Technical University of Denmark

Modelling the influence of yaw using a simple vortex rotor model

Troldborg, Niels; Gaunaa, Mac; Guntur, Srinivas

Published in:
Proceedings of EWEA 2012 - European Wind Energy Conference & Exhibition

Publication date:
2012

Document Version
Publisher's PDF, also known as Version of record

Modelling the influence of yaw using a simple vortex rotor model

Niels Troldborg* Mac Gaunaa* and Srinivas Guntur *
*DTU Wind Energy, Department of Wind Energy, Technical University of Denmark, Risø Campus, DK-4000 Roskilde, Denmark
niet@risoe.dtu.dk

Abstract

A simple analytical rotor model based on vortex theory is briefly presented and used to investigate the main mechanisms for wind turbine rotors operating at yaw misalignment. The overall findings of the model is verified by comparing with an existing model as well as with results obtained using the actuator disc technique combined with full Navier-Stokes computations.

Keywords: Yawed rotor, vortex model, actuator disc

1 Introduction

Wind turbines operate in an unsteady environment where the wind direction changes frequently and therefore they may operate in yaw for longer periods of time. A yaw misaligned wind turbine experiences cyclic loads which reduces its fatigue life time and therefore the influence of yaw on turbine loads needs to be understood and modelled properly. For this reason the effects of yaw misalignment on rotor loads have been studied extensively, both experimentally [1] and numerically [2, 6]. However, in a recent study [7] it was revealed that state-of-the-art engineering models attempting to include the effect of yaw misalignment on wind turbine performance and loads still show important discrepancies when compared to measurements.

In this paper we employ a simple analytical rotor model based on vortex theory to investigate and explain the main mechanisms for yawed wind turbine rotors. The overall findings of the model is verified by comparing both with a model by Glauert [3, 4, 5] and with results obtained using the actuator disc technique combined with full Navier-Stokes computations.

2 Glauert's model for yawed rotors

For a rotor operating at an angle ψ relative to a steady uniform inflow Glauert [3, 4, 5] showed the following relation between the thrust coefficient C_T and the normal induction factor $a = -W_n/V_0$:

$$C_T = 4a\sqrt{\sin\psi^2 + (\cos\psi - a)^2} \tag{1}$$

Here W_n is the induced velocity in the direction normal to disc, V_0 is the free-stream velocity and

$$C_T = \frac{T}{0.5\rho A V_0^2} \tag{2}$$

where T is the trust on the disc and A is the disc area. In equation 1 T and a are averages over the disc. In order to describe the induced velocities over a yaw misaligned rotor disc Glauert proposed the following model [3]

$$a_n(r,\theta) = a\left(1 + (r/R)\tan(\chi/2)\cos(\theta - \theta_0)\right) \tag{3}$$

where $a_n(r,\theta)$ is the local induction factor, a is the mean induced induction factor found from equation 1, χ is the wake skew angle defined as the angle between the velocity in the wake and the rotational axis of the rotor. θ is the azimuthal position and θ_0 is the angle where the blade is deepest into the wake. The wake skew angle is assumed to be constant with the radius and is in the present case computed as

$$\cos\chi = \frac{\cos\psi - a}{\sqrt{(1 - a\cos\psi)^2 + a^2\sin^2\psi}} \tag{4}$$

Having computed $a_n(r,\theta)$ the local power coefficient for a constantly loaded rotor is determined as:

$$C_{P,loc} = C_T\left(\cos(\psi) - a_n(r,\theta)\right) \tag{5}$$

3 A simple vortex model for yawed rotors

The proposed vortex model is a generalization of the model originally developed by Øye [8, 9] where the idealized wind turbine is modelled with an infinite number of blades of constant circulation. In order to further simplify the analysis we assume infinitely high tip speed ratio, such that rotational effects in the wake are avoided, as in classical $1D$ momentum theory [10]. Moreover, we assume steady uniform flow and neglect the expansion of the wake. Besides these assumptions the present model uses the usual potential-flow assumptions, which are an incompressible, irrotational and inviscid flow corresponding to very high Reynolds numbers and low mach-numbers. Effects of drag on rotor performance are not included in the model, so that the local forces per unit span on the blades can be determined using Joukowski's relation.

$$d\boldsymbol{F} = \rho \boldsymbol{V}_{rel} \times d\boldsymbol{\Gamma} \qquad (6)$$

where $d\boldsymbol{\Gamma}$ is the local bound circulation, which is aligned with the blade, $\boldsymbol{V}_{rel}$ is the local velocity relative to the blade and ρ denotes density. Using this relation together with the above assumptions the following can be shown:

$$C_{T,loc} = \frac{\tilde{F}_n}{0.5\rho V_0^2} = \frac{\Gamma\Omega}{\pi V_0^2} \qquad (7)$$

$$C_{P,loc} = C_{T,loc}\left(\cos(\psi) + W_n(r,\theta)/V_0\right) \qquad (8)$$

where $\tilde{F}_n$ signify the local force per unit area in the and normal direction and V_0 denotes the free stream velocity. Γ signifies the total bound circulation on the rotor, ψ is the yaw angle and r and θ is the radial and tangential position where the forces are evaluated. W_n is the induced velocity in the normal direction at the rotor disc.

The fact that the local thrust coefficient is constant (equation 7) is a consequence of the assumption of a very high tip speed ratio. From this assumption it namely follows, that the free-stream velocity component relative to the spinning rotor is negligible, yielding that a rotor with a constant circulation corresponds to a rotor with constant local thrust coefficient.

4 The Navier-Stokes actuator disc model

The actuator disc model used in the present work is described in [12, 16] and was originally developed by Mikkelsen as an extension of the model presented in [11]. The model combines the Navier-Stokes flow solver EllipSys3D [13, 14, 15], with an actuator disc technique where body forces are distributed on a disc representing a wind turbine rotor. As described in [16], the applied disc forces are distributed smoothly on several mesh point to avoid singular behavior.

The steady state flow field is solved in a cartesian computational domain with dimensions $(L_x, L_y, L_z) = (30R, 30R, 100R)$, where L_x, L_y and L_z is respectively the width, height and length of the domain and R is the radius of the disc. The disc is located $15R$ downstream of the inlet. In the region around the rotor disc grid points were distributed equidistantly with a resolution of $R/30$ in order to resolve the strong gradients here. From $1.3R$ to $20.2R$ downstream of the disc the grid was stretched slightly using a relative stretch of 0.27% in the flow direction in order to limit the number of grid point while still keeping a good resolution of the wake. The total number of grid points in the used grid is $18.9 \cdot 10^6$. The reason for having a high resolution of the first $20R$ of the wake is that the present computations originally was designed to study wake deflection as a function of yaw angle [17]. The boundary conditions were as follows: Uniform inflow at the inlet, Neumann conditions at the outlet, symmetry conditions on the top and bottom boundaries and periodic conditions on the sides.

5 Preliminary results

This section presents some results of the simple vortex model and compare them to Glauert's model as well as to predictions from actuator disc simulations. We consider a uniformly loaded disc, (i.e. $C_{T,loc} = C_T$) in a uniform inflow at different yaw angles. Assuming uniform loading as well as uniform and steady inflow is of course very crude in relation to wind turbine rotors but will still highlight the main mechanism of rotors in yaw. In the following, results are presented for thrust coefficients of $C_T = 0.36, 0.64, 0.80$ and 0.89, respectively, and yaw angles of $\psi = 0^o, 5^o, 10^o, 20^o, 30^o, 45^o$ and 60^o, respectively.

Figures 1-2 compares actuator disc and vortex model predictions of $C_{P,loc}$ and $a_n = \cos(\psi) - W_n(r,\theta)/V_0$, respectively for $C_T = 0.64$ and $\psi = 30^o$. Azimuth

$\theta = 0^o$ is horizontal in the direction pointing upstream and $\theta = 90^o$ is vertically upwards. The corresponding comparisons of actuator disc predictions with Glauert's model is shown in Figures 3-4. Note that the predictions at 90^o and 270^o (the two vertical directions) are coinciding.

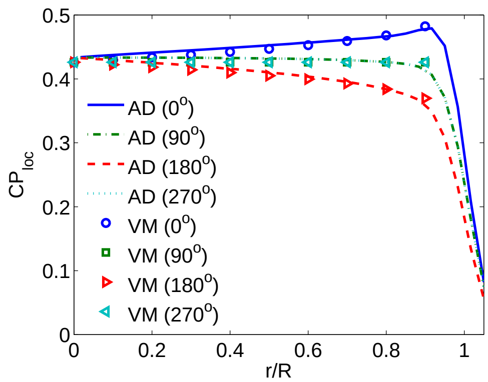

Figure 1: *Local power coefficient predicted using the simple vortex model and actuator disc model, respectively. The thrust coefficient and yaw angle is* $C_T = 0.64$ *and* $\psi = 30^o$.

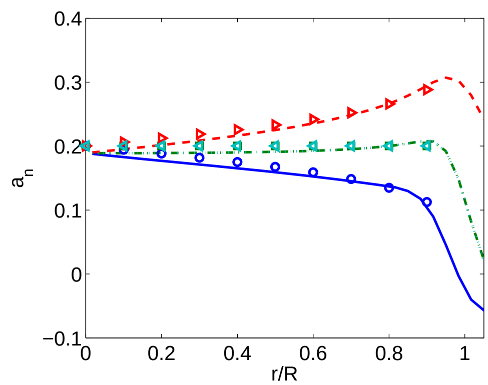

Figure 2: *Normal induction in the rotor plane predicted using the simple vortex model and actuator disc model, respectively. The thrust coefficient and yaw angle is* $C_T = 0.64$ *and* $\psi = 30^o$. *The legend is as in Figure 1*

As seen the predictions of both the proposed vortex model and Glauert's model are in good agreement with the actuator disc computations, which suggests that the two models captures the most important physics of rotors in yaw. It appears that Glauerts model is slightly more accurate in the inner part of the disc, whereas the vortex model is slightly more accurate on the outer part where it captures the nonlinear behavior of both $C_{P,loc}$ and a_n.

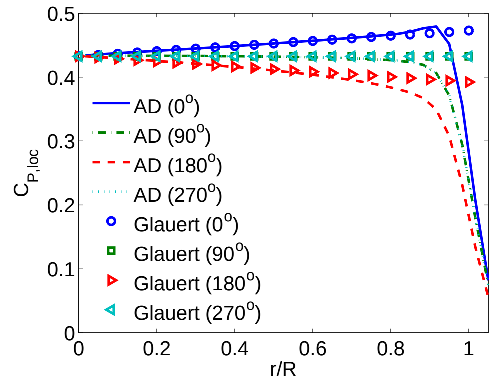

Figure 3: *Comparison of local power coefficient predicted by Glauert's model and the actuator disc model, respectively. The thrust coefficient and yaw angle is* $C_T = 0.64$ *and* $\psi = 30^o$.

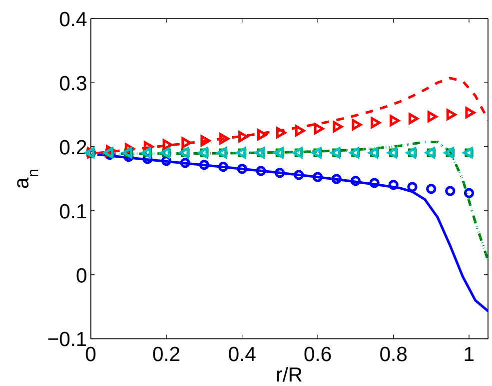

Figure 4: *Normal induction in the rotor plane predicted using Glauert's model and the actuator disc model, respectively. The thrust coefficient and yaw angle is* $C_T = 0.64$ *and* $\psi = 30^o$. *The legend is as in Figure 3*

One of the strengths of the proposed vortex model is its ability to also predicting the induced velocities in the radial and tangential directions. The vortex model predictions of these induced velocity components are compared with actuator disc results in Figures 5 and 6 for the same case as in Figures 1-2. As seen the resemblance between the predictions is good, which further verifies the the proposed vortex model.

Finally, Figures 7-8 shows the global power coefficient as a function of yaw angle predicted by the vortex model and Glauert's model, respectively in comparison with actuator disc results. Generally, both models show good agreement with actuator disc predictions, however, the proposed vortex model show significant discrepancies with increasing yaw angle and thrust coefficient. The reason for this is that the vortex model, in contrast to the model of Glauert,

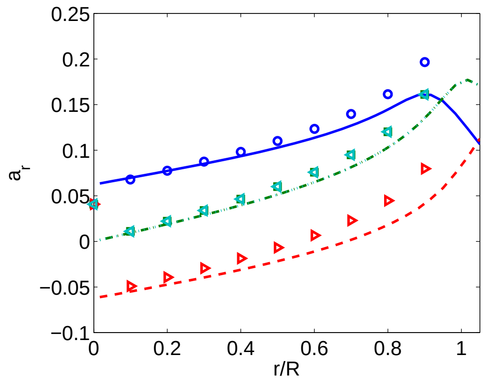

Figure 5: *Spanwise distribution of radial induction, $a_r = W_r/V_0$ (The legend is as in Figure 1); The thrust coefficient and yaw angle is $C_T = 0.64$ and $\psi = 30^o$.*

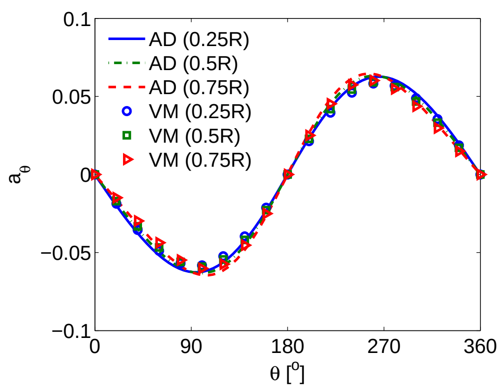

Figure 6: *Azimuthal variation of tangential induction, $a_\theta = W_\theta/V_0$, at $r/R = 0.25, 0.50$ and 0.75. The thrust coefficient and yaw angle is $C_T = 0.64$ and $\psi = 30^o$.*

assumes the average induction in the rotor plane to be constant with yaw angle. It is seen that the model of Glauert performs very well on an integral basis for nearly all yaw angles and loads.

Future work will be directed towards improving the vortex model prediction for high loading and yaw angles as well as to formulate in a form which is suitable for implementation in an aero-elastic code.

6 Conclusions

An analytical model based on classical vortex theory is proposed for predicting performance of uniformly loaded rotors operating in yaw. The capability of the model has been shown by comparing it to Navier-Stokes actuator disc simulations and with predictions of the model by Glauert. The vortex model is

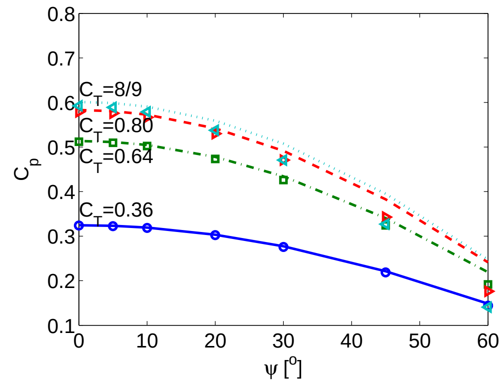

Figure 7: *Global power coefficient as a function of yaw angle at different thrust coefficients. Lines: Actuator disc predictions; Symbols; Vortex model predictions.*

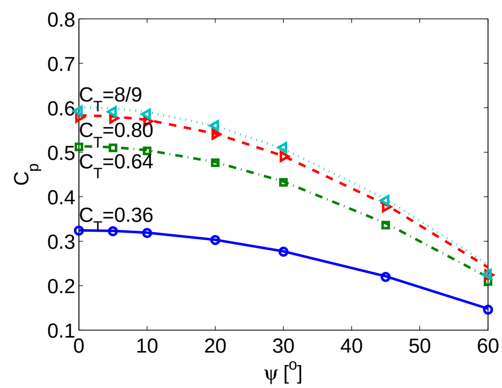

Figure 8: *Global power coefficient as a function of yaw angle at different thrust coefficients. Lines: Actuator disc predictions; Symbols; Glauert's model predictions.*

proven to give predictions of all induction components and power performance, which are in close agreement with the actuator disc simulations at low to medium loads and over a wide range of yaw angles. However, for higher loads and yaw angles the proposed vortex model is less accurate. In these cases the model is outperformed by the model of Glauert, which is shown to be accurate for all load conditions in terms of integral values. However, unlike the proposed vortex model, the model of Glauert does not provide the inductions in the radial and tangential directions and cannot predict the nonlinear behavior of the induced velocity on the outer part of the disc.

References

[1] Haans, W., van Kuik, G. and van Bussel, GJW. Experimentally observed effects of yaw misalignment on the inflow in the rotor plane. Journal of Physics: Conference Series. The Science of Making Torque from Wind. Technical University of Denmark, Lyngby, August 2007.

[2] Sant, T., van Kuik, G. and van Bussel, GJW. Estimating the Angle of Attack from Blade Pressure Measurements on the National Renewable Energy Laboratory Phase VI Rotor Using a Free Wake Vortex Model: Yawed Conditions. Wind Energy 2009, 12:1-32.

[3] Glauert, H. A general theory for the autogyro. Technical Report ARC R & M 786, ARC, 1926.

[4] Glauert, H. The elements of aerofoil and airscrew theory, second edition. Cambridge University Press, Cambridge, England, 1947.

[5] Glauert, H. Airplane propellers. In W.F. Durand, editor, Aerodynamic Theory, chapter Division L, pages 169-360. Dover Publications, New York, NY., USA, 1963.

[6] Shen, WZ. Actuator Line / Navier Stokes Computations for Flows past the Yawed MEXICO Rotor. Wake Conference, held at Gotland University 2011.

[7] Schepers, JG et al. Final report of IEA Task 29, Mexnext (Phase 1): Analysis of Mexico wind tunnel measurements. To be published 2012.

[8] Øye, S. A simple vortex model. In Proc. of the third IEA Symposium on the Aerodynamics of Wind Turbines, ETSU, Harwell, 1990, p. 4.1-4.15.

[9] Madsen, HA, Mikkelsen, R, Johansen, J,Bak, C, Øye, S & Sørensen, NN. Inboard rotor/blade aerodynamics and its influence on blade design. Chapter in Risøtechnical report: Research in aeroelasticity EFP-2005. Ed. Bak, C. Risø-R-1559(EN), Roskilde, Denmark, May 2006. p.19-39.

[10] Hansen, MOL. Aerodynamics of Wind Turbines. Second edition. Earthscan, 2008.

[11] Mikkelsen, R., Actuator Disc Methods Applied to Wind Turbines, MEK-FM-PHD 2003-02, Technical University of Denmark, 2003.

[12] Ivanell, SSA. Numerical Computations of Wind Turbine Wakes, PhD Thesis, Linné Flow Centre, KTH Mechanics, Stockholm, Sweden, 2009.

[13] Michelsen, JA. Basis3D - a Platform for Development of Multiblock PDE Solvers. Report AFM 92-05, Dept. of Fluid Mechanics, Technical University of Denmark, DTU, 1994.

[14] Michelsen, JA., Block Structured Multigrid Solution of 2D and 3D elliptic PDE's, Report AFM 94-06, Dept. of Fluid Mechanics, Technical University of Denmark, DTU, 1994.

[15] Sørensen, NN., General Purpose Flow Solver Applied to Flow over Hills, PhD Dissertation, Risø-R-827(EN), RisøNational Laboratory, Roskilde, Denmark, 1995.

[16] Troldborg, N., Gaunaa, M. and Mikkelsen, R. Actuator Disc Simulations of Influence of Wind Shear on Power Production of Wind Turbines. Torque conference, 2010.

[17] Guntur, S., Troldborg, N. and Gaunaa, M. Application of engineering models to predict wake deflection due to a tilted wind turbine. EWEA, Denmark, Copenhagen, 2012.