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State of the Art of Wind Farm Optimization

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Abstract

In recent years the trend has been to collect wind generators into larger and larger wind farms. As the investments are substantial, the optimization of the wind farm layout plays a major role today. The scope of the present work is to define the state of the art in wind farm optimization. To do so the literature of the last two decades has been analyzed, and the structure of the problem has been defined. The most effective techniques and models used in the past are described. The common pitfalls are listed as well, with the aim to create a blueprint for future development of wind farm optimization tools/softwares. The main findings concern the high dependency of the resulting layout on the objective function chosen, which objective should be as detailed as possible; the energy yield alone has been proven not to be the best function for practical purposes. The need for all-encompassing functions requires the costs to be computed besides the production yield. New strategies have been developed to handle comprehensive objective functions and to reduce long computational times, namely the "two-steps" optimization, which consist of a combination of two algorithms, usually a meta-heuristic and a local search approach. The last point touched by this work highlights the areas where a better understanding is needed and more research should be addressed, like the models for degradation and the solving algorithms used.

1 Introduction

The ambitious target of producing a larger share of energy from wind has led to the proliferation of wind farms in recent years. The initial investment required to set up a wind farm is large, and even larger for an offshore wind park. Therefore, finding the optimal configuration in terms of placement and type of the wind turbines is becoming of major importance. Since 1992 the scientific community has addressed the problem, starting with the pioneering article written by Mosetti et al. After that many authors have written a significant amount of contributions. Two main tracks can be identified: 1) on one hand small works that focused on different optimization algorithms, applied on a simplified standard test case, consisting of a square wind farm subdivided in 10x10 slots. These papers have given an interesting overview on the performances of different algorithms, but due to a lack of coordination, it is unclear whether which is the best algorithm for the wind farm problem. 2) The second track consist of two big projects: OWFLO and TopFarm (and DOWEC to a certain extent) that attempted an all-encompassing description of the optimization of a wind farm. Both of them developed an optimization tool, and applied it to real test cases. These projects have developed smart approaches to keep the complexity and computational time of the problem within reasonable limits–feature that would otherwise grow exponentially.

It is now necessary to review the many contributions to this topic, in order to pinpoint the most effective techniques and further to highlight the areas where a better understanding is needed and more research should be prioritized.

The first step is a clear definition of the problem. To sum up the work done in the last years, the optimization of a wind farm can be defined as the process of *"finding the positions of the wind turbines that maximize the value of some objective function"*. In other words, given a prescribed area, on land or off-shore, the goal is to determine where to place the wind turbines in order to get the maximum output from them.

The classical methodology consists of two steps: the definition of the *objective function* and the choice of the *optimization strategy*. The first part of the present work deals with the definition of the *objective function* and associated *cost models*. The second part focuses on different *optimization algorithms and strategies*. Finally, the discussion focuses on what type of models are the most appropriate for the wind farm problem, and which one should be investigated further for becoming useful future tools to be integrated in the wind farm planning process.

2 Objective Functions

The optimization path begins with defining the objective function, or the criterion that the wind farm has to meet to be considered optimal. The most commonly used functions in literature are *Energy Production*, *Cost of Energy*, *Profit* or a combination of the them.

2.1 Energy production

The first parameter used in literature as optimization objective is the *energy* produced on a prescribed area. Some authors refer to the *annual energy production* (AEP). Some others, although referring to the same physical quantity (energy), use a different nomenclature: *total power production*, P_{tot}, which is misleading. Even if in wind farm literature the words *energy* and *power* are often used as synonyms, this is wrong. *Power*, defined as the energy converted per unit of time and measured in [KW], is in fact the instantaneous output of the wind farm. *Annual energy production* (AEP) on the contrary, is defined as the integral of power over a period of one year and is measured in [KWh]. All that said, the energy production has been used in one study performed in the past, cf [13], where the cost of the installation was assumed a priori. Although not widespread in literature, the *annual energy production* is used in some commercial softwares, for instance *WindFarmer* [17], *OpenWind* or *WindPro* [19]. The only software that seems to be able to optimize the windfarm layout from an economic perspective is *WindFarm*.

2.2 Cost of Energy

A more practical function is the *Cost of Energy* (COE in short), defined as the cost per KWh of energy "produced" [1], which takes into account both the energy output of the wind farm and the cost of the installation, maintenance and disposal. Different definitions have been used for the COE. The simplest one $Cost/P_{tot}$, has been used by Wan et al. [12],[14] and Grady [3]. The focus of their studies was on implementing a new algorithm, and therefore they kept the objective function as simple as possible. In other studies the concept of *interest* has been introduced, leading to a slightly different definition. Elkinton for instance, in the *OWFLO* framework, and Szafron [16] use the LPC, levelized production cost, defined as

$$LPC = \frac{C_{inv}}{a \cdot E_a} + \frac{C_{OM}}{E_a} \tag{1}$$

where C_{inv} denotes the total investment cost, C_{OM} is the annual cost for maintenance and operation of the power plant, E_a is the annual energy production defined previously, and a is the annuity factor.

2.3 Profit

Among the authors that used *profit* as objective function, Réthoré and colleagues built the most comprehensive function within the TopFarm framework [11]. The function they used is the *financial*

[1] It has to be noted that any time that we encounter the words "cost per KW of power produced", two logical errors are contained in this sentence, that should be avoided: the cost is calculated on a say 20 years base and is therefore based on integral quantity, while the Power is defined as an instantaneous value, differential by nature. The ratio between the two is meaningless. A more appropriate definition is "Cost per KW of energy", that involves two integral quantities. The second mistake consist in the the fact that speaking of power produced is against the first principle of thermodynamics. The correct way is "power converted".

balance defined as follows:

$$FB = P_e \cdot TEP - C_D - C_M - (C_f + C_g) \cdot \Big(1 + \Big(\frac{r_l - r_i}{N_L}\Big)\Big)^{X \cdot N_L} \tag{2}$$

where P_e is the price of energy on the assumed market, TEP is the total energy production over the expected lifetime of the wind farm, X. The first term represents then the income from selling the power in the entire lifespan. The second term, C_D, is the *degradation cost* and represents the total loss of value of the wind turbines during their lifespan due to fatigue degradation. The third term, C_M is the *maintenance cost* of the power plant. The last term represents the future value of the part of the investment relating to *foundations*, C_f, and *electrical grid*, C_{fg}, that depends on the number as well as position of wind turbines. In other words the last term accounts for the future value of the money spent to build the infrastructure within to the wind farm, like foundations and electrical connections, etc. .

Despite a general model for the profit is not easy to build, the financial balance here defined only contains the terms dependent on the wind farm layout, that allows to ignore part of the unknown costs, which are assumed to be independent from the layout (e.g. transformer platform installation cost,...).

2.4 Net Present Value

Gonzáles and colleagues [15] suggested a slightly different formulation: the *net present value* of the wind farm, defined as:

$$NPV(\vec{X}) = -I_{WF}(\vec{X}) - C_D(\vec{X}) + V_R(\vec{X}) + \sum_{k=1}^{L} T \frac{N_k(\vec{X})}{(1+r)^k} \tag{3}$$

where I_{WF} represents the initial investment, that include the cost of the generators, the *civil and the electrical infrastructure costs*, C_D accounts for the *decommissioning cost* after a *lifetime* LT and V_R is the *present residual value* of the windfarm after the production period. N_k is the *net cash flow*, defined as the difference between the income resulting from the energy sale and the *operation and maintenance cost* and represents the net incomes produced by the wind farm during the k-th year. The symbol $\vec{X}$ represents the dependency of each term on the actual wind farm layout, clearly indicating that behind each term there is a funciton relating it to the layout that needs to be assessed at each step of the optimization.

2.5 Linear combination of cost and energy production

According to Mosetti and colleagues [9], *"the objective of the optimization is to produce the highest amount of energy at a reasonable cost"*. In their famous article [9] they attempted to keep the objective function as simple as possible. This resulted in the following definition:

$$Obj = \omega_1 \cdot \frac{1}{P_{tot}} + \omega_2 \cdot \frac{Cost}{P_{tot}} \tag{4}$$

where P_{tot} is the *total power produced* per year and ω_1 and ω_2 are two arbitrary weights. Mosetti et al. kept ω_1 small to focus the optimization on the cost reduction. This formulation accounts for *maximum energy production* (obtained setting $\omega_1 = 1, \omega_2 = 0$), *minimum cost of energy* (obtained setting $\omega_1 = 0, \omega_2 = 1$), and all the situation (sub-optimal) in between.

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3 Sub-models

To build the objective function described in section 2 many sub-models are needed. According to the desired level of fidelity and computational resources available, each sub-model can be more or less accurate. In the following the most common models in literature are described.

3.1 Wake models

The energy extracted by a turbine depends on the inflow wind field hitting the rotor. In a wind farm, the flow is generally influenced by the presence of upstream turbines, which decrease the energy content of the air downstream. It is therefore essential to model the wake generated by each turbine in order to get the proper *energy yield* of the plant. Amongst the many models developed to date, only the simplest ones are suitable for the optimization process, to keep the computational time within reasonable limits. Few engineering wake models have been used: the wake model by Jensen [5], for instance, that provides an estimate of the mean velocity drop in a cone behind the turbine rotor. Jensen's model has been used as a fast engineering tool because of its extreme simplicity in many study in the past, where the focus was on the optimization algorithm. However, it underestimate the wind velocity of the downstream flow, leading to a lower energy yield. A more accurate model is for example the semi-empirical stationary wake model proposed by Larsen [7]. It uses the thrust coefficient and the atmospheric turbulence intensity at the wind farm site to estimate the *speed deficit* of an axis-symmetric single wake. It is based on an analytical solution of the thin shear layer approximation of the NS equation. To further increase the accuracy, a unsteady wake model has been developed in 2007 by the same author: the *Wake Meandering Model*, [7], that accounts for the transversal movement of the wake, which is believed to affect significantly the wake. Among the other models, worth mentioning is: the one developed by Ott and colleagues at Risø in 2009 [10] and intended for future use in the TopFarm platform. Is is based on linearized RANS equations, which mimic the full CFD model's behaviour very well in regions where the perturbations are small, like the far field of the wake, but without the high computational cost of a full CFD simulation.

3.2 Cost models

As previously said, modeling the cost is essential to build the *financial balance* or any meaningful objective function. Some of the works analyzed ([9], [12], [8], [3]) used a very simple cost function where the cost of the wind farm only depends on the number of turbines installed, N:

$$Cost = N(2/3 + 1/3 \cdot e^{-0{,}00147N^2}) \tag{5}$$

assuming that the price is decreasing with the count of wind turbine purchased. The reason for using this very simple empirical relation for the cost resides primarily in the wish of most of the authors to compare their results with the first work done by Mosetti. In fact most of the literature focuses on the application of different optimization engines to the same test case used by Mosetti, back in 1992.

OWFLO and TopFarm are the only two works where a proper *cost model* has been developed. The costs that have been accounted for are:

- cost of foundation
- cost of electrical grid
- cost of civil infrastructures
- cost of maintenance
- cost of degradation
- cost of decommissioning

The list includes the models used in both projects but it should be noted that the approach used in TopFarm and OWFLO is slightly different and they focus on different parts of the cost function. Large use of empirical correlations or statical regressions has been done, drawwing heavily on the previous DOWEC project. Only the main features or innovative ideas from both projects are here described.

3.2.1 TopFarm approach

First, all the cost models developed in the TopFarm project are based on two simple ideas: **relevance** and **relative cost basis**.

The first one derives from the definition of the optimization problem whose goal is to find the position of the wind turbines, identified by a vector $\vec{X}$, that maximizes the objective function. The costs C present a fixed component, $Cost_{fixed}$, and a variable one, that depends on the wind turbines positions, $Cost(\vec{X})$. Only the latter is considered, therefore *relevant* and modeled. The costs that are not influenced by the actual wind farm layout, like the cost of planning of the wind farm, the cost of the civil infrastructure connecting the wind farm with the surrounding roads and the price of the electrical connection to the main grid, are considered *irrelevant* and not modeled in the TopFarm framework.

The second idea stems from the way optimization algorithms work. All of them seek to find the best layout comparing different solutions and only the *relative difference* between the two layouts is relevant for the optimization process. Then only the *relative cost* is taken into account.

With these two simple assumptions Therefore the total *absolute cost* is not modeled, but can be be calculated a posteriori.

The costs accounted for in TopFarm that are modeled in an original way are described in the following.

3.2.2 TopFarm degradation cost

Degradation is mainly due to fatigue, i.e. millions of cycles of alternating loads that wear the turbine components. The calculation of these loads in run-time mode, during the optimization is unfeasible due to the huge computational time needed. The solution adopted by Réthoré and colleagues has been to create off-line a very accurate database that can be used in run-time mode by the optimization routine. Several aeroelastic simulation have been run for 600s to assess the loading on the downwind turbine due to the wake of the upwind one. 11 different wind cases, 4 turbulence intensity levels, 11 inflow angles and 13 different spacings between the to generators have been investigated. In addition 44 different combinations of turbulence intensity and wind speed have been analyzed for the stand-alone condition. All the data have been used to calculate the *lifetime equivalent fatigue loads*, L_{eq}

The cost of (fatigue) degradation is accounted assuming a *linear writing off*. For a particular structural member the fatigue degradation cost is presumed proportional to the *mean accumulated equivalent fatigue load*:

$$CD = p\,\frac{L_a}{L_d} \tag{6}$$

where p is the price for the component, L_a represents the *accumulated load* and L_d the *design equivalent fatigue load*, previously calculated with the off-line aeroelastic calculations.

3.2.3 OWFLO approach

The OWFLO project attempts instead to minimize the *cost of energy*, COE and focuses on the *total costs*, unlike TopFarm. The impressive work done by Elkinton consists in a preliminary design of the various components to assess, as final stage their *absolute cost*. The starting point has been a model for the *rotor nacelle assembly*, RNA, which has been used as input for the tower design and three different kind of foundations: monopiles, tripods and gravity bases. The submodels have been validated against the few cost data available in literature, where possible, showing fair accuracy. The same process has been repeated to determine the total *cost of the electrical grid*.

4 Optimization Algorithms and strategies

The optimization problem can be mathematically stated as finding the global maximum (or minimum) of the objective function, Π, defined as $\Pi = \Pi(\vec{x})$ where $\vec{x}$ is the vector describing the coordinates of all the generators in the wind farm. Different approaches have been tried in the past; we list them here, and we try to highlight pros and cons of all of them.

4.1 Gradient methods

Gradient methods (also called "hill climbing") are based on the evaluation of derivatives (or generalized derivatives) of the objective function $\Pi = \Pi(\vec{x})$ in each point $\vec{x}$. This class of methods present the drawback of converging quickly to local maxima/minima. To find a global maximum/minimum they need to start pretty close to it. Therefore the starting point has to be determined in another way. The only case in which the hill climbing methods have been applied is in the extensive analytical work done by Elkinton [6] on a very limited number of wind turbines, namely two.

When dealing with multi-megawatt plants, a large number of turbines needs to be installed, and the dimension of the vector $\vec{x}$ increases rapidly. Finding the best configuration for the wind farm thus involves then a an objective function defined on a high dimensional space. A gradient based method can still be used but it has to start close to the global optimum, which makes it appropriate in a **two steps strategy**, as refinement algorithm. The best algorithms to navigate through large spaces without getting lost are the so-called meta-heuristic methods. Meta-heuristics are algorithms which find the optimum for a problem by iteratively trying to improve a candidate solution with regard to a quality parameter. They will be analyzed in the following.

In order to compare the different methods many of the authors used the same, standard problem as a test case. They have all dealt with the a square area with a fixed kind of wind turbine and three prescribed wind conditions: uniform wind from one direction, the same wind intensity from all directions and a multi-directional wind rose.

4.2 Genetic algorithms

The most common *meta-heuristic* models used in literature belong to the class of *genetic algorithms*, (GA in the following), used, among others by Mosetti [9], Grady [3], Wan [12], Wang [14] and Szafron [16]. The method is inspired by the principles of genetics and natural selection/evolution and is often chosen because of the good quality of the solutions.

The first step of this technique consists in subdividing the domain in square cells each with a side length of *n* rotor diameters. That results in a tidy grid of 10x10 cells. Such choice is necessary to implement the GA in a simple way. In fact, doing so the wind turbines can only be placed in the center of each cell and the degrees of freedom (DOF in the following) necessary to describe the system drop from 200 (2 DOF per turbine) to 100 (only one DOF to describe each turbine). Their positions can be efficiently defined by a vector 100x1. Then every possible wind farm layout is described by a string of ones and zeros, the ones identifying the cells where there is a wind turbine. The method starts with a random population of individuals (=layouts). The objective function is computed for all the individuals and the values obtained are compared. The layouts with the highest scores are kept while the others are recombined by the mechanism of mutation and crossover.

Wang and colleagues [14] investigated the possibility of using equilateral triangles instead of squares cells to subdivide the domain. Triangles allow in fact to better exploit the space guaranteeing the same spacing from one turbine to all of the closest ones. They also investigated the influence of the grid orientation on the optimization results.

One drawback of using a GA is that there is no complete freedom in the placement of the turbines. Different solution have been tried to compensate for this.

4.3 Viral algorithms

Similar to the *genetic algorithms*, this class is inspired by the replication mechanisms of viruses. Once a first attempt population of solution is set, the layouts with the best fit values are replicated according to the lytic cycle, which is a direct way of propagating a good solution within the evolutionary process. Within the lytic cycle, in fact, the virus injects its own nucleic acids into a host cell, which mistakenly copies the viral acids instead of its own. In the next step the viral DNA forms new viruses inside the cell which then brake the cell membrane to infect other cells. The layouts with the lowest objective function value follow the lysogenic cycle, that means that they form a subpopulation which need to be recombined in order to reach higher values of fit, before they end in the high part of the ranking and can be replicated in a direct way (lytic one). This path has been tried, among others by Ituarte et al. [4].

4.4 Particle Swarm algorithms

Particle swarm optimization algorithms (also called PSO), are inspired by the behavior of birds in a swarm or fishes in a school. This kind of algorithm starts with an initial guess population of candidate solutions. Each solution corresponds to a specific wind farm layout. So the starting point is a collection of layouts, generated randomly. In jargon the candidate solutions (layouts) are called particles and the population, swarm. The particles can move around in the search-space. In practice every layout of the collection is seen as a bird/fish that can move around according to a few simple rules. The particles are guided in their movement by their own best known position and the whole swarm best known position. When solutions with a best fitness value are discovered, they become the new attraction poles for the entire swarm. The process is repeated until no further improvement is achievable. Thinking of the wind farm layouts as particles of the swarm is quite difficult because we naturally tend to associate the turbines to particles and the wind farm to the swarm, which is wrong. In reality the whole wind farm layout is a bird/fish that is moved around by the algorithm, where “moved around” means that the position of the wind generators within the layout is changed.

4.5 Greedy Heuristic algorithms

Greedy heuristic algorithms can be described in this way. First, an initial layout is guessed. Then, three different operations are performed on the guess layout: add a turbine, remove each turbine (one at a time), and move each turbine (up to prescribed distance). After each modification, the objective function is evaluated and the layout with the highest value is kept as best candidate solution. As this method consists of small perturbation on a initial layout, there is a risk that the algorithm gets stuck in the closest local maximum/minimum. To avoid this some turbines positions can be perturbed by a random distance. Eventually, when a specified number of consecutive perturbations fails to improve the solution, the layout found is considered optimal.

5 State-of-the-art and trend concerning Objective Functions

ENERGY YIELD, FINANCIAL BALANCE , NET PRESENT VALUE AND COST OF ENERGY

Among the Objective functions, the simplest one is the energy output, that is widely used in commercial softwares. It should be noted, however, that the mere energy production is not the most relevant objective function from an application point of view. The reason for that can be easily seen in a idealized scenario. Let us consider a simplified offshore case, in which the wind speed (unidirectional wind) and the water depth increase with the distance form the shore. If we tried to maximize the energy production we would obtain a cluster or a line of wind turbines located as far as possible from the shore, where the wind resource is the largest. This would imply installing the turbines in deep water, which would increase the cost of the foundations, of the vessels for their installation, of the connection cables and of the maintenance. Taking this simple case to the extreme, it is clear that we will eventually reach a point, where it is not convenient any more to install the wind turbines. The cost of the installation outweighs the advantage of harvesting energy.

This example demonstrates clearly the necessity for a balance between the *energy yield* of the wind farm and its *cost*, which is turn has lead to the development of more comprehensive objective functions. It can be said, that today, the most convenient objective functions for wind farm developers are the *cost of energy*, *COE*, the *net present value*, *NPV*, and the *financial balance*, *FB*.

One possible drawback of using profit is its dependency on the *price of energy*, P_e, whose value on the market has to be assumed and acts as a weighting function for the *total energy production*, TEP, cf. eq. 2.

This shows that the final layout, that as we previously said depends on the objective function, can be influenced by an assumption on a some temporal parameters, namely the *price of energy*. The same qualitative consideration holds for the *rate of interest*. However, it is unclear what the quantitative impact on a real case layout is and a sensitivity study should be done in the future.

Regarding renewable energy incentives and taxation, they might influence the layout of the wind farm but are neglected because are peculiar to the country or region where the wind farm has to be built and may vary over time.

Another drawback of the profit function resides in its intrinsic complexity. All the cost terms: *electric grid cost*, *maintenance cost*, *foundation cost* and *degradation cost* need to be assessed for each layout at each step of the iterative optimization. This turns out to be very demanding from a computational point of view if complex models are used for the cost. An alternative solution is to use empirical correlations, which are not so accurate because of the scarcity of informations. This is a point where big uncertainties have been shown to enter the model

The temptation to leave out or to oversimplify some of the cost models is not a good idea. In fact, Réthoré and colleagues - the first ones that took into consideration the *degradation cost* - discovered that this term is a relevant component of the financial balance. According to their simulations this component can contribute with up to 30% of the total relative balance. Neglecting one component of the total cost can lead to changes in the optimal layout. We can state, as a general rule, that all the costs of the wind farm: *electric grid cost*, *maintenance cost*, *foundation cost* and *degradation cost* must be modeled in order the method to converge to a realistic optimal configuration.

COST AND ENERGY PRODUCTION

Regarding the *linear combination of cost and energy production*, described by eq. 4, it has only been used by Mosetti and colleagues, because of its simplicity and the freedom given by the two weighting functions ω_1 and ω_2. The focus of the optimization can be either on the cost or on the energy production. The price for this freedom is very high though, the physical meaning of the whole function is unclear and it should therefore be avoided in future.

An additional consideration concerns the influence of the choice of the Objective function on the wind farm layout. The tendency is to get higher turbine density if the sole energy output is used as criterion, while when introducing the costs in the objective function, more coarser layouts are found.

Summary

For all the reasons explained above, at present the only objective functions that seem suitable to determine realistic wind farm layouts are the *cost of energy*, *COE*, the *financial balance*, *FB*, and the *net present value*, *NPV*.

Possible alternatives that could be investigated in future are for instance:

- MAXIMUM ENERGY PER UNIT AREA, to obtain the maximum extraction of energy form a given area, measured in [MWh/km^2],

- MAXIMUM PROFIT PER UNIT AREA [€/km^2],
- MINIMUM NOISE AND/OR ENVIRONMENTAL IMPACT, for the maximum social acceptance of the installation.
- MINIMAL FLUCTUATION IN POWER OUTPUT for a better integration in the electrical grid reducing the electrical fluctuations of the wind farm cause by the layout, when the wind changes direction.

It can be said, however, that all the suggested functions can also be used as optimization constraints using any another objective function.

5.1 Commercial software

A special note has to be done concerning the commercial softwares. Most of them are also able to perform a financial balance, but they still use the Energy production as Objective function in the optimization process and assess the Profit or the Cost of Energy in a subsequent module. Some of them do not seem able to perform an integrated optimization in which the target is either the *cost of energy* or the *profit*, which the authors believe are the most relevant ones. Some authors, [1] for instance, showed that the layout that optimizes the energy yield is very different form the one with the minimum cost of energy. These results rise doubts on the quality of the solutions found in this way.

6 State-of-the-art and trend concerning Optimization methods and strategies

Looking at the literature from an historical point of view some interesting considerations ca be done. After the pioneering work by Mosetti, that suggested the simplified square wind farm, many authors dealt with the same problem applying different optimization techniques. Swarm optimization, simulated annealing and viral algorithms have been used and the results compared with the original work by Moetti and colleagues. Mosetti subdivided the domain in square cells, each with a side of 5*D* (D = turbine diameter) that resulted in a tidy grid of 10x10 cells. Such choice represents a smart implementation o the genetic algorithm. It allows in fact to reduce the degrees of freedom and to include implicitly the constraint on the domain geometry. Any domain shape can in fact easily be mapped in this way. The reduction of the number of DOF is on the other hand a very good feature in the wind farm optimization problem. The price to pay with the "discrete domain" (checked) is that there is no complete freedom in the placement of the wind turbines that are constrained to sit in predefined positions. This lack of freedom could lead to a non-optimal solution and some strategies have been tried to bypass the issue. Wan et al. [14], have for example have tried the particle swarm with a penalty function. This technique allows the wind turbines to occupy virtually any position in the field. The drawback with the PSO is that the number of turbines in the wind farm has to be set as input. The PSO optimizes the position of a given number of wind turbines but does not answer the question (non-trivial) "how many wind turbines?" on the given area. The difference, not always noted in literature, between the GA and the PSO is that while the first determines both the number and the position of the turbines within the field, the later only finds optimal position of the generators, given their number. For this reason the PSO should not be used as a stand-alone algorithm.

An alternative strategy worth to try on the classical square area wind farm, is the following. As first step a genetic algorithm to determine both the number and the position of the generators. Then the constraint due to presence of the grid could be relaxed to refine the layout. In other words, the layout found with the GA (in terms of number and position of WT) becomes the starting point for a PS algorithm that will change only the the turbines mutual positions. This approach is inspired by the *tandem* work done by Grady [3] and Wan [13]. In fact, according to the latter, the *constraint relaxation* leads to an improvement on the power production of about 5% with the same total cost.

A third way has been attempted to “gain some freedom” using a grid and a GA. Wang et al. [14], subdivided the domain in equilateral triangles instead of squares, in which the turbines can be placed in the vertexes. This technique has been proven to be more effective in finding better layouts. The reason may be in the fact that the available space is used in a more efficient way.

Regarding the application of a *gradient based method*, it is worth to mention the work done by Lackner and Elkinton [6]. One of the nicest features of their work is that they created a map of the energy content in the field due to wakes interactions. This is an invaluable tool to understand the physics behind the mathematical problem. Unfortunately this is only possible in the 2 turbines case.

Even though all the other methods have proven some advantages over the GA, the literature on them is so small that is not possible to evaluate them.

The stategy that seem to be the most promising today is the one developed in TopFarm and OWFLO that can be named “two step strategy”. Two algorithms are applied in tandem, the first is a meta-heuristic, able to search on a huge solution space and to move towards the global maximum. The second is either a *greedy heuristic* or a *linear programming* algorithm, that refines the solution looking in the neighborhoods. This strategy has shown to give the best results when compared to the 2 algorithms applied singularly, without a big additional computational time if compared to the meta-heuristic applied alone.

7 Conclusions

Some major conclusions can be drawn. The first one concerns the *objective function*: the work done in the last two decades has largely demonstrated the necessity of a detailed objective function. In fact, the initial approach of considering the sole *energy production* as optimization target gives partial results from an application point of view. To get a real optimal layout, it is necessary to use a target function that accounts for both *energy yield* and *cost*.

A central role is played by the *energy yield* of the wind farm, which depends on the velocity drop in the wake. The choice of an appropriate *wake model* is therefore very important. Many models have been suggested in the past, with different levels of accuracy and complexity, but only few of them seem to be suitable for optimization problem.

The model for the cost represents another critical component in any objective function. Despite the clever attempts to bypass the scarcity of information done in the TopFarm, OWFLO and DOWEC projects, this remains a point where uncertainty is introduced in the models. Future research should focus on it in order to get more meaningful results.

From an application point of view a new standard has been set by the TopFarm and OWFLO projects, which represent the state-of-the-art in this field today. They both adopted a *two step optimization strategy* that consists of two algorithms working in tandem. The first is usually a meta-heuristic, able to search on a huge solution space and to move towards the global maximum. The second is a local search algorithm, to refine the solution by looking in the neighborhoods.

One lesson learned looking at 20 years of literature is that the most successful technique consists in using a range of models of increasing complexity with different algorithms, as suggested in [11] for instance. This strategy, called “multi-fidelity approach” represents a trade-off between good quality solutions and a reasonable computational time.

A possible direction for the future could be the the creation of a framework consisting of a simplified (but realistic) test case to be used as a benchmark for analitical/numerical work. Different objective functions, algorithms and strategies can be tested to point out, in a clear way, the most suitable for every specif case.

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