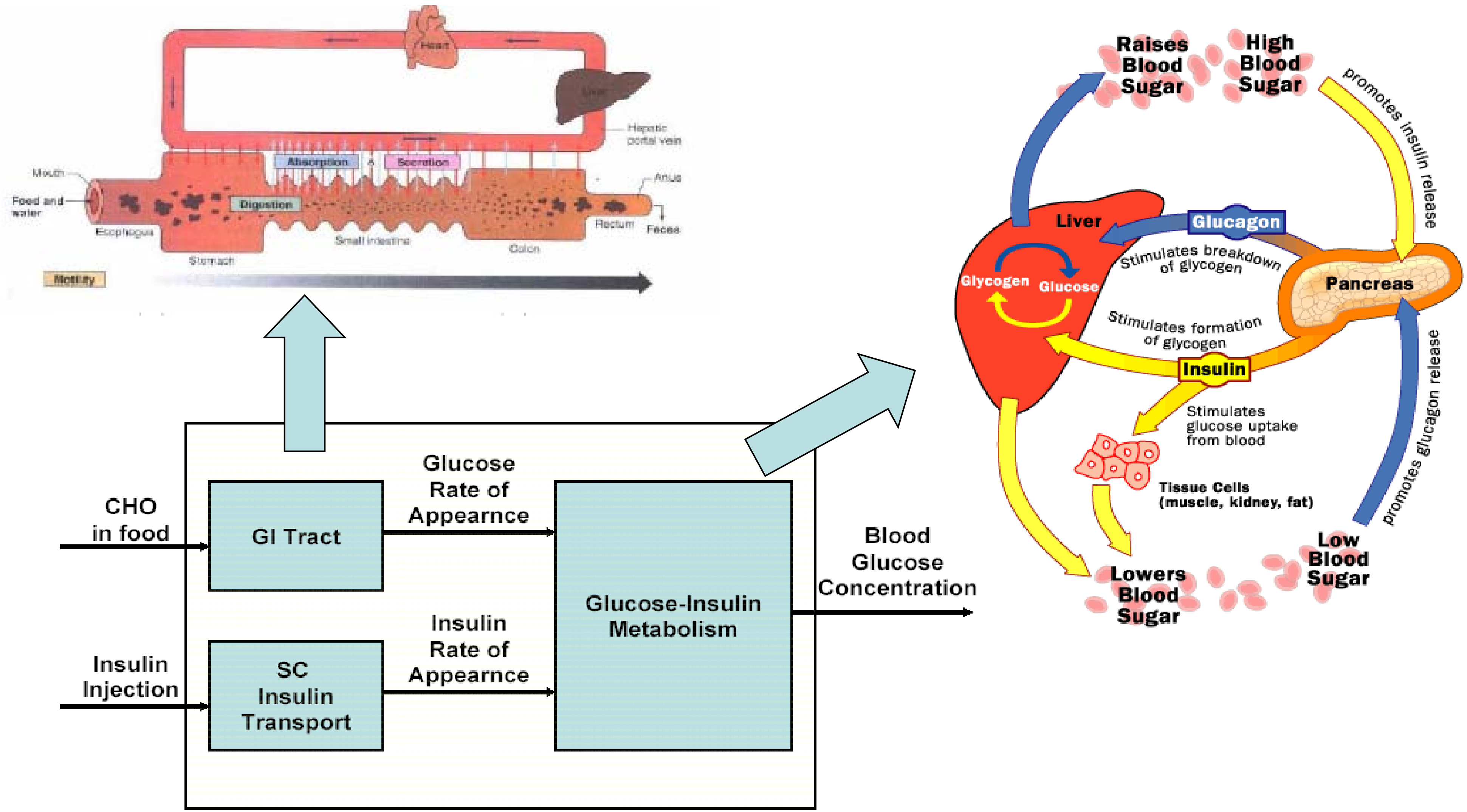
Insulin Administration Systems



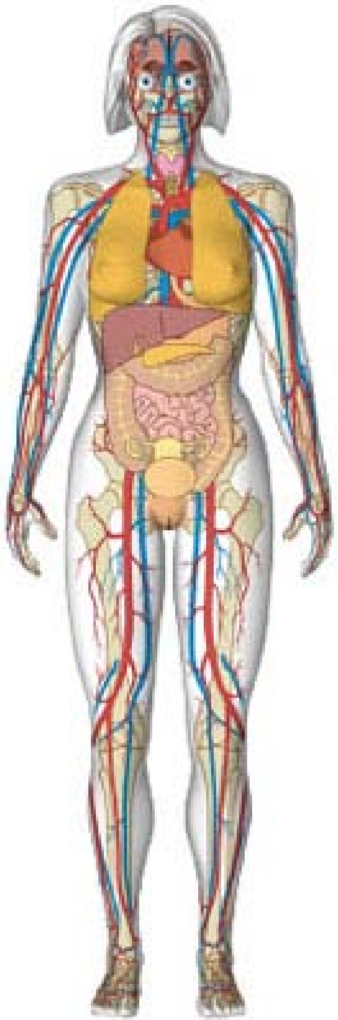
Systems Diagram

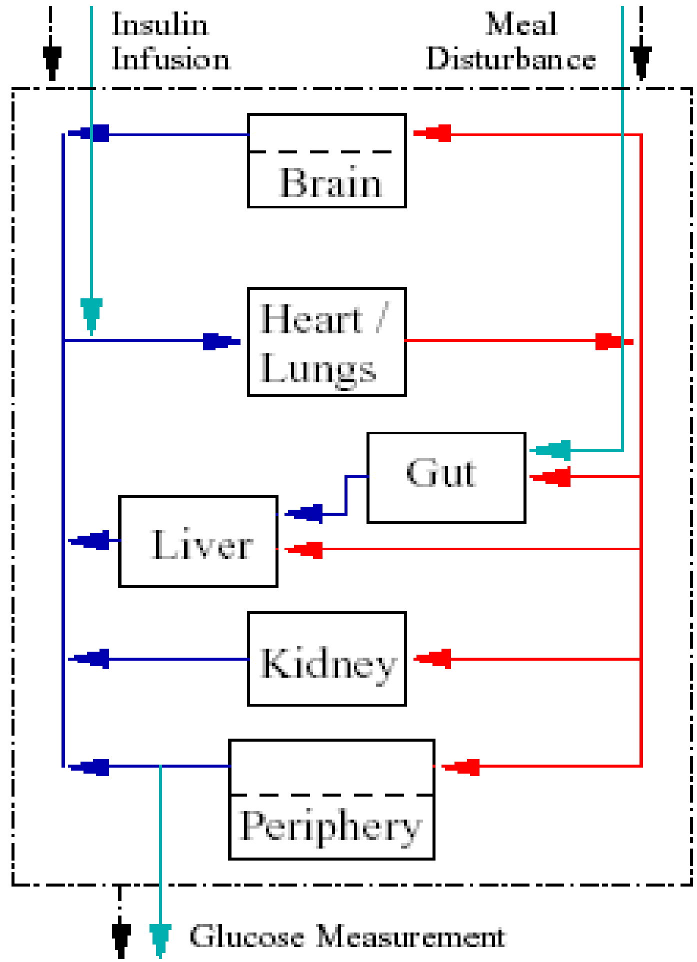


Physiological Models – The Virtual Patient

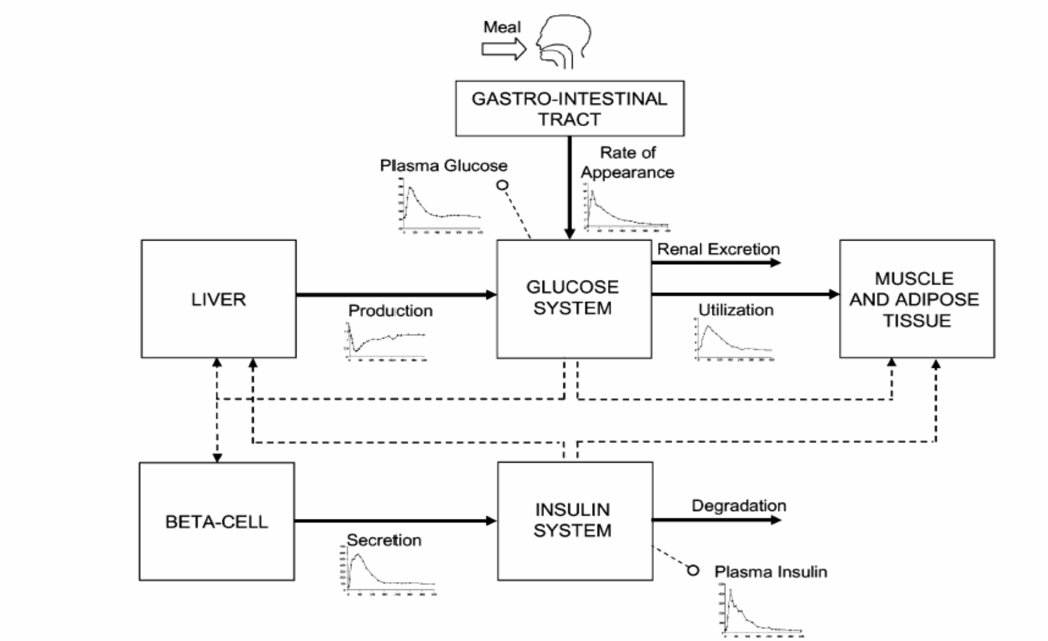


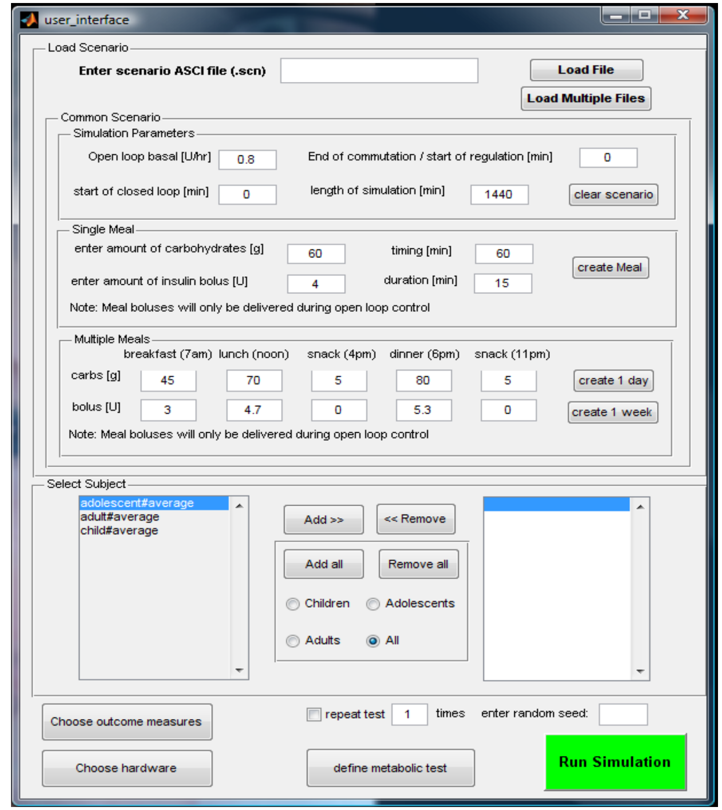
Physiological Models – A Compartment Model





The Cobelli Model – and the Virtual Clinic





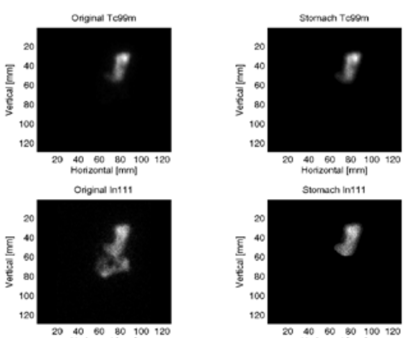
- Developed by
  - University of Virginia
  - University of Padova
  - Juvenile Diabetes Research Foundation
- 30 virtual patients in the version that we have a license to
- The full version including 300 patients approved to substitute animal tests by FDA

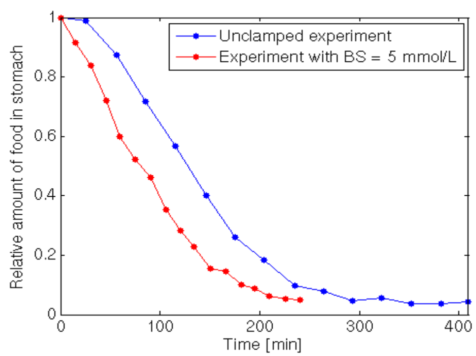
Clinical Trials

- Gastric emptying trials (left)
- Clinical trials collecting data for modeling (right)

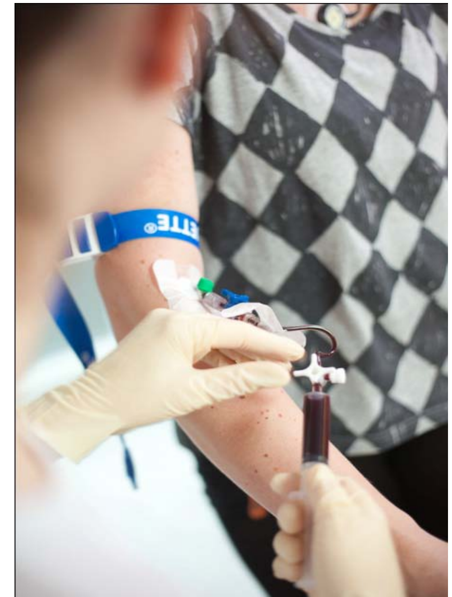


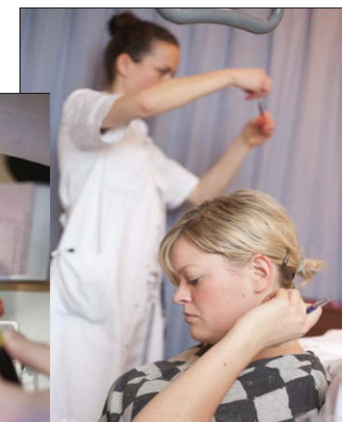


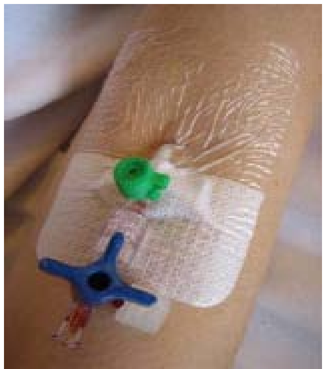









Example of a Clinical Experiment



8:00
Insert
lines

10:00
Meal/
bolus

12:30
Exercise

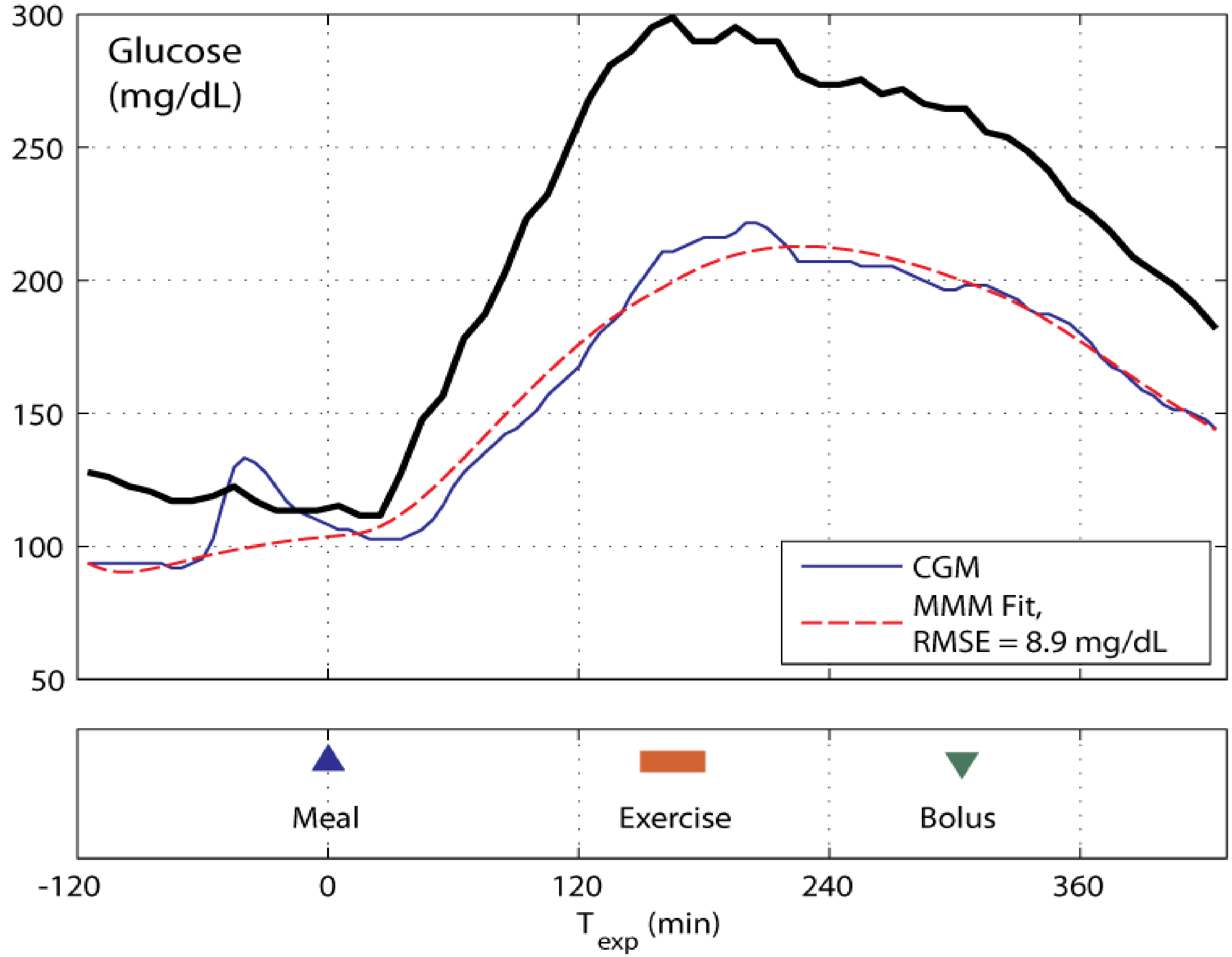
15:00
Snack

Identification Results

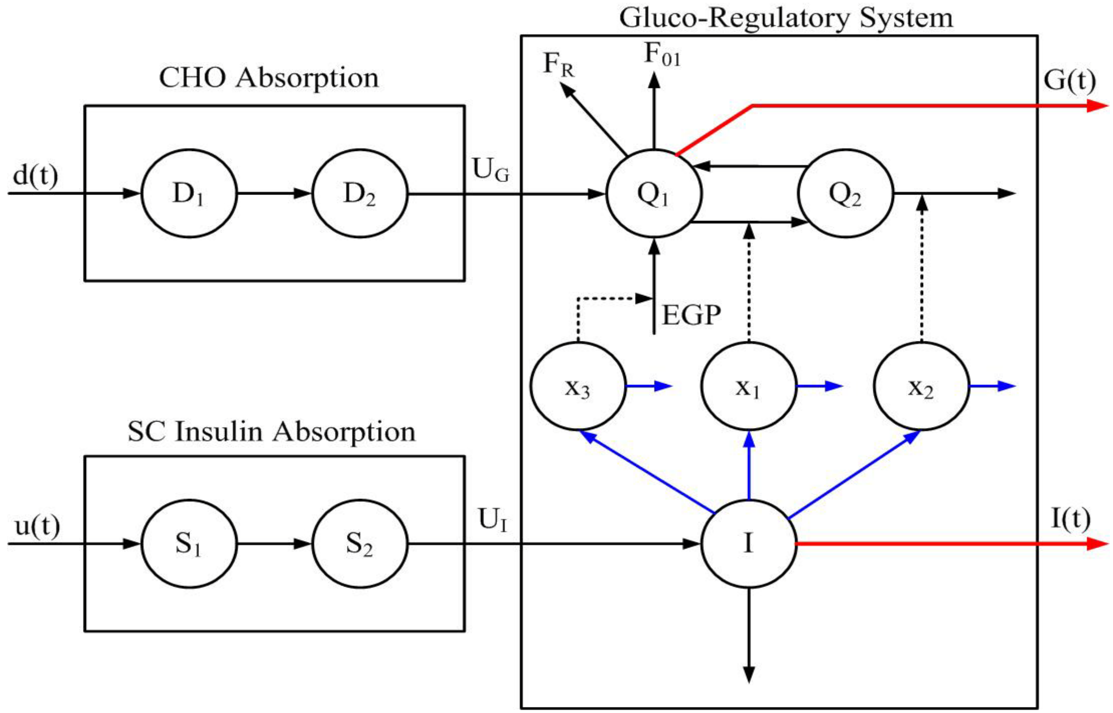
- Representative model fit

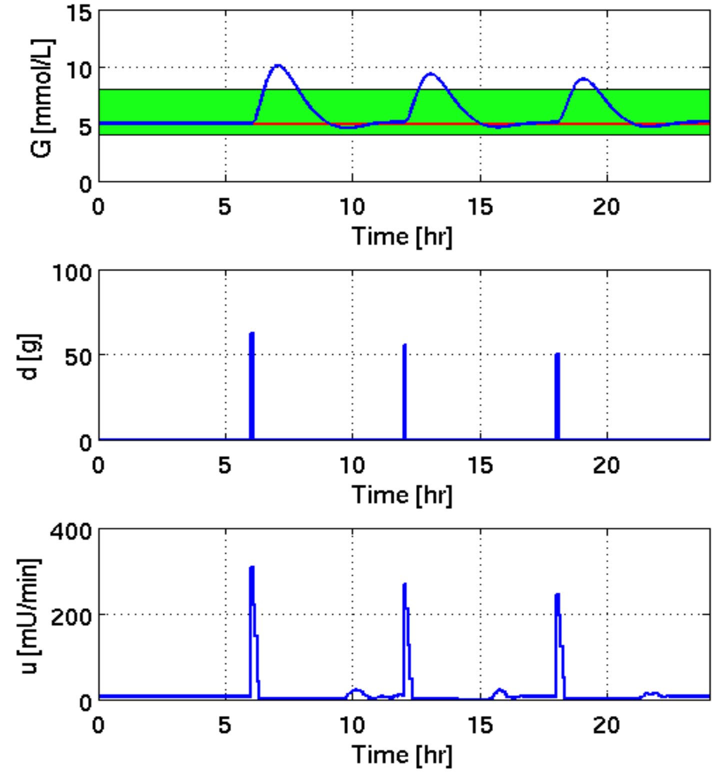
- Very accurate model fit

- **Note:**
  Model is fit to the CGM signal. The intravenous glucose concentration is also shown (thick line) for comparison



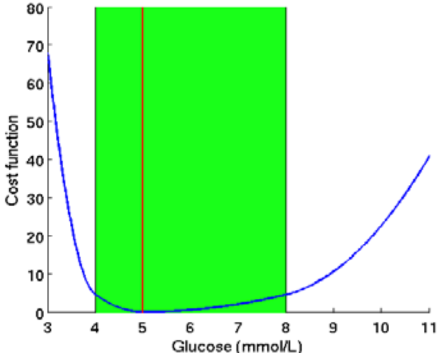
The DTU Type 1 Diabetes Model





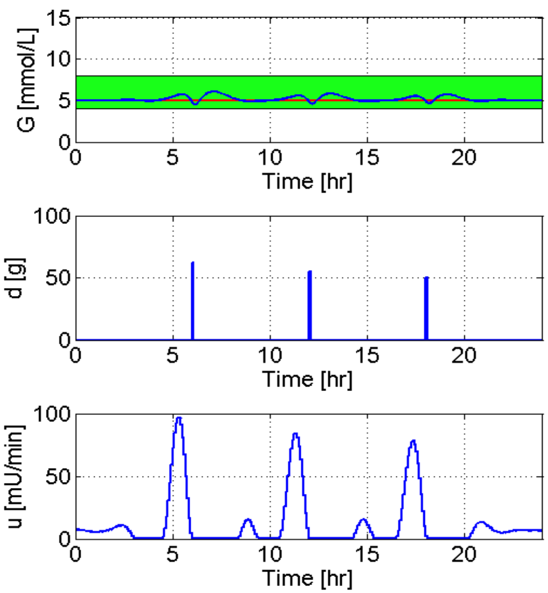
$$d\boldsymbol{x}(t) = f(x(t), u(t), d(t))dt + \sigma d\boldsymbol{\omega}(t)$$
$$\boldsymbol{y}(t_k) = g(\boldsymbol{x}(t_k)) + \boldsymbol{v}(t_k)$$

Insulin Administration Strategies

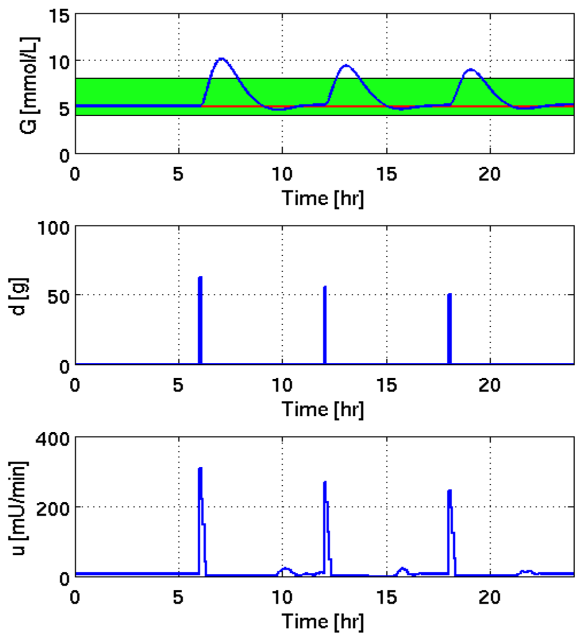


$$
\begin{aligned}
\min \quad & \phi = \int_{t_k}^{t_k+T} g(x(t), u(t))dt \\
s.t. \quad & x(t_k) = \hat{x}_{k|k} \\
& \dot{x}(t) = f(x(t), u(t), d(t)) \quad t \in [t_k,\, t_k + T] \\
& c(x(t), u(t)) \geq 0 \qquad\qquad\; t \in [t_k,\, t_k + T]
\end{aligned}
$$

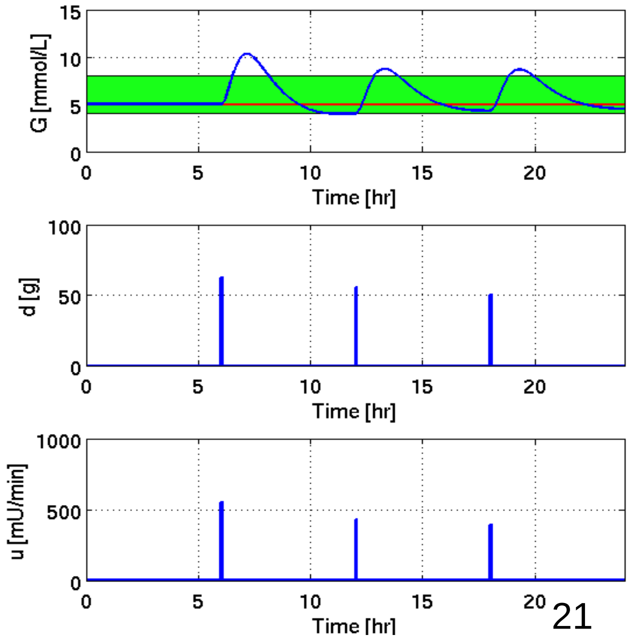
NMPC
Pre-meal Insulin Allowed



NMPC
No Pre-meal Insulin

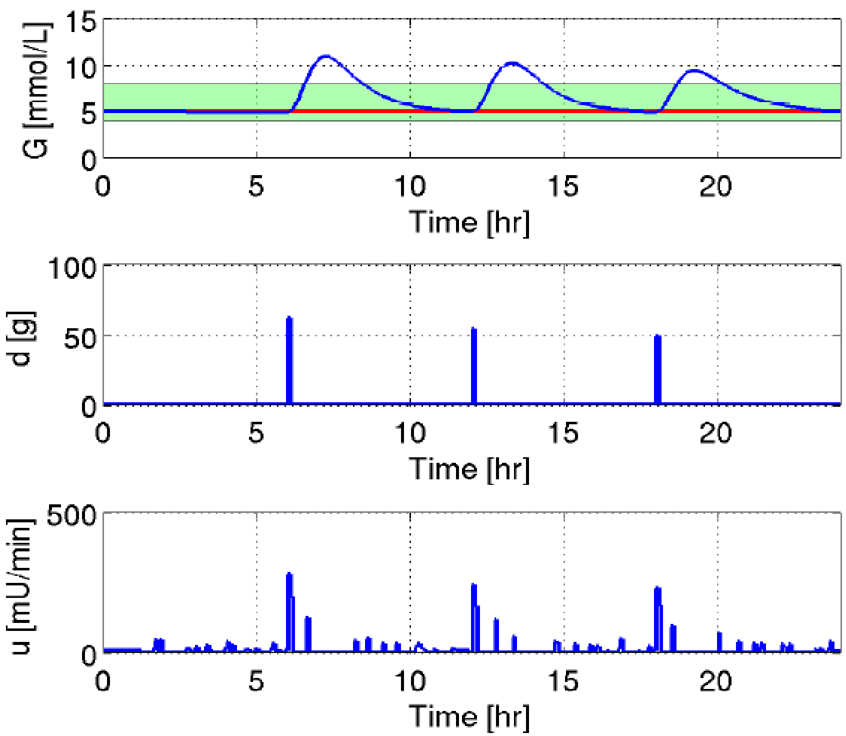


MDI (Pen Based) Insulin Treatment

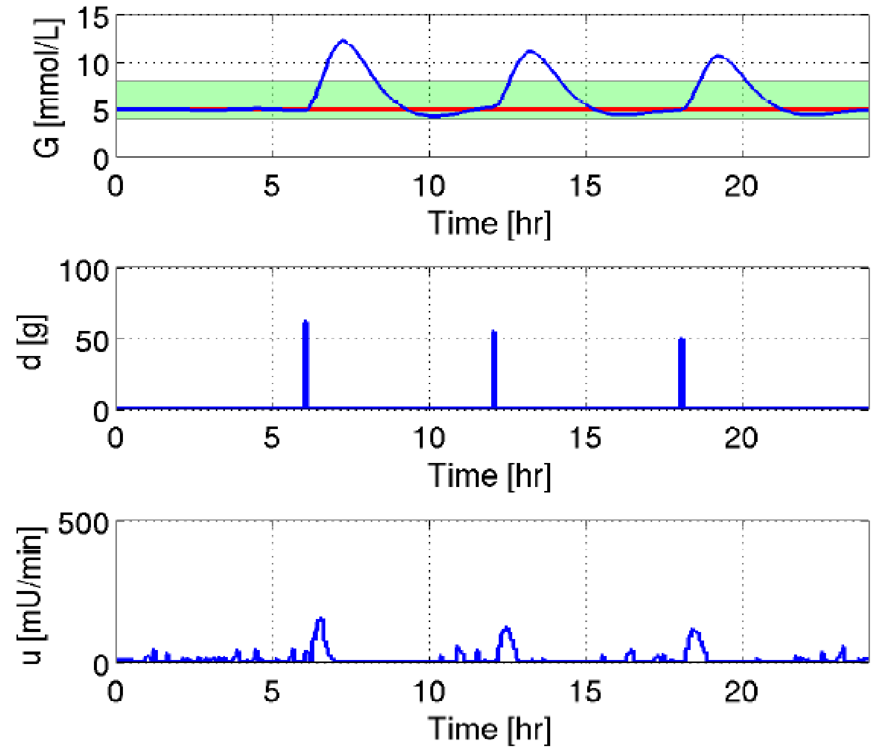


Closed-Loop Studies by NMPC

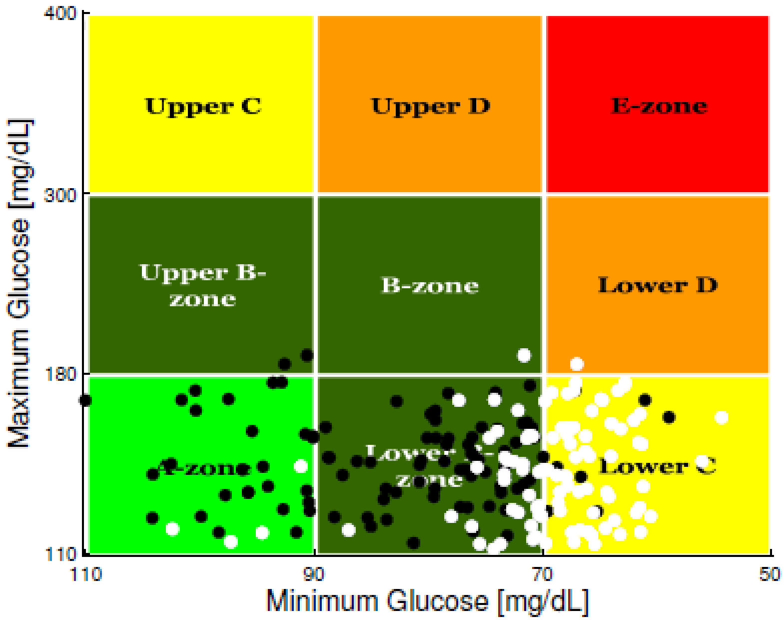
Meals announced at meal-time



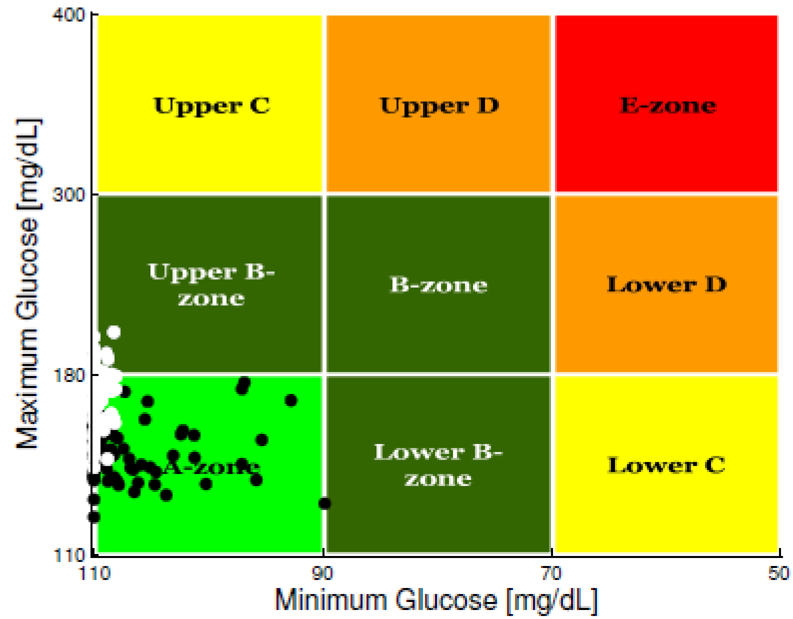
Meals not announced



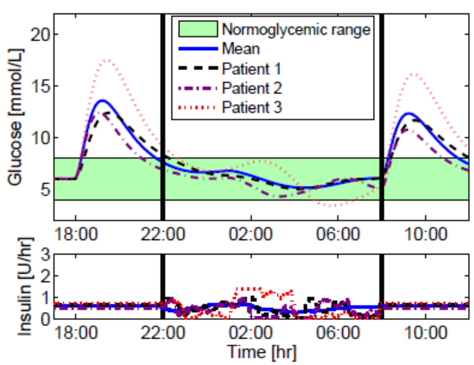
Overnight Stabilization – A Cohort



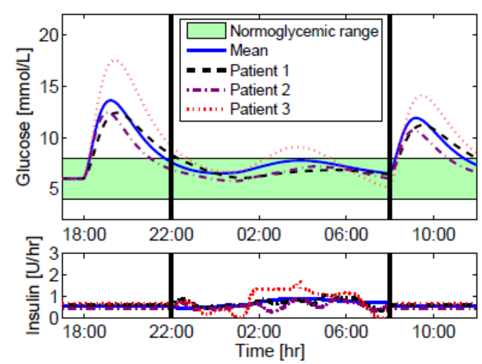
(a) Insulin sensitivity increases by 30%



(b) Insulin sensitivity decreases by 30%

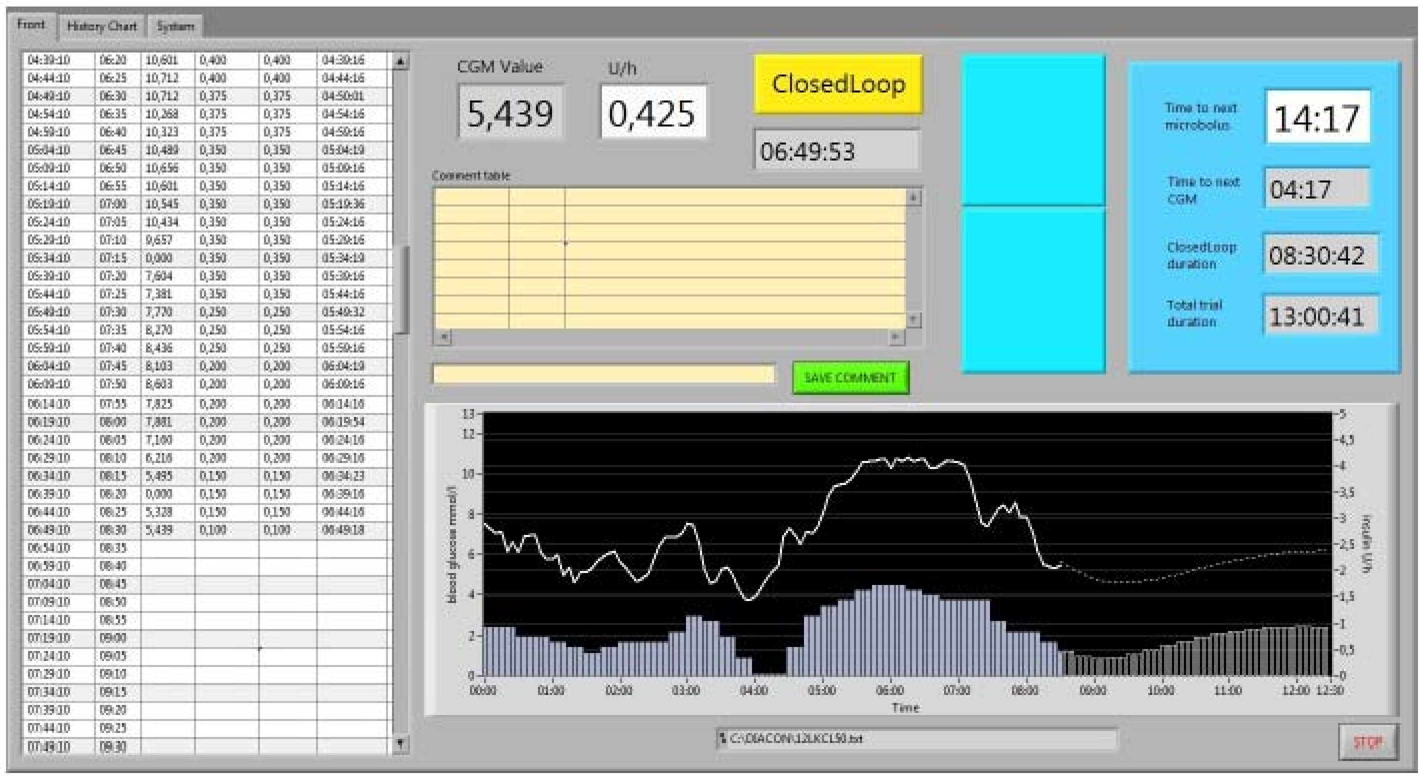


(a) Insulin sensitivity increases by 30%



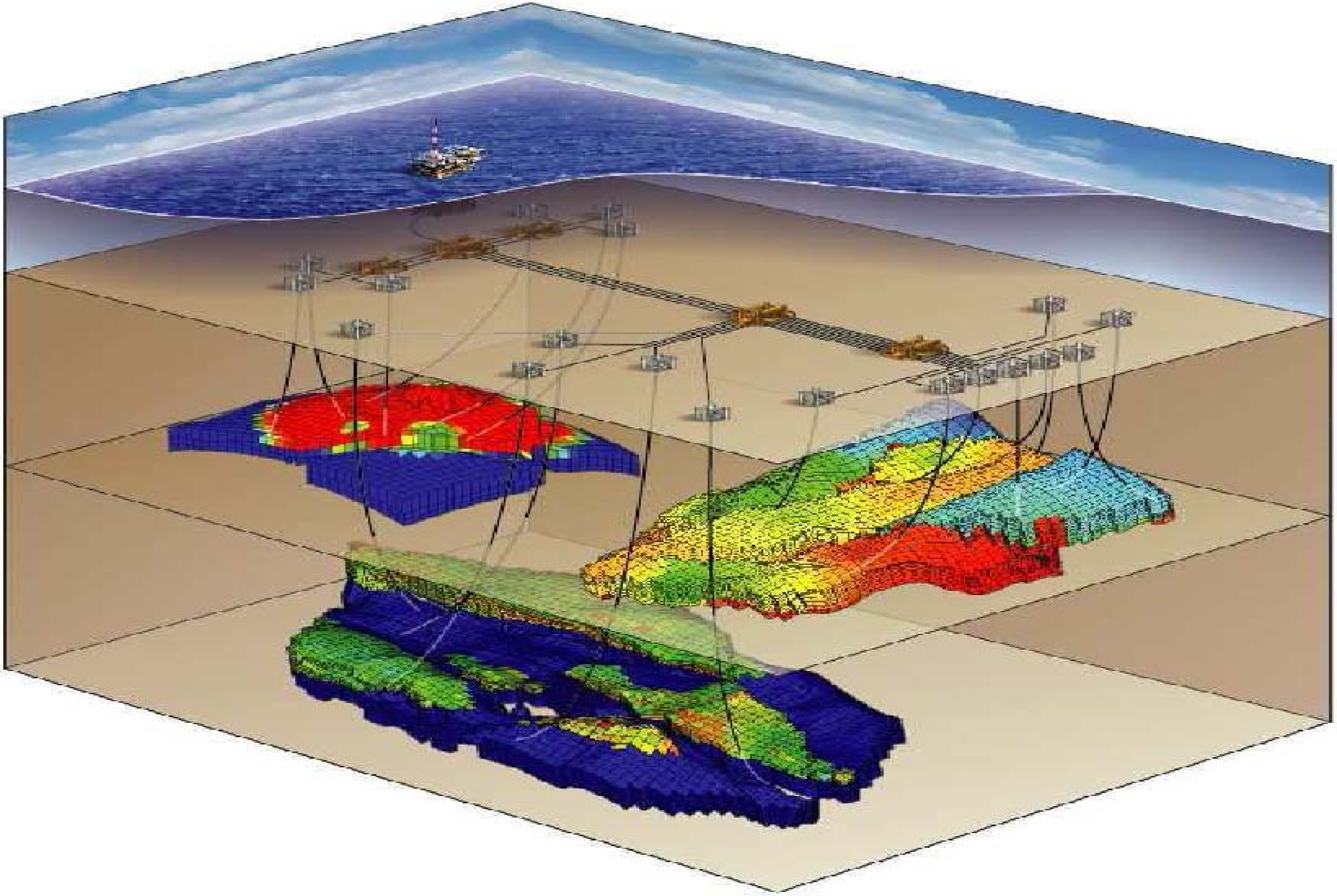
(b) Insulin sensitivity decreases by 30%

Clinical Closed Loop Studies

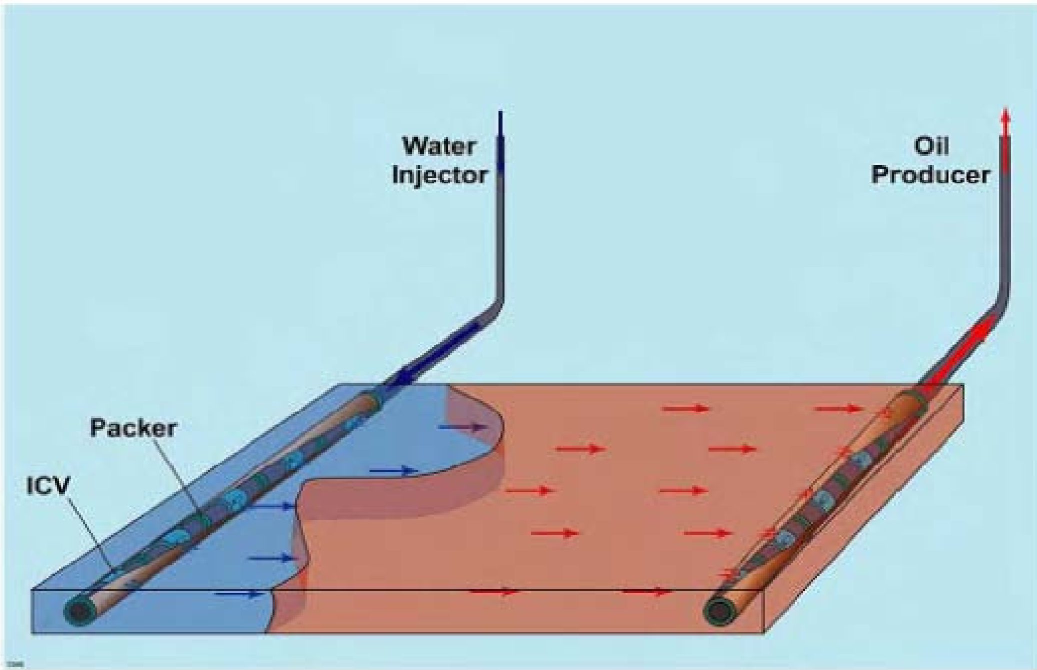


MODEL PREDICTIVE CONTROL FOR MANAGEMENT OF OIL RESERVOIRS

An Offshore Oil Reservoir



Mathematical Model



The mass conservation of water and oil

$$\frac{\partial}{\partial t} C_w(P_w, S_w) = -\nabla \cdot \mathbf{F}_w(P_w, S_w) + Q_w$$
$$\frac{\partial}{\partial t} C_o(P_o, S_o) = -\nabla \cdot \mathbf{F}_o(P_o, S_o) + Q_o$$

The mass concentrations

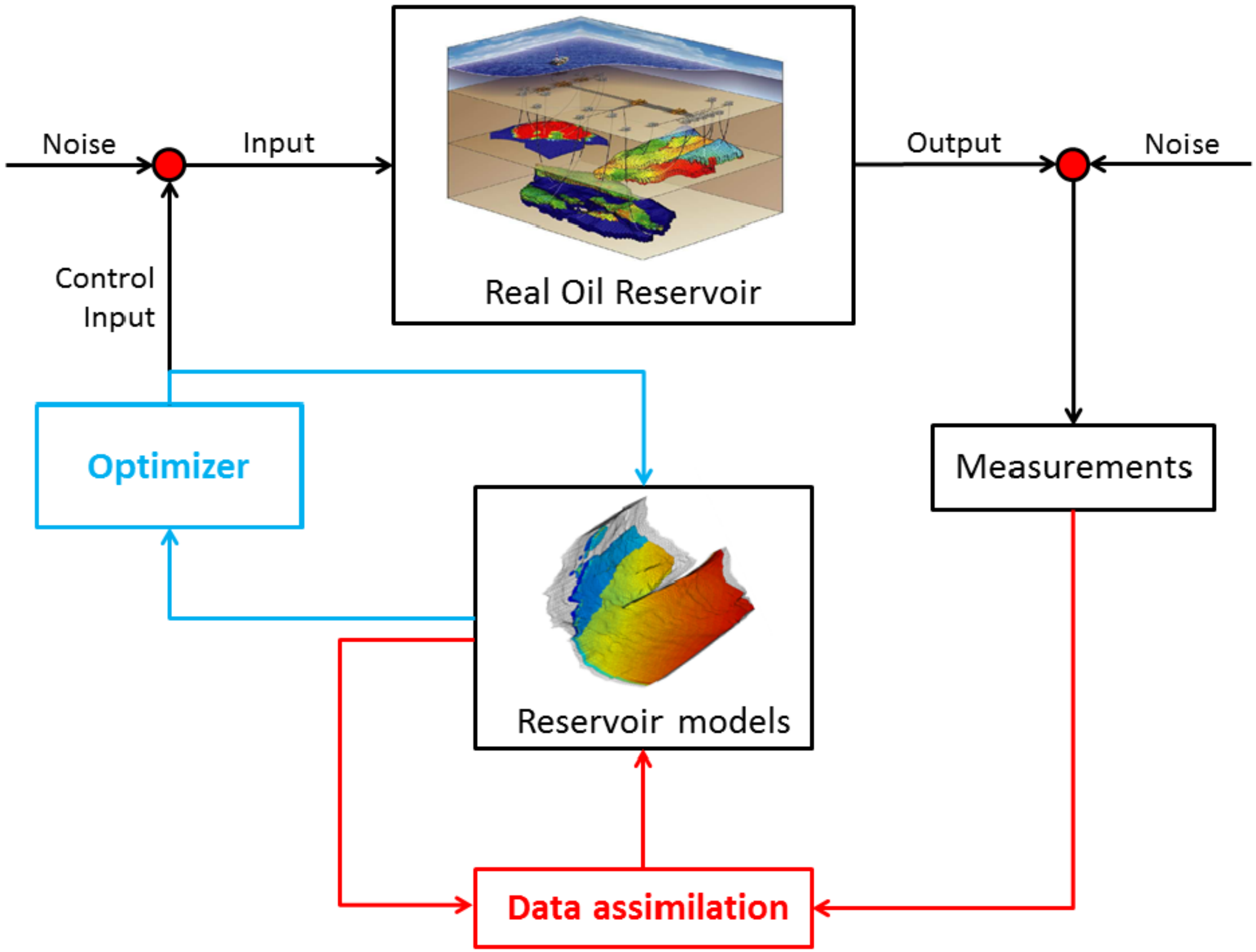
$$C_w = \phi \rho_w(P_w) S_w$$
$$C_o = \phi \rho_o(P_o) S_o$$

Fluxes through the porous medium

$$\mathbf{F}_w = \rho_w(P_w) \mathbf{u}_w(P_w, S_w)$$
$$\mathbf{F}_o = \rho_o(P_o) \mathbf{u}_o(P_o, S_o)$$

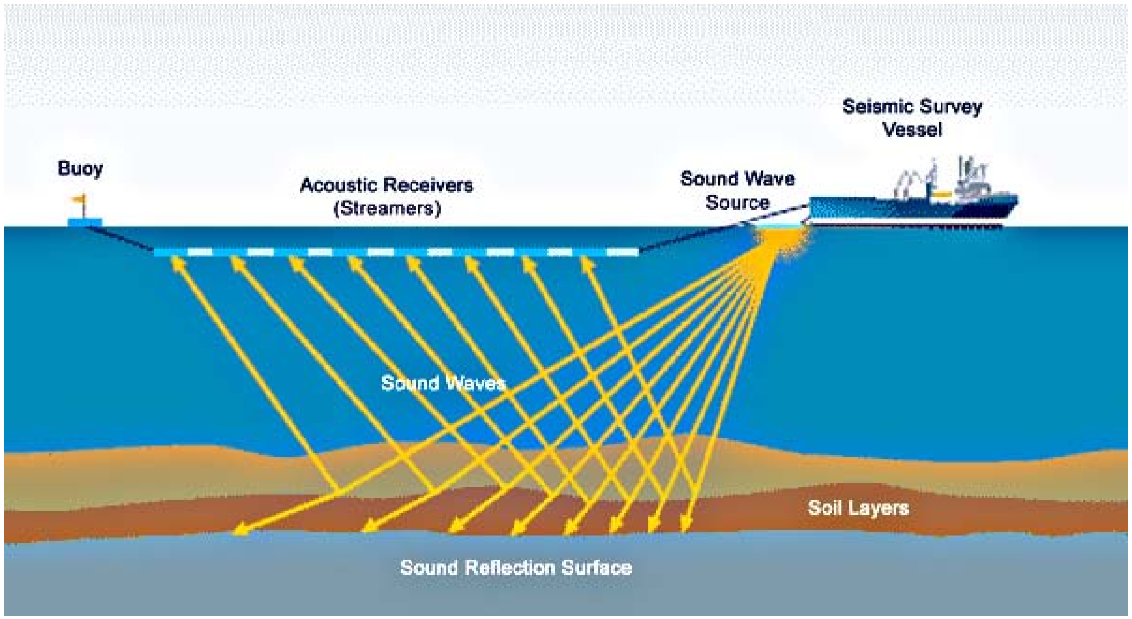
$$\frac{d}{dt} g(x(t)) = f(x(t), u(t))$$

Closed-Loop Reservoir Management

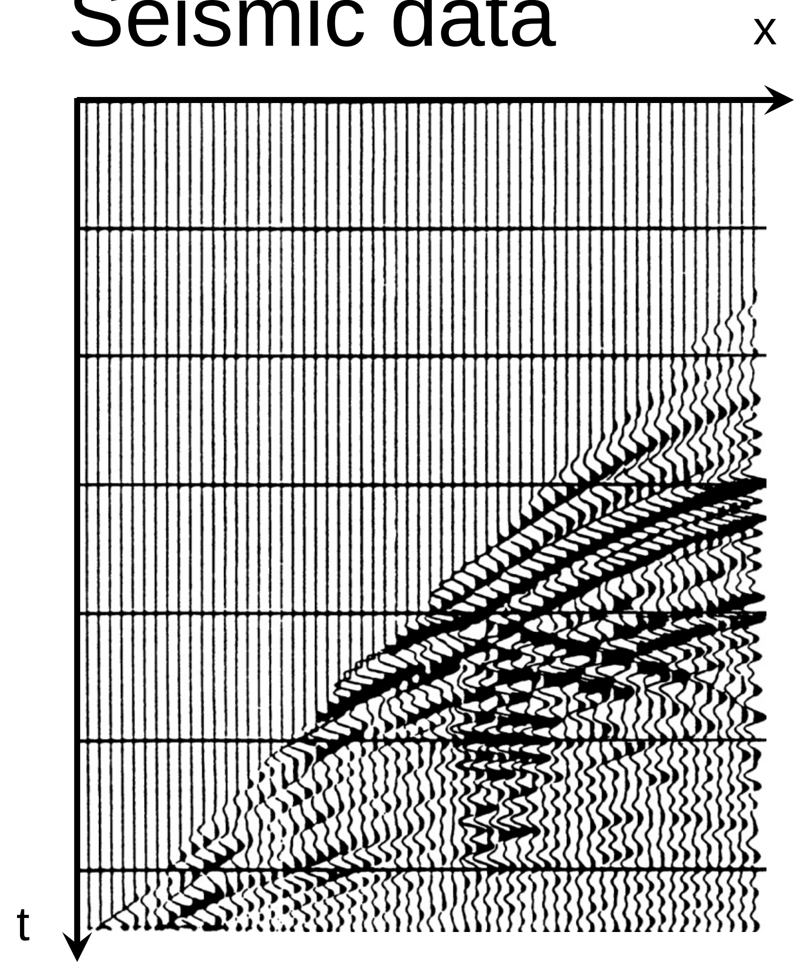


Applied Seismology: Subsurface Imaging

Experimental setup

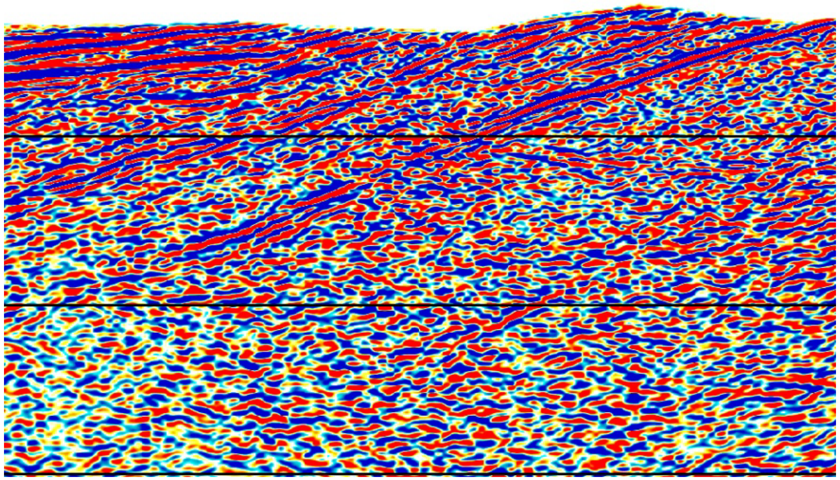


Seismic data

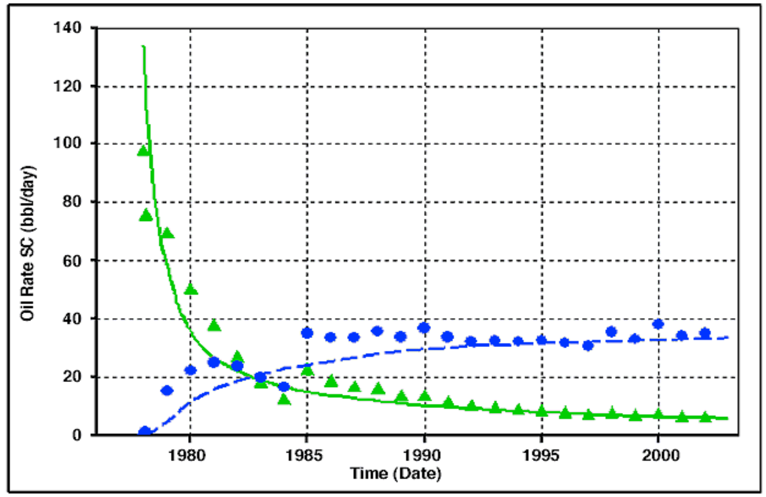


The Computational Challenge: Integrated Inversion of Seismic and Production Data

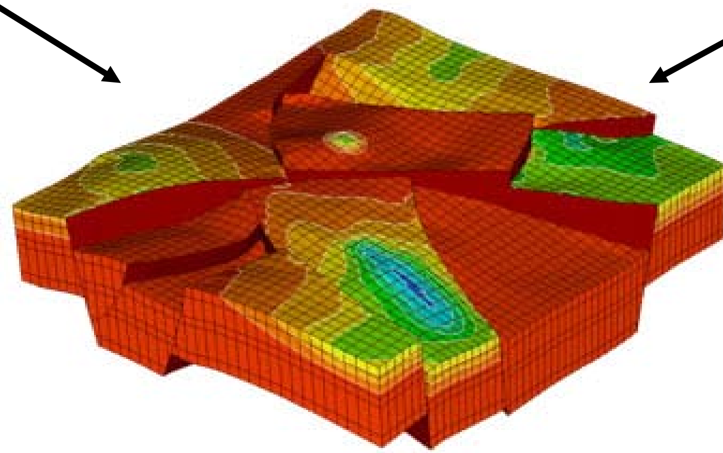
Seismic data



Oil production data



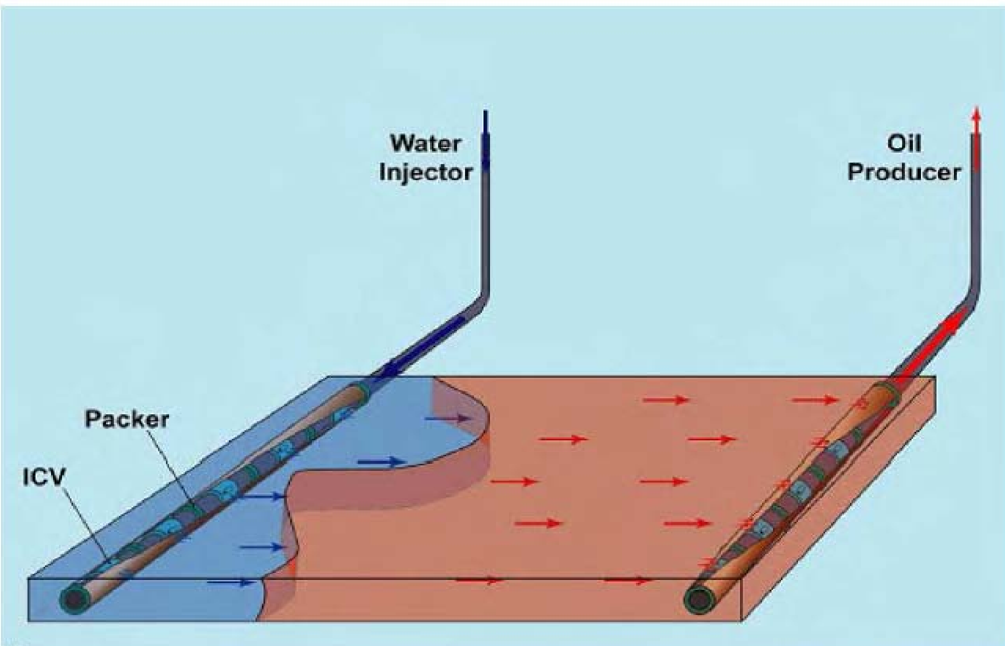
Seismic inversion

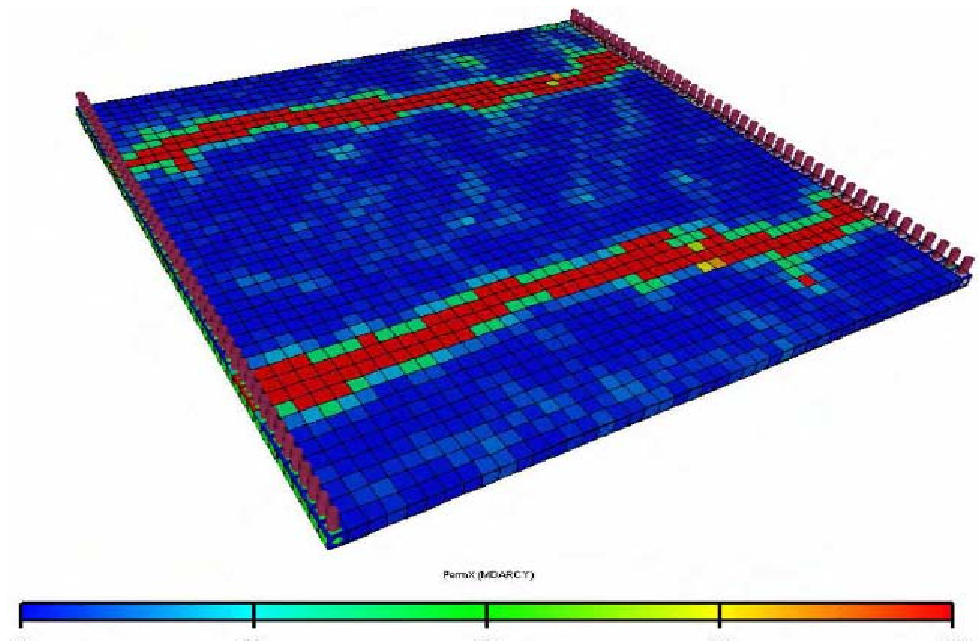


History matching

History Matching (State and Parameter Est) + Seismic Data

=> Permeability Field





Optimization Problem

$$
\begin{aligned}
\min_{[x(t),u(t)]_{t_0}^{t_f}} \quad & \int_{t_0}^{t_f} J(x(t),u(t))dt \\
\text{s.t.} \quad & \frac{d}{dt} g(x(t)) = f(x(t),u(t)) \\
& x(t_0) = x_0 \\
& u_{min} \leq u(t) \leq u_{max} \\
& -u_{min}^{\Delta} \leq \frac{d}{dt} u(t) \leq u_{max}^{\Delta}
\end{aligned}
$$

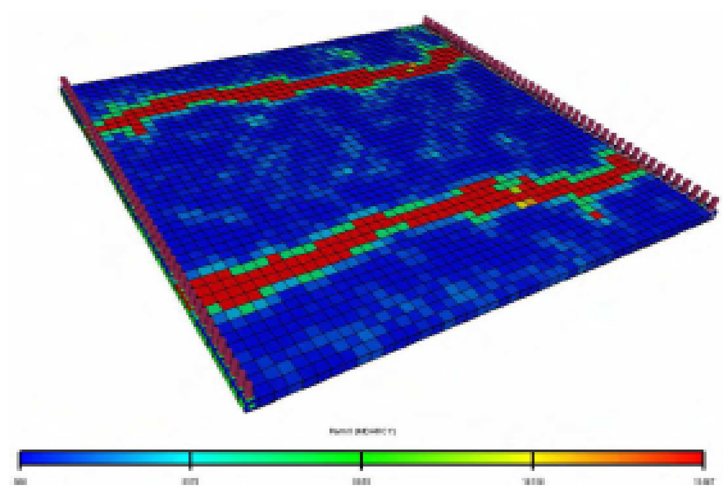
- Single-Shooting and SQP
- ESDIRK methods tailored for this problem
- Gradients computed by the adjoint method

Objective Function is the Net Present Value

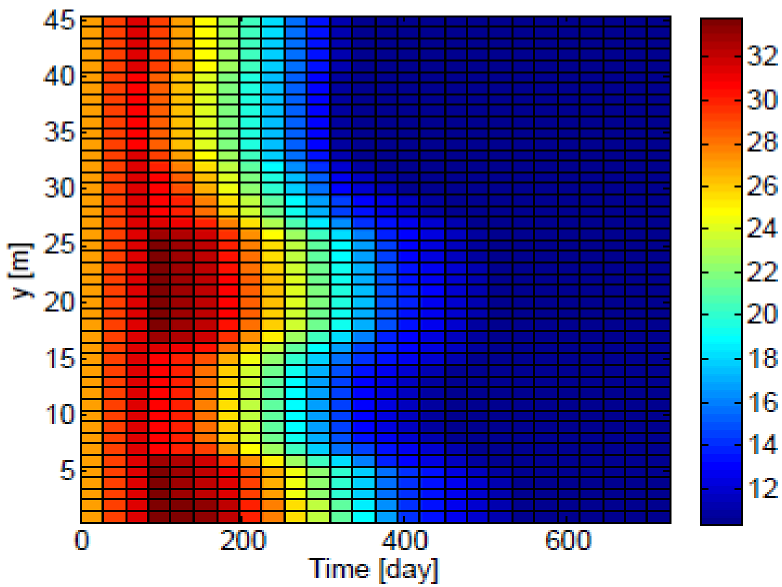
$$NPV = \sum_{k=0}^{N-1} \left[\frac{h_k}{(1-d)^{t_k/\tau}} \left(\sum_{j=1}^{N_{pro}} \left[r_{op} Q_{k,j}^{op} - r_{wp} Q_{k,j}^{wp} \right] - \sum_{j=1}^{N_{inj}} r_{wi} Q_{k,j}^{wi} \right) \right]$$

Adjust the injectors and producers by taking the oil price, the water injection cost and the water separation cost into account.

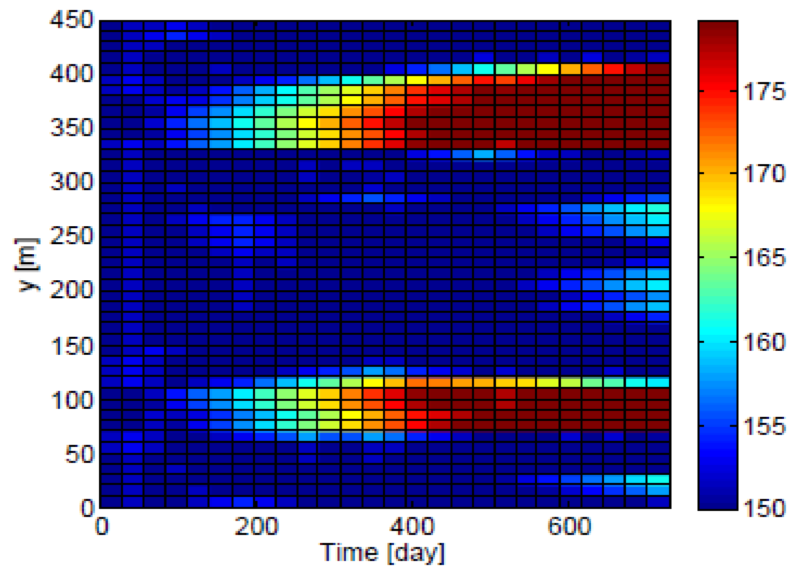
- Rate constrained injector on the left.
- BHP constrained producer to the right.
- 2×45 individually controllable sections.
- Max. 2 PV's injected over two years.
- Injection rates and BHP's are updated once a month.



Optimal Oil Field Development - MVs

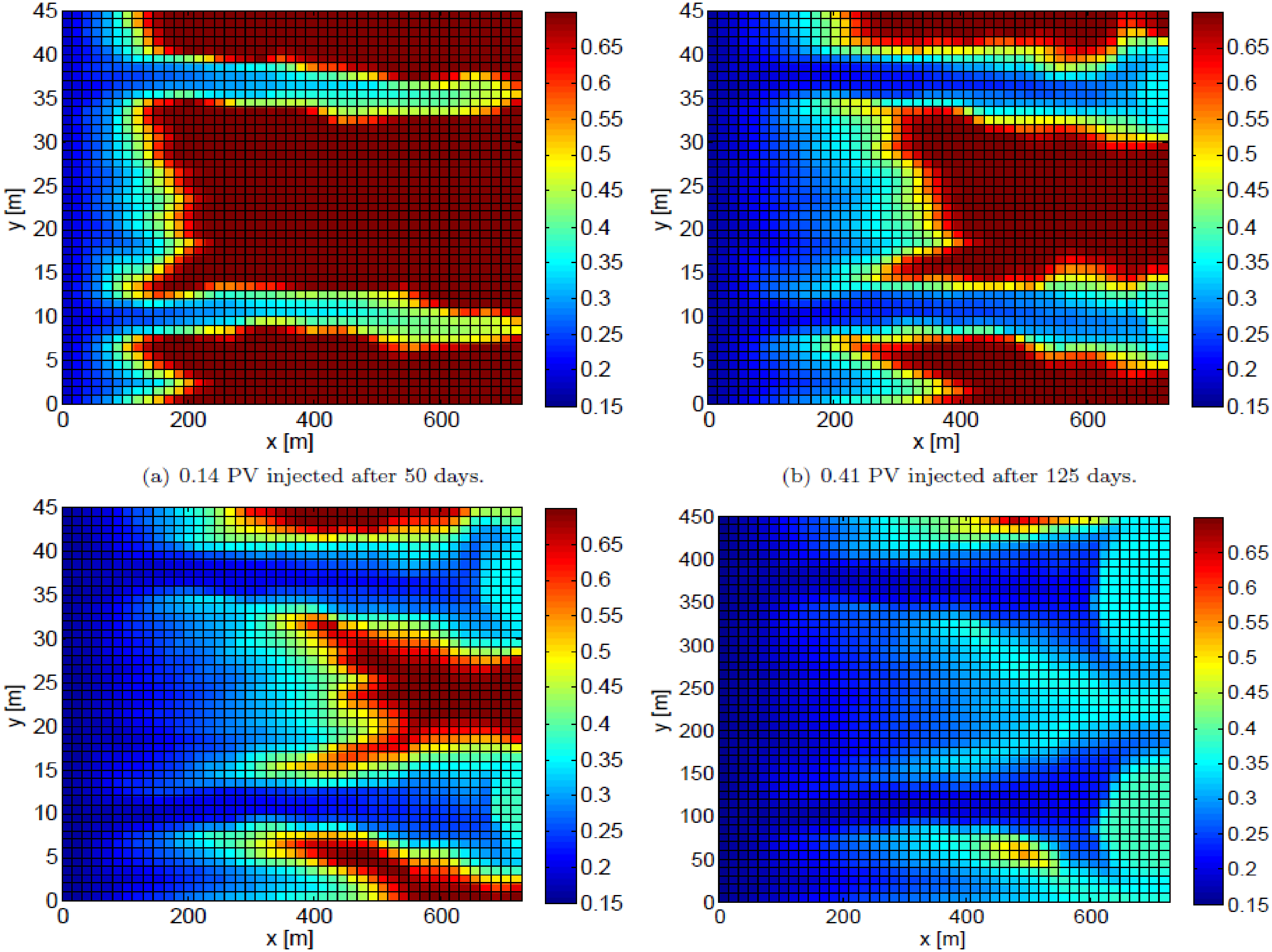


(a) Individual injection rates (m^3/day) of 45 injectors.



(b) Individual BHP's (atm) of 45 producers.

Optimal Oil Field Development - CVs



(a) 0.14 PV injected after 50 days.

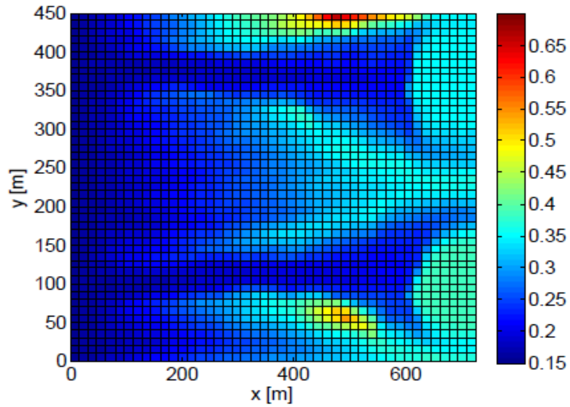
(b) 0.41 PV injected after 125 days.

(c) 0.64 PV injected after 200 days.

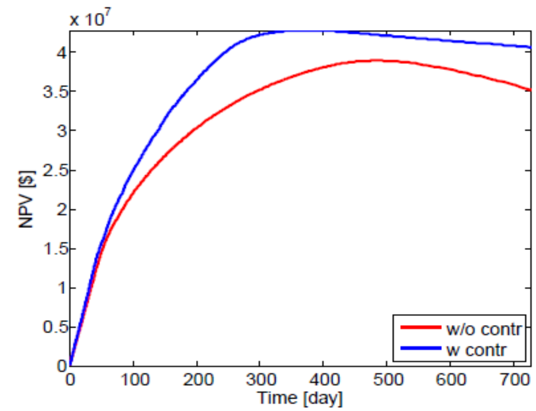
(d) 1.00 PV injected after 374 days.

Oil Field Development Using NMPC

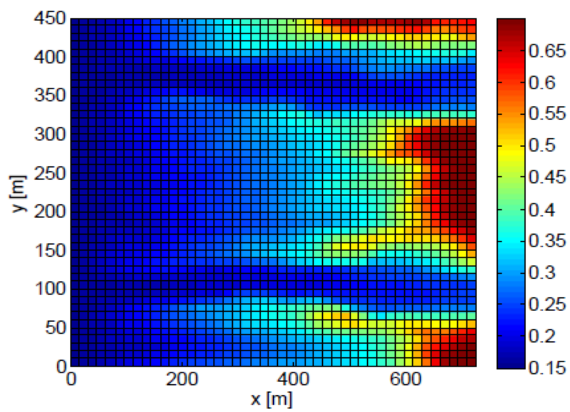
With control strategy, 374 days:



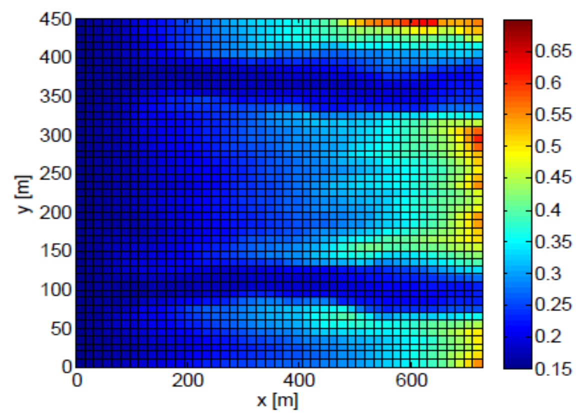
Net present value over 2 years:



No control strategy, 374 days:

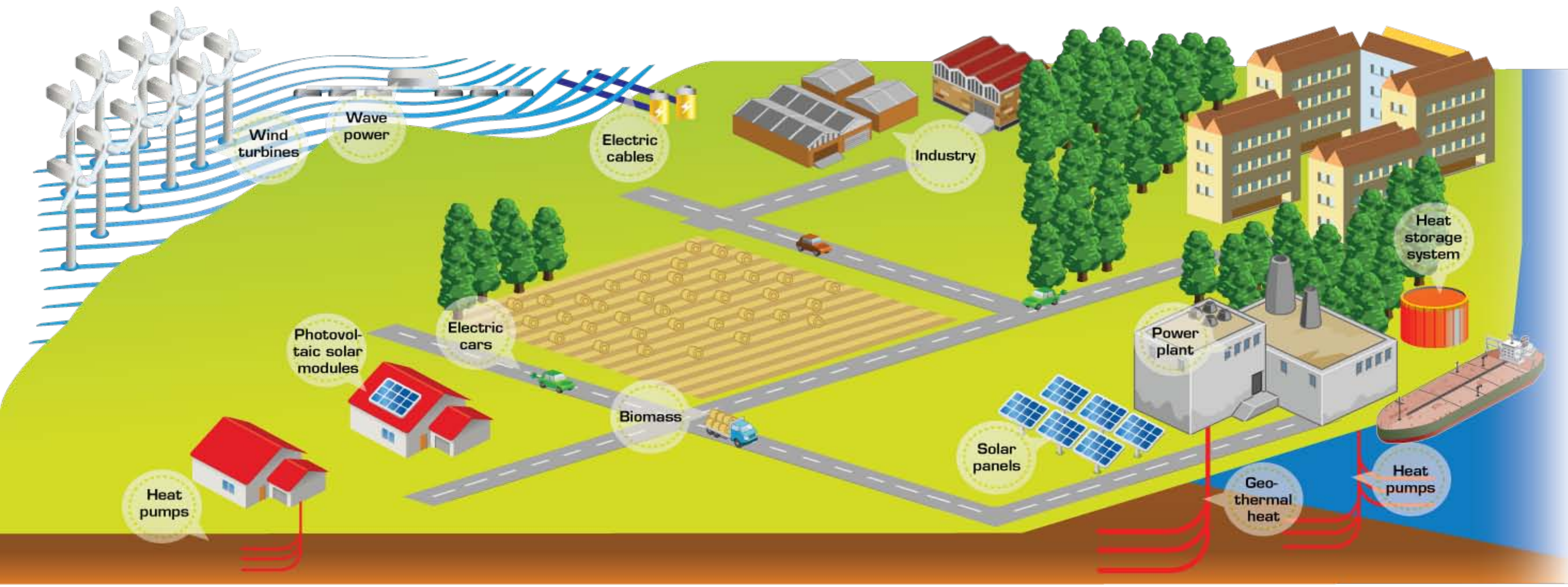


No control strategy, 484 days:

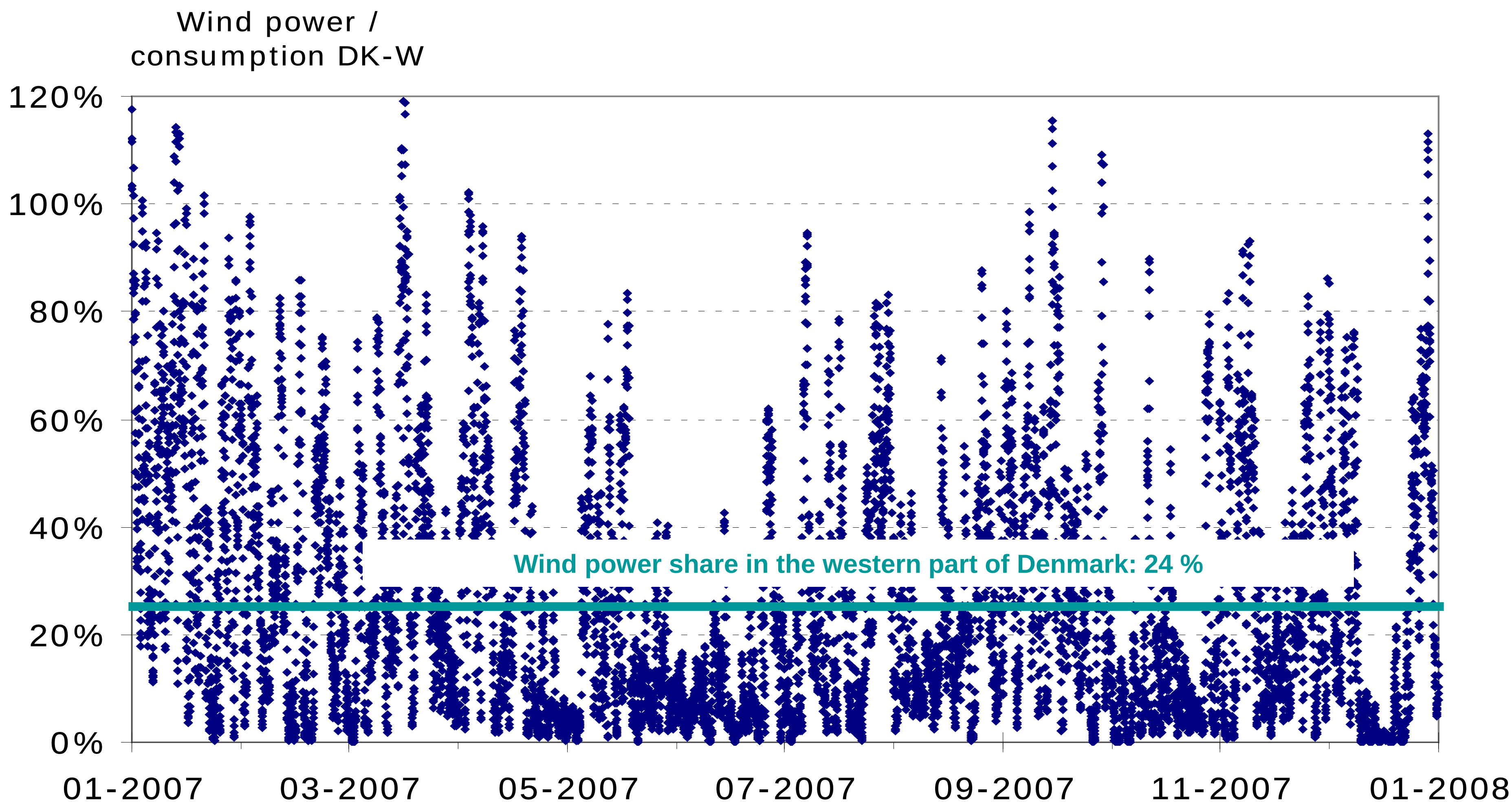


MODEL PREDICTIVE CONTROL OF SMART ENERGY SYSTEMS

Smart Energy Systems

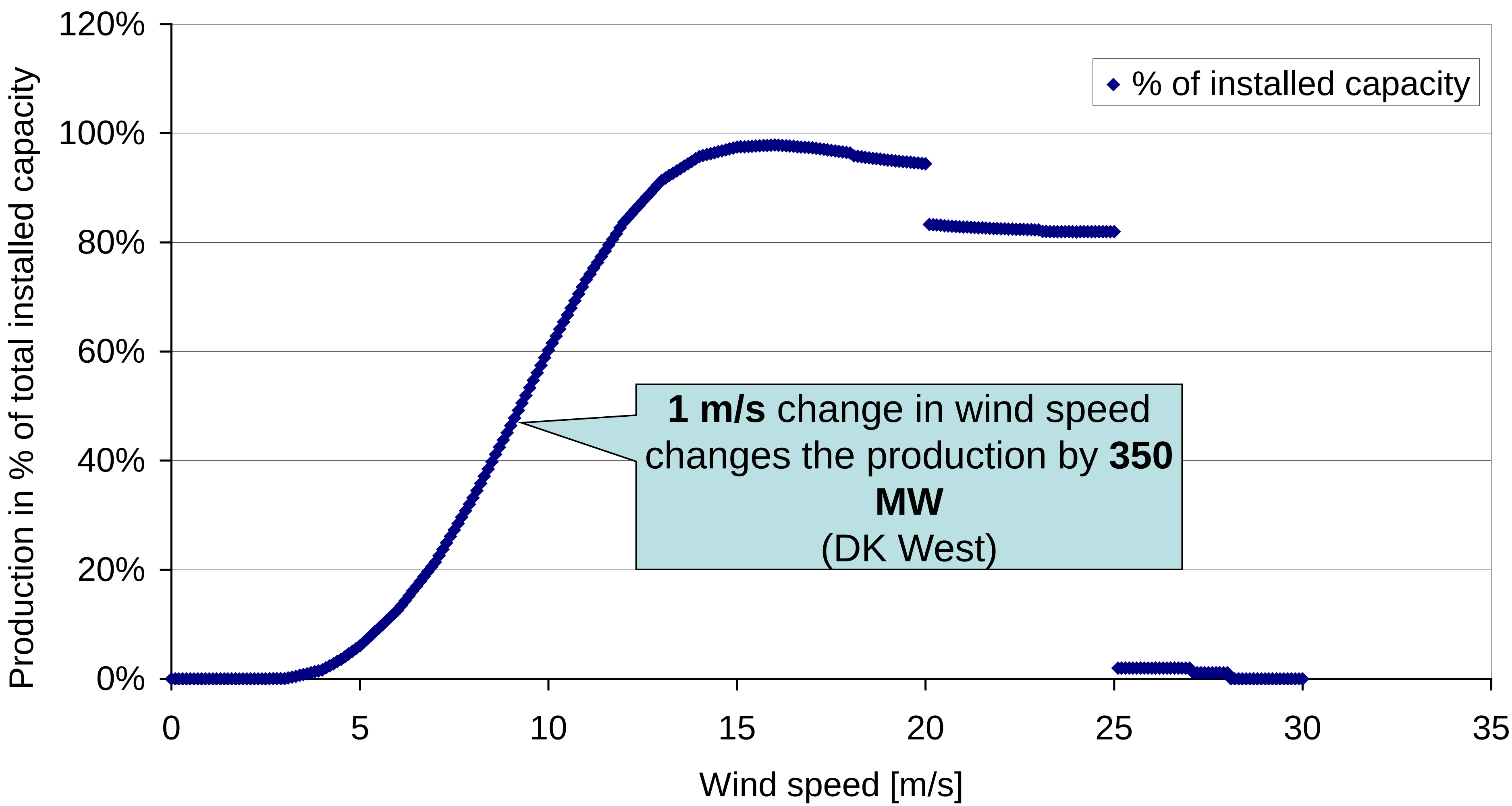


Wind Power

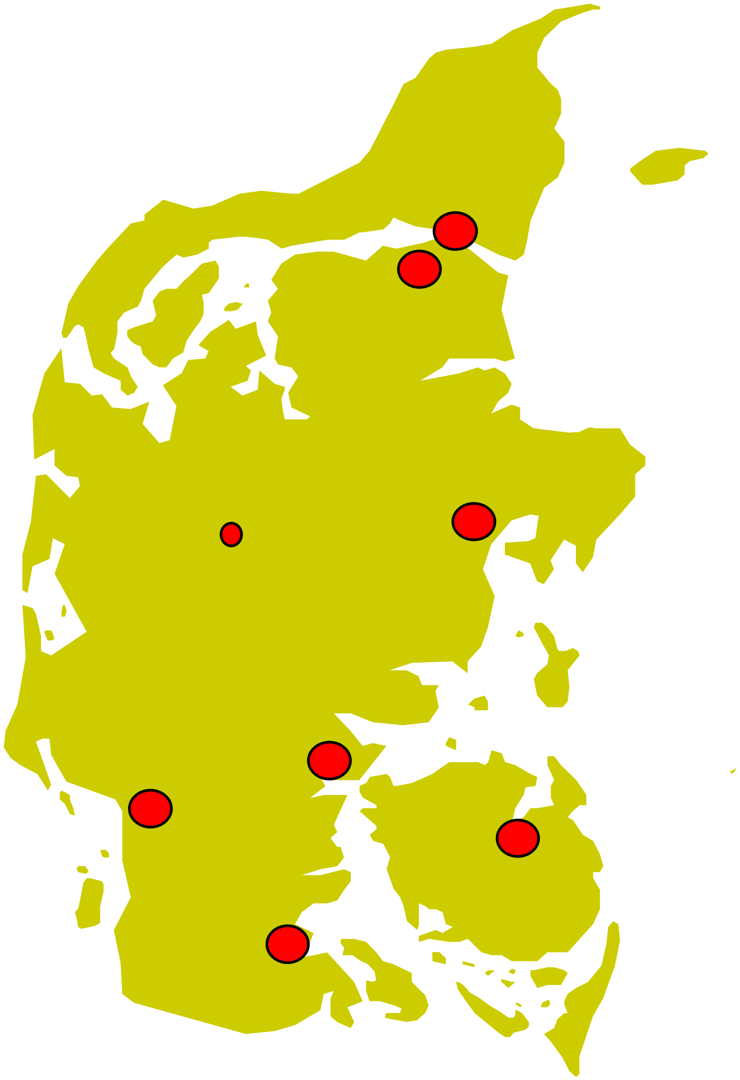


Wind power /
consumption DK-W
120%
100%
80%
60%
40%
20%
0%
01-2007
03-2007
05-2007
07-2007
09-2007
11-2007
01-2008
Wind power share in the western part of Denmark: 24 %

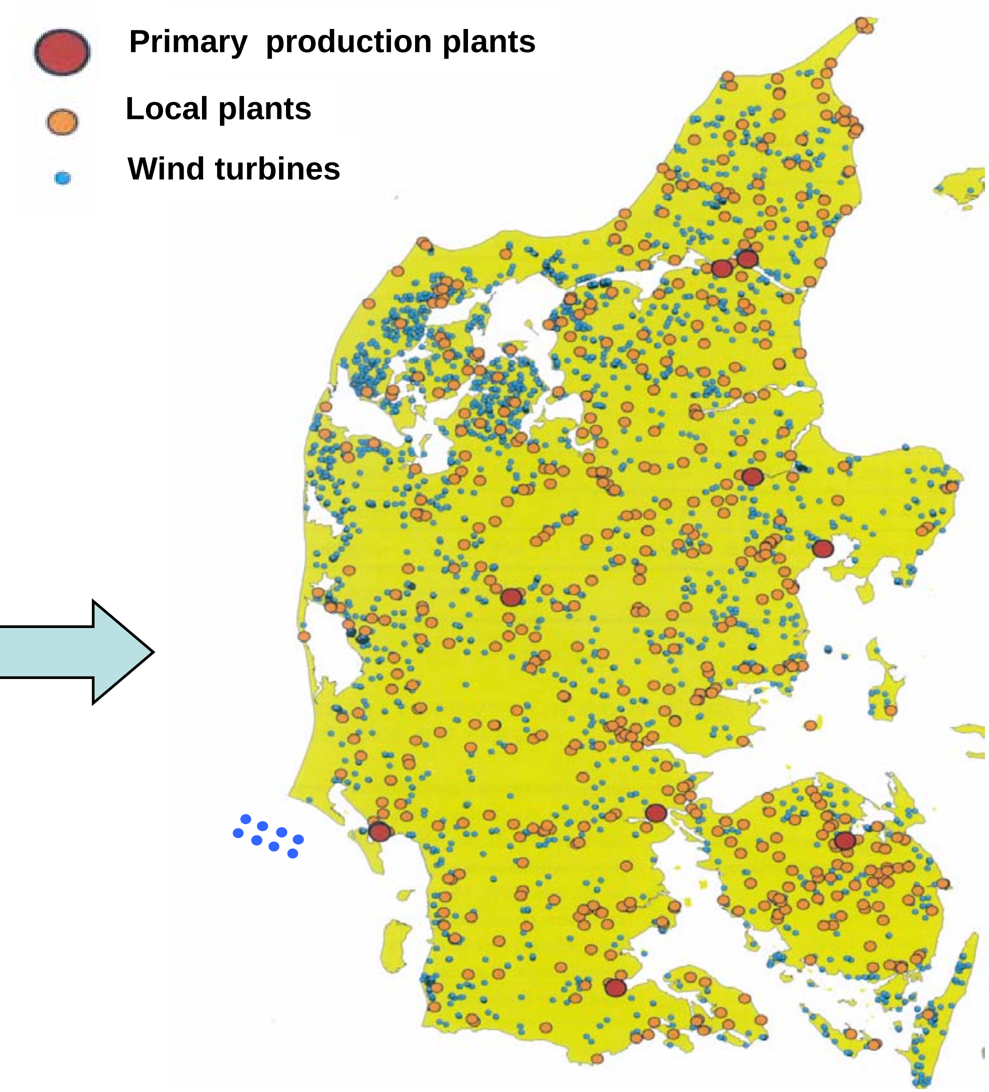
Wind Power Production and Imbalances



Power System Development – DK West

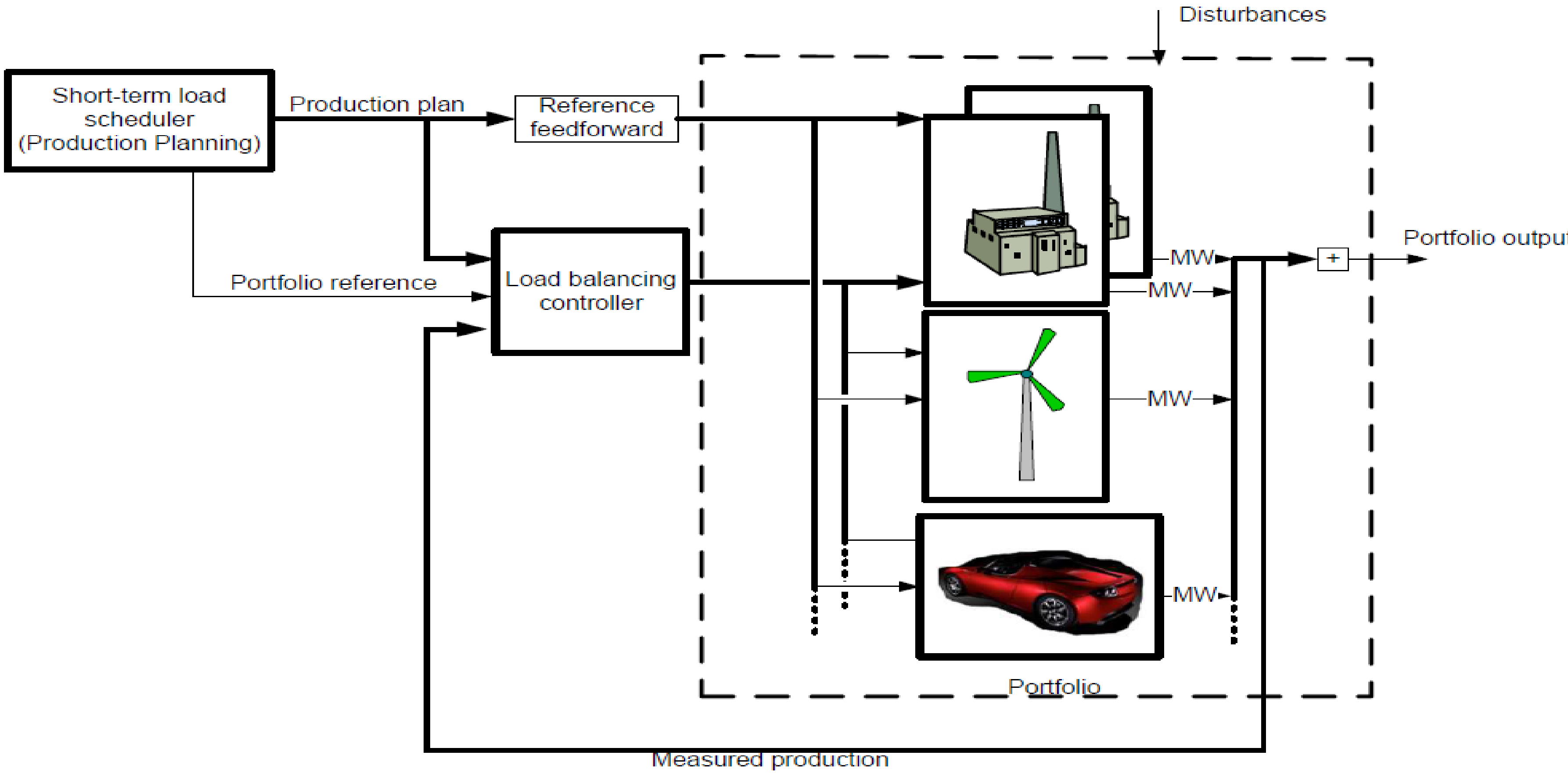


Centralized system of the mid 1980s

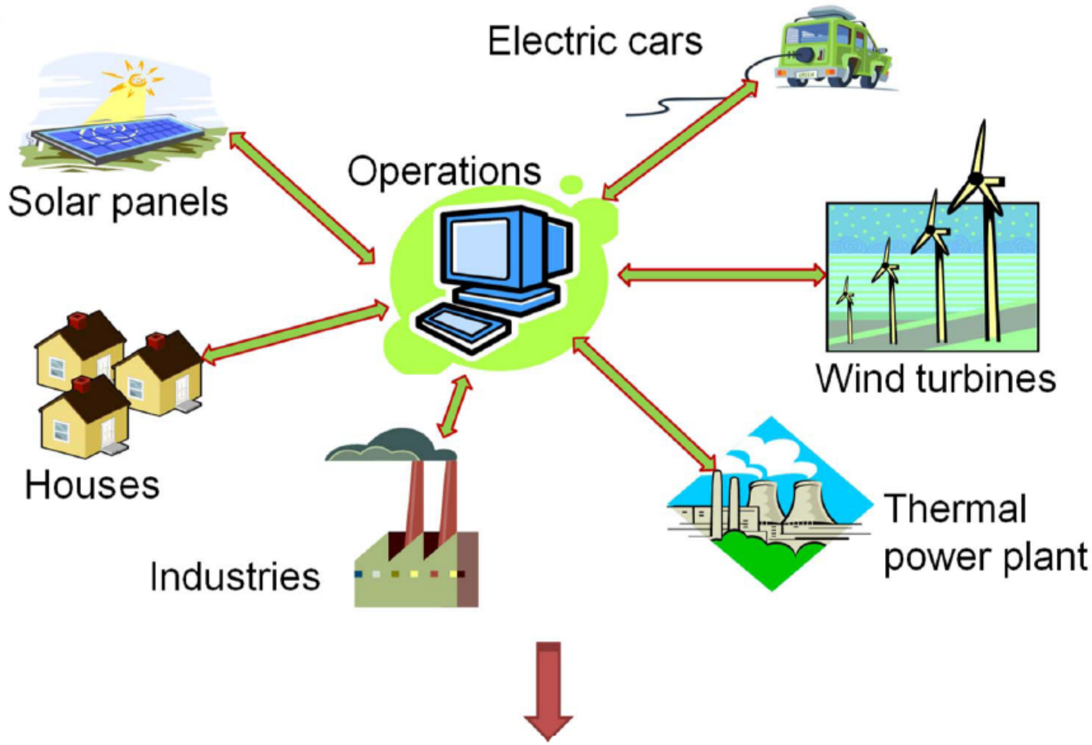


More decentralized system of today

Load Balancing Controller



Models of Energy Components



$$x_{k+1}^i = A_i x_k^i + B_i u_k^i + E_i d_k^i$$
$$y_k^i = C_i x_k^i$$

Controllable Power Generators

Each controllable power generator has an allowable operating range

$$u_{\min} \leq u_k \leq u_{\max}$$

and limits on how fast you can change the desired production

$$\Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max}$$

The unit production cost for power plant #i is: c_i.
Therefore, the total production cost over a given period is

$$\phi = \sum_{k=0}^{N-1} \sum_{i=1}^{n_u} c_i (u_k)_i = \sum_{k=0}^{N-1} c' u_k$$

MPC for Economic Power Portfolio Optimization

The portfolio power generation problem can be stated as

$$
\begin{aligned}
\min_{\{u_k\}_{k=0}^{N-1}} \quad & \phi = \sum_{k=0}^{N-1} c' u_k \\
s.t. \quad & x_{k+1} = A x_k + B u_k + E d_k && k = 0, 1, \ldots, N-1 \\
& y_k = C x_k && k = 1, 2, \ldots, N \\
& u_{\min} \leq u_k \leq u_{\max} && k = 0, 1, \ldots, N-1 \\
& \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} && k = 0, 1, \ldots, N-1 \\
& y_k \geq r_k && k = 1, 2, \ldots, N
\end{aligned}
$$

The parameters for this problem are

- Initial state, x_0, and previous decision, u_{-1}
- Predicted loads on non-controllable generators (e.g. wind speed on wind turbines): $\{d_k\}_{k=0}^{N-1}$
- Predicted power demand: $\{r_k\}_{k=1}^{N}$

Soft Economic MPC

It may not always be possible to meet the power demand. Therefore, we relax the MPC problem

$$\min_{\{u_k, v_{k+1}\}_{k=0}^{N-1}} \quad \phi = \sum_{k=1}^{N} \rho v_k + \sum_{k=0}^{N-1} c_k' u_k$$

$$\begin{aligned} s.t. \quad & x_{k+1} = Ax_k + Bu_k + Ed_k && k = 0, 1, \dots, N-1 \\ & y_k = Cx_k && k = 1, 2, \dots, N \\ & u_{\min} \leq u_k \leq u_{\max} && k = 0, 1, \dots, N-1 \\ & \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} && k = 0, 1, \dots, N-1 \\ & y_k \geq r_k - v_k && k = 1, 2, \dots, N \\ & v_k \geq 0 && k = 1, 2, \dots, N \end{aligned}$$

by introduction of the slack variables, v_k. ρ is selected sufficiently large, such that the power demand is met whenever possible.

Simple Test Example

Power Generator #1: Slow and Cheap

$$Y_1(s) = \frac{1}{(\tau_1 s + 1)^3} U_1(s) \qquad \tau_1 = 20 \qquad c_1 = 1$$

$$\begin{aligned} 0 \leq u_k &\leq 10 \\ -1 \leq \Delta u_k &\leq 1 \end{aligned}$$

Power Generator #2: Fast and Expensive

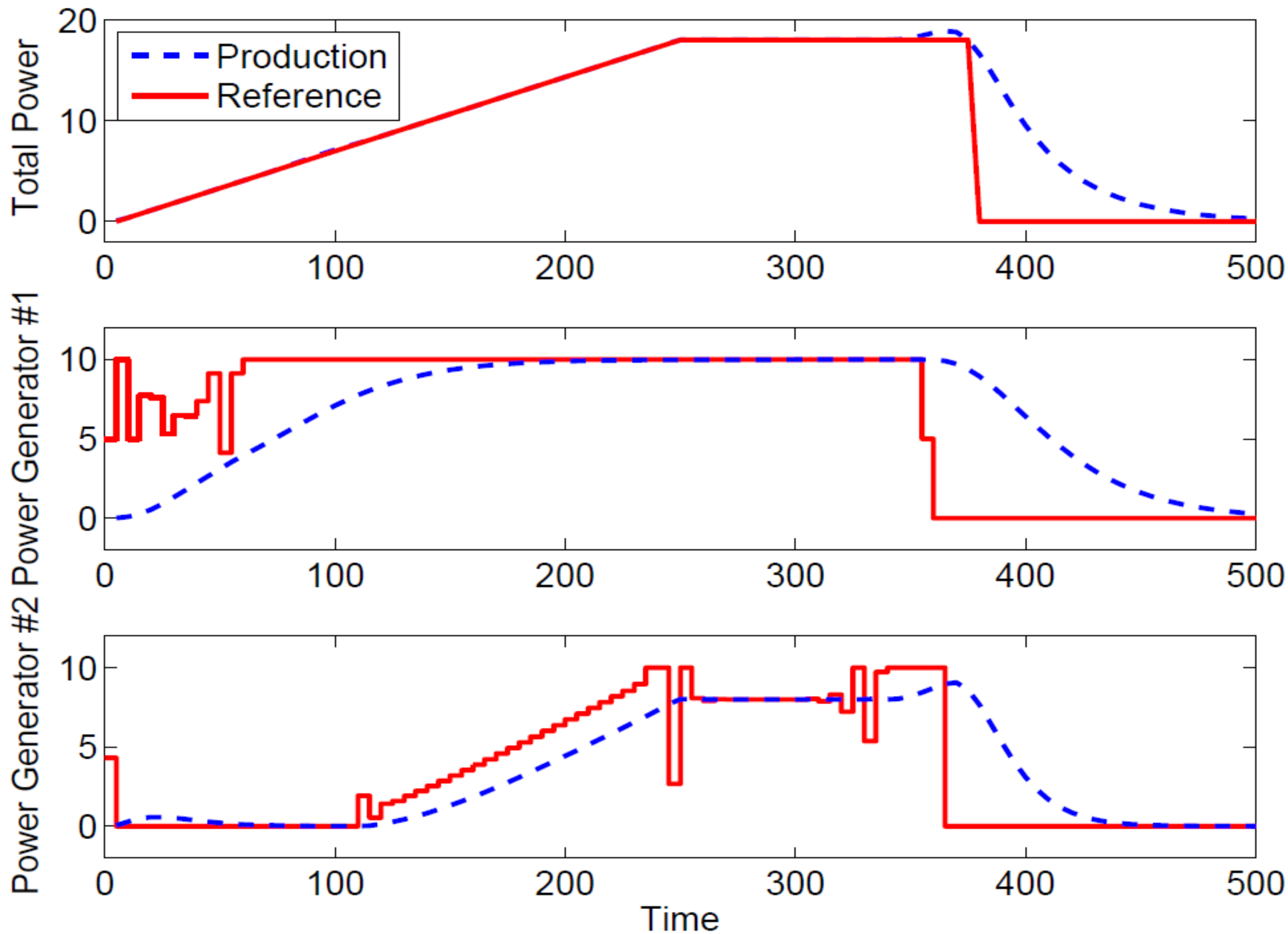
$$Y_2(s) = \frac{1}{(\tau_2 s + 1)^3} U_2(s) \qquad \tau_2 = 10 \qquad c_2 = 2$$

$$\begin{aligned} 0 \leq u_k &\leq 10 \\ -3 \leq \Delta u_k &\leq 3 \end{aligned}$$

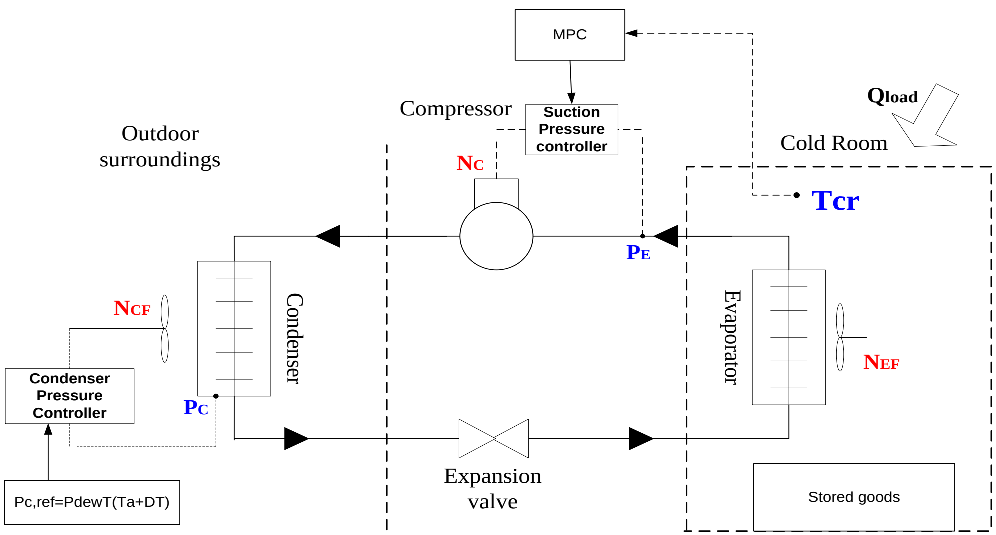
Total production: $Y(s) = Y_1(s) + Y_2(s)$

Penalty for producing less than sold: $\rho = 1.0 \cdot 10^5$

Optimal Production Profiles



Cooling Houses – A Flexible Consumer



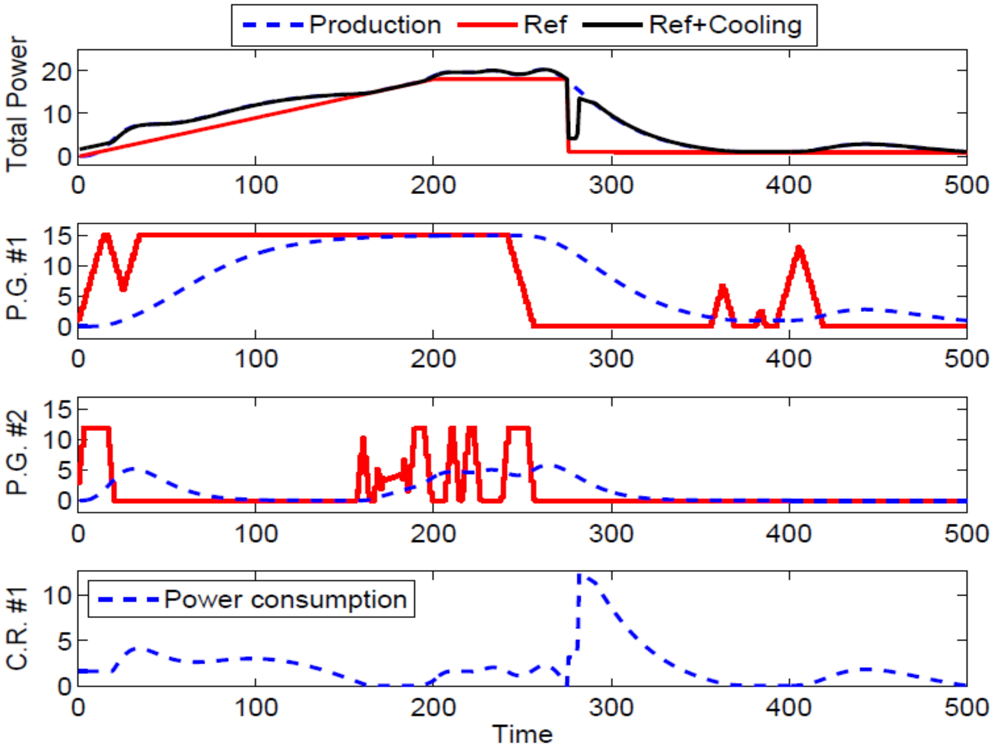
Cooling system used in refrigerators and cooling houses

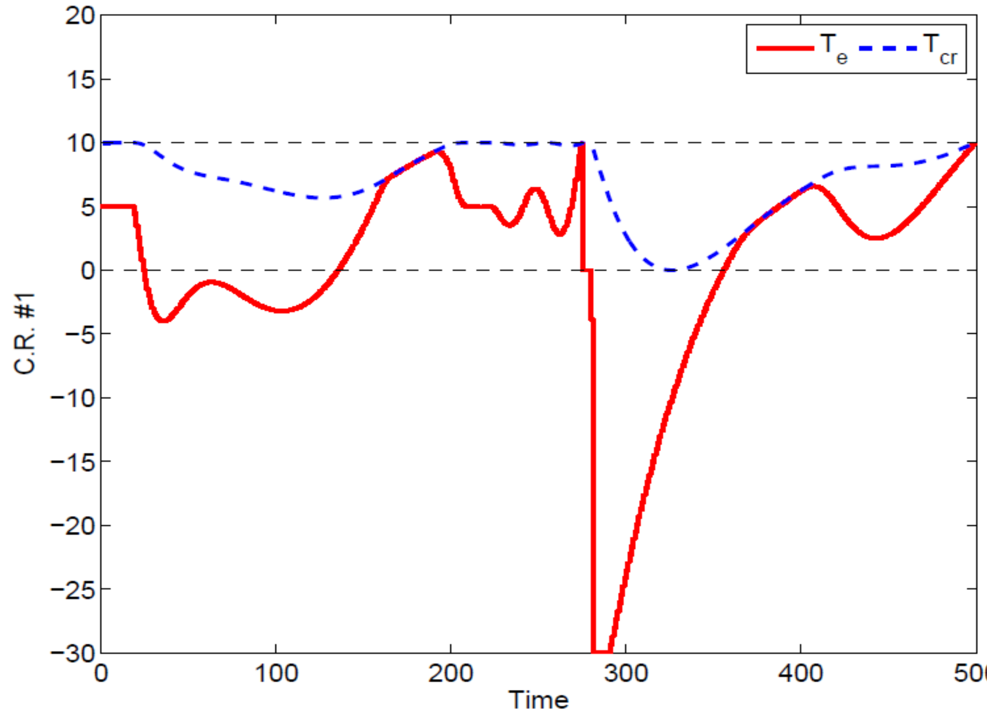
- **Motivation:**
- Refrigeration and air-conditioning consume substantial amounts of energy.
- E.g. up to 80% of energy consumed by supermarkets goes to refrigeration.

- **Methods:**
- Economic MPC:
  Minimize the cost of cooling subject to temperature limits
- Predictions of weather, energy prices and load profiles.
- Implementation on industrial hardware.

FPGA

Power Management
2 Power Plants and 1 Cooling House

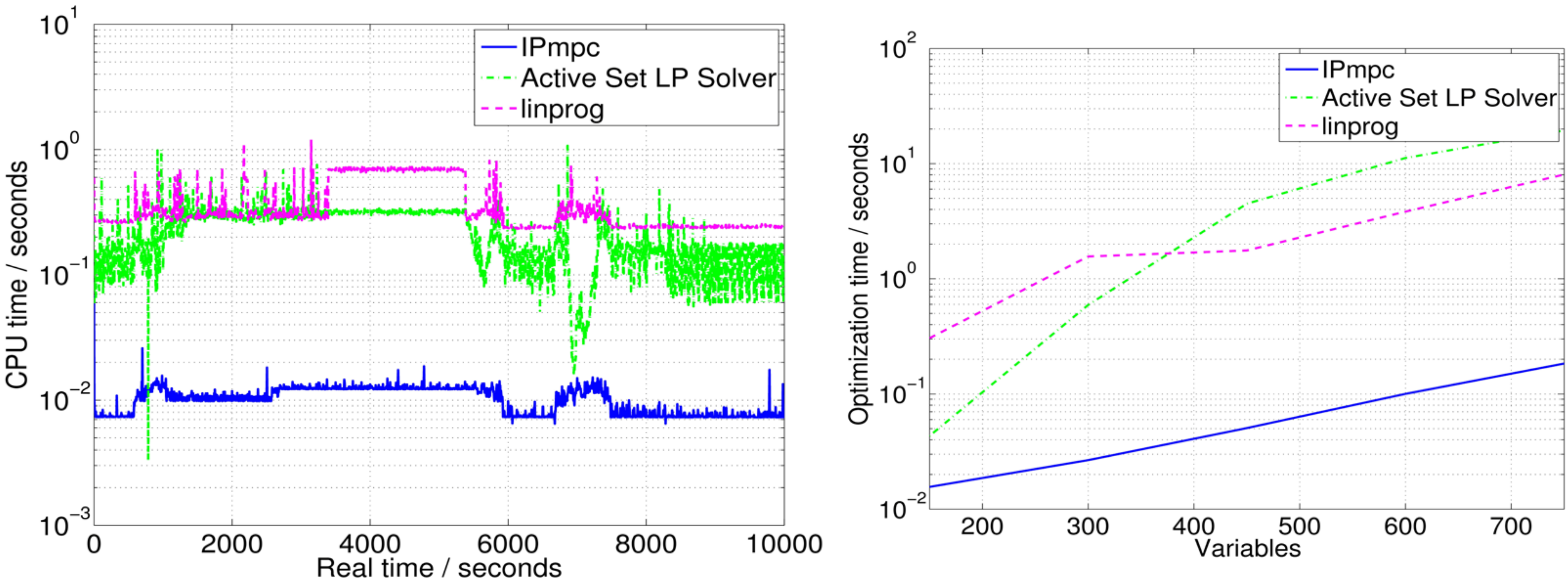




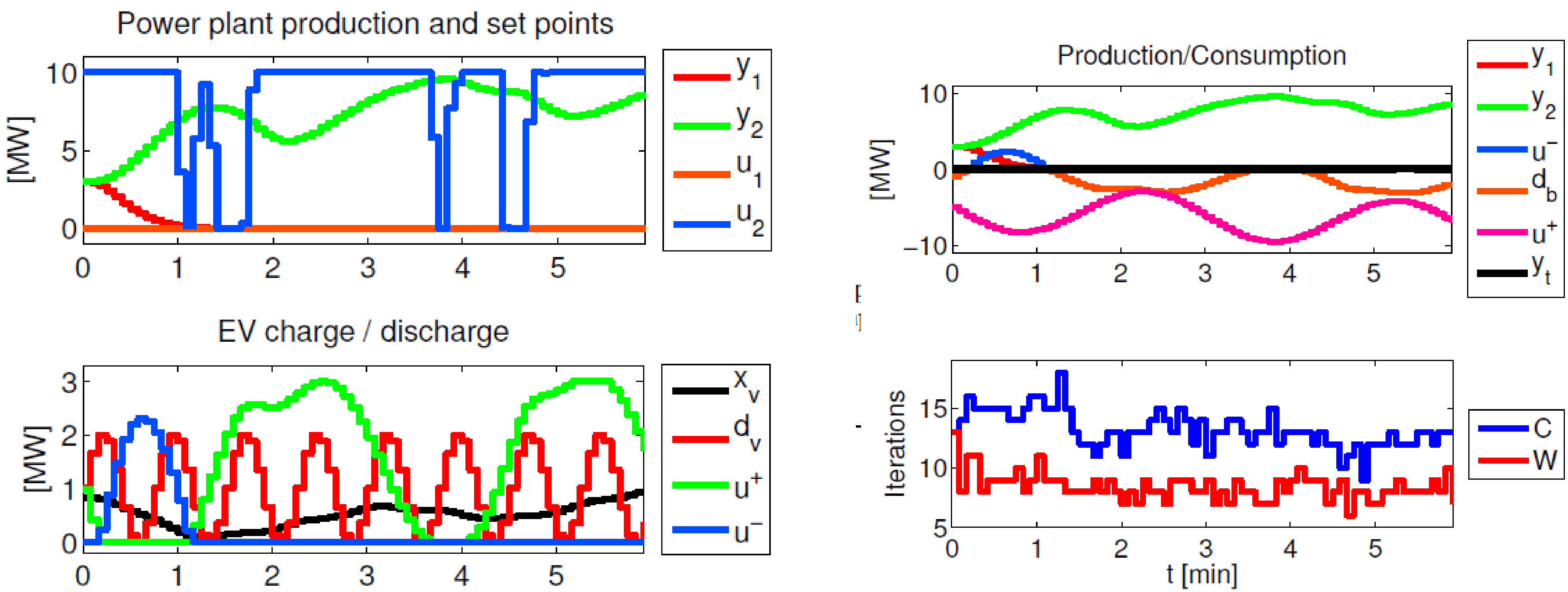
- Economic Optimizing MPC
- Minimize power consumption and costs without lowering the cooling quality
- Load shifting utilizing the thermal capacity of the cooling house

Online Computing Time

Tailored Primal-Dual Interior Point Algorithm



Warm Start
Interior Point Methods



Accelerating LP Algorithms with GPUs

- Interior Point Methods are used for solving mathematical optimization problems eg:

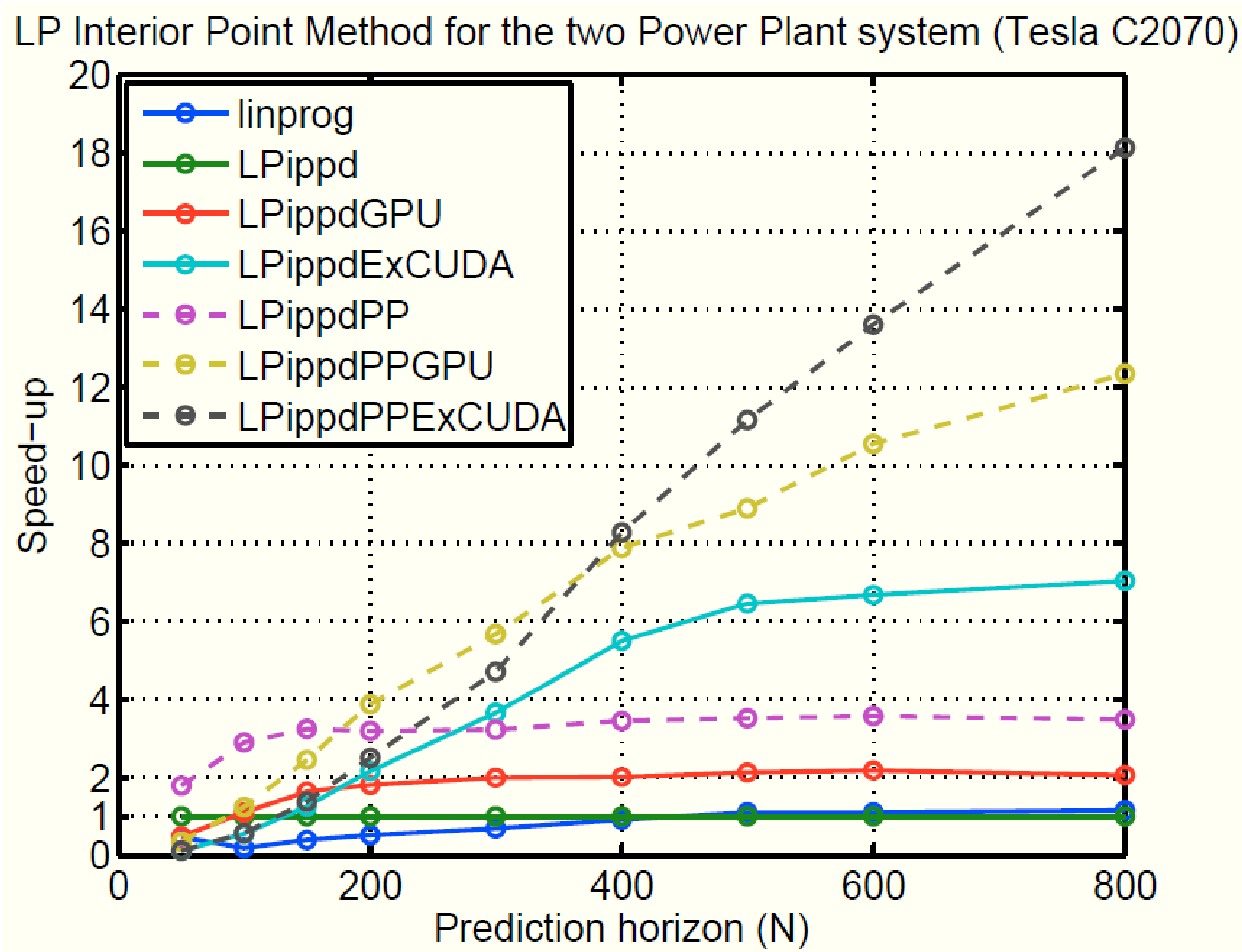
$$\begin{aligned} \min \quad & \phi = g'x \\ s.t. \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

- Basic direct algorithm:
  - While not converged
    - Form Hessian matrix: $H = A'DA$
    - Factorize Hessian: $L = chol(H)$
    - Compute affine step
    - Compute corrector step
    - Take step
    - Compute residuals
- The main workload is the formation of the Hessian followed by the factorization.
- GPUs are very good at matrix-multiplication.

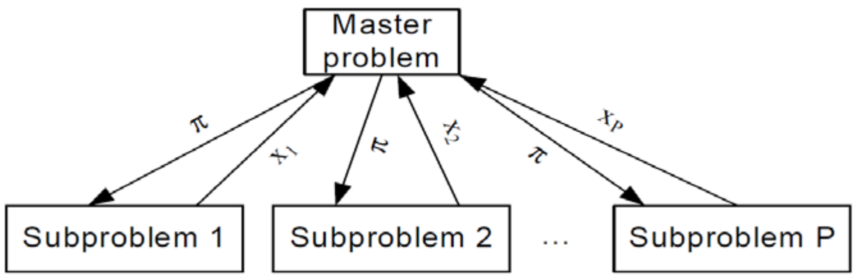
Implementation Details

- MATLAB CPU implementation as reference: LPippd
- MATLAB GPU implementation (MATLAB R2011b): LPippdGPU
- C + CUDA implementation: LPippdExCUDA
  - Uses available libraries for standard operations.
  - CUBLAS used for matrix operations.
  - MAGMA used for Cholesky factorization.
  - CUDA kernels implemented for additional operations such as computing step lengths.

Speed-Up with GPU

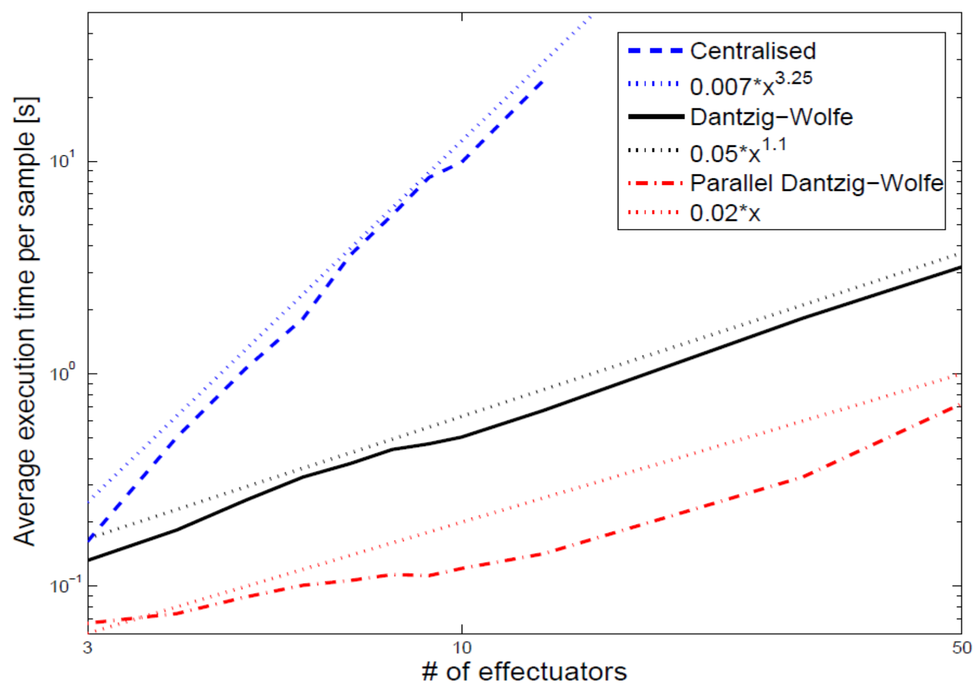


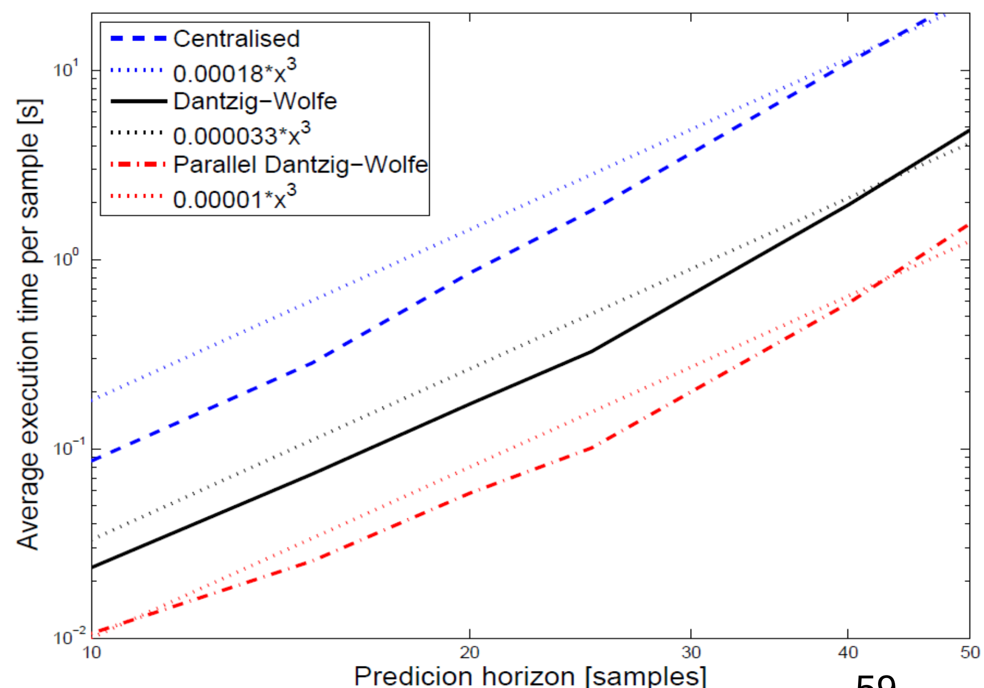
Dantzig-Wolfe Decomposition



$$\min_{\mathbf{z}_i} \quad \phi_i = \left[\mathbf{c}_i - \mathbf{F}_i'\pi\right]' \mathbf{z}_i$$

$$s.t. \quad \mathbf{G}_i \mathbf{z}_i \geq \mathbf{h}_i$$





Uncertainty

Smart energy systems are subject to uncertainty:

- Demand / Price / Weather

$$\Downarrow$$

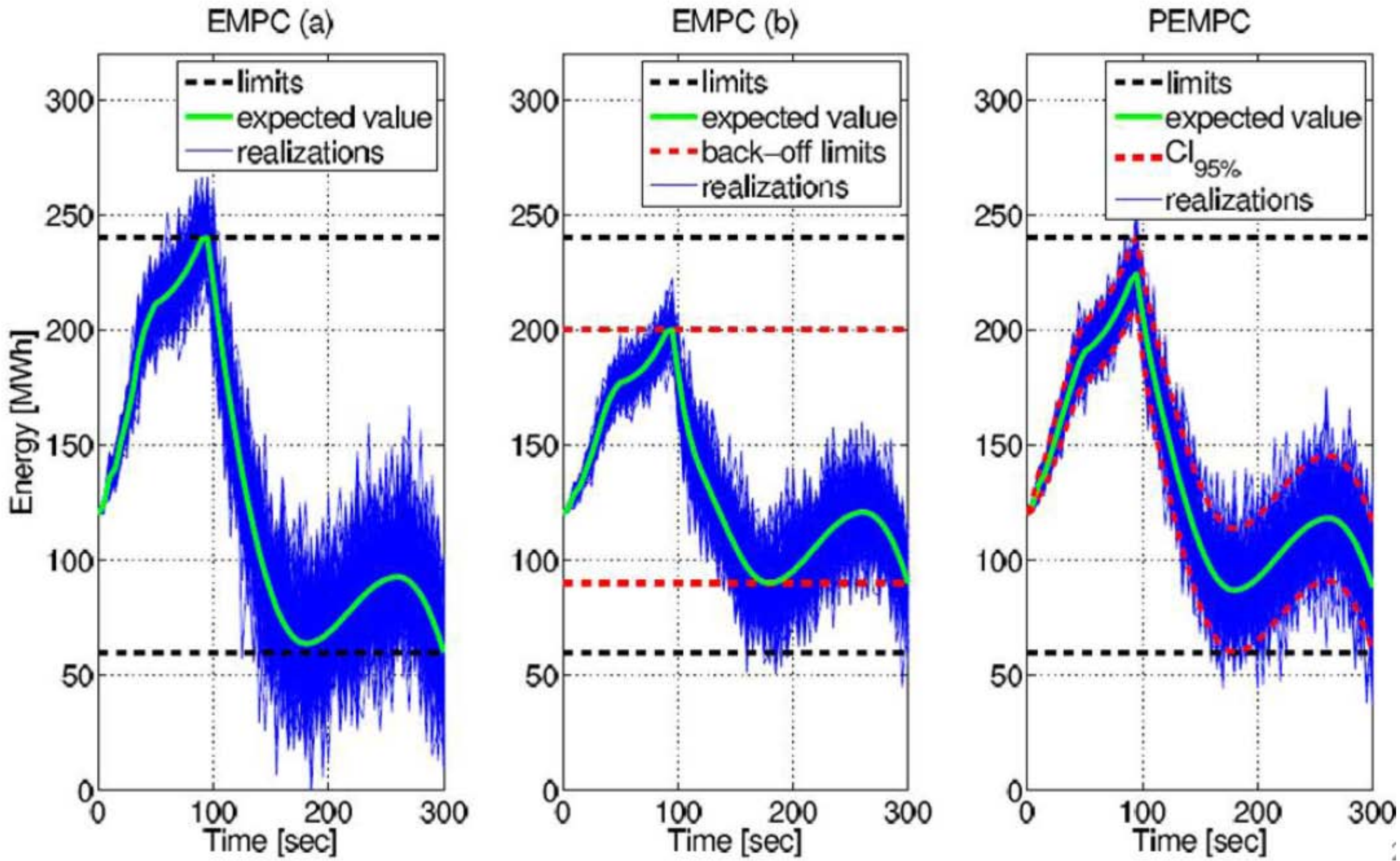
- Uncertainty on g and b $\implies$ LP

$$\min_x \mathbf{g}^T x$$
$$s.t \quad Ax \geq \mathbf{b} \qquad j = 1, ..., m$$
$$x \in \Re^n$$

- Uncertainty on g, A and b with Gaussian distribution $\implies$ SOCP

$$\begin{aligned} &\min_x \mathbf{g}^T x \\ s.t \quad &\mathbf{A}_{[j,:]} x \geq \mathbf{b}_j \qquad j = 1, ..., m \\ &x \in \Re^n \end{aligned} \quad \implies \quad \begin{aligned} &\min_x g^T x \\ s.t \quad &Ax \geq b \qquad j = 1, ..., m \\ &x \succeq_{K^n} 0 \end{aligned}$$

Uncertainty Scenarios



The Extended LQ Problem

$$\min_{\{u_k, x_{k+1}\}_{k=0}^{N-1}} \quad \phi = \sum_{k=0}^{N-1} l_k(x_k, u_k) + l_N(x_N)$$

$$s.t. \quad x_{k+1} = A_k x_k + B_k u_k + b_k \quad k \in \mathcal{N}$$

with $\mathcal{N} = \{0, 1, \ldots, N-1\}$ and stage costs defined by

$$l_k(x_k, u_k) = \frac{1}{2} \begin{bmatrix} x_k \\ u_k \end{bmatrix}' \begin{bmatrix} Q_k & M_k' \\ M_k & R_k \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \begin{bmatrix} q_k \\ s_k \end{bmatrix}' \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \rho_k$$

$$l_N(x_N) = \frac{1}{2} x_N' P_N x_N + p_N' x_N + \gamma_N$$

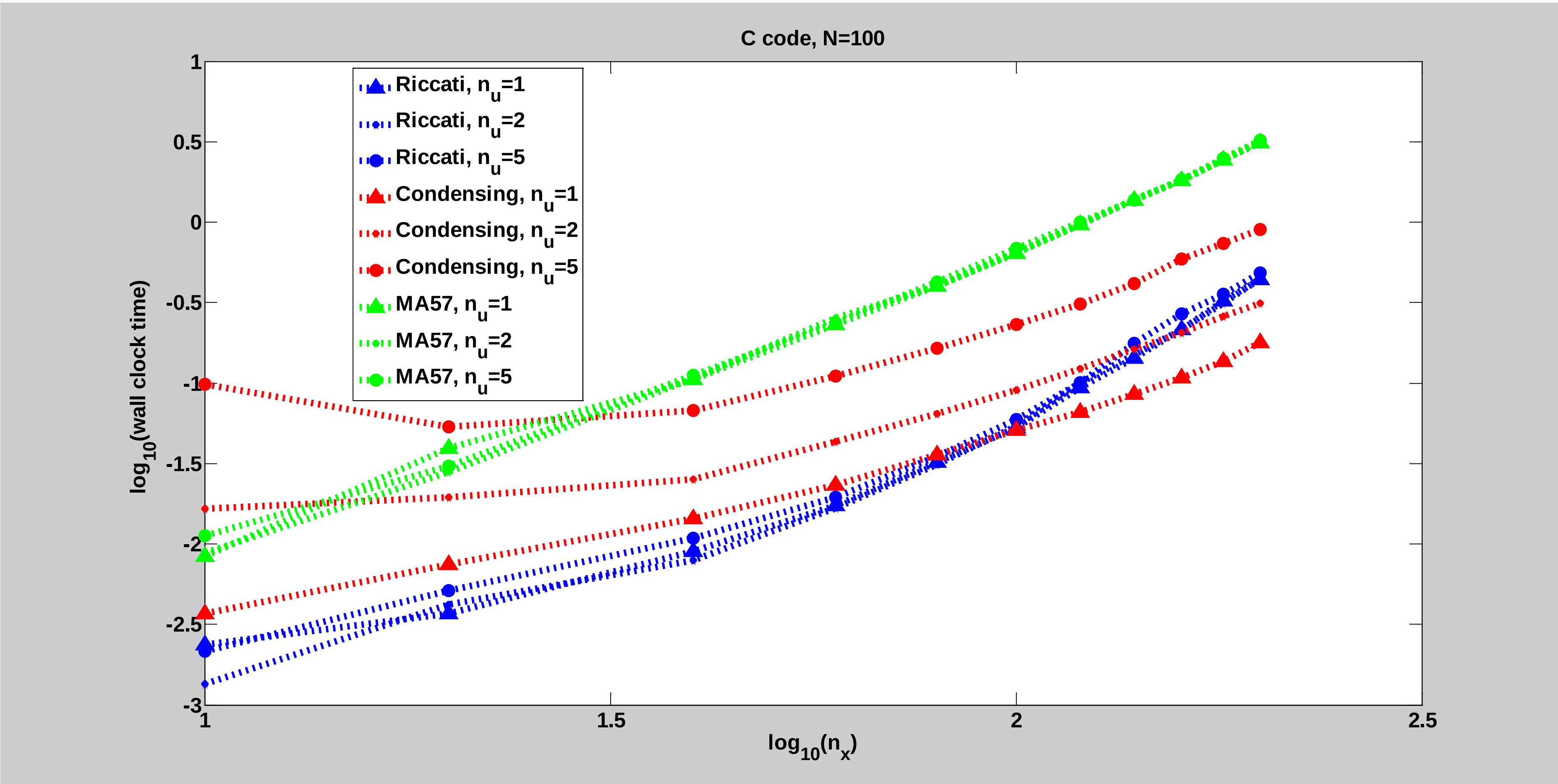
KKT System for the Extended LQ Problem

$$
\left[\begin{array}{cccccc|ccc}
R_0 & & & & & & B_0' & & \\
 & Q_1 & M_1' & & & & -I & A_1' & \\
 & M_1 & R_1 & & & & & B_1' & \\
 & & & Q_2 & M_2' & & & -I & A_2' \\
 & & & M_2 & R_2 & & & & B_2' \\
 & & & & & P_3 & & & -I \\
\hline
B_0 & -I & & & & & & & \\
 & A_1 & B_1 & -I & & & & & \\
 & & & A_2 & B_2 & -I & & &
\end{array}\right]
\left[\begin{array}{c}
u_0 \\ x_1 \\ u_1 \\ x_2 \\ u_2 \\ x_3 \\ \hline \pi_1 \\ \pi_2 \\ \pi_3
\end{array}\right]
= -
\left[\begin{array}{c}
\tilde{s}_0 \\ q_1 \\ s_1 \\ q_2 \\ s_2 \\ p_3 \\ \hline \tilde{b}_0 \\ b_1 \\ b_2
\end{array}\right]
$$

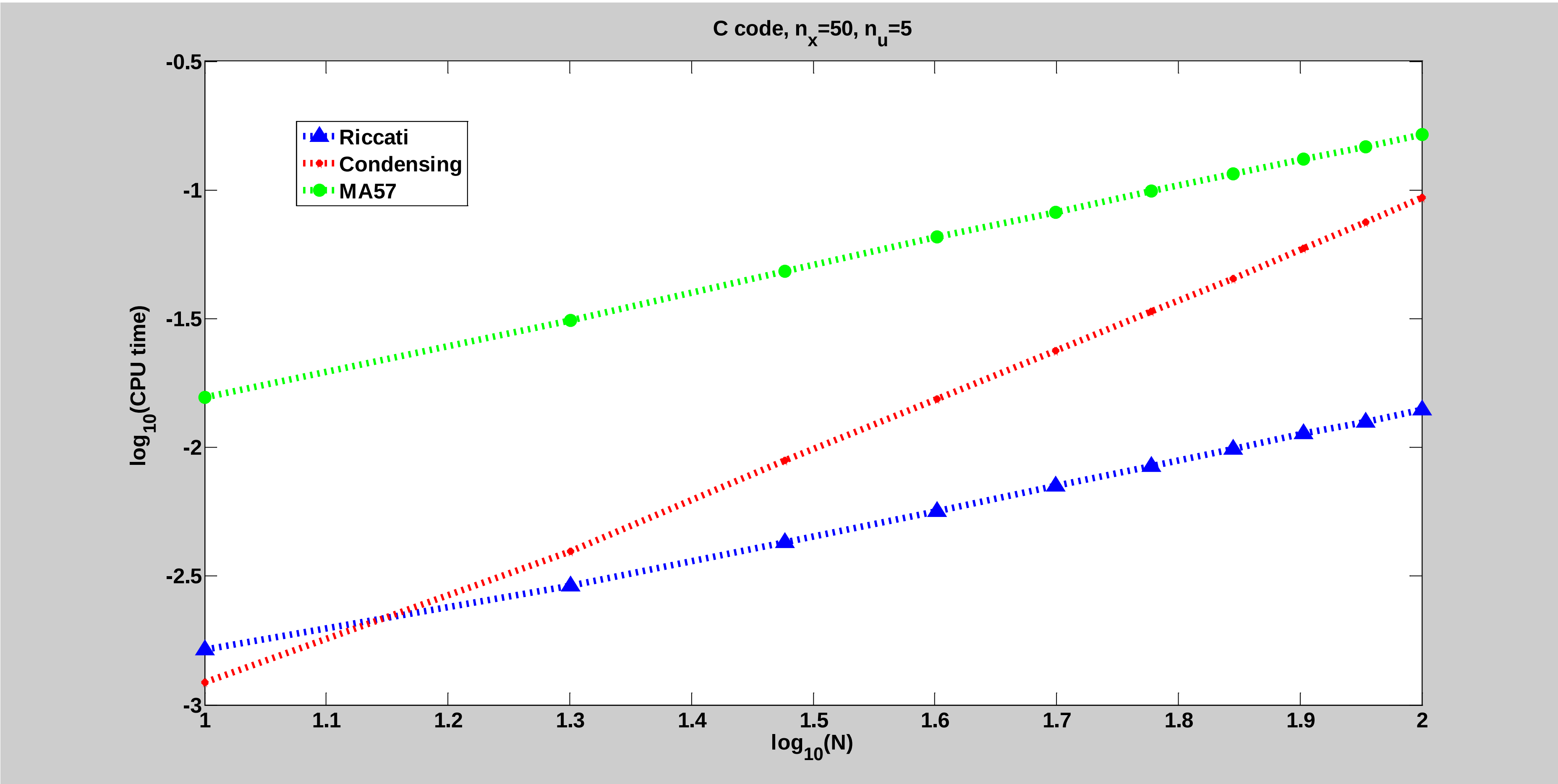
KKT System for the Extended LQ Problem

$$
\left[\begin{array}{cc|ccc|ccc|c}
R_0 & B_0' & & & & & & & \\
B_0 & 0 & -I & & & & & & \\
\hline
 & -I & Q_1 & M_1' & A_1' & & & & \\
 & & M_1 & R_1 & B_1' & & & & \\
 & & A_1 & B_1 & 0 & -I & & & \\
\hline
 & & & & -I & Q_2 & M_2' & A_2' & \\
 & & & & & M_2 & R_2 & B_2' & \\
 & & & & & A_2 & B_2 & 0 & -I \\
\hline
 & & & & & & & -I & P_3
\end{array}\right]
\left[\begin{array}{c}
u_0 \\ \pi_1 \\ \hline x_1 \\ u_1 \\ \pi_2 \\ \hline x_2 \\ u_2 \\ \pi_3 \\ \hline x_3
\end{array}\right]
= -
\left[\begin{array}{c}
\tilde{s}_0 \\ \tilde{b}_0 \\ \hline q_1 \\ s_1 \\ b_1 \\ \hline q_2 \\ s_2 \\ b_2 \\ \hline p_3
\end{array}\right]
$$

Numerical Results



Numerical Results



Summary

- There is a need for HPC software supporting MPC
  - QP, LP, SOCP algorithms
  - ODE/PDE solvers equipped with sensitivity computing abilities (adjoints)
  - SQP / NLP algorithms
  - Efficient direct sparse and iterative methods for KKT systems (even when they are ill-conditioned)
- The number of potential applications is very large

Center for Energy Ressources Engineering (CERE) Consortium

www.cere.dtu.dk

Novo Nordisk A/S, Hvidovre Hospital, Medtronic
Danfoss A/S, DONG Energy A/S, Vestas A/S, GEA Process Engineering A/S
The Danish Strategic Research Council
The Danish Research Council for Production and Technology Sciences