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Vibrations of axially moving strings with in-plane oscillating supports

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Abstract

For a traveling string moving in the plane we analyze analytically the transverse vibrations arising from oscillation of the string supports. Of special interest is the excitation typical of roller chain drives, where meshing between chain and sprockets cause both noise and vibration. Considering a uniform, heavy string moving at subcritical speed with prescribed endpoint motion, and ignoring longitudinal inertia, one obtains a continuous, nonlinear, gyroscopic, parametrically and externally excited system. By employing a single-mode approximation, using velocity dependent mode shapes, the system response is approximated using the method of multiple scales. Vibrations from support oscillations characteristic of roller chain drives are investigated. Conclusions about critical values for chain drive parameters such as pretension and meshing frequency are sought and identified.

INTRODUCTION

Research in the field of axially moving strings has been motivated by applications like chain saws, belt drives, roller chains and fibre winding. Fluctuation of transport speed and string tension both leads to parametric excitation. Usually mono-frequency excitation has been considered, but meshing impacts, attached machinery and crankshaft powered drives may introduce multifrequency excitation [1]. The discrete nature of chain drives introduce effects known as polygonal action [2]. In this work we recognize that polygonal action leads to combined non-smooth longitudinal and transverse excitation, corresponding to parametric and direct excitation, respectively, of transverse string vibration. A model is presented capable of including these effects by prescribing the positions of the string endpoints in the plane. The formulation utilizes velocity dependent mode shapes and analysis of nonlinear effects is done using the method of multiple scales. For a comprehensive review of research on transverse vibrations of axially moving strings see e.g. [3].

MATHEMATICAL MODEL

Figure 1a shows meshing between a roller chain and a sprocket. Relative velocity between the impact roller and the tangential velocity of the sprocket causes impact, which is a significant source of noise and vibration. A mathematical model is formulated in order to analyze transverse vibration arising from this type of excitation. Figure 1b shows a uniform string with mass per unit length ρA, axial stiffness EA moving with constant velocity V between supports positioned in the inertial coordinate system (X, Y) at $\left(\tilde{x}_0(\tilde{t}), \tilde{y}_0(\tilde{t})\right)$ and $\left(\tilde{x}_L(\tilde{t}), \tilde{y}_L(\tilde{t})\right)$. Transverse and longitudinal deformations are given by (U, W) in (X, Y)-directions, respectively.
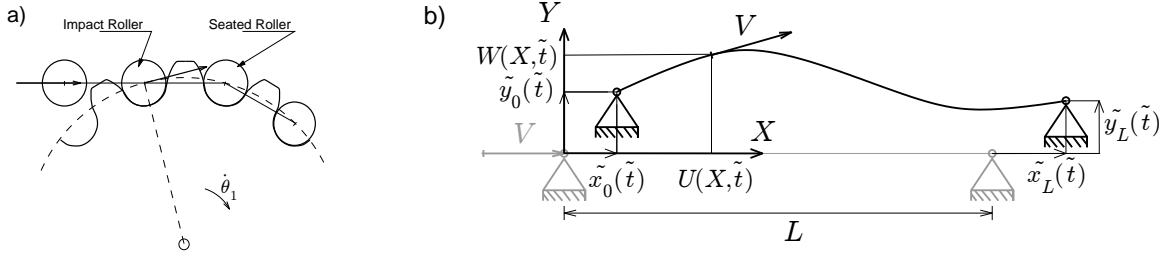
Figure 1. a) Meshing between chain and sprocket. The discrete nature of a chain leads to non-smooth contact between the chain roller and sprocket. b) Tensioned string traveling longitudinally between two supports moving in the plane. Shown in grey is the undeformed string moving between stationary supports.

The equation of motion is formulated using Hamilton's principle. Since finite amplitude vibration response during resonant excitation is of interest, the mathematical model is formulated using the approximate nonlinear strain measure

$$\epsilon(X, T) = U_X + \frac{1}{2} W_X^2. \tag{1}$$

In a roller chain, wear affects chain tension, and therefore pretension $P_0 > 0$ is an important parameter, defined here as the tension of the undeformed stationary string. The potential energy for the string can be formulated using the approximate strain measure and the pretension. Kinetic energy is formulated using the transverse- and longitudinal velocity components of a string element $V_1 = V(1 + U_X) + U_{\tilde{t}}$ and $V_2 = VW_X + W_{\tilde{t}}$, valid for small $|U_X|, |W_X|$, where $(\cdot)_{X,T}$ denotes partial differentiation with respect to X, T, respectively. Requiring the virtual displacements to be kinematically admissible means that their variation is zero at the supports where motion is specified, i.e. $\delta U|_0^L = \delta W|_0^L = 0$. Therefore, virtual work at the supports become zero and Hamilton's principle can be applied in its standard form. The actual work done by the reaction forces are not zero, the system is non-conservative, and the support conditions classify as rheonomic. The non-dimensional equations of motion governing longitudinal and transverse vibration become, respectively,

$$\tilde{u}_{tt} + 2v\tilde{u}_{xt} + v^2 u_{xx} - \mu(u_x + \tfrac{1}{2}\tilde{w}_x^2)_x = 0, \tag{2}$$

$$\tilde{w}_{tt} + 2v\tilde{w}_{xt} + v^2 w_{xx} - \left[\left(1 + \mu(\tilde{u}_x + \tfrac{1}{2}\tilde{w}_x^2)\right)\tilde{w}_x\right]_x = 0 \tag{3}$$

with dimensionless parameters

$$x, \tilde{u}, \tilde{w} = \frac{X, U, W}{L}, \quad t = \sqrt{\frac{P_0}{\rho A L^2}}\tilde{t}, \quad \bar{v} = V \Big/ \sqrt{\frac{P_0}{\rho A}}, \quad \mu = \frac{EA}{P_0} \tag{4}$$

and inhomogeneous support conditions

$$\tilde{u}(0, t) = x_0(t), \quad \tilde{u}(1, t) = x_1(t), \quad \tilde{w}(0, t) = y_0(t), \quad \tilde{w}(1, t) = y_1(t), \tag{5}$$

$$x_0, x_1, y_0, y_1 = \left(\tilde{x}_0, \tilde{x}_L, \tilde{y}_0, \tilde{y}_L\right)L^{-1}. \tag{6}$$

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In the case where transverse wavespeed $v_0 = \sqrt{P_0/\rho A}$ is much lower than longitudinal wave speed $c = \sqrt{E/\rho}$, changes in tension $N(x,t) = 1 + \mu(\tilde{u}_x + \frac{1}{2}\tilde{w}_x^2)$ propagates nearly instantaneously and dynamic tension can be approximated as being independent of x. Utilizing this by requiring $N_x = 0$ leads to $\epsilon_x = 0$, and integration of (1) from 0 to x with respect to x and use of the support conditions leads to a solution of (2), thus neglecting longitudinal inertia. The strain independent of x is found by integrating (1) over the interval. Support conditions for (3) are made homogeneous by introducing the transformation $\tilde{w}(x,t) = w(x,t) + y_0(t)(1-x) + y_1(t)$. Inserting this and the time dependent strain into (3) yields

$$w_{tt} + 2vw_{xt} - (1 - \bar{v}^2)w_{xx} - \mu\Big[p(t) + \frac{1}{2}\int_0^1 \tilde{w}_x^2 dx\Big]w_{xx} = f(x,t) \qquad (7)$$

where

$$p(t) = x_1(t) - x_0(t) + \frac{1}{2}\big(y_0(t) + y_1(t)\big)^2, \qquad (8)$$

$$f(x,t) = -y_{0,tt}(t)\big(1-x\big) - y_{1,tt}(t) - 2v\big(y_{1,t}(t) - y_{0,t}(t)\big) \qquad (9)$$

and boundary (support) conditions for (7) are $w(0,t) = w(1,t) = 0$. Equation (7) governs the transverse motion of the string. It is a second order partial differential equation which is non-linear due to effects of longitudinal stretching, parametrically excited from longitudinal support motion and directly excited due to transverse support motion.

ANALYSIS

Linear solution

The analysis of (7) is carried out using eigenvalues and eigenvectors obtained from the corresponding linear unforced system; $\lambda_n = i\omega_n = in\pi(1 - v^2)$, $\psi_n = \frac{1}{n\pi\sqrt{1-v^2}}e^{in\pi vx}\sin(n\pi x)$, where $i = \sqrt{-1}$ is the imaginary unit [5]. Velocity dependent eigenpairs are chosen because the (non-dimensional) transport speed $\bar{v}$ depends on pretension, which may decrease significantly due to chain wear, thus affecting chain tension. Furthermore, the Coriolis acceleration is proportional to $\bar{v}$ and therefore has an increased significance as pretension is reduced.

Nonlinear perturbation analysis

An approximate solution of (7) valid for small nonlinearity and parametric excitation is determined using a single term Galerkin approximation based on the n'th (complex) mode. As is customary, [1], the system is analyzed in state space formulation, with $\mathbf{v} = \{w_t, w\}^T$. Introducing the excitation vector $\mathbf{q} = \{f, 0\}^T$ and using the standard notation [5] the equation of motion (7) becomes

$$\mathbf{A}\mathbf{v}_t + \mathbf{B}\mathbf{v} + \varepsilon\mathbf{C}(\mathbf{w})\mathbf{v} = \mathbf{q} \qquad (10)$$

where the non-standard nonlinear matrix operator is defined as

$$\mathbf{C} = \begin{bmatrix} 0 & Q\big(p(t) + N(w_x)\big) \\ 0 & 0 \end{bmatrix} \quad \text{with} \quad Q = -\mu\frac{\partial^2}{\partial x^2}, \quad N(w_x) = \frac{1}{2}\int_0^1 w_x^2 dx. \qquad (11)$$

For $\varepsilon \ll 1$ an approximate solution of (10) is sought using the method of multiple scales.

RESULTS

The aim of the analysis is to obtain approximate analytical solutions for the frequency response and resulting stability properties. Periodic support motion is analyzed by decomposing parametric and direct excitation terms into spectral components using (truncated) Fourier series. Solutions are sought for support motion such as

$$x_0 = r_1 \sin(\Omega t), \; y_0 = r_1 \cos(\Omega t) \quad x_1 = r_2 \sin(\Omega t + \varphi), \; y_1 = r_2 |\cos(\Omega t + \varphi)|. \qquad (12)$$

Load case (12) represents the typical case where a chain roller experiences smooth motion when it enters the free span at $x = 0$ (looses contact with a sprocket) and impact as it leaves the free span at $x = L$ (gets in contact with the sprocket). Two events generally happening out of phase. In this case, a one term Fourier decomposition of (8) gives frequency components of both Ω and 2Ω. This combined with direct excitation (9) of frequency Ω may lead to parametric amplification. Similar types of support motion introduce cases of combination resonance. Using such examples, relevant conclusions for safe operation of roller chain drives are sought, e.g. critical excitation- frequencies, motion patterns and pretension effects.

Physical approximations will be tested against simulation software for detailed chain drive simulation. If possible, results will also be tested using commercially available simulation software for analyzing chain drives. Mathematical approximations will be tested by applying numerical continuation.

CONCLUSION

This work is in progress. So far a model has been established for theoretically analyzing the dynamics of a moving string fixed between supports undergoing simultaneous transverse and longitudinal motion. The method of multiple scales will be used for analyzing the nonlinear system to obtain analytical results useful for understanding the dynamics of roller chain drives.

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