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## A Semi-Discretization Approach to Generalized Beam Theory and Analytical Solutions of the Generalized Column Equations

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### ABSTRACT

A generalized beam theory can be formulated based on the assumption that the displacements can be described as a sum of displacement fields. These displacement fields are each assumed to be separable into the products of functions of the local transverse coordinates and functions of the axial coordinate  $z$ . Thus in a single displacement field as shown in Fig. 1 the transverse displacements are described by the product of a transverse displacement mode  $w_n(s)$ ,  $w_s(s)$  and a function  $\psi(z)$  of the axial coordinate. Further more due to shear constraints the related axial warping of the transverse displacement is described by the product of the related warping function  $\Omega(s)$  and the derivative  $\psi'(z)$  of the axial function. To establish these displacement

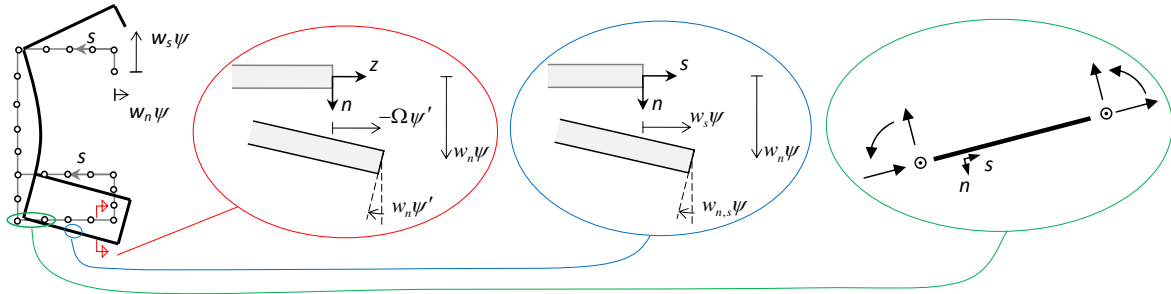


Figure 1: Displacement assumptions and discretization of the cross section in a straight elements.

fields the thin-walled cross section is discretized in elements in which the displacement modes and warping functions are interpolated. Introducing the constraining assumptions of beam theory the remaining degrees of freedom of the interpolated functions are  $\tilde{\mathbf{v}}_w$ . Thus the thin-walled beam has been semi-discretized and the governing differential equilibrium equations for determination of the transverse displacement modes  $\tilde{\mathbf{v}}_w$  and the axial variation  $\psi(z)$  takes the following form

$$\tilde{\mathbf{K}}^\sigma \tilde{\mathbf{v}}_w \psi'''' - \left[ \tilde{\mathbf{K}}^\tau + \lambda \tilde{\mathbf{K}}^0 \right] \tilde{\mathbf{v}}_w \psi'' + \tilde{\mathbf{K}}^s \tilde{\mathbf{v}}_w \psi = 0 \quad (1)$$

in which the matrices  $\tilde{\mathbf{K}}^\sigma$ ,  $\tilde{\mathbf{K}}^\tau$ ,  $\tilde{\mathbf{K}}^0$  and  $\tilde{\mathbf{K}}^s$  correspond to axial stiffness, shear stiffness, initial stress influence and transverse stiffness respectively. The magnitude of the initial stress is governed by the  $\lambda$  factor. The semi-discretization approach treated in this paper is developed in [1–3]. In the classic stability theory the solution functions  $\psi(z) = e^{\xi z}$  are normally assumed

to be trigonometric functions,  $\psi(z) = e^{i\mu z} = \sin \mu z = \sin(n\pi z/L)$ , in order to satisfy suitable simple boundary conditions. These solutions are illustrated as the conventional half wavelength buckling curves or so called cross section signature curves in the upper left part of Fig. 2. On the other hand seeking general solutions to the differential equations it is necessary to fix the initial stress level and thus perform calculations with fixed values of  $\lambda$ . Furthermore it is necessary to reduce the order of the differential equations and introduce a state vector with twice the number of dof. Through solution of the related linear eigenvalue problem of double size the state space displacement solutions are identified. The eigenvalues  $\xi$  are functions of the initial stress level and correspond to complex solution length scales ( $\pi/\xi$ ) plotted in the upper right part of Fig. 2. The changes in solution modes and length scales are shown in the lower part of the figure.

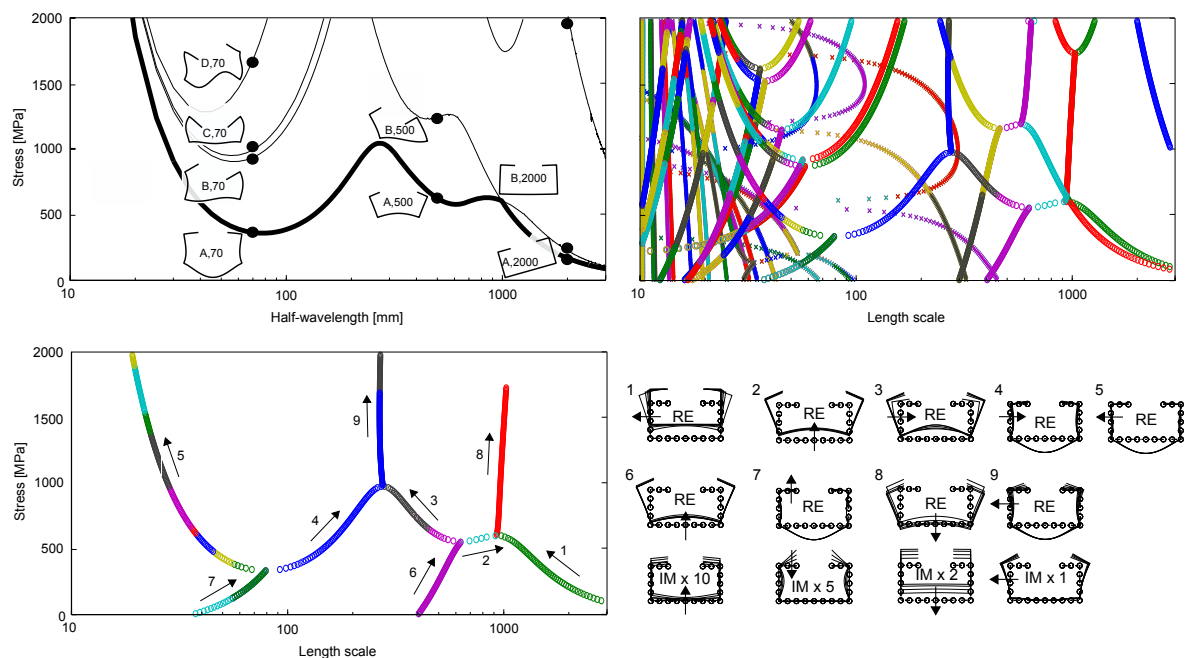


Figure 2: Signature curves, solution length scale curves and solution mode development.

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