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# Stochastic flow forecasting with calibrated radar rainfall input

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# Contents

# 1 Introduction

Radar rainfall measurements are expected to improve the results of runoff models in the urban drainage context as they provide improved information on the spatial rainfall distribution and can further be used to create short term radar forecasts. Conversion between reflectivity values provided by the C-band radars and rainfall intensities is commonly problematic as the conversion relationship is not unique but changes with the rainfall characteristics. The radar measurements are therefore commonly adjusted to rain gauge observations using mean field bias. We consider an approach for online radar rainfall calibration developed at Aalborg University and evaluate the quality of runoff forecasts generated with these inputs. Forecasts are generated assuming future rainfall to be known.

# 2 Methods

## 2.1 Data and Catchments

We consider two catchments in the Copenhagen area. The Ballerup catchment has a total area of approx. 1300 ha. It is mainly laid out as a separate system but has a small combined part. Further, rainfall dependent infiltration and misconnection strongly influence the runoff from this area. The catchment was extensively used for the development of stochastic flow forecasting models ([1]).

The Damhusåen catchment is located close to Ballerup but drains to a different treatment plant. We consider the northern part of the catchment with a total area of approx. 3000 ha. The catchment is laid out as a combined sewer system and a multitude of CSO's is located in the area. Flow measurements are available at the outlets of both catchments in 5 min resolution.

A C-band radar is operated by the Danish Meteorological Institute (DMI) in Stevns approx. 45 km south of the considered catchments. The spatial resolution of the radar pixels is 2x2 km. The provided radar data are rain intensities derived using the Marshall Palmer relationship. The coefficients have been adjusted such that the average rainfall depth observed by the radar during the calibration period matches selected gauge measurements ([6]). We consider an area of 9x11 pixels that covers the whole Copenhagen area (figure 1).

Within the catchments a multitude of online rain gauge measurements is available from the Danish SVK network. The gauges marked red in figure 1 are used to adjust the radar data. Only few of the available gauges are used for this purpose as one objective for using radar rainfall data is to derive rain intensities from as few ground measurements as possible. To make results comparable, we use the same gauges that are used for radar calibration in a real time control project in the Copenhagen area ([2]). A reference simulation is performed where flow forecasts are generated using rain gauge measurements as an input. The gauges for these simulations were selected w.r.t. to their location to the catchment and by analyzing cross correlation between flow and rainfall observations and are marked in Figure 1.

We have selected a 3-month period of measurements from 25/06/2010 until 29/09/2010 for this study. The period contains several rain events that can be considered relevant for control applications in urban drainage systems (Figure 2).

A modeling time step of 10 min is adopted corresponding to the resolution of the provided radar data. The flow and rain gauge data are averaged to match this time step.

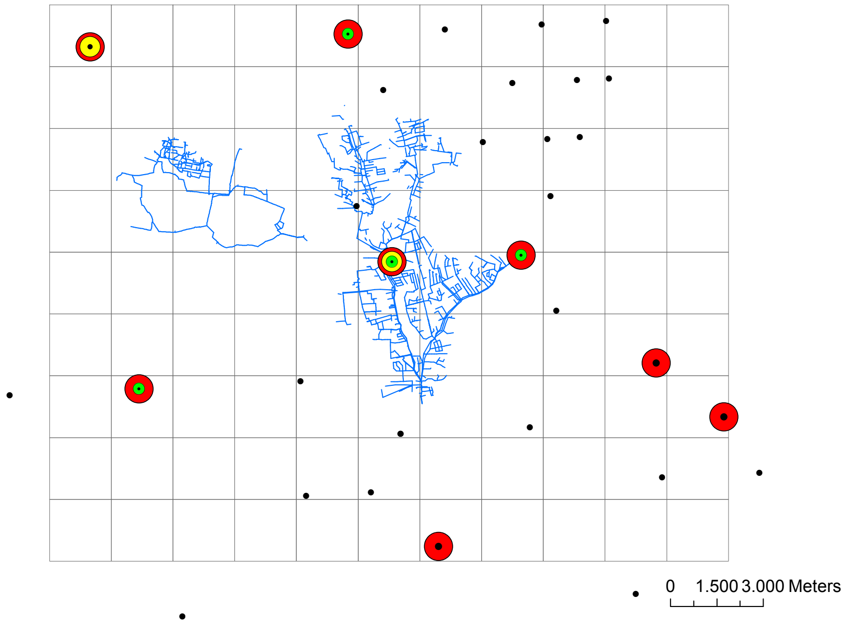


Figure 1: Calibration area with C-band radar pixels, Ballerup (left) and Damhusåen (right) catchments, rain gauges in the area (black dots), gauges used for radar calibration (red), gauges used as input for reference simulations with gauges for Ballerup (yellow) and Damhusåen (green) catchments

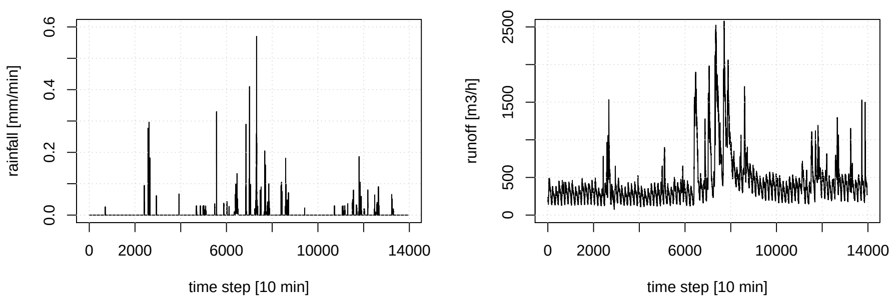


Figure 2: Mean area rain intensities observed by gauges (left) and observed flow (right) for the Ballerup catchment during calibration period 25/06-29/09/2010

## 2.2 Radar Rainfall Measurements

C-band radar measurements from the DMI radar in Stevns were provided by Aalborg University. The data are converted to rainfall measurements in a two-step calibration procedure. A detailed description of the approach can be found in [6].

- In a first, 'static' step the radar reflectivities $Z$ are converted to rainfall intensities $R$ using the Z-R-relationship $Z = a \cdot R^b$. The coefficients $a$

and $b$ are determined such that during the calibration period the average rainfall height from the radar data matches that from the gauge data. We refer to these data as statically calibrated data.

- In a second, 'dynamic' step the radar rain intensities are again adjusted to match the rain gauge observations at every time step. These data are referred to as dynamically calibrated data.

The measurements are used as model input in the form of mean area rainfall for the considered catchment. As the radar observations are noisy, we consider area rainfall levels below 0.008 mm/min equal to 0.

## 2.3 Stochastic Runoff Forecasting

### 2.3.1 Model structure

We generate flow forecasts using stochastic greybox models. These are simple models with physically interpretable parameters that, in addition to the physical process description, contain a stochastic term. The models are laid out as state space models, i.e. the model consists of two parts. The system equations describe the process whereas the observation equation relates model predictions and observations. The separation into these two parts also allows to distinguish between model and observation errors.

We apply a simple model consisting of a cascade of 2 reservoirs to predict runoff from the two catchments. The model was developed and tested on the Ballerup catchment ([1]). For the bigger and more structured Damhusåen catchment the model structure is most likely too simple. However, the aim of this study is to investigate the effect of different rainfall inputs on the forecast quality, not to obtain the best forecast quality for a specific catchment.

Equation (1) shows the structure of the system equations. The reservoir volumes or system states $S_1$ and $S_2$ are described by differential equations depending on sealed area $A$, rainfall input $P$, mean dry weather flow $a_0$ and travel time constant $K$. The model predictions are noisy and the uncertainty is described by a Wiener process $d\omega_t$ with incremental variances $\sigma(S_{1,t})$ and $\sigma(S_{2,t})$ where the process uncertainty depends on the current state values. We refer to [1] for a detailed description of the model setup.

$$d\begin{bmatrix} S_{1,t} \\ S_{2,t} \end{bmatrix} = \begin{bmatrix} A \cdot P + a_0 - \frac{1}{K} S_{1,t} \\ \frac{1}{K} S_{1,t} - \frac{1}{K} S_{2,t} \end{bmatrix} dt + \begin{bmatrix} \sigma(S_{1,t}) \\ \sigma(S_{2,t}) \end{bmatrix} d\omega_t \tag{1}$$

In the observation equation the predicted outflow from the reservoirs is combined with the variation in the dry weather flow $D$ and set into relation with the observed flow $Q$. The observations are uncertain, so an observation error $e_t$ with variance $\sigma_e$ is included.

$$log(Q_k) = log(\frac{1}{K} S_{2,k} + D_k) + e_k \tag{2}$$

An extended Kalman filter routine is used to update the model states from the observations at every time step ([3]). As the variance of the state errors depends on the system states themselves, a Lamberti transform is applied to obtain numerical stability in the filter routine ([1]).

### 2.3.2 Estimation

Parameter estimation of the forecasting models was in previous works ([1],[5]) based on maximizing the likelihood of the one step ahead flow predictions. We consider this criterion non-optimal with respect to obtaining forecasts over longer horizons. Evaluating only the one-step ahead prediction forces the state updating in the Kalman filter to stay as close to the observations as possible. At the same time we can observe a tendency of the physical model parameters to be insignificant. As a result, the model is unsuitable to create predictions for longer horizons.

When generating forecasts for real time control, the control decisions are based on the expected runoff volume over the prediction horizon. We therefore select this variable for a modified objective function and select the model parameters in such a way that the best prediction interval for the expected runoff volume is obtained. The quality of the prediction interval is evaluated using the skill score function described in section 2.3.3.

Considering a forecast with lead time 100 min in steps of $\Delta t$=10min, the expected runoff volume over this horizon $V_k$ can be derived by summing up the flow predictions starting from time step $k$ for $i$ =1 to 10 time steps into the future.

$$V_k = (\sum_{i=1}^{10} Q_{k+i}) \cdot \Delta t \tag{3}$$

The 10 step ahead flow predictions are generated in the Kalman filter setup together with an estimate of the prediction variance $\sigma_{k+i}$ for every horizon. Considering a 95% prediction interval of the flow values, we can also derive a prediction interval for the runoff volumes by summing up the lower and higher flow prediction boundaries. The 97.5 % quantile of the standard normal distribution $n_{0.975}$ is used to define the flow prediction intervals.

$$V_{k,up} = (\sum_{i=1}^{10} Q_{k+i} + n_{0.975} \cdot \sigma_{k+i}) \tag{4}$$

$$V_{k,low} = (\sum_{i=1}^{10} Q_{k+i} - n_{0.975} \cdot \sigma_{k+i}) \tag{5}$$

Using the above prediction interval, the skill score function can be computed by comparing to the actually observed runoff volumes (section 2.3.3). We evaluate the skill score function only during wet weather periods as these are the main focus of control systems. The dry weather parameters of the model are estimated from a 14 day dry weather period at the beginning of the calibration period and then fixed. We consider wet weather periods when the observed runoff volume clearly exceeds the dry weather peak. In the Ballerup catchment this is the case, when more than 1000 $m^3$ runoff volume are observed during the forecast horizon of 100 min. In the Damhusåen catchment we set the treshhold to 3300 $m^3$.

### 2.3.3 Forecast Evaluation

When evaluating stochastic flow forecasts, we need to consider the quality of prediction intervals rather than a mean error between prediction and observation. A set of criteria for forecast evaluation has been proposed in [5]. The criteria used for evaluation are width (or sharpness) of the prediction intervals and the percentage of observations not contained in the intervals. We use the following criteria:

- Reliability ($Rel$) percentage of observations not contained in a 95% prediction interval. Ideally, this value corresponds to 5%, lower values suggest an overfitted model, higher values an unreliable model
- Sharpness ($Sh$) - average width of the 95% prediction interval
- Skill score ($Sk$)

  $$Sk = Sh + \frac{2}{0.05 \cdot N} \sum_i (U_i + L_i) \tag{6}$$

  where $N$ is the number of wet weather observations and $U_i$ and $L_i$ are the distances of the $i$-th observation from the upper / lower 95% prediction interval (over-/ undershoots). $U_i$ and $L_i$ are 0 if the observation is contained in the prediction band.

We compute these criteria for a runoff volume prediction interval as described above. Only wet weather periods are considered in the computation of the evaluation criteria. Informatively, we also provide the root mean square error ($RMSE$) between predicted and observed runoff volumes.

# 3 Results

## 3.1 Rainfall Inputs

Figure 3 shows the cumulated rainfall heights from different input sources as mean area rainfall over the two catchments. We observe that the rainfall depths obtained from the radar data are clearly lower than that of the gauge measurements. As the radar data are calibrated to the rain gauge observations, we would expect them to give similar rainfall values with the dynamically calibrated data following the ground measurements somewhat more closely than the statically calibrated data.

In figure 3 we can clearly see that this is not the case. On the contrary, the ground measurements are underestimated by both radar datasets in both catchments with the dynamically calibrated data deviating even stronger than the statically calibrated data. We suspect a problem in the calibration or handling of the radar data but cannot track the issue at this point.

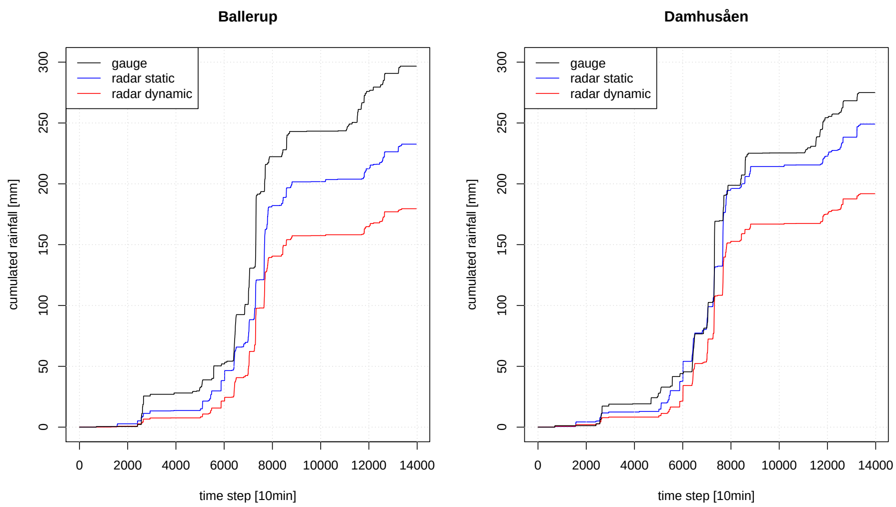


Figure 3: Cumulated rainfall heights during the calibration period as mean area rainfall for the two catchments

## 3.2 Runoff forecasts

Table 1 evaluates the quality of runoff forecasts obtained for the two catchments with different rainfall inputs. As described in section 2.3.3, the shown values relate to predicted runoff volumes in $m^3$ over a predicition horizon of 100 min. Future rainfall values are assumed known when generating the runoff forecasts.

We see that in both catchments the best runoff predictions are obtained using raingauge input. Using radar rainfall input, the runoff predictions are either unreliable (Damhusåen catchment) or the prediction intervals are wider (Ballerup). In both catchments there is no clear difference in the quality of runoff forecasts generated with statically and dynamically calibrated radar in-

Table 1: Evalution of 100 min volume forecasts based on different rainfall inputs for Ballerup and Damhusåen catchments. Only wet weather periods are evaluated.

| | Ballerup | | | | Damhusåen | | | |
|---|---|---|---|---|---|---|---|---|
| | $Rel$ | $Sh$ | $Sc$ | $RMSE$ | $Rel$ | $Sh$ | $Sc$ | $RMSE$ |
| Gauge | 5% | 1095 | 1465 | 116 | 4% | 9543 | 11777 | 1587 |
| Radar stat | 5% | 1145 | 1545 | 100 | 9% | 6564 | 15181 | 1018 |
| Radar dyn | 5% | 1100 | 1542 | 101 | 9% | 6763 | 14799 | 1009 |

put. These results are in agreement with the analysis of cumulated rainfall heights above.

We see that the prediction intervals obtained for the Damhusåen catchment are generally wider which leads to clearly worse skill scores as compared to the Ballerup catchment. The reasons for these results are the generally higher flows in this catchment (mean dry weather flow Damhusåen approx. 900 $m^3$/h, Ballerup approx. 300 $m^3$/h) and the too simple structure of the forecast model. When tuning the prediction model to this catchment, further effects such as overflows will most likely need to be considered in the model structure.

Last, comparing the results in table 1 to those obtained in [4], we can clearly observe an improvement of the runoff forecast quality when using the volume prediction skill score as an objective function rather than minimizing the mean error of the volume predictions. The improvement results mainly from the more reliable prediction intervals, i.e. the prediction intervals are not predicted too narrow as was the case in [4].

Volume forecasts for two events in the Ballerup catchment are shown in figures 4 to 6.

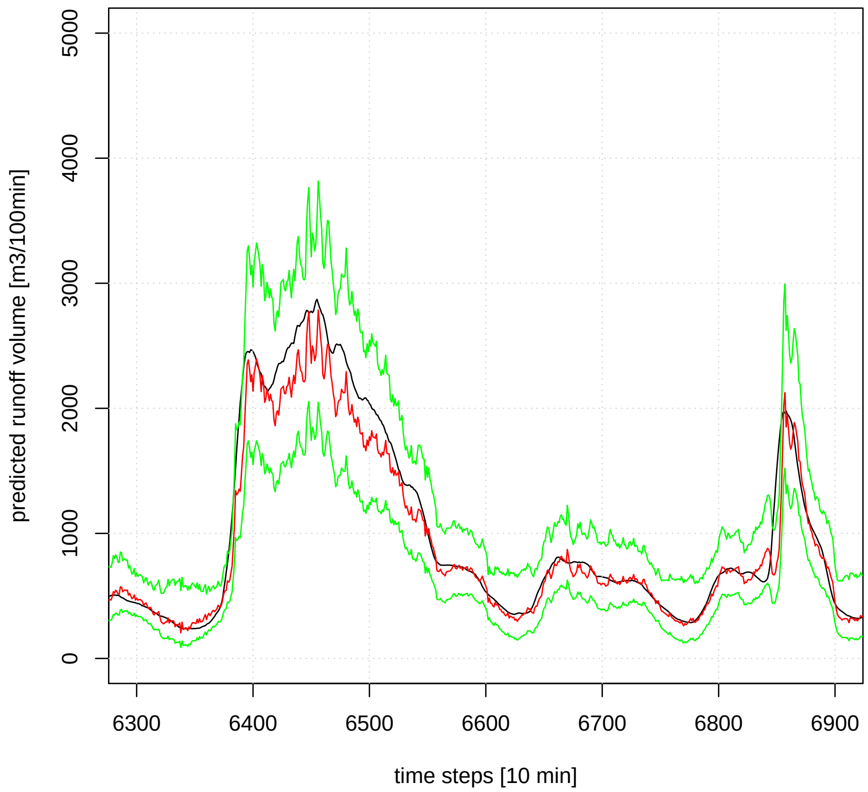


Figure 4: Forecasts of runoff volume over 100min in the Ballerup catchment for two events uing rain gauge input. Includes observed (black) and predicted (red) volume as well as upper and lower 95% predicition bounds (green)

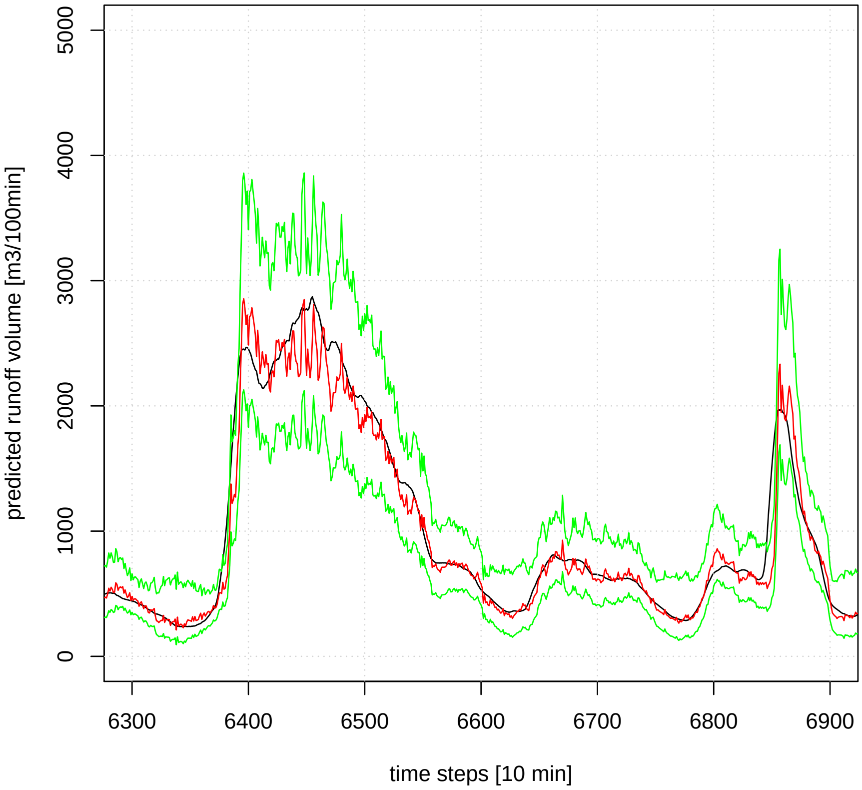


Figure 5: Forecasts of runoff volume over 100min in the Ballerup catchment for two events using statically calibrated radar rainfall input. Includes observed (black) and predicted (red) volume as well as upper and lower 95% predicition bounds (green)

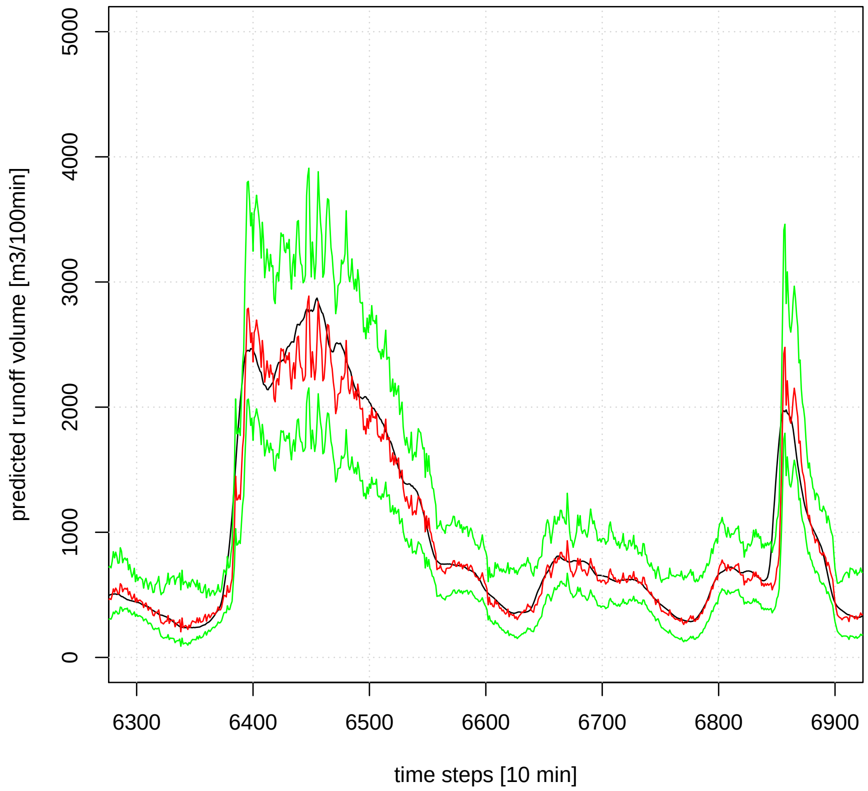


Figure 6: Forecasts of runoff volume over 100min in the Ballerup catchment for two events using dynamically calibrated radar rainfall input. Includes observed (black) and predicted (red) volume as well as upper and lower 95% predicition bounds (green)

# 4 Conclusions

We have analysed the suitability of different rainfall inputs for generating stochastic runoff predictions. For both considered catchments using rain gauge measurements as input to the forecasting models yields the best results in terms of stochastic forecast quality.

In the smaller Ballerup catchment, the prediction intervals generated with different rainfall inputs are similar but slightly sharper when using rain gauge measurements. In the Damhusåen catchment, prediction intervals of similar width are generated for all inputs. However, these are less reliable when using radar rainfall input. We cannot identify a clear difference in predicitve quality between runoff forecasts generated with statically and dynamically calibrated radar rainfall measurements.

The runoff forecasting results are in line with the cumulated rainfall heights shown in figure 3, where the radar rainfall measurements clearly underestimate the ground measurements. From the quality of the different runoff forecasts we conclude, that the mean area rainfall is better represented by the gauge measurements and that, with the currently available data, gauge data are preferable to use as input for runoff predictions. We do, however, point out that the radar offers potential for generating rain forecasts which is not present in the gauge measurements and should, with improved radar calibration, lead to improved runoff forecasts.

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