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Macroscale mechanical domain parametric amplification: superthreshold pumping and optimal excitation parameters

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Abstract

This work investigates theoretically and experimentally the phenomenon of parametric amplification in a macroscale mechanical context, using a base-excited tilted cantilever beam as the model object. It demonstrates that an optimum mix between selected excitation parameters exists, that parametric amplification is possible for the second vibration mode, that the detuned case is phase lag insensitive, and that superthreshold pumping changes the gain/phase lag relationship, the phase lag range for which amplification and attenuation is realized, the optimum phase lag, and the attainable gain.

INTRODUCTION

Parametric Amplification (PA) is obtained by *pumping* (adding parametric oscillations to) externally driven harmonic oscillations [3]. Subthreshold pumping, i.e. pumping below the linear instability threshold (the transition between the stable and unstable regime wrt. parametric resonance), may be beneficial, e.g. by lifting a weak signal from the noise floor, effectively increasing the signal-to-noise ratio. The primary quantity of interest is the *gain*, which is the ratio between the stationary vibration amplitudes of the pumped and unpumped system. To advance the insight into the phenomenon of PA, the present work investigates theoretically and experimentally how various factors influence the gain, under subthreshold as well as superthreshold pumping conditions.

MODEL SYSTEM

As a representative model system we consider a base-excited cantilever beam. The combined scenario of direct and parametric excitation is realized by tilting the cantilever beam wrt. the line of excitation x as depicted in Fig. 1(a). The experimental setup is shown in Fig. 1(b,c). A non-dimensional third-order nonlinear equation of motion for the amplitude $z(t)$ of the first transverse vibration mode of a cantilever beam subjected to parametric and direct excitation, imposed at the base with perfect tuning (i.e. 2:1 ratio of parametric and direct excitation frequencies, also referred to as a *degenerate* case), can be written [2]:

$$\ddot{z} + \varepsilon 2\zeta\dot{z} + z + \varepsilon\Omega^2\lambda\sin\alpha\left(\hat{A}\cos\left(\Omega\tau+\phi\right) + 4\hat{B}\cos\left(2\Omega\tau\right)\right)z + \varepsilon\rho z^3 + \varepsilon\mu\left(\ddot{z}z^2 + \dot{z}^2 z\right)$$
$$+\varepsilon\frac{1}{2}\Omega^2\gamma\sin\alpha\left(\hat{A}\cos\left(\Omega\tau+\phi\right) + 4\hat{B}\cos\left(2\Omega\tau\right)\right)z^3 = \varepsilon\Omega^2\eta\cos\alpha\left(\hat{A}\cos\left(\Omega\tau+\phi\right) + 4\hat{B}\cos\left(2\Omega\tau\right)\right), \tag{1}$$

where the base-excitation along direction x, and its components $(\hat{u}_p, \hat{v}_p)$ along and transverse to the beam axis, respectively, is:

$$\hat{x}_p = \hat{A}\cos(\Omega\tau+\phi) + \hat{B}\cos(2\Omega\tau), \quad \hat{u}_p = \hat{x}_p\sin\alpha, \quad \hat{v}_p = \hat{x}_p\cos\alpha, \tag{2}$$

and where ε bookmark small terms, $\hat{A}$ is the *direct amplitude*, $\hat{B}$ the *pump amplitude*, ζ the damping ratio, and η, λ, γ, μ and ρ are mode shape integration constants.

STEADY-STATE MODEL RESPONSE

Theoretical predictions

Employing the method of multiple scales yields an algebraic equation for the first-order approximate stationary value for the first-mode vibration amplitude a of z:

$$\left[\left(-\sigma+\cos(2\psi)\,\beta(a)+\frac{1}{8}(3\rho-2\mu)\,a^2\right)^2+\left(\zeta+\sin(2\psi)\,\beta(a)\right)^2\right]a^2$$
$$=\left(\frac{1}{2}\eta\cos(\alpha)\,\hat{A}\,(\sigma+1)^2\right)^2,\quad \beta(a)=\frac{1}{4}\sin(\alpha)\,\hat{B}\left(16\lambda+3\gamma a^2\right)(\sigma+1)^2, \tag{3}$$

where $\sigma = \Omega - 1$ is the detuning from direct resonance and $\psi{=}\psi(\phi)$ is a phase lag between the direct and parametric excitation. A comparison with a steady-state response obtained in [2] employing the method of averaging, and direct numerical integration of (1) using a fourth-order Runge-Kutta method, is depicted in Fig. 2(a), alongside a linear response which is found letting $\rho{=}\mu{=}\gamma{=}0$ in (3). The linear response is given for an unpumped and a pumped state, of which the latter yields a higher amplitude, demonstrating that PA increases the gain $G{\equiv}a_{\text{pumped}}/a_{\text{unpumped}}{=}a/\,a|_{\hat{B}=0}$. For perfect excitation tuning (σ=0) all methods yield identical results. The accuracy obtained by multiple scales appears better compared to averaging but is within the same order of magnitude.

Optimal excitation parameters

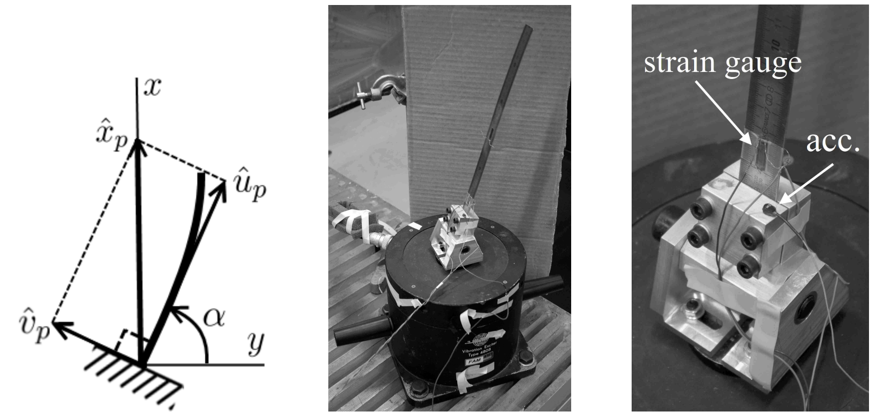
Direct numerical integration of the linearized version of (1), to find G for various combinations of damping coefficient ζ, direct amplitude $\hat{A}$, pump amplitude $\hat{B}$, and tilt angle α in the case of perfect tuning (σ=0) is illustrated in Fig. 2(b). It appears that larger $\hat{B}$ increases G but also the maximizing $\alpha = \alpha_{\text{opt}}$. This is also the case when reducing ζ.

PA of the second vibration mode

To the authors knowledge, it has not been experimentally demonstrated that PA of the second vibration mode is possible. The frequency equation for the second vibration mode is similar to (3) in structure, and can be solved for the amplitude a of that mode similarly. Fig. 3(a,b) reveals an increase in a, both at the free end and approximately midway of the experimental cantilever beam when utilizing PA. The largest relative increase in a for these two points yields the largest gain G. However, it might not be the most appropriate point at which one would like to harvest G due to aspects such as implementation in the physical setup.

Superthreshold pumping and detuned PA

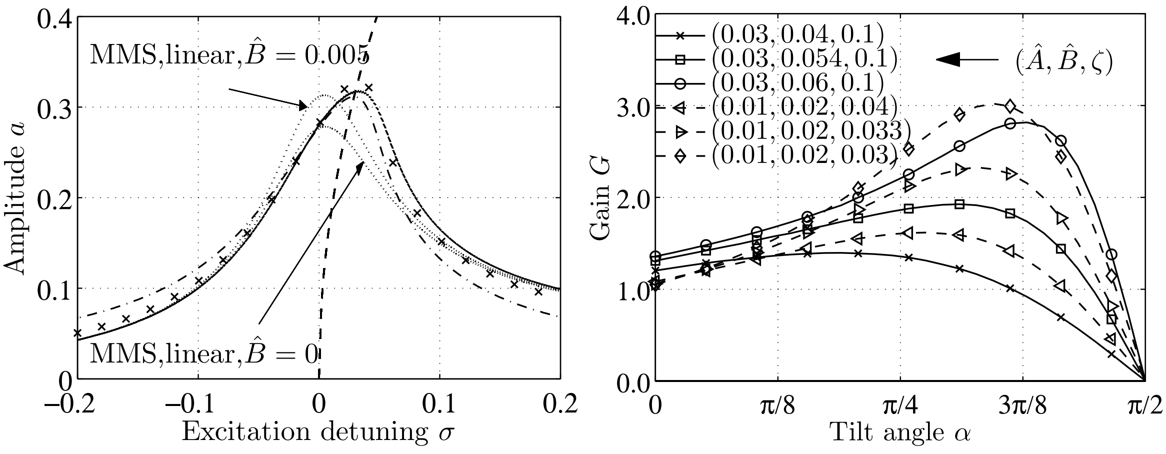
Superthreshold pumping produced experimental results as depicted in Fig. 3(c), where subthreshold pumping is shown for reference. The gain G appears, respectively, symmetric and asymmetric wrt. phase lag when utilizing small-medium and medium-large amplitudes a. This is also reported in [4], but for subthreshold pumping. The larger a appear to produce a larger G over a larger range of phase lags, thus increasing the applicability range of PA. Superthreshold pumping increases the signal,



(a) (b) (c)

Figure 1. Model system; (a) schematic; (b,c) cantilever beam, vibration exciter, fixture, and measurement sensors.

but also increases the noise floor [1]. Hence, this approach may not be appropriate for e.g. nano- and microelectromechanical systems. Parametric *attenuation* becomes more challenging to exploit for superthreshold pumping, because the optimum phase lag changes, the phase lag range causing significant attenuation becomes smaller, and the sensitivity wrt. changes in phase lag increases. Thus, the experiment confirms that the perfectly tuned case is phase lag sensitive, e.g. [2, 3], but also indicates that the optimum phase lag can change. Also, superthreshold pumping makes it possible to increase G further, thus partly overcoming the nonlinear saturation effects present for subthreshold excitation [2, 4]. To the authors knowledge, it has not been shown experimentally that the macroscale detuned system is phase lag insensitive. Experimental results for two detuned systems produce the results given in Fig. 3(d), demonstrating that detuned PA can be phase lag insensitive. The perfectly tuned system yielded a higher amplitude than the detuned system, except around parametric attenuation. In an industrial setting, being phase lag insensitive would be advantageous.



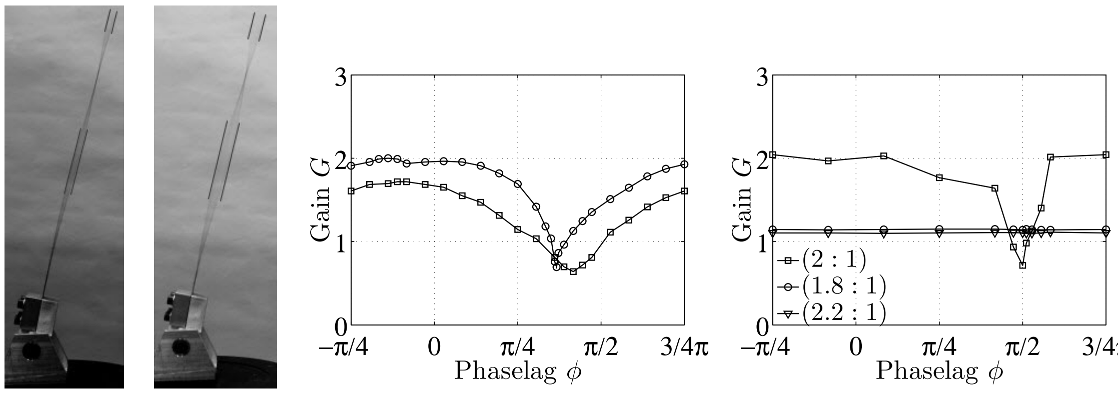
(a) (b)

Figure 2. Theoretical results; (a) stationary first-mode amplitude vs. excitation detuning. Direct numerical integration ($\times$), averaging (–·–·), multiple scales (——) and backbone (– –) of (1), ζ=$\hat{A}$=0.05, $\hat{B}$=0.005; (b) gain vs. tilt angle for various combinations of $\hat{A}, \hat{B}, \zeta$ with perfect excitation tuning (σ=0) employing direct numerical integration of (1). For (a) and (b): ϕ=ϕ_{opt}=$-\pi/4$, α=$\pi/4$.

CONCLUSIONS

It was found that an optimal mix exists between the pump amplitude, damping coefficient, and tilt angle. This is relevant for the class of systems which can be represented by a cantilever beam (e.g. high towers and helicopter blades) exposed to PA. It was demonstrated that PA is possible for the second vibration mode. PA within the parametric instability region yields 1) an asymmetric gain/phase lag relationship 2) a broader phase lag range for which the gain is realized, whereby the domain of applicability increases 3) a narrower phase lag range for which attenuation is realized 4) a higher gain even though nonlinear saturation effects reduce it 5) a change in the optimum phase lag. It was confirmed experimentally that the detuned case can be phase lag sensitive.

The work is in progress, currently involving experimental tests for subthreshold pumping and optimal tilt angle, and theoretical energy considerations for PA.



(a) (b) (c) (d)

Figure 3. Experimental results; second mode resonant beam vibration with (a) direct excitation or (b) direct and parametric excitation for $\alpha\approx{}^{4\pi}/_{9}$. (——): max. transverse deflection; (c,d): gain at the first direct resonance; (c) perfectly tuned system (σ=0) with small-medium (□) and medium-large (○) amplitudes, respectively, well below and above operation threshold; (d) perfectly tuned vs. detuned systems.

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