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Available fluid codes for turbulence study

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Available fluid codes for turbulence study

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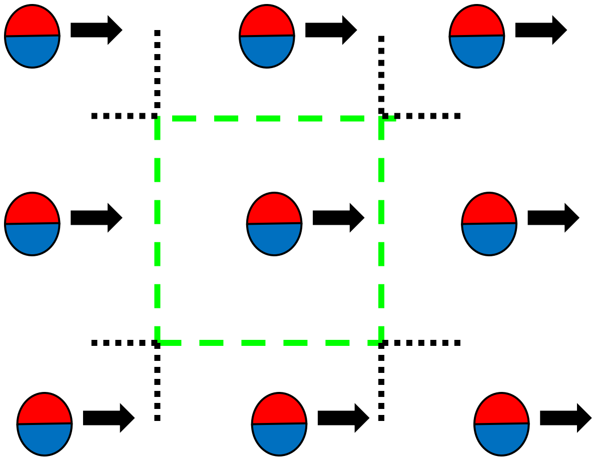
Periodic codes (spectral)

- Probably the simplest codes to make
- Easy and fast to develop, a master study
- Easy, fast and compact to run, a bachelor study
- Can use fairly high number of modes on a single CPU: 2048x2048
- Can reach fairly high Reynolds numbers: Re(2D) = UL/μ<20.000
- Can fairly easy be parallelized using MPI: linear speedup using 100 CPUs on a 2048x2048 grid
- Note that the domain is infinite with a periodic restriction!



Periodic codes (spectral)

- Solutions are expanded into Fourier modes (global)

$$\begin{pmatrix}\omega(x,y,t)\\ \psi(x,y,t)\end{pmatrix} = \sum_m \sum_n \begin{pmatrix}\omega_{mn}(t)\\ \psi_{mn}(t)\end{pmatrix} \exp\left(\frac{2\pi imx}{L_x}\right) \exp\left(\frac{2\pi iny}{L_y}\right)$$

- Vorticity equation

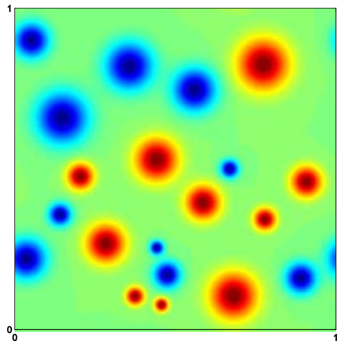
$$\frac{\partial\omega}{\partial t} + J(\psi,\omega) = \nu\nabla^2\omega \Rightarrow \forall(mn) : \frac{\partial\omega_{mn}}{\partial t} + [\psi,\omega]_{mn} = \nu\nabla^2\omega_{mn}$$

$$[\psi,\omega]_{mn} = FT\left\{\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}\right\}_{mn}$$

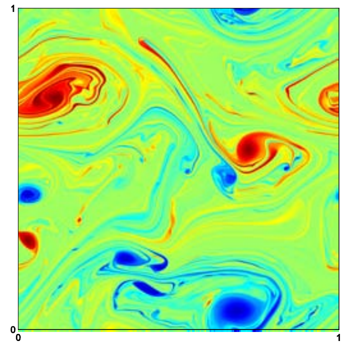
- Fast Fourier Transformation will take 75 % of computational time
- De-aliasing scheme, zero pad the largest 1/3 of the modes
- The Poisson equation is trivial: $k^2\psi_k = \omega_k$

The Risøs Euler code

Time = 0.0

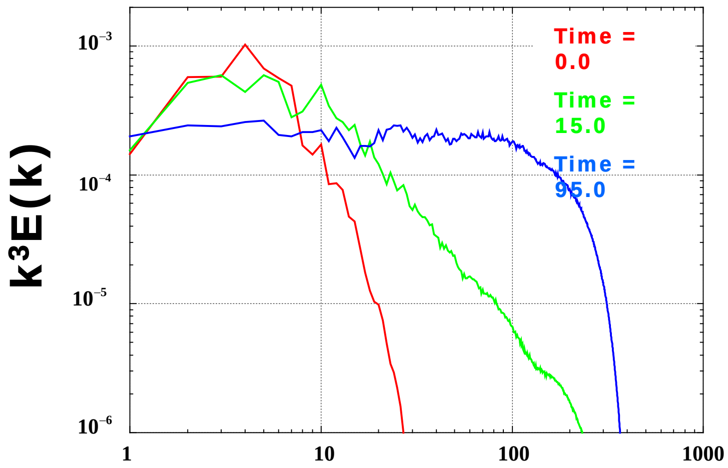


Time = 95.0



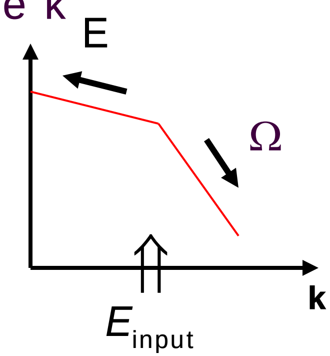
Kuznetsov et al POP **19, 105110 2007**

$$E(k) = \sum_{m,n} u_{mn}^2(t) + v_{mn}^2(t)$$



Inverse cascade :

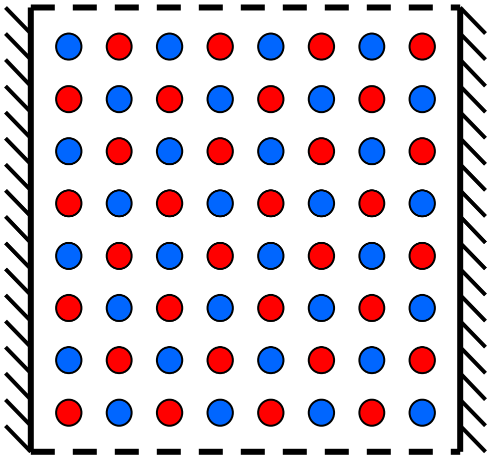
$$\boxed{\Omega_k = k^2 E_k} \Rightarrow \begin{cases} E \rightarrow \text{small k} \\ \Omega \rightarrow \text{large k} \end{cases}$$



- 1024x1024 points
- 512x512 modes
- de-aliased removes upper 1/3
- $K_{max}=340$
- Taken account for viscosity leave us with approximately 2 decades!

Solid boundaries

Periodic



Periodic

- Finite difference in x, Fourier expansion in y
- Multiplication simple, derivatives complex

$$\frac{\partial f_i}{\partial x} \square \frac{f_{i+1}\Delta f_{i-1}}{2dx}$$

$$\frac{\partial^2 f}{\partial x^2} \square \frac{f_{i+1} - 2f_i + f_{i-1}}{(\Delta x)^2}$$

Banded matrix, solved by Gauss elimination

$$\underline{\underline{A_k}} = \frac{1}{\Delta x^2}\begin{pmatrix} -1 & 1 & & & & & \\ 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & & \\ & & & \ddots & & & \\ & & & 1 & -2 & 1 & \\ & & & & 1 & -2 & 1 \\ & & & & & 1 & -1 \end{pmatrix}$$

Poisson equation

$$\nabla^2 \phi = \omega \Rightarrow$$

$$\forall k : \frac{\partial^2 \phi_k}{\partial x^2} - k^2 \phi_k = \omega_k \Rightarrow$$

$$\forall k : \underline{\underline{A_k}}\ \underline{\phi_k} = \underline{\omega_k} + \underline{BC}$$

Diffusion equation

$$\frac{\partial \omega}{\partial t} = \nabla \cdot (D(\vec{x}, t)\nabla \omega) + \cdots \Rightarrow$$

$$(1 - \Delta t \nabla \cdot (D(\vec{x}, t)\nabla))\omega(t + \Delta t) \approx \omega(t) + \cdots$$

Generally gives a complicated matrix, has to be solved by iteration (Petsc)

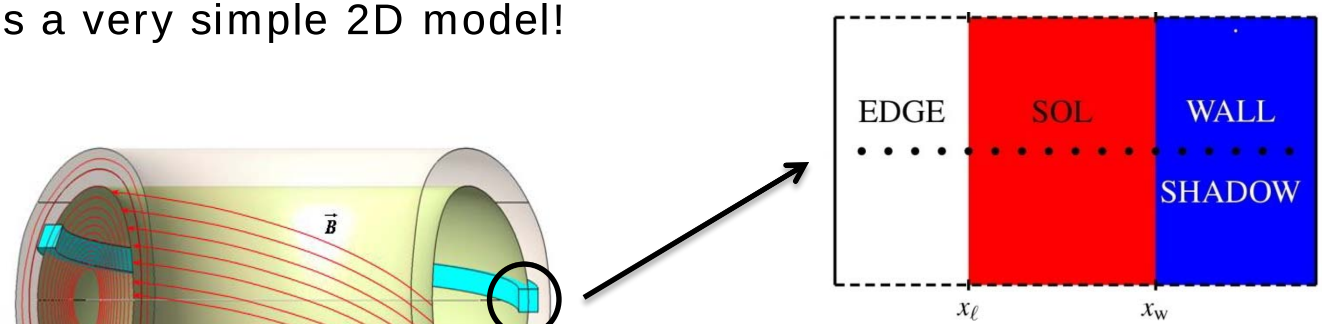
$D = D_0$: Helmholtz equation

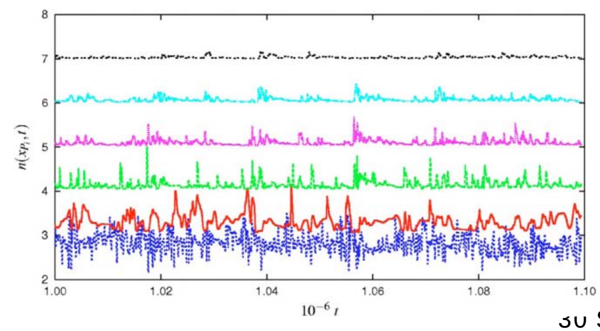
$$(1 - \Delta t D_0 \nabla^2)\omega(t + \Delta t) = \omega(t)$$

banded matrix, Gauss elimination possible

Finite difference, ESEL

- Global model with self-consistent profiles
- Simulation domain include both edge, SOL and limiter shadow regions
- 2D approximation; parallel loss mechanism modeled by a parameterize loss term
- Input; basic plasma parameters
- Test on TCV, JET, ASDEX with reasonable results
- Collisional diffusion coefficients and parallel loss terms from first principal
- It is a very simple 2D model!





Several millions time point

Finite difference, ESEL

Interchange model

$$\frac{dn}{dt} + n\square(\phi) - \square(Tn) = \Lambda_n$$

$$\frac{dT}{dt} + \frac{2}{3}T\square(\phi) - \frac{7}{3}T\square(T) + \frac{2}{3}\frac{T^2}{n}\square(\mathrm{n}) = \Lambda_T$$

$$\frac{d\omega}{dt} + \square(nT) = \Lambda_\omega$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{B} b \times \nabla\phi \cdot \nabla$$

$$\frac{1}{B} = 1 + \frac{r_o + \rho_s x}{R_0}$$

$$\square(B,f) = \frac{\partial B}{\partial x}\frac{\partial f}{\partial y} - \frac{\partial B}{\partial y}\frac{\partial f}{\partial x} \qquad \left(= \frac{\rho_s}{R_0}\frac{\partial f}{\partial y} \right)$$

$$\Lambda_\alpha = -\frac{\alpha}{\tau_{\square,\alpha}} + D_{\perp,\alpha}\nabla_\perp^2\alpha$$

Subsonic advection:

$$\tau_{\square,n} \square \tau_{\square,\omega} \square \frac{M_\square c_s}{L_p}, \; M_\square = 0.5$$

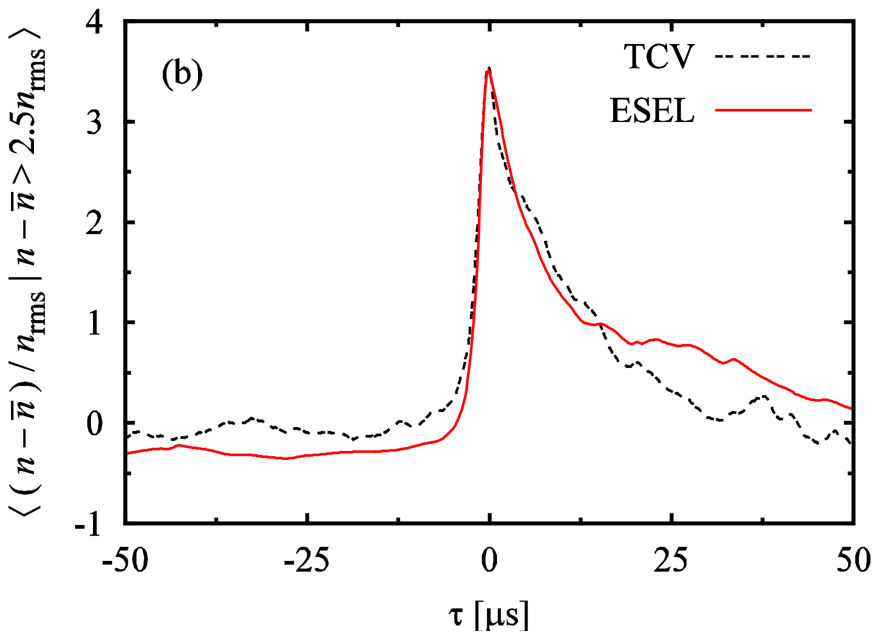
Spitzer-Harm diffusion:

$$\tau_{\square,T} \square \frac{3L_p^2}{2\chi_{\square e}}$$

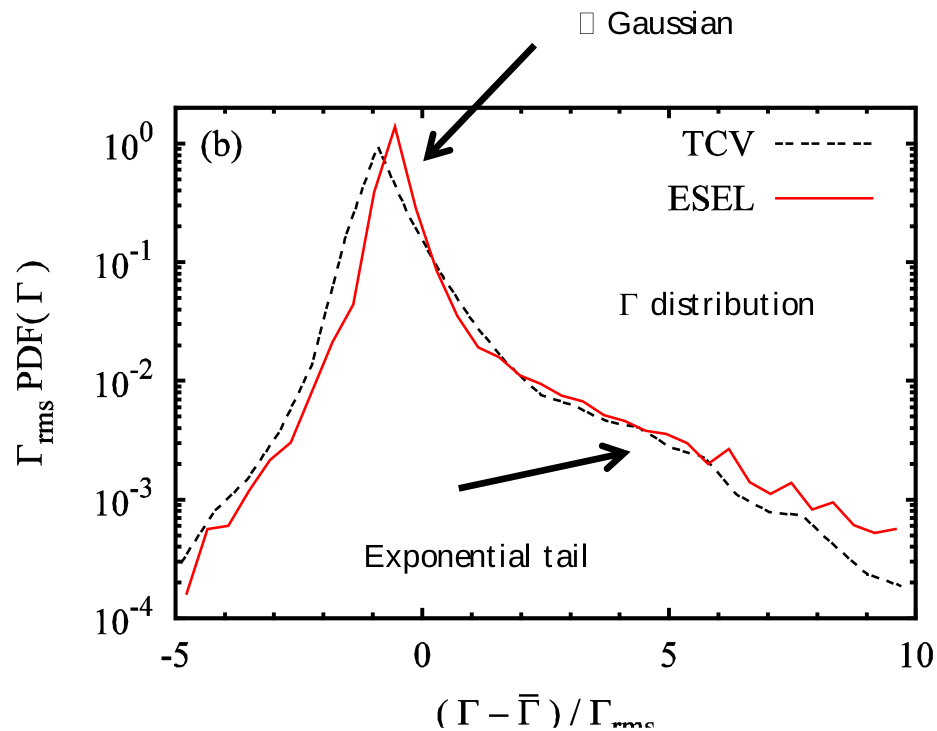
Collisional SOL limit

$$10 < \upsilon_e^* = \frac{L_\square}{\lambda_e} < 80$$

TCV-ESEL **Comparison**



Conditionally averaged density blob structure



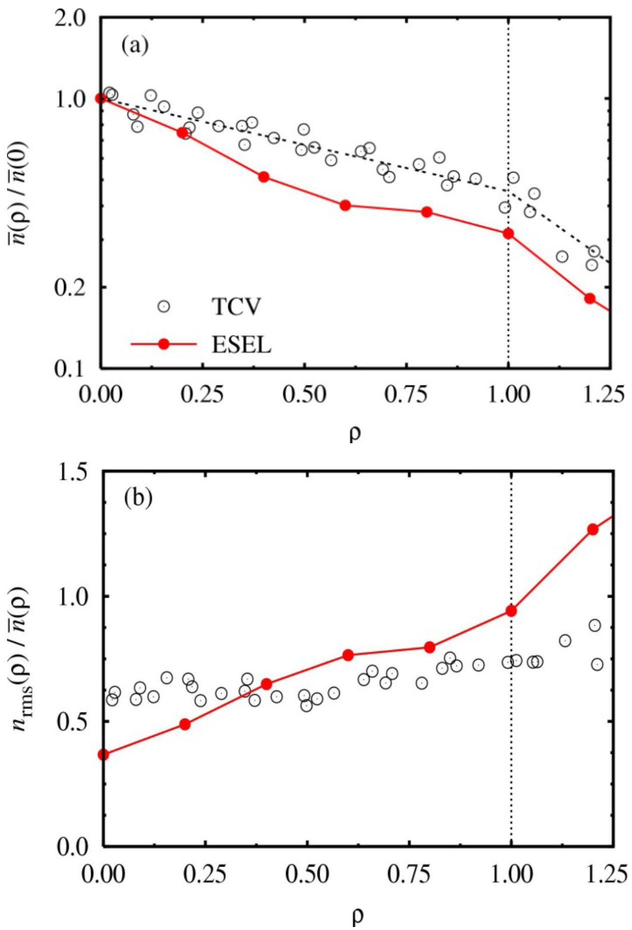
PDF of particle density flux

Direct comparison with experimental results from the TCV-Tokamak, Lausanne: excellent quantitative agreement

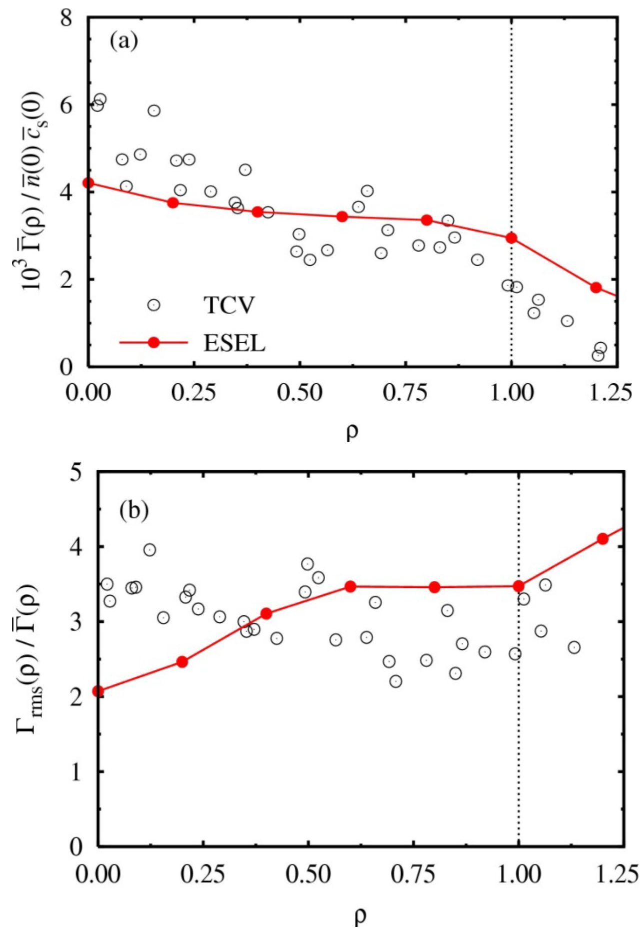
Garcia *et al.* PPCF **48**, L1 (2006)

TCV-ESEL Comparison

Density profile and relative fluctuations



Particle flux profiles



Good agreement between experiment and turbulence simulations

Garcia *et al.* PPCF **48**, L1 (2006)

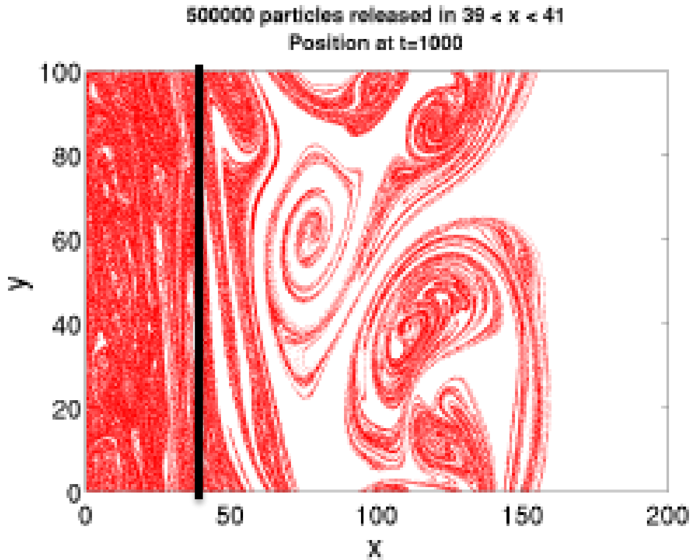


Fig 4: Particles released inside LCFS, $t = 1000$

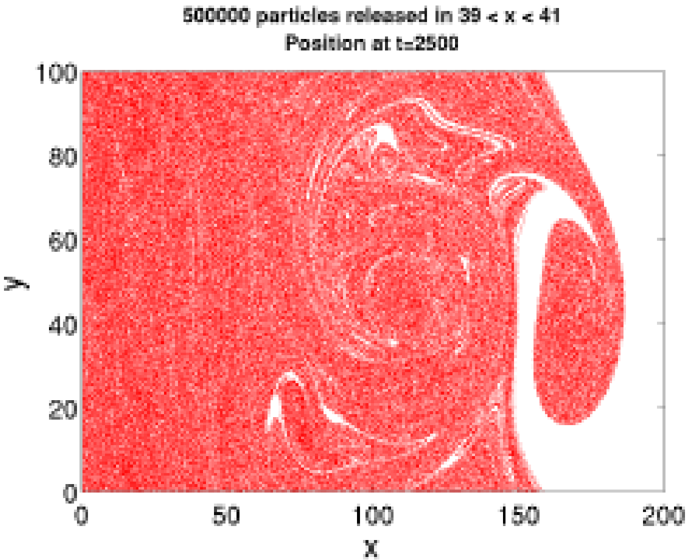
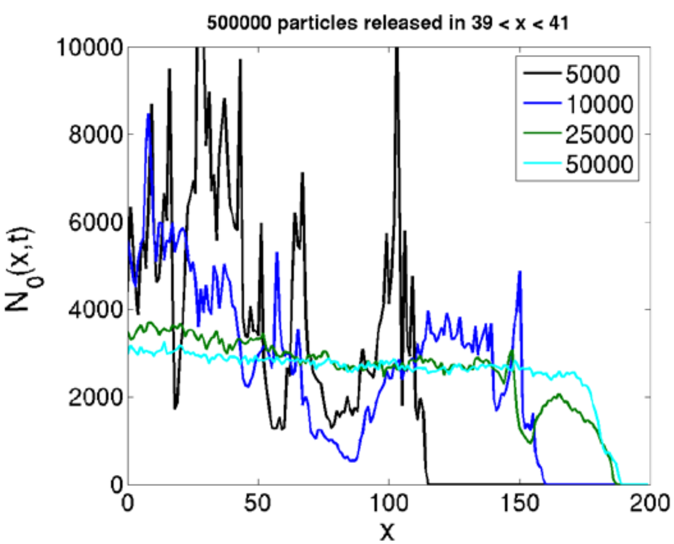


Fig 5: Particles released inside LCFS, $t = 2500$

Passive particles

$$x(t) = x(0) + \int_0^t v(x,t)dt$$

Summer student S. Boudaux, (2005)



Solid boundaries - Disk

- Radial points are cosine distributed
- Chebyshev polynomials calculated via cosine transformation
- The Poisson equation decouples in ϴ, a series of banded 1D problem to be solved in r
- r=0 should be a regular point
- As r->0 the grid spacing in ϴ decreases: 1/(2πr)
- Even thought these scales are well below the viscosity scale they are extremely unstable and have to be removed manually (zero pad).

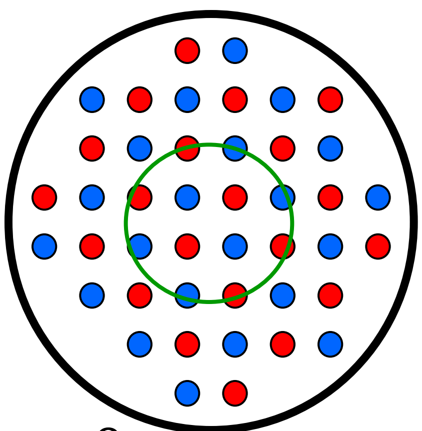
$$\begin{pmatrix} \omega(r,\theta,t) \\ \vec{u}(r,\theta,t) \\ \psi(r,\theta,t) \end{pmatrix} = \sum_m \sum_n \begin{pmatrix} \omega_{mn}(t) \\ \vec{u}_{mn}(t) \\ \psi_{mn}(t) \end{pmatrix} T_m(r) \exp(-in\theta)$$

$$r \in [-1:1], \theta \in [0:2\pi]$$

$$T_n(x) = \cos(n\cos^{-1}(x)) = \cos(nz)$$

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

$$T_n(-1) = (-1)^n, \; T_n(1) = 1$$



$$\vec{u}_{\partial D} = 0$$

Poisson equation

$$\nabla^2 \phi = \omega \Rightarrow$$

$$\forall k : \frac{\partial^2 \phi_k}{\partial r^2} + r\frac{\partial \phi_k}{\partial r} - k^2 \phi_k = r^2 \omega_k \Rightarrow$$

$$\forall k : \underline{\underline{A_k}}\, \underline{\phi_k} = \underline{\omega_k} + \underline{BC}$$

$$\underline{\underline{A_k}} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ x & x & x & x & x & & & & & \\ & x & x & x & x & x & & & & \\ & & x & x & x & x & x & & & \\ & & & & & \ddots & & & & \\ & & & & & & \ddots & & & \\ & & & & & x & x & x & x & x \\ & & & & & & x & x & x & x \\ & & & & & & & x & x & x \end{pmatrix}$$

Energy Spectrym: $E(r,n) = \frac{r}{2}\sum_n u_n^2(r,t) + v_n^2(r,t)$

Spectral versus finite difference

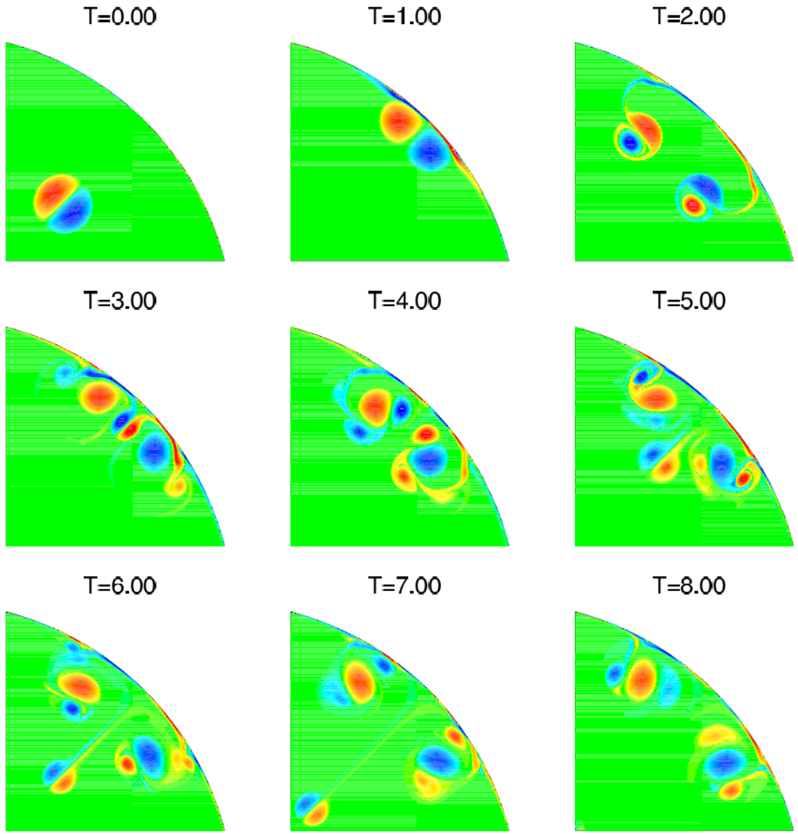


FIG. 5.4. *Time evolution of the vorticity field for the interaction of a Lamb-dipole with a no-slip wall. The spectral scheme has been used with $M = N = 1024$ and $Re = 2.000$. Notice that only a part of the computational domain is displayed.*

Close-up

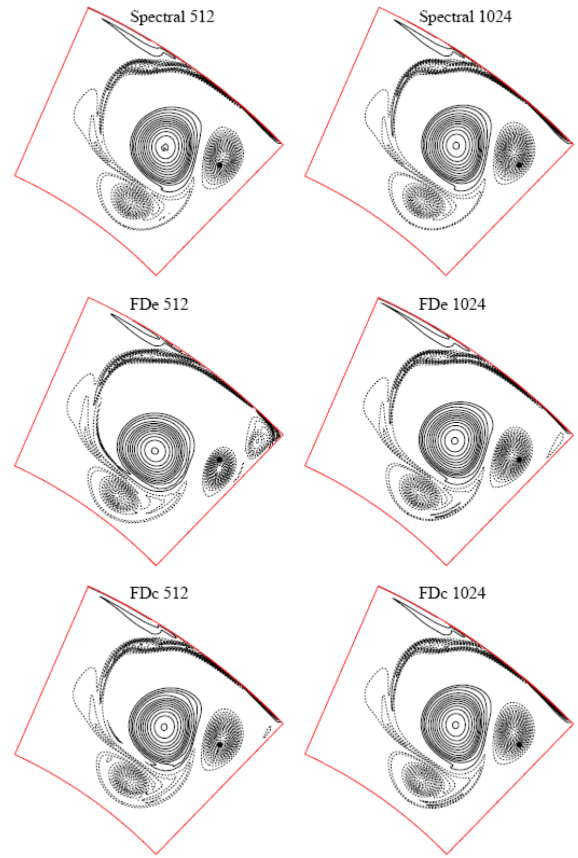


FIG. 5.6. *A close-up of vorticity contours for runs with the same parameters as in Figure 5.4 at $T = 4.0$ and $Re = 2.000$. Top: Spectral scheme. Middle: Arakawa scheme using an equidistant radial grid. Bottom: Arakawa scheme using cosine distributed radial grid points. Left resolution 512 and right resolution 1024. The dot in each frame locates the position where the time development is compared; see Figure 5.7.*

(V. Naulin and A.H. Nielsen **25**, 104–126 SIAM J. SCI. COMPUT 2003)

Spectral versus finite difference

Solve the vorticity equation with solid boundaries in annulus geometry

$$\frac{\partial \omega}{\partial t} + \left[\omega, \psi\right] = \nu \nabla^2 \omega, \quad \nabla^2 \psi = -\omega, \quad \vec{u}\,|_{\partial D} = 0$$

We used a spectral code (Chebyshev-Fourier expansion) and finite difference code (cosine distributed radial points). A Lamb dipole

$$\omega = \begin{cases} \dfrac{2\lambda U}{J_0(\lambda R)} J_1(\lambda r)\cos(\theta) & , r \le R \\ 0 & , r > R \end{cases}$$

was used as initial condition and let it interact with the outer wall for different Reynolds numbers, Re=UL/v.

Conclusion:

- Spectral schemes are more accurate than FD using the same resolution BUT
- Using the same computer power we can obtain similar results for the two different schemes

Spectral

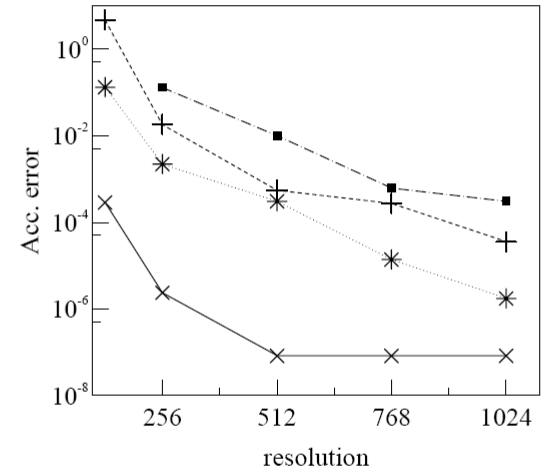


Fig. 5.9. *Integrated error calculated from (5.2) versus resolution for the spectral scheme. Reynolds number: 200 (solid line), 1.000 (dotted line), 2.000 (dashed line), 4.000 (dashed-dotted line).*

Finite difference

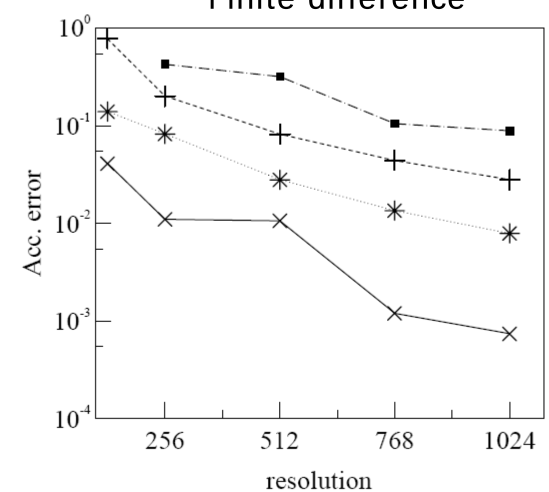


Fig. 5.10. *Integrated error calculated from (5.2) versus resolution for the FDc scheme. Reynolds number: 200 (solid line), 1.000 (dotted line), 2.000 (dashed line), 4.000 (dashed-dotted line).*

(V. Naulin and A.H. Nielsen **25**, 104–126 SIAM J. SCI. COMPUT 2003)

DIESEL

- Global version of “ESEL”
- Covers the full toroidal domain

$$\frac{\partial n^i}{\partial t} + \left\{\phi, n^i\right\} = \mu_n \nabla_\perp^2 n^i + c_s \nabla_\parallel n$$

$$\frac{\partial \omega^i}{\partial t} + \left\{\phi^i, \omega^i\right\} = \left\{\frac{1}{B}, n^i\right\} + \mu_\omega \nabla_\perp^2 \omega^i + V_A \nabla_\parallel \omega$$

$$\omega = \frac{1}{B}\nabla_\perp^2 \phi \quad , \quad B(r,\theta) = \frac{B_0}{1 + \frac{x}{R}\cos\theta}$$

$$c_s \nabla_\parallel n^i = \begin{cases} \frac{c_s}{L_\parallel}\left(n^{i+1}(r,\theta+\Delta\theta) - n^i(r,\theta)\right) & \text{if } \mathrm{n}^{\mathrm{i}} \geq n^{i+1} \\ \frac{c_s}{L_\parallel}\left(n^{i-1}(r,\theta-\Delta\theta) - n^i(r,\theta)\right) & \text{otherwise} \end{cases}$$

$$c_s \nabla_\parallel \omega^i = \begin{cases} \frac{c_s}{L_\parallel}\left(\omega^{i+1}(r,\theta+\Delta\theta) - \omega^i(r,\theta)\right) & \text{if } \left|\omega^{\mathrm{i}}\right| \geq \left|\omega^{\mathrm{i+1}}\right| \\ \frac{c_s}{L_\parallel}\left(\omega^{i-1}(r,\theta-\Delta\theta) - \omega^i(r,\theta)\right) & \text{otherwise} \end{cases}$$

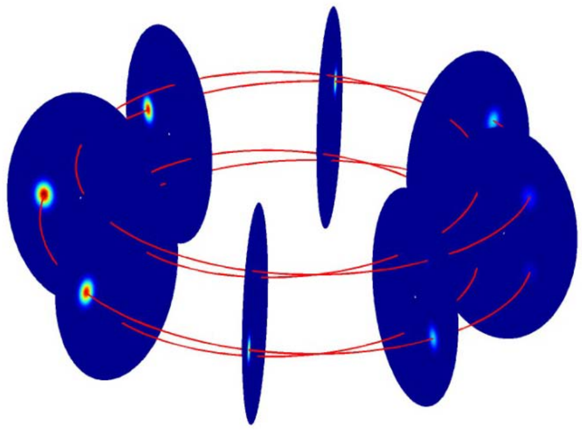
$$\Delta\theta = \frac{2\pi}{q n_{\mathrm{drift}}}$$

- Normalisation[4], space and time $x \rightarrow \frac{x}{a}$, $t \rightarrow \gamma t$
- Interchange growth rate: $\gamma = \sqrt{\frac{2}{aR}} c_s$
- c_s in normalized unites : $c_s \rightarrow \sqrt{\frac{R}{2a}}$

DIESEL

- Global model using full toroidal geometry on closed magnetic field lines
- Model, at present, based on a simple interchange model, see e.g. [1,2]
- n_{drift} 2-D drift planes each covering the full cross section of the torus
- In the above equations $i \in [1;n_{drift}]$ and denotes the particular drift plane
- Parallel numerical code, based on spectral expansion of the solutions
- Scale linearly at least upto 100 CPU using 1024x2048 pr. drift plane
- Estimated maximum number of CPU is above 1.000!? (to be tested)
- The drift planes are separated toroidally by $L_{\parallel} = 2\pi R / n_{drift}$
- Parallel velocitites are parameterized using c_s and V_A
- q enters in the two parallel terms:

Vorticity



Density