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Quantum-dot nano-cavity lasers with Purcell-enhanced stimulated emission

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We present a rate equation model for quantum-dot light-emitting devices that take into account Purcell enhancement of both spontaneous emission and stimulated emission as well as the spectral profile of the optical and electronic density-of-states. We find that below threshold the β-factor in a quantum-dot nanolaser depends strongly on the pump. For quantum dots with linewidth comparable to that of the cavity, we then show that an otherwise non-lasing device can lase due to Purcell enhancement of the stimulated emission. Finally, we compare the rate equation model to a microscopic model and obtain good agreement. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.3697702]

The quest for higher modulation speed and reduced energy consumption inevitably leads to miniaturization and integration of optical components. In this context, semiconductor-based light-emitting devices compatible with integration have received increasing attention, and in particular the quantum dot (QD) nano-cavity laser[1] appears as a promising candidate to achieve high efficiency, high speed, and a small footprint.

It is well-known that the radiative carrier lifetime in such devices can be decreased via the Purcell effect[2] or increased using a photonic bandgap.[3] The spontaneous emission (SpE) rate in a nano-cavity with a high quality (Q) factor and a low mode volume V is enhanced by the Purcell factor[2] given by $F = 6Q/(\pi^2 V)$. A necessary condition for this enhancement is that the linewidth of the cavity is large[4] compared to that of the emitter, and correct modeling of Purcell enhancement thus requires a proper treatment of the electronic density-of-states (eDOS) as well as the effect of homogenous broadening.

The modeling of the dynamics of nano-cavity light-emitting devices has generally followed two paths. In microscopic models,[5] light emission from discretized energy levels in, e.g., a QD is studied, and the details of the photonic environment are usually limited to a light-matter coupling constant and a photon lifetime. Here, the quantization of the electromagnetic field leads to a photon emission rate proportional to $F(n_p + 1)$, where n_p is the photon number, and thus Purcell enhancement of both SpE and stimulated emission (StE) is unavoidably included. With the same argument, the inclusion of the Purcell enhancement in the rate equation model employed by the laser community has been suggested.[8] This work simply assumed that the necessary condition was fulfilled and did not treat the eDOS. On the experimental side, cavity-enhanced StE was observed with lasing in rhodamine6G-ethanol droplets[7] and in polymer microrings,[6] materials with insufficient gain for lasing in the absence of gain enhancement.

Recently, a high direct modulation speed was experimentally demonstrated[9] in a photonic crystal-based quantum-well nano-cavity laser, an observation unexplainable using the standard[10] rate equation model without Purcell enhancement. The initial modeling[9,11] considered only Purcell enhanced SpE and did not take into account the profile of the eDOS. We subsequently presented a detailed model[12] for Purcell enhanced SpE with a proper treatment of the eDOS. In the quantum well case, the initially predicted[9,11] enhancement is limited[12] by a saturation arising from the violation of the necessary condition for Purcell enhancement, and for the QD case, homogeneous and inhomogeneous broadening are main limitations[13] for Purcell enhanced SpE.

In this work, we extend our previous rate equation model to include Purcell enhancement of the StE while maintaining the detailed treatment of the eDOS. We study the SpE β-factor and show that, with proper inclusion of the eDOS, the β-factor depends strongly on the carrier density. We demonstrate that lasing can occur for a device having insufficient gain to reach threshold in absence of Purcell enhancement. Finally, we verify our rate equation results by comparing to a microscopic model.[5]

We model the dynamics of a QD ensemble in an optical cavity using the rate equations neglecting non-radiative recombination

$$\frac{dN}{dt} = J - R_c - R_b, \quad (1)$$

$$\frac{dN_p}{dt} = \Gamma R_c - \frac{N_p}{\tau_p}, \quad (2)$$

where N is the carrier density and J is the injection current density, $R_c = R_{c,sp} + R_{c,st}$ is the total radiative decay density rate to the cavity including both SpE and StE contributions, R_b is the background rate, N_p is the photon density as measured at the cavity frequency, Γ is the confinement factor, $\tau_p = Q/\omega_c = \delta\omega_c^{-1}$ is the photon lifetime and ω_c is the cavity resonance frequency. While Eqs. (1) and (2) are identical to textbook versions, we compute the radiative decay rates using a detailed model[10,12] taking into account the interaction between the cavity mode, the optical background, the QD transition and the wetting layer as well as Purcell enhancement of the SpE and StE on an equal footing. SpE is thus calculated directly and there is no need for the introduction of a phenomenological β-factor. The radiative decay rates are determined using

$$R_{c,sp} = \int\int \rho_{o,c}(\hbar\omega)\rho_e(E)f_2(1 - f_1)B_{21}\hbar\omega L\hbar d\omega dE \quad (3a)$$

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$$R_{c,st} = \int\int \rho_{o,c}(\hbar\omega)\rho_e(E)N_p V(f_2 - f_1)B_{21}\hbar\omega L\hbar d\omega dE. \quad (3b)$$

Here, the eDOS, ρ_e, includes a Gaussian distribution of QD transition energies centered at the cavity frequency with standard deviation σ representing the inhomogeneous broadening. A wetting layer with bandgap $\hbar\omega_{WL}$ is also included. The homogeneous broadening is modeled using a Lorentzian function, L, with a full-width half-maximum of γ. The valence and conduction band Fermi functions f_1 and f_2 are computed from the carrier density, and B_{21} is the Einstein B coefficient. Replacement of the cavity optical density-of-states (oDOS), $\rho_{o,c}$, with the background oDOS, $\rho_{o,b}$, in the right hand side of Eq. (3a) gives the background rate R_b. The oDOSs are modeled as

$$\rho_{o,c} = \frac{F}{B_{21}\hbar\omega\tau_{21}}\frac{(\delta\omega_c)^2}{4(\omega - \omega_c)^2 + (\delta\omega_c)^2} \quad (4a)$$

$$\rho_{o,b} = \frac{n^3\omega^2}{\pi^2 c^3}[H(\omega - \omega_u) + H(\omega_l - \omega)] \\ + \rho_{o,cb}H(\omega - \omega_l)H(\omega_u - \omega). \quad (4b)$$

The differential recombination time, τ_{21}, is given by $\tau_{21}^{-1} = \hbar\omega_c B_{21}\rho_{o,Bulk}(\hbar\omega_c)$ and is chosen so that the bulk spontaneous emission time, τ_{sp}, is recovered for the oDOS $\rho_{o,Bulk}$ in a bulk material, i.e., $R_{sp}(N_{tr}) = N_{tr}/\tau_{sp}$ at transparency N_{tr}. H is the Heaviside function, n is the material refractive index, and $\delta\omega_{PBG} = \omega_u - \omega_l$ is the width of the imperfect photonic bandgap which includes a background $\rho_{o,cb}$. The oDOS and eDOS profiles are illustrated schematically in Fig. 1.

In Eq. (3b), the StE is proportional to $n_p = N_p V$, the number of photons in the cavity mode, and experiences Purcell enhancement due to the change of the optical DOS compared to that of a homogeneous medium just like the SpE. This description of the SpE and StE is appropriate when the cavity linewidth $\hbar\delta\omega_c$ is comparable to or larger than the total broadening of the emitter, including both homogeneous and inhomogeneous broadening. When $\hbar\delta\omega_c \ll \gamma$, a more elaborate model taking into account reversible energy exchange should be used. In this work, we assume a low inhomogeneous broadening to enable comparison with the mi-

TABLE I. Parameters used in the calculations.

Parameter	Value
Cavity resonance ($\hbar\omega_c$)	0.8 eV
Q	10 000
Photonic bandgap ($\delta\omega_{PBG}$)	0.2 ω_c
Refractive index (n)	3.4
SpE recombination time (τ_{sp})	1 ns
Differential recombination time (τ_{21})	125 ps
Homogeneous broadening (γ)	5 μeV
Inhomogeneous broadening (σ)	100 μeV
Wetting layer bandgap ($\hbar\omega_{WL}$)	1 eV
Temperature	100 K
Number of QDs (N_{QD})	100
QD volume (V_{QD})	4000 nm^3

croscopic model which does not include inhomogeneous broadening, and we consider a homogeneous broadening corresponding to cryogenic temperature in order to obtain a substantial Purcell enhancement.

The SpE β-factor is given by

$$\beta(N) = \frac{R_{c,sp}}{R_{c,sp} + R_b}. \quad (5)$$

In the microscopic model only a single optical mode is included explicitly together with a constant β-factor. It should be noted that this is not a restriction of the model, but for numerical reasons only. However, our rate equation model takes into account the full spectral dependence of the cavity and background oDOSs. The emission at a particular frequency depends on the carrier occupation at that frequency and both densities of states, and the β thus becomes a function of N. We initially study β as function of carrier density using the parameters listed in Table I and using two sets of values for $\rho_{o,cb}$ and F. The β, computed in the REs directly from Eq. (3a), the corresponding equation for R_b and Eq. (5) as function of N, is shown in Fig. 2. For a low carrier density, the SpE is dominated by background emission due to the overlap between the low-energy tail of the eDOS and the background oDOS outside the bandgap. As the pump rate increases, the relative contribution to the cavity mode initially increases. For a very large carrier density the Fermi energy reaches the wetting layer, the radiative decay into the background increases and β is again reduced. Obviously, these findings depend strongly on the details of the oDOS
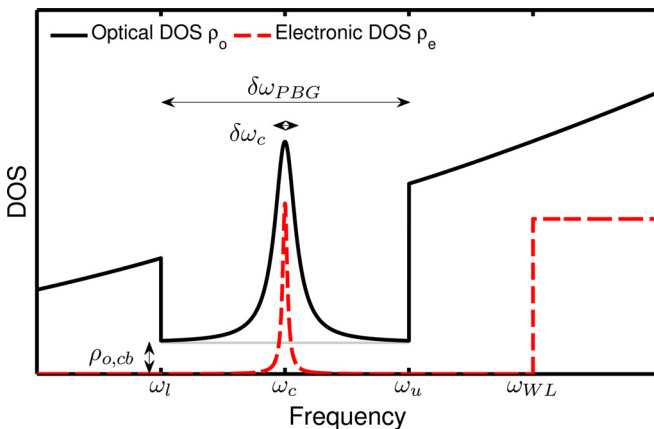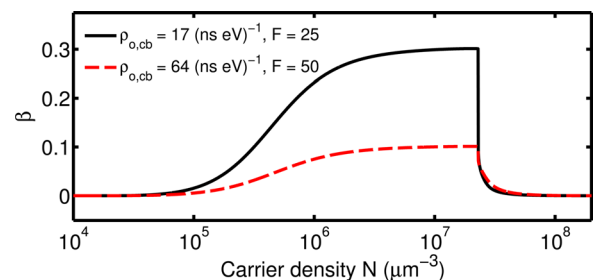

FIG. 1. Schematic of the oDOS ρ_o including cavity and background contributions and the eDOS ρ_e including the QD transition aligned with the cavity and a wetting layer contribution.

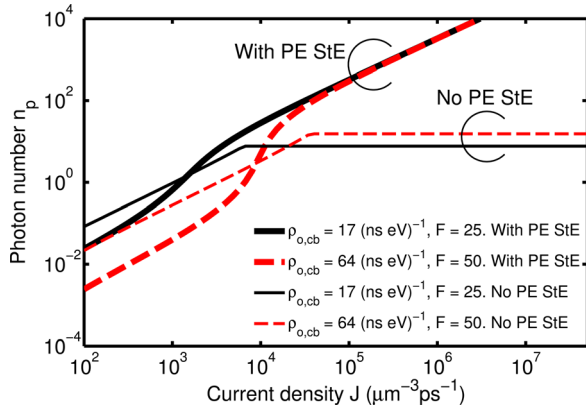FIG. 2. The β-factor as function of carrier density N for the two systems considered.

FIG. 3. Light-current characteristics of QD nano-cavity systems with parameters sets $(\rho_{o,cb} = 17$ (ns eV)$^{-1}$, $F = 25)$ and $(\rho_{o,cb} = 64$ (ns eV)$^{-1}$, $F = 50)$ with and without Purcell enhanced (PE) StE.
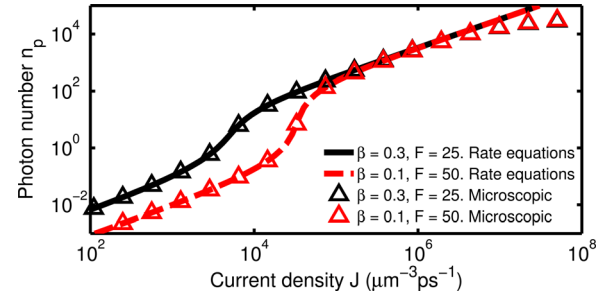

FIG. 4. Light-current characteristics of QD nano-cavity systems with parameters sets $(\beta = 0.3, F = 25)$ and $(\beta = 0.1, F = 50)$ computed using the rate equations and the microscopic model.

and eDOS and therefore a proper inclusion of both is important.

This analysis shows that the approximation of the β-factor to a constant is generally poor. Typical nano-cavity lasers operate at carrier densities of 10^6 to 10^7 μm^{-3} and in this regime the β varies by 20%. The description of the radiative decay using contributions such as $F\beta$ for the Purcell enhanced cavity mode contribution and $(1 - \beta)$ for the background is thus inaccurate, and we avoid such terms in our formalism by computing the R_c and R_b rates directly. We can, afterwards, extract a carrier-dependent β-factor using Eqs. (3a) if desired. This is not necessary in our formalism, and we only do so to enable subsequent comparison with a microscopic model.

For a carrier density of $N_{th} \approx 10^7 \mu m^{-3}$, corresponding to the threshold in the following calculations, β factors of 0.1 and 0.3 are obtained for the two parameter sets.

We now compute the light-current characteristics using our model. Calculations of n_p with and without Purcell enhanced StE are presented in Fig. 3. Here, we thus consider only the cavity contribution to light emission and not the background contribution R_b. Below threshold, the systems including Purcell enhanced StE also experience cavity-enhanced absorption, and for a weak pump the photon numbers are thus lower than in the absence of Purcell enhanced StE. For the system with $\rho_{o,cb} = 17$ (ns eV)$^{-1}$, a Purcell factor $F = 25$ is sufficient to reach lasing, whereas for $\rho_{o,cb} = 64$ (ns eV)$^{-1}$ a larger value of $F = 50$ is chosen. However, in the absence of Purcell enhanced StE the gain provided by the QD ensemble is insufficient to obtain lasing. We here stress that a necessary condition for the Purcell enhanced StE is that the linewidth of the emitter is narrow compared to that of the cavity. The flattening of the curves do not indicate a threshold but simply a saturation of the photon number as the Fermi energy moves past the QD transition linewidth and starts feeding the wetting layer. We observe that proper inclusion of Purcell enhancement is necessary to correctly predict the laser threshold in nano-cavity devices.

We have compared our results from the rate equation model with those obtained using a microscopic approach.[5] This model is based on the Schrödinger equation including all relevant many-body interactions, and all processes are derived from first principle.[5] In the microscopic model, the coupling to the background is treated using a carrier-independent β-factor. In our comparison presented in Fig. 4, we employ the β values of 0.1 and 0.3 determined previously at threshold for the two parameter sets. Instead of computing the background emission R_b in the rate equation modeling, we dynamically vary R_b to keep β fixed to these values. This procedure is not ideal but necessary to perform the comparison. Inspecting Fig. 4, we observe that the agreement between the two models is nearly perfect for both parameters sets except for a very strong pump where Pauli blocking limits the photon number in the microscopic model. The good agreement between the two models indicates that the Purcell enhanced StE is correctly implemented in our rate equation model.

In conclusion, we have proposed a rate equation formalism which takes into account the Purcell enhancement of both the SpE and StE and includes a proper treatment of the eDOS. We have established the conditions for Purcell enhancement of the emission rate and we have shown that the SpE β-factor widely used in rate equation models can generally not be approximated to a constant, that the inclusion of the Purcell enhanced StE in QD nano-cavity lasers is necessary to correctly predict the onset of lasing and that the emission rates should instead be computed using a model taking into account the spectral profiles of the optical and electronic DOS. A comparison of the rate equation model with a microscopic model shows very good agreement, indicating that our formalism is correct.

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