Technical University of Denmark

# Application of engineering models to predict wake deflection due to a tilted wind turbine

**Guntur, Srinivas; Troldborg, Niels; Gaunaa, Mac**

*Published in:*
Proceedings of EWEA 2012 - European Wind Energy Conference & Exhibition

*Publication date:*
2012

*Document Version*
Publisher's PDF, also known as Version of record

# Application of engineering models to predict wake deflection due to a tilted wind turbine

Srinivas Guntur, Niels Troldborg, Mac Gaunaa
Wind Energy Department, DTU, DK-4000 Roskilde, Denmark
srgu@risoe.dtu.dk

## Abstract

It is a known fact that the power produced by wind turbines operating inside an array decreases due to the wake effects of the upstream turbines. It has been proposed previously to use the yaw mechanism as a potential means to steer the upstream wake away from downstream turbines, however such a mechanism introduces control complications due to changing wind directions. Deflecting the wake in the vertical direction using tilt, on the other hand, overcomes this challenge. In this paper, the feasibility of steering wake is explored in a simple uniform inflow case. This is done by trying to model the wake deflection as a function of the yaw/tilt angle and the rotor thrust, initially using the momentum and vortex theories. Thereafter, a relatively more promising empirical model based on a set of actuator disc CFD computations is proposed. Finally, comments are made on the feasibility of wake control using yaw/tilt.
Keywords: Wind turbines, yaw, tilt, wake deflection.

## 1 Introduction

Experimental measurements from the Horns Rev wind farm in Denmark showed a power loss of around 40% between the first and the last turbine over a serial array of five wind turbines [3]. Of others, Medici et al. presented a study involving active yaw control of wind turbines in a wind farm [1-4]. One notable conclusion from their study is that although there is a reduction in power due to yaw, the net gain in power from the whole wind farm is positive. However, this control mechanism is sensitive to wind direction and the relative placement of the downstream turbine which makes it a challenge to implement. The approach of deflecting the wake in the vertical direction by using tilt is equivalent to yawing the turbine but benefits from being insensitive to changes in the wind direction. To analyse the feasibility of such a concept, one needs a method to trace the path of the wake in the far wake region as a function of tilt angle, rotor thrust coefficient, and the downstream distance. Of others, one such attempt has been made in [1] for instance, modelling the path of the far wake from a yawed turbine. In the current work, an attempt has been made to derive an analytical expression for the wake deflection using the momentum and vortex theories, to be compared with actuator disc CFD computations. In addition, based on the knowledge gained from the CFD computations, an empirical model for wake deflection is also proposed. The aim of this work is to study the potential for using yaw/tilt as a means for deflecting the wake away from downwind turbines. In this preliminary study we only consider a turbine in uniform inflow and therefore there is no difference between yaw and tilt.

## 2 Analytical Model

The schematic of the problem is shown in figure (1). The turbine is tilted at an angle $\psi$ in a uniform steady flow with a velocity $v_1$ far upstream. The methodology adopted here is similar to the original works on classical blade element momentum (BEM) method, e.g. see [13]. The aim here is to obtain an expression for the wake path function $f(x)$ as defined in figure (1). In view of this, the following two assumptions are made:

**a.** First, we assume that the change in the momenta in the $y$ and the $x$ directions are proportional to the rotor thrust force components in the respective directions:

$$\tan\psi = \frac{T_y}{T_x} = \frac{\dot{m}(V_{2y})}{\dot{m}(V_1 - V_{2x})} = \frac{2a_y V_1}{V_1 - (1-2a_x)V_1} \Rightarrow \tan\psi = \frac{a_y}{a_x}, \quad (1)$$

where $a_x$ and $a_y$ are the induction in the $x$ and $y$ directions, respectively, defined as $a_x = 1 - \frac{V_x(x)}{V_1}$ and $a_y = \frac{V_y(x)}{V_1}$.

**b.** Second, we assume that the following is true for all $x > 0$:

$$\tan\psi = \frac{a_y}{a_x} = \frac{a_y(x)}{a_x(x)}. \quad (2)$$

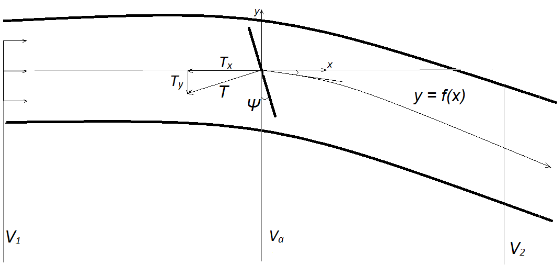


Figure 1: Schematic of the simplified system.

In order to obtain the axial induction as a function of downstream distance $x$, a wind turbine vortex model is used and is described in the following section.

# 3 Vortex model

The basic version of the vortex model was suggested by Øye [12]. This original model, with uniform circulation along the rotor was generalized to arbitrary circulation and sheared inflow by Troldborg and Gaunaa in [11]. The same authors extended the uniformly loaded case to include also the effects of yaw/tilt in [10] for determination of the effects of misalignment on both integral and local power production and also the induced velocity field at the rotor disc. Here only the key elements of the model will be described briefly. The interested reader is referred to the references for a thorough description of the model [10–12].

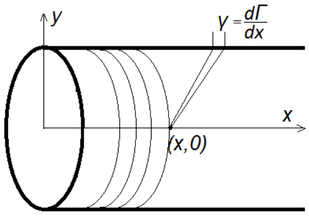


Figure 2: Schematic of a rotor placed at $x = 0$ with a vortex sheet of total strength $\Gamma$ in its wake.

The simple vortex model assumes the bound vorticity to be constant along the actuator disc. An actuator disc implies that the number of blades is infinite, and furthermore we consider the case where the tip speed ratio tends to infinity, such that wake rotational effects are not present. By use of Joukowski's equation with the assumptions above it can be shown that both the integral and local power coefficients are equal, and that their value is proportional to the product of the bound vorticity and the tip speed ratio. Since in this case the tip speed ratio is tending to infinity, the bound vorticity on the disc must tend to zero in such a way that the product of the two is constant. Since the disc extends from radius zero and the magnitude of the bound vorticity, $\Gamma$, tends to zero, we have that the trailed root vortex lies in the direction of the wake and has the magnitude $\Gamma$, which as discussed above tends to zero. Therefore in this case we have effectively only a vorticity sheet shed from the outer radius of the actuator disc, which can then be assumed to be an infinitely wound vortex solenoid. Then, if the vortex strength per unit length is defined as $\gamma = \frac{d\Gamma}{dx}$ then, from [11]

$$\frac{\gamma}{V_1} = 1 - \sqrt{1 - C_T}. \tag{3}$$

The induction at the center line, by the Biot-Savart law is,

$$d\,v_i(x) = \frac{d\Gamma}{2R}\frac{1}{(1 + (x/R)^2)^{3/2}} \tag{4}$$

which can be written as

$$v_i(x) = \frac{\gamma}{2R}\int_{-x}^{\infty}\frac{1}{(1 + (x/R)^2)^{3/2}}dx \tag{5}$$

which, upon solving yields,

$$v_i(x) = \frac{\gamma}{2}\left(1 + \frac{x}{\sqrt{R^2 + x^2}}\right). \tag{6}$$

(6) and (3) give,

$$v_i(x) = \frac{V_1(1 - \sqrt{1 - C_T})}{2}\left(1 + \frac{x}{\sqrt{R^2 + x^2}}\right), \tag{7}$$

and therefore,

$$V(x) = V_1 - \frac{V_1(1 - \sqrt{1 - C_T})}{2} \times \left(1 + \frac{x}{\sqrt{R^2 + x^2}}\right) \tag{8}$$

$$\Rightarrow a_x(x) = \left(1 - \frac{V(x)}{V_1}\right) = \frac{(1 - \sqrt{1 - C_T})}{2}\left(1 + \frac{x}{\sqrt{R^2 + x^2}}\right) \tag{9}$$

Under the assumption that (9) will also hold in a tilted case, from (9) and (2),

$$a_y(x) = \left(1 - \frac{V(x)}{V_1}\right) = \frac{(1 - \sqrt{1 - C_T})}{2}\left(1 + \frac{x}{\sqrt{R^2 + x^2}}\right)\tan\psi. \tag{10}$$

Using (9) and (10), path of the wake center ($f(x)$ in figure (1)) can be obtained. A test case is shown in figure (3).

## 4 The Navier-Stokes actuator disc model

The actuator disc model used in the present work is described in [6, 11] and was originally developed by Mikkelsen as an extension of the model presented in [5]. The model combines the Navier-Stokes flow solver EllipSys3D [7–9], with an actuator disc technique where body forces are distributed on a disc representing a wind turbine rotor. As described in [11], the applied disc forces are distributed smoothly on several mesh point to avoid singular behaviour. The steady state flow field is solved in a Cartesian computational domain with a total of $18.9 \cdot 10^6$ grid points distributed with a high density in the region of the wake.

Using this CFD model, computations were performed for the test cases of $C_T = \{0.36, 0.64, 0.80, 0.89\}$, and tilt $\psi = \{5°, 10°, 20°, 30°, 45°\}$, where the thrust coefficient is defined as

$$C_T = \frac{T}{\frac{1}{2}\rho A U_\infty^2}. \quad (11)$$

# 5 Results

Figure (3) shows a comparison between $f(x)$ obtained using the model from momentum and vortex theories, and that from the CFD computations, and clearly they do not agree well. There are two possibilities to explain this behaviour: Either assumption (**b.**) is invalid, or the simple analytical model cannot capture the effect of pressure gradients and turbulent fluctuations and other effects present in a real flow, which cause the wake path to be different from what is derived using the analytical means.

On the other hand, analysis of the CFD data interestingly reveals that scaling the deflection $y(x)$ as

$$\tilde{y}(x) = \frac{y(x)}{C_T \tan\psi} \quad (12)$$

results in an approximate self similarity between the different test cases as shown in figure (4). This self similarity has a relation

$$\begin{aligned} \tilde{y}(x) &= 0.24x \\ \Rightarrow y(x) &= 0.24x C_T \tan\psi \end{aligned} \quad (13)$$

and is denoted as the *empirical model* in figure (4). Figures (6), (7) and (8) show this linear empirical model applied to cases $C_T = 0.36$, $C_T = 0.64$ and $C_T = 0.80$, respectively, for tilt angles $\psi = \{5°, 10°, 20°, 30°\}$, and compared against CFD data. As can be seen from the figures, it seems that the model works reasonably well for low tilt angles. As the tilt angle is increased, the deviation between the model and the CFD data starts to grow, for example figure (5) shows an extreme case of $C_T = 0.89$, $\psi = 45°$.

In any case, the CFD data reveals that the deflection of the wake obtained using even the highest of the $\psi$ and $C_T$ values is only menial, compared to what would be required to steering the wake away from downstream turbines. In addition, wake meandering due to the atmospheric turbulence (which is not taken into account here) would make the control of the wake even more difficult.

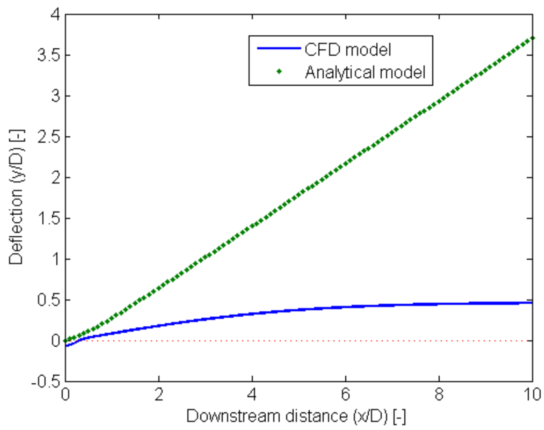


Figure 3: Wake center as a function of downstream distance from the rotor generated for a test case of tilt $\psi = 30°$, and $C_T$ = 0.64.

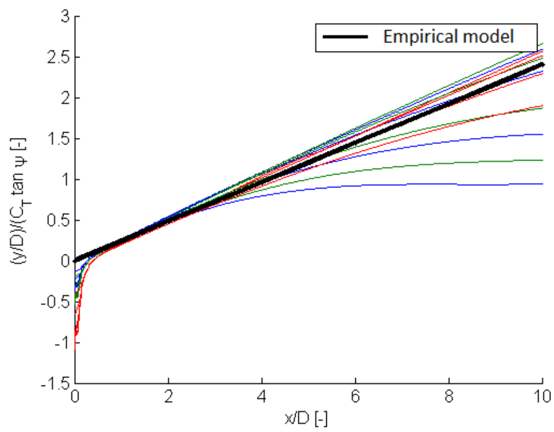


Figure 4: Self similarity between the all curves generated for $0.36 < C_T < 0.80$, and $5° < \psi < 30°$

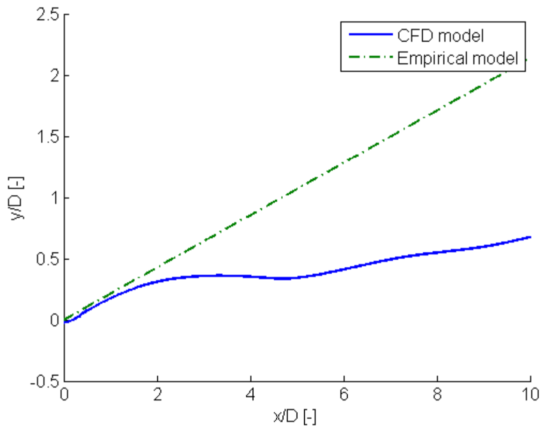


Figure 5: The empirical model and CFD data, for $C_T = 0.89$ and $\psi = 45°$

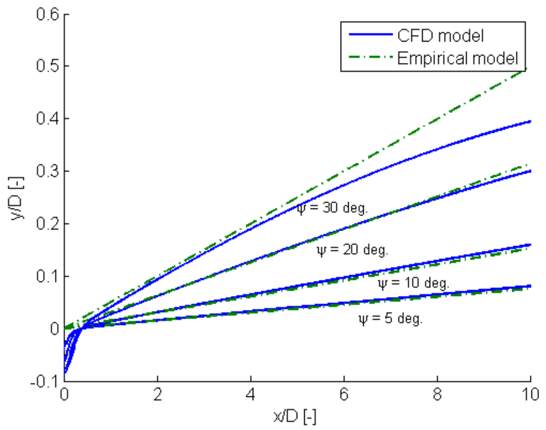


Figure 6: Empirical model compared with CFD data, $C_T$ = 0.36.

# 6 Conclusion

The following conclusions can be drawn from the work presented above:

- The ability of the proposed empirical model to predict the wake path from a tilted wind turbine is explored by comparing with results from an actuator disc CFD model. At this point, the model is believed to give a reasonable estimate for the wake path function for small tilt angles.

- In theory, it is possible to steer the wake

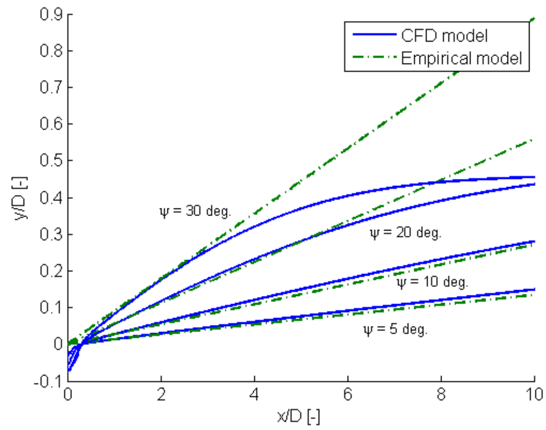


Figure 7: Empirical model compared with CFD data, $C_T$ = 0.64.

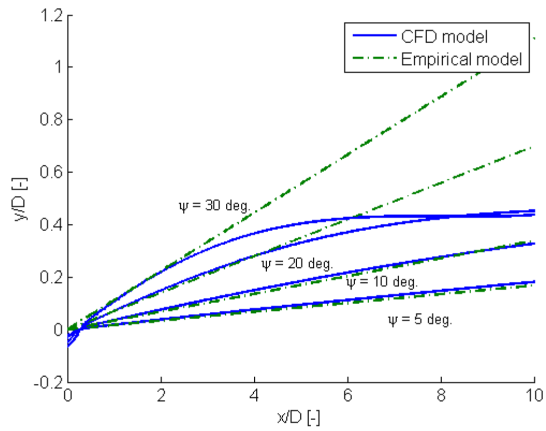


Figure 8: Empirical model compared with CFD data, $C_T$ = 0.80.

away from a downstream turbine, however the amount of yaw and the distance between the turbines required to accomplish this seem to suggest that this concept is rather impractical.

- The idea is to be able to control the wake away from the downstream turbines. Meandering and turbulence effects may dominate at larger downstream distances, which makes the control of the turbine based solely on the current concept a challenge.

## Acknowledgements

A part of this work was funded by the European Commission Marie-Curie (FP7) project *SYSWIND*. The authors would also like to acknowledge the Center for Computational Wind Turbine Aerodynamics and Atmospheric Turbulence (COMWIND), DTU wind energy, Denmark.

## References

[1] Jimenez, A., Crespo, A., and Migoya, E. Application of a LES technique to characterize the wake deflection of a wind turbine in yaw. *Journal of Wind Energy* (**13**), pp. 559-572, 2010.

[2] Madsen, H.A., Larsen, G.C., Larsen, T.J., Troldborg, T., Mikkelsen, R. Calibration and Validation of the Dynamic Wake Meandering Model for Implementation in an Aeroelastic Code. *Journal of Solar Engg.*(**132**), 2010.

[3] Barthelmie, R.J., Rathmann, O., Frandsen, S.T., Hansen, K., Politis, E., Prospathopoulos, J., Rados, K., Cabezn, D., Schlez, W., Phillips, J., Neubert A., Schepers, J.G., and van der Pijl, S.P. Modelling and measurements of wakes in large wind farms. *Journal of Physics: Conference Series* (**75**), 2007.

[4] Medici, D. *Wind Turbine Wakes - Control and Vortex Shedding*. Technical report, KTH Mechanics, RIT, Sweden, 2007.

[5] Mikkelsen, R., *Actuator Disc Methods Applied to Wind Turbines*. MEK-FM-PHD 2003-02, Technical University of Denmark, 2003.

[6] Ivanell, S.S.A. *Numerical Computations of Wind Turbine Wakes*. PhD Thesis, Linné Flow Centre, KTH Mechanics, Stockholm, Sweden, 2009.

[7] Michelsen, J.A. *Basis3D - a Platform for Development of Multiblock PDE Solvers*. Report AFM 92-05, Dept. of Fluid Mechanics, Technical University of Denmark, DTU, 1994.

[8] Michelsen, J.A., *Block Structured Multigrid Solution of 2D and 3D elliptic PDE's.* Report AFM 94-06, Dept. of Fluid Mechanics, Technical University of Denmark, DTU, 1994.

[9] Sørensen, N.N., *General Purpose Flow Solver Applied to Flow over Hills*. PhD Dissertation, Risø-R-827(EN), RisøNational Laboratory, Roskilde, Denmark, 1995.

[10] Troldborg, N. and Gaunaa, M. *Modelling the influence of yaw using a simple vortex rotor model*. EWEC 2012, Copenhagen. Extended abstract submitted.

[11] Troldborg, N., Gaunaa, M. and Mikkelsen, R. *Actuator Disc Simulations of Influence of Wind Shear on Power Production of Wind Turbines*. Torque conference, Crete, Greece, 2010.

[12] Øye, S. *A simple vortex model* In Proc. of the 3rd IEA Symposium on the Aerodynamics of Wind Turbines, ETSU, Harwell, 1990, p. 4.1-4.15.

[13] Hansen, M.O.L., *Wind Turbine Aerodynamics*. James & James, UK, 2000.