Technical University of Denmark

The two-dimensional finite-difference neutron diffusion programme TVEDIM

Forskningscenter Risø, Roskilde

Publication date:
1977

Document Version
Early version, also known as pre-print

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Risø-M-1919

Risø-M-1919

Title and author(s)

The two-dimensional finite-difference neutron diffusion programme TVEDIM

by

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11 pages + tables + 7 illustrations

Date March 1977

Department or group

Department of Reactor Technology

Group's own registration number(s)

RT-10-14

GKK/rj

Abstract

The B6700 algol programme TVEDIM is described. Based on the observation that the source iteration technique with a nearly complete solution of the group equations is not usually optimal, a partial inversion is accomplished by means of a frontal elimination technique. Too stringent limitations on blocksize due to stability problems are avoided by using double precision artihmetic where advantageous.

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F 25-300

ISBN 87-550-0450-4

INTRODUCTION

The programme TVEDIM deals with the same problem as the programme TWODIM (ref. 1). They differ mainly in the iterative scheme used for solution of the resulting difference-equations (multiline vs. oneline SOR) and in the discretization-method (interrelation of fluxvalues at mesh-corners and meshcenters, resp.).

At some places the following discussion is cut short, when the material has already been dealt with in ref. 1.

The present report also contains a list of errors in ref.1.

THE PROBLEM

The set of elliptic partial differential equations

$$- \nabla \cdot (D_g \nabla \phi_g) + \Sigma_{gg}\phi_g + \alpha\Sigma_{p,g}\phi_g =$$

$$= \sum_{g' \neq g} \Sigma_{gg'}\phi_{g'} + \lambda X_g \sum_{g'} (\nu\Sigma_f)_{g'}\phi_{g'} + S_g$$

$$(1 \leq g \leq ng), \qquad (1)$$

with boundary-conditions

$$- D_g \frac{\partial}{\partial n} \phi_g = \gamma_{gg}\phi_g - \sum_{g' \neq g} \gamma_{gg'}\phi_{g'} \quad (1 \leq g \leq ng) \qquad (2)$$

(note that the off-diagonal terms in the γ-matrix have opposite signs here and in ref. 1) at interfaces between ordinary regions and boundary regions, are considered for 2-dimensional geometries xy, rz, and rθ.

There are 3 types of problems,

1) $\lambda = 1/K_{eff}$ is a criticality-parameter (= eigenvalue) ($S_g = \Sigma_{p,g} = 0$ for all g),

2) α is a criticality-parameter (λ fixed, $S_g = 0$ for all g),

3) Source-problem ($S_g \neq 0$ for some g, λ fixed, $\Sigma_{p,g} = 0$ for all g).

Similar techniques are used for solving these problems.

METHOD OF SOLUTION

The usual mesh-corner 5-point difference equations are solved by a block relaxation procedure in which each block corresponds to a number of consecutive lines in the reactor-model. The line-direction and an upper bound for the number of lines in a block are user-specified. Since, for instance, interior boundary regions are permitted, the actual number of lines in a particular block (this number is found by the programme) may be less.

It is well known that a somplete inversion of the group-equations often does not result in the minimal number of outer iterations. Experience shows that the optimal number of lines in a block (often about 5) is usually even less than the maximal number permitted by stability requirements.

TVEDIM is not always faster than TWODIM, in particular not for small, easy problems. But due to its extra parameter, the line-number, it has proved more flexible in use.

In both TWODIM and TVEDIM one inner iteration is performed per outer iteration. The eigenvalue is calculated by means of a neutron-balance. This method is not always locally convergent; i.e. there does not always exist a neighbourhood of the solution, so that convergence can be obtained for an initial flux-vector arbitrarily chosen from this neighbourhood. If the eigenvalue were calculated by a variational expression one would obtain local convergence, and even, with suitable precautions, in the selfadjoint case (one-group-theory) global convergence. In the present case we can linearize relative to the solution to get a criterion for local convergence. The simplest subcase for which an example of divergence does not seem to be known is a one-group problem in one dimension, where the equations are taken in their natural order, and the Jacobi splitting is used. The existence of a counterexample to the conjecture in ref. 3 would strongly indicate the possibility of divergence for the subcase discussed above, while a proof of the conjecture would constitute a proof of convergence.

INPUT

The following description is partly taken from ref. 2.

The input differs from that for TWODIM (ref. 1) in that it contains specification of linenumber and buckling, and another set of control parameters.

Name of inputfile: d2di

Name of restart-(disk-)file: tworest

Input-list

Name	type	meaning
problem	integer	problem-number
geom	integer	geometry, 1=xy, 2=rz, 3=rθ
cmx	-	number of material mesh regions in x-direction
cmy	-	- - - - - - y -
ncp	-	number of materials ("boundary materials" inclusive)
nb	-	number of boundary materials
ndb	-	- - - - with γ diagonal
ng	-	number of energy groups
nthg	-	- - thermal groups (possibly with upscattering)
type	-	calculation-type, 1=λ, 2=α, 3=source
nrec	-	number of recalculations (with mesh refinement)
prev	-	if prev>0 initial flux and eigenvalues are read from the disk-file tworest (coarse mesh only). For every abs (prev) iterations eigenvalues and fluxiterate are written on tworest (finest mesh only)
rela	-	direction of lines, 2 = x constant, 3 = y constant
mx	integer	number of regions (finest mesh) for x-direction
my	integer	- - - - - - y -
lines	integer	maximal number of lines in a block
xc	array [0:cmx]	material mesh x line coordinates

Name	type	meaning
fmx	integer array [1:cmx]	for each material mesh in x-direction number of coarse mesh regions
facx	integer array [1:cmx]	(only for nrec>0)mesh refinement factor for each material mesh in x-direction (applied for each recalculation)
yc	array [0:cmy]	as xc, but for y-direction; for $r\theta$ - geometry θ must be in radians
fmy	integer array [1:cmy]	as fmx, but for y-direction
facy	integer array [1:cmy]	(only for nrec>0); as facx, but for y-direction
cpnc	integer array [0:cmx+1, 0:cmy+1]	map of material numbers (including interior and exterior boundary materials), each logical record consisting of a row (cmx+2 numbers), starting with the row corresponding to the last read y-values. Note that a complete rectangular set of numbers must be specified. Those numbers which are meaningless (for instance in case of large internal controlareas or corner-regions) should be boundary material numbers. Large elements in the γ-matrices give stabilityproblems. However, the "mathematical" boundary-condition: $\phi_g = 0$ for all g, can be obtained by setting the corresponding material number in cpnc equal to zero.
busq	real	buckling; the group absorption + removal crosssections Σ_{gg} are modified by addition of D_g * busq.
f	integer	format-indicator for the following multi-group-coefficients: 1=6e12,5, otherwise free format

The following set of records specifies data for each material. As regards notation see eq. (1).

name	type	meaning	
k	integer	material number or 0; what data are expected after this, depends on k	
gad	array [1:ng]	diagonal γ-matrix (for $0<k\leq ndb$)	
gam	array [1:ng, 1:ng]	γ-matrix (for $ndb<k\leq nb$)	
dd	array [1:ng]	D_g	(for $nb<k\leq ncp$)
ss	array [1:ng, 1:ng]	$\Sigma_{g,g'}$	(for $nb<k\leq ncp$)
fs	array [1:ng]	χ_g	(for $nb<k\leq ncp$)
nsf	-	$(\nu\Sigma_f)_g$	(for $nb<k\leq ncp$)
rr	-	reactionrate-(usually fission-) cross sections (see sample output)	(for $nb<k\leq ncp$)
pp	-	(only for type $\neq$ 1) for type = 2: $\Sigma_{p,g}$ for type = 3: S_g	(for $nb<k\leq ncp$)
kb	integer	(k=0 only)	For material numbers $m\varepsilon$[kb,ke] the data are set equal to those just read
ke	-	(k=0 only)	For material numbers $m\varepsilon$[kb,ke] the data are set equal to those just read

The number of non-zero k-values plus the sum of the numbers (ke-kb+1) must equal ncp.

The following control-variables (rm to kappa incl., not epsm) have default-values, obtained, if the input-value is negative. They are given in parentheses in the following list.

name	type	meaning
rm	integer	maximal iteration number (50)
eps	real	the iterations stop, when the maximal estimated error in a flux-coordinate is less than eps times maximal flux (0.0005)
epsm	-	if epsm>0 omega will be adjusted (only in combination with an extrapolation) to maximise estimated convergence-rate.
epse	-	the calculated extrapolation - factors must agree within epse (relative error) for three consecutive iterations, before extrapolation is attempted (0.1)
omega	array [0:nrec]	relaxation factors (1.2)
lamb	real	$\lambda = 1/K_{eff}$ (1)
alfa	-	α (1)
kappa	-	κ = upper bound for α (1)
printer	integer	for printer<2 no proper output. For 2≤printer<9 flux- and reactionrate-distributions are printed, while for printer ≥9 the print-out contains also progress-report for the iterations.

If not all reaction-rate crosssections are zero then for each calculation (nrec + 1 times in all) the following set of input parameters must be specified:

name	type	meaning
powerout	integer	powerout <-2 gives no reaction-rateprint-out; if powerout = -1 the reaction-rate-distribution is calculated and printed for the earlier specified grid (initially the coarse grid); if powerout = -2 a new grid must be specified (see below)

name	type	meaning
facxp	integer array [0:mx]	(for powerout=-2) mx is the number of regions in the current mesh; facxp[n]=1 means that x-interface-line number n+1 in the current mesh shall also be an interface-line in the reaction-rate-grid
facyp	integer array [0:my]	(for powerout = -2) as facxp (mutatis mutandis)

Output

As seen in the sample-output the input is printed immediately after it has been read.

The amount of output is governed by the input-parameter printer, as described above, and may contain progress-report for the iterations, flux- and reaction-rate-(usually power-density-) tables.

REFERENCES

1. G.K. Kristiansen, "The finite-difference neutron diffusion programme TWODIM", Risø-M-1891 (1976).

2. G.K. Kristiansen, Risø, Denmark, unpublished work (1973)

3. G.K. Kristiansen, "A matrix convergence problem", problem 77-14*, SIAM Review, to appear.

LIST OF ERRORS IN REF. 1

1) p. 7, line 5 from bottom, should read,
   rela integer direction of lines in SLOR,
   2 = x constant, 3 = y constant

2) p. 11, top, the first 8 lines should be replaced by,
   If not all reaction-rate-crosssections are zero, then for each calculation (nrec + 1 lines in all) the following set of input parameters must be specified:

name	type	meaning
powerout	integer	powerout <-2 gives no reaction-rate-printout; if powerout = -1 the reaction-rate-distribution

TVEDIM sample output

```
PROBLEM
      1
   GEOM   CMX   CMY   NCP    NH   NOB    NC NTHG TYPE NREC PREV RELA
      1.    3.    3.    5.    2.    1.    2.   1.   1.   1.   0.   2.
   MX     MY
     16.   16.

16 LINES
 XC
  0.          6.000E+00  7.000E+00  8.000E+00
 FMX
        6.          1.          1.
 FACX
        2.          2.          2.
 YC
  0.          6.000E+00  7.000E+00  8.000E+00
 FMY
        6.          1.          1.
 FACY
        2.          2.          2.
 COMPOSITION NUMBERS
    1.    2.    2.    2.    2.
    1.    3.    3.    3.    2.
    1.    5.    6.    3.    2.
    1.    4.    5.    3.    2.
    1.    1.    1.    1.    1.
 COMPOSITIONS
 BUSQ:   0.
 EFORMAT=    0
1
EXTPL:   0.          0.
2
  GAM:   2.000E-01   0.          1.000E-01  5.000E-02
3
    D:   2.000E+00   3.000E-01
   SS:   3.000E-02   0.          3.000E-02  1.000E-02
   FS:   1.000E+00   0.
  NSF:   0.          0.
RR/SF:   0.          0.
4
    D:   1.000E+00   4.000E-01
   SS:   2.000E-02   0.          1.000E-02  1.000E-02
   FS:   1.000E+00   0.
  NSF:   0.          5.000E-02
RR/SF:   0.          2.000E-02
0
    5     6
```

RM	EPS	EPSM	EPSF
50	1.0E-04	0.	1.0E-01

OMEGA:

1.50 1.50

LAMB	ALFA	KAPPA
1.00[illegible]E+00	1.[illegible]E+00	1.[illegible]E+[illegible]

PRINTER= 10.

KEFF CALCULATION

ITNO	RESIDUAL	RATIO	LAMBDA	KEFF
1	1.733878600E+[illegible]	1.000000000E+00	1.172276916E+00	8.530469398E-01
2	1.67927590[illegible]E+[illegible]	-4.846069574E-01	1.333727749E+00	7.497782072E-01
3	[illegible].62235104[illegible]E-[illegible]1	-[illegible]E-01	9.435763704E-01	1.059685324E+00
4	[illegible].3955[illegible]82[illegible]E-01	-[illegible] 164[illegible]E-01	1.169602714E+00	8.549911757E-01
5	2.85799[illegible]E-01	1.[illegible]E+00	1.035993232E+00	9.[illegible]43[illegible]7475E-01
6	1.300108543E-01	-5.431477361E-01	1.094157275E+00	9.139448073E-01
7	6.42416200[illegible]E-02	-5.133006651E-01	1.070288513E+00	9.343275097E-01
8	3.57[illegible]443[illegible]E-02	-4.012297673E-01	1.079475295E+00	9.272[illegible]79533E-01

EXTRAPOLATION FACTOR = 6.7557E-01 RELAXATION FACTOR = 1.5000E+00

ITNO	RESIDUAL	RATIO	LAMBDA	KEFF
9	6.7159327[illegible]E-[illegible]	1.000000000E+00	1.075976911E+00	9.293979731E-01
10	1.595110[illegible]E-[illegible]	-2.172915562E-01	1.076534177E+00	9.289068767E-01
11	8.28[illegible]E-04	-2.29712[illegible]4E-01	1.075535131E+00	9.289[illegible]0534E-01
12	5.840489[illegible]E-04	-5.626710127E-01	1.076577336E+00	9.288596380E-01
13	3.0277127[illegible]E-04	-6.451204027E-01	1.076518434E+00	9.289[illegible]4611E-01

COMPOSITION NUMBERS

Y \ X	500.	1500.	2500.	3500.	4500.	5500.	6500.	7500.
7500.	3.	3.	3.	3.	3.	3.	3.	3.
6500.	5.	5.	5.	5.	5.	5.	6.	3.
5500.	4.	4.	4.	4.	4.	4.	5.	3.
4500.	4.	4.	4.	4.	4.	4.	5.	3.
3500.	4.	4.	4.	4.	4.	4.	5.	3.
2500.	4.	4.	4.	4.	4.	4.	5.	3.
1500.	4.	4.	4.	4.	4.	4.	5.	3.
500.	4.	4.	4.	4.	4.	4.	5.	3.

FLUX

GROUP 1.

Y \ X	0.	.1000	.2000	.3000	.4000	.5000	.6000	.7000	.8000.
8000.	5250	.5213	.5101	.4916	.4658	.4333	.3957	.3565	.3230.
7000.	5851	.5810	.5697	.5460	.5192	.4825	.4390	.3920	.3565.
6000.	7070	.7021	.6872	.6622	.6265	.5791	.5133	.4390	.3957.
5000.	8031	.7975	.7804	.7513	.7094	.6530	.5791	.4925	.4333.
4000.	8773	.8711	.8520	.8197	.7728	.7094	.6265	.5192	.4658.
3000.	9323	.9251	.9051	.8703	.8197	.7513	.6622	.5480	.4916.
2000.	9703	.9632	.9417	.9051	.8520	.7904	.6872	.5687	.5101.
1000.	9926	.9851	.9632	.9255	.8710	.7975	.7021	.5810	.5213.
0.	9999	.9925	.9703	.9323	.8773	.8031	.7070	.5851	.5250.

GROUP 2.

Y \ X	0.	.1000	.2000	.3000	.4000	.5000	.6000	.7000	.8000.
8000.	9636	.9604	.9511	.9361	.9163	.8930	.8680	.8426	.8176.
7000.	9421	.9397	.9327	.9216	.9075	.8916	.8756	.8613	.8426.
6000.	9246	.9227	.9174	.9092	.9001	[illegible]	.8739	.8756	.8680.
5000.	9151	.9147	.9107	.9047	.8979	.8913	.8836	.8916	.8930.
4000.	9133	.9121	.9093	[illegible]	[illegible]	.8979	.8991	.9075	.9163.
3000.	9135	.9127	.9105	.9075	.9050	.9047	.9072	.9216	.9361.
2000.	9150	.9145	.9126	.9105	[illegible]	.9127	.9174	.9327	.9511.
1000.	9153	.9157	.9147	.9127	.9122	.9147	.9227	.9397	.9604.
0.	9158	.9161	[illegible]	.9135	[illegible]	.9161	.9246	.9421	.9636.

Let $\varphi_k^{(i)}$ be coordinate k of flux-iterate i,

$\|\varphi\|_\infty = \max_k |\varphi_k|$, and $\|\varphi\|_1 = \sum_k |\varphi_k|$.

Then

$$\text{residual}^{(i)} = \frac{\|\varphi^{(i)} - \varphi^{(i-1)}\|_\infty}{\|\varphi^{(i)}\|_\infty},$$

$$\text{ratio}^{(i)} = \frac{\|\varphi^{(i)} - \varphi^{(i-1)}\|_1}{\|\varphi^{(i-1)} - \varphi^{(i-2)}\|_1} \cdot \text{sign}\left(\frac{\varphi_j^{(i)} - \varphi_j^{(i-1)}}{\varphi_j^{(i-1)} - \varphi_j^{(i-2)}}\right),$$

where $|\varphi_j^{(i-1)} - \varphi_j^{(i-2)}| = \|\varphi^{(i-1)} - \varphi^{(i-2)}\|_\infty$.

For $\text{ratio}^{(i)} \to \text{mys}$ we have

extrapolation factor $= 1/(1-\text{mys})$.

```
REACTION RATE DENSITY
 Y
 I
 I     AVERAGE 10000
 I
7000.     5116  50[illegible]1  [illegible]  2401
5000.    10063 [illegible]  [illegible]  [illegible]
3000.    10[illegible]  [illegible]  [illegible]2  5031
1000.    10[illegible] 10035 10[illegible]9  5116

---- X   1000. 3000. 5000. 7000.
EL.TIME=     1[illegible].2  PROC.TIME=      6.[illegible]  I/O.TIME=      1.5
```

ITNO	RESIDUAL	RATIO	LAMBDA	KEFF
1	7.251003[illegible]33E-02	1.000000000E+00	1.095843364E+00	9.125391754E-01
2	[illegible]184E-02	1.[illegible]91781[illegible]E+00	1.089334123E+00	9.179319910E-01
3	2.752802591E-02	-5.[illegible]71293325E-01	1.082530407E+00	9.237613812E-01
4	1.695345860E-02	5.4[illegible]1377332E-01	1.077659306E+00	9.279370525E-01
5	1.340504710E-02	-6.112954285E-01	1.073398399E+00	9.316252232E-01
6	6.4257[illegible]07E-03	-5.[illegible]5470[illegible]6E-01	1.077301444E+00	9.2824[illegible]2981E-01
7	4.000811334E-03	-4.024520632E-01	1.076679299E+00	9.287916733E-01
8	1.913421060E-03	1.000000000E+00	1.079418971E+00	9.272333906E-01
9	1.403708832E-03	-3.[illegible]99954922E-01	1.079075398E+00	9.275789074E-01
10	8.526355225E-04	-6.[illegible]263638[illegible]E-01	1.073196496E+00	9.274747263E-01
11	4.921497570E-04	-9.[illegible]17976285E-01	1.077898683E+00	9.277309789E-01
12	3.304543[illegible]93E-04	-4.[illegible]3121 1E-01	1.077893992E+00	9.277350935E-01
13	1.439769635E-04	-4.[illegible]7361187CE-01	1.077821546E+00	9.277073737E-01
14	3.55300772[illegible]E-05	-7.[illegible]15694971E-01	1.077871620E+00	9.277342717E-01
15	7.940655584E-05	-5.[illegible]5214562E-01	1.077859788E+00	9.2775[illegible]8433E-01
16	3.318015583E-05	-9.[illegible]51013374E-01	1.077826117E+00	9.277371874E-01
17	1.871940661E-05	-3.754163754E-01	1.077838064E+00	9.277401183E-01

COMPOSITION NUMBERS

Y																
7750.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.
7250.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.
6750.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	6.	6.	3.	3.
6250.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	6.	6.	3.	3.
5750.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
5250.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
4750.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
4250.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
3750.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
3250.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
2750.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
2250.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
1750.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
1250.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
750.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
250.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.	3.	3.
---- X	250.	750.	1250.	1750.	2250.	2750.	3250.	3750.	4250.	4750.	5250.	5750.	6250.	6750.	7250.	7750.

FLUX

GROUP 1.

Y \ X	0.	500.	1000.	1500.	2000.	2500.	3000.	3500.	4000.	4500.	5000.	5500.	6000.	6500.	7000.	7500.	8000.
8000.	5239	5222	5202	5155	5090	5007	4905	4785	4647	4492	4322	4140	3949	3755	3569	3396	3231
7500.	5526	5510	5481	5432	5364	5276	5168	5042	4896	4733	4553	4359	4154	3947	3747	3568	3396
7000.	5841	5831	5800	5749	5676	5584	5470	5335	5192	5008	4815	4604	4379	4146	3918	3747	3569
6500.	6448	6436	6442	6385	6306	6204	6078	5928	5755	5557	5334	5086	4810	4501	4146	3947	3755
6000.	7064	7051	7014	6953	6866	6754	6616	6451	6259	6038	5787	5502	5179	4910	4379	4154	3949
5500.	7575	7562	7522	7456	7362	7241	7092	6912	6703	6460	6182	5864	5502	5086	4604	4359	4160
5000.	8027	8013	7971	7900	7800	7670	7510	7318	7091	6829	6527	6182	5787	5334	4815	4553	4322
4500.	8424	8410	8365	8290	8184	8047	7877	7672	7431	7151	6829	6461	6038	5557	5008	4733	4492
4000.	8771	8755	8708	8629	8516	8374	8195	7980	7726	7431	7091	6703	6259	5755	5182	4896	4647
3500.	9069	9053	9004	8922	8806	8656	8459	8245	7980	7672	7318	6913	6451	5928	5336	5042	4785
3000.	9322	9305	9255	9170	9050	8895	8702	8469	8195	7877	7510	7092	6616	6078	5470	5168	4905
2500.	9533	9515	9463	9375	9253	9093	8895	8555	8374	8047	7671	7241	6754	6204	5584	5276	5007
2000.	9702	9685	9632	9543	9417	9253	9050	8806	8518	8186	7800	7362	6866	6306	5677	5364	5090
1500.	9833	9815	9761	9671	9543	9375	9170	8922	8629	8290	7900	7455	6953	6386	5749	5432	5155
1000.	9926	9907	9853	9761	9632	9463	9255	9004	8708	8365	7971	7522	7015	6442	5800	5481	5202
500.	9981	9962	9907	9815	9685	9515	9305	9053	8755	8410	8013	7562	7051	6476	5831	5510	5230
0.	9999	9981	9926	9833	9702	9533	9322	9069	8771	8425	8027	7575	7064	6488	5841	5520	5239

GROUP 2.

Y \ X	0.	500.	1000.	1500.	2000.	2500.	3000.	3500.	4000.	4500.	5000.	5500.	6000.	6500.	7000.	7500.	8000.
8000.	9632	9624	[illegible]	9561	9507	9432	[illegible]	9264	9150	9047	8928	8804	8679	8552	8425	8294	8156
7500.	9544	9537	9516	9482	9434	9374	9303	9222	9133	9037	8937	8834	8731	8628	8525	8414	8294
7000.	9412	9406	9388	9358	9318	9267	9207	9140	9066	8989	8909	8829	8752	8670	8610	8525	8425
6500.	9309	9303	9288	9262	9227	9183	9132	9074	9012	8944	8883	8821	8763	8714	8678	8628	8552
6000.	9234	9229	9216	9193	9163	[illegible]	9031	8972	8931	8908	8877	8831	8791	8763	8752	8731	8679
5500.	9182	9178	9166	9147	9120	9089	9050	9010	8967	8926	8887	8854	8831	8821	8829	8834	8804
5000.	9148	9145	9134	9117	9095	9067	9035	9001	8957	8925	8907	8887	8877	8883	8909	8937	8928
4500.	9129	9125	9116	9102	9082	[illegible]	9032	9004	8977	8953	8935	8926	8928	8948	8989	9037	9047
4000.	9119	9117	9108	9096	9079	9059	9037	9015	8994	8977	8967	8967	8991	9012	9066	9133	9160
3500.	9118	9115	9106	9097	9083	9065	9048	9030	9015	9004	9001	9009	9032	9074	9140	9222	9264
3000.	9121	9119	9113	9103	9091	9076	9061	9048	9037	9032	9035	9050	9081	9131	9207	9303	9357
2500.	9127	9126	9120	9112	9101	9087	9076	9066	9059	9055	9067	9088	9125	9183	9267	9374	9439
2000.	9135	9133	9128	9121	9111	9101	9091	9083	9079	9082	9094	9120	9163	9227	9318	9434	9507
1500.	9142	9140	9136	9129	9121	9112	9103	9097	9096	9102	9117	9146	9193	9262	9358	9481	9561
1000.	9148	9146	9142	9135	9128	9122	9113	9109	9109	9116	9134	9165	9216	9288	9388	9516	9600
500.	9152	9150	9146	9140	9133	9126	9119	9115	9117	9125	9145	9179	9229	9303	9406	9537	9624
0.	9153	9152	9148	9142	9135	9127	9121	9118	9119	9129	9148	9182	9234	9309	9412	9544	9632

```
 REACTION RATE DENSITY
   Y
   I
   I     AVERAGE 10000
   I
 7000.     5115  5031  4897  2402
 5000.    10069  9961  9925  4897
 3000.    10038  9934  9951  5031
 1000.    10068 10038 10069  5115

 ---- X  1000. 3000. 5000. 7000.
EL.TIME=    61.0  PROC.TIME=    33.0  I/O.TIME=      3.2
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