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Turbulence measurements using six lidar beams

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Abstract

The use of wind lidars for measuring wind has increased significantly for wind energy purposes. The mean wind speed measurement using the velocity azimuth display (VAD) technique can now be carried out as reliably as the traditional instruments like the cup and sonic anemometers. Using the VAD technique the turbulence measurements are far from being reliable. Two mechanisms contribute to systematic errors in the measurement of turbulence. One is the averaging of small scales of turbulence due to the volume within which lidars measure wind speed. The other is the contamination by the cross components of the Reynolds stress tensor, which arises because, in a VAD scan the lidar beams are combined to obtain different components of the wind field. In this work we demonstrate theoretically, how the contamination by the cross components can be avoided by using the measured variances of the line-of-sight velocities of six lidar beams. Under certain assumptions the volume averaging can then be avoided using the ensemble averaged line-of-sight Doppler velocity spectra. In this way, we can then in principle measure the true turbulence using six lidar beams.

1 Introduction

If say we had a perfect instrument that could measure the wind speed then we could measure all scales of turbulence without having to worry about any flow distortions. We imagine that such a dream instrument would have infinitely small measuring volume and would stably float at different points in space. Unfortunately, we do not live in a 'Harry Potter' world that have magic wands to virtually do anything, simply with a wave of a wand. We then have to deal with real-world instruments that have a finite measuring volume and a certain geometry. At present the best available instruments to measure turbulence are the sonic anemometers. These instruments need to be mounted on a meteorological mast (met-mast), and care has to be taken to avoid flow distortions due to the mast and the instrument itself. Met-masts are expensive structures, particularly as their sizes increase with the increasing height of the wind turbines ($>$ 150 m). With rapid expansion of offshore wind energy the cost of installing a met-mast in water increases manifold. If wind turbines would have been confined to only onshore sites and their sizes would have remained relatively small (say up to 50 m) then the wind energy industry would have been content with traditional met-mast anemometry. With new developments, the motivation to look for alternative ways of measuring wind speed has grown manifold. Fortunately, lidars are being investigated in their ability to measure wind speeds.

For wind energy purposes, lidars have been used only recently (since 2006). For meteorological studies they have been used for quite a long time (since 1960s). For wind energy purposes their utility in measuring the mean wind speed has been verified in several studies [Kindler et al., 2007, Peña et al., 2009, Smith et al., 2006]. In all these studies the velocity azimuth display (VAD) technique was employed in data processing. Turbulence measurements are however subjected to large systematic errors. Sathe and Mann [2012], Sathe et al. [2011] explain in detail these systematic errors. Two sources of systematic errors that render imprecise turbulence measurements are:

1. Averaging due to the large sample volume within which lidars measure wind speed

2. Contamination by the cross components of the Reynolds stress tensor

The first source of systematic error results in reducing the amount of measured turbulence. The second source of systematic error results in increasing the amount of measured turbulence. The combined result of these two contrasting effects depends on the type of the lidar (continuous wave or pulsed), height of the measurement, and atmospheric stability [Sathe et al., 2011]. In this work we demonstrate theoretically, how in principle it is possible to counter these sources of systematic errors.

2 Theoretical Considerations

At first it is useful to see mathematically how the cross components of the Reynolds stress tensor (R_{ij}) contribute in the measurement of turbulence using the VAD scan.

According to Sathe et al. [2011],

$$\langle \mathrm{v}'_m \mathrm{v}'_n \rangle_{\text{lidar}} = \int \Phi_{ij}(\boldsymbol{k}) X_i{}^m(\boldsymbol{k}) X_j^{*n}(\boldsymbol{k})\, d\boldsymbol{k}, \qquad (1)$$

where $\boldsymbol{v} = (u, v, w)$ is the wind field, $\langle \mathrm{v}'_m \mathrm{v}'_n \rangle_{\text{lidar}}$ is the component of the Reynolds stress tensor, $\Phi_{ij}(\boldsymbol{k})$ is the three dimensional spectral tensor, $\boldsymbol{k} = (k_1, k_2, k_3)$ is the wave vector, $\int d\boldsymbol{k} \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_1 dk_2 dk_3$, $X_i{}^m(\boldsymbol{k})$ is the weighting function for the m^{th} component of the wind field and $*$ denotes complex conjugation. In Eq. (1) the Einstein summation is clearly evident on the right hand side.

2.1 How to get rid of contamination by the cross components of the Reynolds stress tensor?

Instead of using a VAD scan, Eberhard et al. [1989] proposed using variances of the line-of-sight velocities. From simple geometrical considerations we then get,

$$\begin{aligned}\langle {v'_r}^2 \rangle = {} & \langle {u'}^2 \rangle \sin^2\phi \cos^2\theta + \langle {v'}^2 \rangle \sin^2\phi \sin^2\theta \\ & + \langle {w'}^2 \rangle \cos^2\phi + 2\langle u'v' \rangle \sin^2\phi \sin\theta \cos\theta \\ & + 2\langle u'w' \rangle \sin\phi \cos\phi \cos\theta \\ & + 2\langle v'w' \rangle \sin\phi \cos\phi \sin\theta \end{aligned} \qquad (2)$$

where $\langle {v'_r}^2 \rangle$ is the line-of-sight velocity variance, $\mathrm{v}'_i \mathrm{v}'_j$ are the components of the Reynolds stress tensor for $i, j = 1..3$, θ is the azimuth angle and ϕ is the half-opening angle. Thus, if we orient the lidar beams at certain θ and ϕ then $\langle {v'_r}^2 \rangle$ is only a function of six unknown components of R_{ij}. If we denote $\boldsymbol{\Sigma} = \left(\langle {u'}^2 \rangle, \langle {v'}^2 \rangle, \langle {w'}^2 \rangle, \langle u'v' \rangle, \langle u'w' \rangle, \langle v'w' \rangle \right)$ as a matrix of six unknown components of R_{ij} and $\boldsymbol{S} = \left(\langle {v'_{r_1}}^2 \rangle, \langle {v'_{r_2}}^2 \rangle, ..., \langle {v'_{r_6}}^2 \rangle \right)$ as a matrix of the variances of line-of-sight velocities, then mathematically we can write (from Eq. 2),

$$\boldsymbol{\Sigma} = \boldsymbol{M^{-1} S} \qquad (3)$$

where, $\boldsymbol{M}$ is a 6×6 matrix of the coefficients of $\boldsymbol{\Sigma}$. In principle we then need six lidar beams to measure R_{ij}. However, choosing any random combination of lidar beams would introduce random errors in the measurement of R_{ij}. In order to minimize the random errors in the measurement of R_{ij}, Sathe [2012] derived the objective function,

$$\text{Minimize } f(\boldsymbol{X}) = Tr\left(\begin{bmatrix} \frac{7}{8} & \frac{1}{8} & 0 & 0 & 0 & 0 \\ \frac{1}{8} & \frac{7}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{M}^{-1} \boldsymbol{M}^{-1T} \right) \qquad (4)$$

subject to constraints,

$0^\circ \le \theta_i|_{i=1..6} \le 360^\circ$

$0^\circ \le \phi_i|_{i=1..6} \le 45^\circ$

where $\boldsymbol{X} = (\theta_i, \phi_i)|_{i..6}$. By optimizing Eq. (4), we get the optimum configuration as given in table 1. Substituting the values of θ_i, ϕ_i from table 1 in Eq. (3)

Table 1: Optimum configuration

i	1	2	3	4	5	6
θ_i	0	72	144	216	288	288
ϕ_i	45	45	45	45	45	0

we can then estimate $\boldsymbol{\Sigma}$ from the measurements of $\boldsymbol{S}$. We thus obtain R_{ij} without any contamination by the cross components.

2.2 How to get rid of volume averaging?

To get rid of volume averaging Mann et al. [2010] derived an expression for a continuous wave lidar such that ensemble averaged Doppler spectra is used instead of individual Doppler spectra. Mathematically, it is given as,

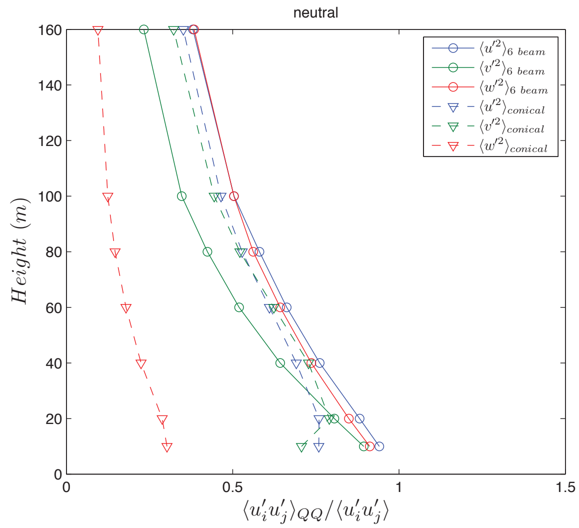
$$\langle S(v_r) \rangle = \int_{\infty}^{\infty} \varphi(s) p(v_r(s)) ds, \qquad (5)$$

where $\langle S(v_r) \rangle$ is the ensemble averaged Doppler spectra, $\varphi(s)$ is the weighting function, and $p(v_r(s))$ is the probability density function of v_r at position s. By assuming a reasonable $p(v_r(s))$ (e.g. Gaussian) we obtain $\langle S(v_r) \rangle$ as a function of unfiltered line-of-sight velocity variance σ. Using the measurements we can then fit Eq. (5) and obtain σ.

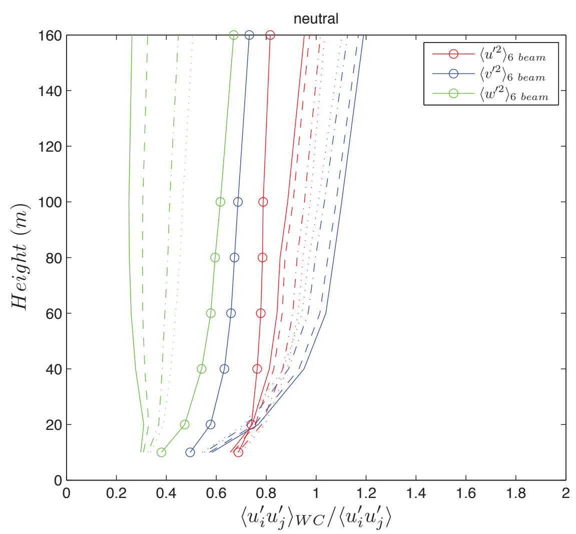
3 Results

Using the theoretical considerations from section 2, we compare the theoretical calculations of the ratio of variances measured by lidar (continuous wave and pulsed) and that of a point measurement. Owing to lack of measurements we could not get rid of volume averaging. We make the comparisons for three stability conditions, neutral, stable and unstable. The ZephIR manufactured by

Natural Power is used as a continuous wave lidar, and the WindCube manufactured by Leosphere is used as a pulsed lidar. Fig. 1 shows the comparison of the ratio of the variances measured by lidar to that of the point measurement using the VAD scan and the six beam method. We define systematic error as the deviation of the lidar measured turbulence with respect to the true turbulence. Thus, the farther the ratio is from one the more the systematic error. At first it is clearly seen that the variances are attenuated significantly for both lidars, particularly for the w component using the VAD scan. Since the six beam method uses one vertical beam the attenuation is reduced significantly. For the continuous wave lidar the systematic errors increase with height, whereas for the pulsed lidar they decrease with height using both measurement techniques. This is because the measurement volume of the continuous wave lidar increases quadratically with height resulting in greater averaging of turbulence. For a pulsed lidar the measurement volume is constant with height. Furthermore it is interesting to note that using a VAD scan for the pulsed lidar there is hardly any systematic error at greater heights. This is because the averaging of turbulence in the measurement volume is compensated by the contamination by the cross components of R_{ij}. Informally, we can say that the pulsed lidar measures the right turbulence for the wrong reasons. Therefore one should be careful in using the turbulence measurements by the VAD scan-

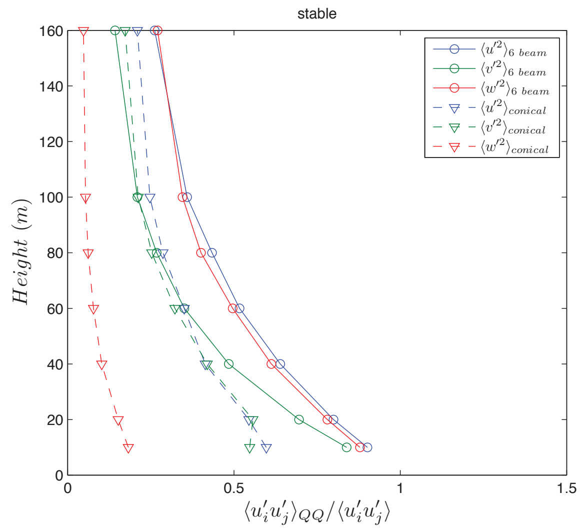


(a) ZephIR

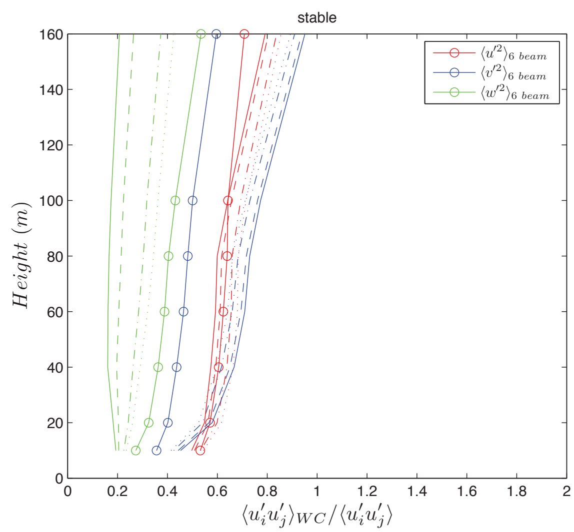


(b) WindCube

Figure 1: Ratio of ZephIR (QQ) and Windcube (WC) variances with respect to the point variances under neutral conditions using the six Beam approach and the VAD (conical) scan

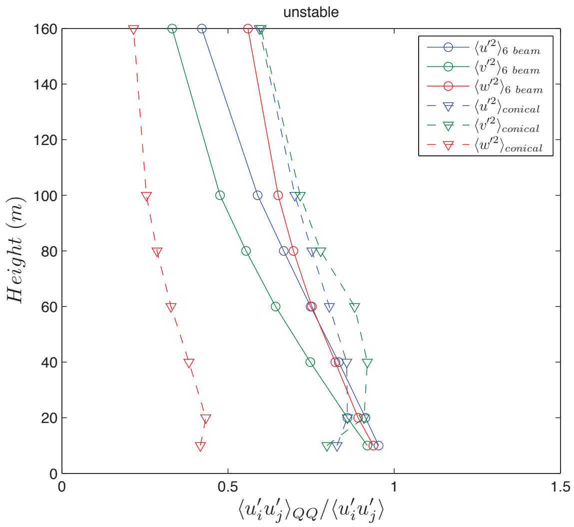


(a) ZephIR

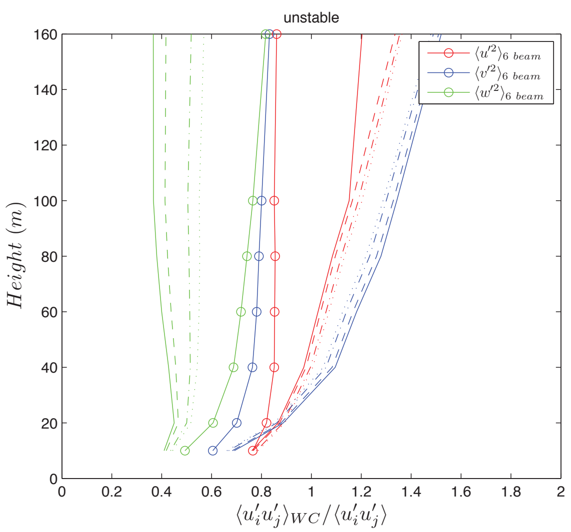


(b) WindCube

Figure 2: Same as Fig. 1 but under stable conditions



(a) ZephIR



(b) WindCube

Figure 3: Same as Fig. 1 but under unstable conditions

ning technique even if the systematic errors seem negligible. Its implications are clearly seen when we study turbulence spectra [Sathe and Mann, 2012]. For the six beam method the only systematic errors are due to the volume averaging. This is much more evident when we observe Fig. 3 for the pulsed lidar, where we see overestimation of turbulence using the VAD scan. This is because under unstable conditions turbulence scales are large and there is not much averaging in the measurement volume. However, the contamination by the cross components results in measuring more turbulence. For the six beam method we can see that there is no overestimation of turbulence. Under stable conditions (Fig. 2), we observe much more averaging of turbulence due to smaller turbulence scales than under neutral and unstable conditions.

4 Conclusions

The VAD scan should not be used for turbulence measurements since they result in obvious systematic errors; volume averaging and contamination by the cross components. Thus, even if we had a perfect lidar where there was no volume averaging there will always be contamination by the cross components of the Reynolds stress tensor. The six beam method in principle does not suffer from the contamination by the cross components, and has the potential to measure precise turbulence, if and when we get rid of the volume averaging.

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