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RISØ NATIONAL LABORATORY

METALLURGY DEPARTMENT

CREEP PROPERTIES OF DISCONTINUOUS FIBRE COMPOSITES WITH PARTLY CREEPING FIBRES

by

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Abstract

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In a previous report (RISØ-M-1810) we analyzed the creep properties of discontinuous fibre composites with non-creeping fibres. In the present report this analysis is extended to include the case of discontinuous composites with partly creeping fibres. It is shown, that the creep properties of the composite at a given strain rate, $\dot{\varepsilon}_c$, depend on the creep properties of the matrix at a strain rate higher than $\dot{\varepsilon}_c$, and on the creep properties of the fibres at $\dot{\varepsilon}_c$. The composite creep law is presented in a form which permits a graphical determination of the composite creep curve. This can be constructed on the basis of the matrix and the fibre creep curves by vector operations in a log $\dot{\varepsilon}$ vs. log σ diagram. The matrix contribution to the creep strength can be evaluated by a simple method.

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CONTENTS

1. INTRODUCTION

In a previous report (Bilde-Sørensen, Böcker Pedersen and Lilholt 1975 - in the following called BBL) we discussed the creep properties of discontinuous fibre composites with non-creeping fibres. We concluded that the creep curve for fibre composites with non-creeping fibres to a good approximation can be obtained from the matrix creep curve by a simple displacement of the latter curve in a log $\dot{\varepsilon}$ vs. log σ diagram. The direction of displacement is such that the transition from a power law to an exponential law occurs at a lower strain rate for the composite than for the unreinforced matrix. This has important practical consequences for the prediction of the creep strength of fibre composites with non-creeping fibres. It is therefore of interest to investigate the implications of extending the displacement vector analysis to include also discontinuous fibre composites with (partly) creeping fibres.

A list of the symbols used in the analysis is presented on page 18.

2. THEORY

2.1. The physical model

We shall base our analysis on the model for composites with partly creeping fibres proposed by Kelly and Street (1972). They consider a single fibre element, and the fibres are assumed to creep in the central part, $0 \leq z < z_c$,[*] where the strain rate of the fibres, $\dot{\varepsilon}_f$, equals that of the composite, $\dot{\varepsilon}_c$. In the range $z_c \leq z \leq \frac{\ell}{2}$ the fibres are assumed to be rigid. This approach neglects the creep rate transient in the fibres up to the point $z = z_c$, where $\dot{\varepsilon}_f$ becomes equal to $\dot{\varepsilon}_c$. Kelly and Street pointed out, that this approach is a good approximation provided that the stress sensitivity of the fibre is large, since the transient then would be very steep.

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[*] Because of the symmetry about the midpoint of the fibre element we only consider positive values of z in the analysis.

In order to carry through the displacement vector analysis, it is necessary to introduce a specific creep law. As in our previous report (BBL) we employ a power law

$$\dot{\varepsilon} = \dot{\varepsilon}_o \left(\frac{\sigma}{\sigma_o}\right)^n \tag{1}$$

at intermediate stresses and an exponential law

$$\dot{\varepsilon} = \dot{\varepsilon}_o' \exp\left(\frac{\sigma}{\sigma_o'}\right) \tag{2}$$

at high stresses.*)

Kelly and Street's model gives the following expressions for the shear strain rate in the matrix:

$$\dot{\gamma} = \frac{\dot{\varepsilon}_c (z-z_c)}{h} \quad \text{for } z \geq z_c \tag{3}$$

and

$$\dot{\gamma} = 0 \qquad\qquad \text{for } 0 \leq z < z_c \tag{4}$$

When the shear values $\dot{\gamma} = \frac{3}{2}\,\dot{\varepsilon}$ and $\sigma = 2\tau$ (BBL) are inserted into the exponential law of eq. (2), we find, with $h = \alpha d$,

$$\tau = \frac{\sigma_o'}{2} \ln\left(\frac{2\;\dot{\varepsilon}_c (z-z_c)}{3\;\dot{\varepsilon}_o'\;\alpha d}\right) \tag{5}$$

The stress distribution in the fibre is found from the integral

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*) A numerical analysis carried through with a hyperbolic sine law leads to the same results.

$$\sigma_f = \frac{4}{d} \int_z^{\ell/2} \tau \, dz$$

$$= \frac{2\sigma_o'}{d} \left[\left(\frac{\ell}{2} - z_c\right) \ln \frac{2\ell_c\left(\frac{\ell}{2}-z_c\right)}{3\ell_o' \, e\alpha d} - (z-z_c) \ln \frac{2\ell_c(z-z_c)}{3\ell_o' \, e\alpha d}\right] \qquad (6)$$

The maximum fibre stress, $\sigma_{f,max}$, is found by setting $z = z_c$:

$$\sigma_{f,max} = \frac{\sigma_o'(\ell-2z_c)}{d} \ln \frac{\ell_c(\ell-2z_c)}{3\ell_o' \, e\alpha d} \qquad (7)$$

The average stress in the fibre is found from

$$\bar{\sigma}_f = \frac{2}{\ell}\left(z_c \cdot \sigma_{f,max} + \int_{z_c}^{\ell/2} \sigma_f \, dz\right) \qquad (8)$$

Since

$$\int_{z_c}^{\ell/2} \sigma_f \, dz = \frac{\sigma_o'(\ell-2z_c)^2}{4d} \ln \frac{\ell_c(\ell-2z_c)}{3\ell_o' \sqrt{e} \, \alpha \, d} \qquad (9)$$

we find

$$\bar{\sigma}_f = \frac{\sigma_o'(\ell-2z_c)}{2d} \left[\left(1 - \frac{2z_c}{\ell}\right) \ln \frac{\ell_c(\ell-2z_c)}{3\ell_o' \sqrt{e} \, \alpha \, d} + \frac{4z_c}{\ell} \ln \frac{\ell_c(\ell-2z_c)}{3\ell_o' \, e \, \alpha \, d}\right] \qquad (10)$$

In a treatment analogous to the above derivations, but based on the power law, the following expressions can be derived for $\sigma_{f,max}$ and $\bar{\sigma}_f$ (BBL):

$$\sigma_{f,max} = \frac{\sigma_o(\ell-2z_c)}{d} \cdot \frac{n}{n+1} \cdot \left(\frac{\dot{\varepsilon}_c(\ell-2z_c)}{3\dot{\varepsilon}_o\ \alpha d}\right)^{1/n}$$

$$\cong \frac{\sigma_o(\ell-2z_c)}{d} \cdot \left(\frac{\dot{\varepsilon}_c(\ell-2z_c)}{3\dot{\varepsilon}_o\ e\alpha\ d}\right)^{1/n} \qquad (11)$$

and

$$\bar{\sigma}_f = \frac{\sigma_o(\ell-2z_c)}{2d} \left[\left(1 - \frac{2z_c}{\ell}\right)\left(\frac{\dot{\varepsilon}_c(\ell-2z_c)}{3\dot{\varepsilon}_o\ \sqrt{e}\ \alpha\ d}\right)^{1/n} + \frac{4z_c}{\ell}\left(\frac{\dot{\varepsilon}_c(\ell-2z_c)}{3\dot{\varepsilon}_o\ e\ \alpha\ d}\right)^{1/n}\right] \qquad (12)$$

In eq. (11) the factor $\frac{n\ e^{1/n}}{n+1}$ has been set equal to 1. The actual value of the factor varies, e.g. from 1.047 for n = 3 to 1.009 for n = 7.

These results can be rewritten in a generalized form in the same way as could the results obtained for composites with non -creeping fibres. A matrix law, $\sigma = f(\dot{\varepsilon})$, which can be approximated by a power law at intermediate stresses and by an exponential law at high stresses leads in the case of composites with partly creeping fibres to the following expression for the maximum fibre stress:

$$\frac{\sigma_{f,max}}{\rho_{eff}} = f\left(\frac{\dot{\varepsilon}_c\rho_{eff}}{3\ e\ \alpha}\right) \qquad (13)$$

In eq. (13) the effective aspect ratio, $\frac{\ell-2z_c}{d}$, has been called ρ_{eff} (Kelly and Street (1972) used the term STAR - stress transfer aspect ratio - for this ratio).

The composite creep strength is found from

$$\sigma_c = V_f \bar{\sigma}_f + (1-V_f)\,\sigma_m$$

$$\simeq V_f \bar{\sigma}_f$$

where the last approximation is valid provided the additive matrix term can be neglected (BBL). With this approximation and with eqs. (10) and (12) we obtain the composite creep law in the generalized form

$$\frac{2\,\sigma_c}{V_f \rho_{eff}} = \left(1 - \frac{2z_c}{\ell}\right) f\left(\frac{\dot{\varepsilon}_c \rho_{eff}}{3\sqrt{e}\,\alpha}\right) + \frac{4z_c}{\ell}\, f\left(\frac{\dot{\varepsilon}_c \rho_{eff}}{3\,e\,\alpha}\right) \qquad (14)$$

Since $(1 - \frac{2z_c}{\ell}) = \frac{\rho_{eff}}{\rho}$, eq. (14) can be rewritten

$$\frac{2\sigma_c}{V_f\,\rho_{eff}} = \frac{\rho_{eff}}{\rho}\, f\left(\frac{\dot{\varepsilon}_c\,\rho_{eff}}{3\sqrt{e}\,\alpha}\right) + 2\left(1 - \frac{\rho_{eff}}{\rho}\right) f\left(\frac{\dot{\varepsilon}_c\,\rho_{eff}}{3\,e\,\alpha}\right) \qquad (15)$$

In order to apply eq. (15) it is necessary to find ρ_{eff}. Since $\dot{\varepsilon}_f = \dot{\varepsilon}_c$ at $\sigma = \sigma_{f,max}$, ρ_{eff} can be found by operations in a log $\dot{\varepsilon}$ vs. log σ diagram by the help of eq. (13). Eq. (15), however, involves addition of two functions and therefore cannot be interpreted in terms of vector operations in a log $\dot{\varepsilon}$ vs. log σ diagram. In the formulation given by eqs. (13) and (15) the creep stress, σ_c, must thus be calculated numerically from eq. (15).

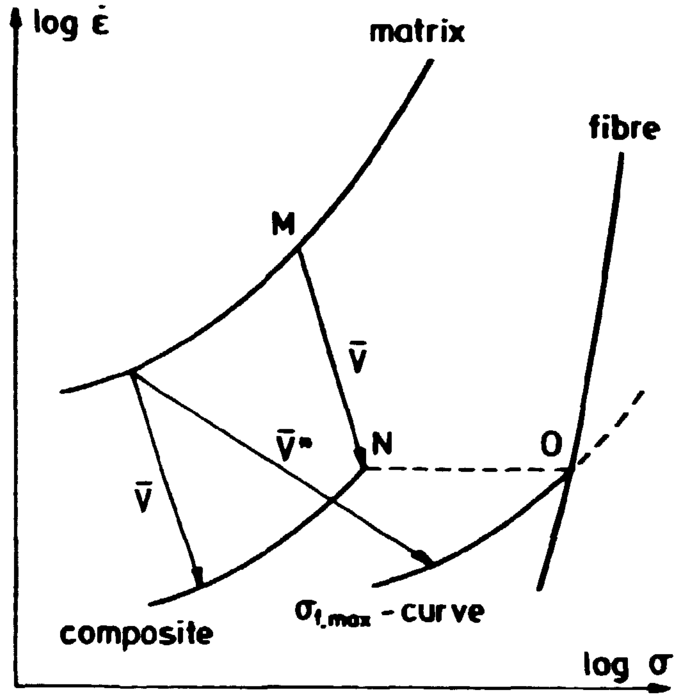

FIGURE 1. Determination of the strain rate at which the fibres begin to creep. The vector $\bar{V}$ has the components $(\log(\frac{1}{2}\rho V_f)$, $\log(\frac{3\sqrt{e\alpha}}{\rho}))$, and the vector $\bar{V}^*$ has the components $(\log\rho$, $\log(\frac{3e\alpha}{\rho}))$.

A great simplification is obtained if eq. (15) is approximated by

$$\frac{2\sigma_c}{V_f\ \rho_{eff}} = \left(2 - \frac{\rho_{eff}}{\rho}\right) f\left(\frac{\dot{\varepsilon}_c\ \rho_{eff}}{3\sqrt{e}\ \alpha}\right) \tag{16}$$

It is noticed, that this expression degenerates to the expression for non-creeping fibres (BBL) for $\rho_{eff} = \rho$. In the appendix it is shown, that the more simple form of eq. (16) in most cases is a sufficiently good approximation. Eq. (15) can also be rearranged in an exact, but more complicated form if the local slope of the matrix creep curve at $\sigma = f(\frac{\dot{\varepsilon}_c\ \rho_{eff}}{3\sqrt{e}\ \alpha})$ is introduced. This is discussed in the appendix.

With the use of the set of equations (13) and (16), the composite creep curve can readily be constructed in a log $\dot{\varepsilon}$ vs. log σ diagram on basis of the matrix and the fibre creep curves.

2.2. The composite creep curve

We shall first discuss how to determine the strain rate at which the fibres start creeping. The necessary operations are illustrated in fig.1.

Eq. (13) states that the maximum fibre stress $\sigma_{f,max}$ is related to the matrix stress σ by the equation

$$\log \sigma_{f,max} = \log \sigma + \log \rho_{eff} \tag{17}$$

and that the composite creep rate $\dot{\varepsilon}_c$ (at $\sigma_{f,max}$) is related to the matrix creep rate $\dot{\varepsilon}$ (at σ) by

$$\log \dot{\varepsilon}_c = \log \dot{\varepsilon} + \log \left(\frac{3\ e\alpha}{\rho_{eff}}\right) \tag{18}$$

For non-creeping fibres ρ_{eff} equals ρ, so the $\dot{\varepsilon}_c$ vs. $\sigma_{f,max}$ curve for non-creeping fibres can be obtained by displacing the matrix curve by a vector $\bar{V}^*$ with components $(\log \rho,\ \log(\frac{3e\alpha}{\rho}))$.

The fibres can only be non-creeping if $\sigma_{f,max}$ is lower than the stress needed to induce creep in the fibres. The fibres are therefore only non-creeping as long as the $\dot{\varepsilon}_c$ vs. $\sigma_{f,max}$ curve lies to the left of the fibre creep curve. The intersection between the two curves, O, thus gives the composite creep rate at which the fibres begin to creep.

For composite creep rates lower than the creep rate at O, the composite creep curve can therefore be obtained by displacing the matrix curve by the vector $\bar{V}$ with components $(\log(\frac{1}{2}\rho V_f),\ \log(\frac{3\sqrt{e}\alpha}{\rho}))$, as described in our previous report (BBL).

The point N in fig. 1, at the same creep rate as point O, is the point where creep in the fibres begin to influence the composite creep behaviour. We note that N has the corresponding point M on the matrix curve.

When the fibres are creeping, the composite curve is governed by both the matrix and the fibre curves. In this case, the composite curve must be constructed point by point. Such a construction is shown in fig.2, where the point E on the composite curve, corresponding to the matrix point A, is determined.

We begin by determining ρ_{eff} with the help of eq. (13) (and the expanded forms eqs. (17) and (18)). A comparison with eq. (16) shows, that a point $(\sigma_c, \dot{\varepsilon}_c)$ on the composite curve has the corresponding point $(f(\dot{\varepsilon}),\ \dot{\varepsilon})$ on the matrix curve, whereas a point with the same $\dot{\varepsilon}_c$ on the $\sigma_{f,max}$-curve $(\sigma_{f,max},\ \dot{\varepsilon}_c)$ has the corresponding point $(f(\frac{\dot{\varepsilon}}{\sqrt{e}}),\ \frac{\dot{\varepsilon}}{\sqrt{e}})$ on the matrix curve. We therefore start from the point B on the matrix curve, which has a strain rate $\sqrt{e}$ times lower than that at A. From B a vector BC is drawn with components $(0,\ \log(3e\alpha))$, and from C a vector CD is drawn with components $(\log \rho,\ -\log \rho)$.

The vector CD intersects the fibre creep curve at D'. The stress coordinate of D' is that maximum fibre stress, which will induce a fibre creep rate $\dot{\varepsilon}_f$, equal to the composite creep rate $\dot{\varepsilon}_c$. The components of the vector BD' are thus $(\log \rho_{eff},$

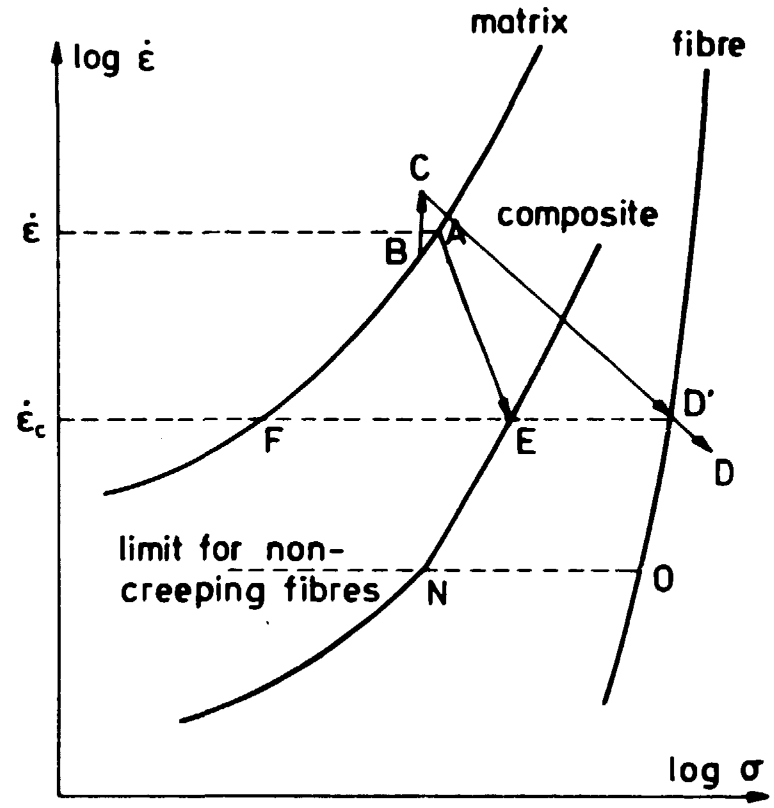

FIGURE 2. Construction of the composite creep curve from the matrix and the fibre creep curves. The vectors have the following components: BC(0, log (3 eα)), CD (log ρ, - log ρ), CD' (log ρ_{eff}, - log ρ_{eff}), AE (log ($\frac{1}{2}$ ρ_{eff} V_f (2 - $\frac{\rho_{eff}}{\rho}$)), log ($\frac{3\sqrt{e}\alpha}{\rho_{eff}}$)). The point B on the matrix curve has a creep rate $\sqrt{e}$ lower than that of point A.

$\log(\frac{3e\alpha}{\rho_{eff}})$) so that the value of ρ_{eff} can be read off from the diagram *)

Eq. (16) now states that the composite creep rate, $\dot{\varepsilon}_c$, is related to the matrix creep rate, $\dot{\varepsilon}$, by the equation

$$\log \dot{\varepsilon}_c = \log \dot{\varepsilon} + \log \left(\frac{3\sqrt{e}\alpha}{\rho_{eff}}\right) \tag{19}$$

and that the composite stress, σ_c, is related to the matrix stress, σ, by the equation

$$\log \sigma_c = \log \sigma + \log \left(\frac{1}{2}\rho_{eff}\, V_f\left(2 - \frac{\rho_{eff}}{\rho}\right)\right) \tag{20}$$

In order to obtain the point E on the composite creep curve, corresponding to point A, it thus remains to draw a vector AE with the components

$$\left(\log \left(\frac{1}{2}\, \rho_{eff}\, V_f\left(2 - \frac{\rho_{eff}}{\rho}\right)\right),\ \log \left(\frac{3\sqrt{e}\ \alpha}{\rho_{eff}}\right)\right)$$

---

*) The construction <u>cannot</u> be made using a single vector BD with components ($\log \rho$, $\log \frac{3e\alpha}{\rho}$). It is obvious that BD will not intersect the fibre curve in D', but in a different point, say D''. The reason for the two-vector construction is, that 1/q of a vector with components (a, b) is $(\frac{a}{q}, \frac{b}{q})$. CD' thus has components $(\frac{1}{q}\log\rho, -\frac{1}{q}\log\rho)$ which is interpreted as $(\log\rho_{eff}, -\log\rho_{eff})$ for the reasons given in the main text. A hypothetical vector BD'' cannot be given such an interpretation.

The composite creep curve can be drawn, when a sufficient number of points on this curve has been found by the outlined procedure.

2.3. The matrix contribution

We note, that the composite curve in the previous section was constructed under the assumption, that the additive matrix term is negligible compared with the $V_f\bar{\sigma}_f$-term in the equation

$$\sigma_c = V_f\bar{\sigma}_f + (1-V_f)\sigma_m \tag{21}$$

where σ_m is the (average) stress in the matrix, when the whole composite creeps at the creep rate $\dot{\varepsilon}_c$.

The importance of the matrix contribution can easily be checked in the way described below. The prescribed method furthermore allows us to include the matrix term in the composite creep strength.

The method is valid for discontinuous composites with rigid fibres as well as composites with creeping fibres, and it is therefore of general applicability.

We introduce a factor k of such a magnitude that

$$V_f\bar{\sigma}_f = k\ (1-V_f)\sigma_m \tag{22}$$

Combined with eq. (21) this gives

$$\sigma_c = V_f\bar{\sigma}_f\left(\frac{1+k}{k}\right) \tag{23}$$

We further find

$$\frac{(1-V_f)\sigma_m}{\sigma_c} = \frac{1}{1+k} \tag{24}$$

It will be remembered that the curve we have constructed on the basis of eq. 16, really is a curve of $\dot{\varepsilon}_c$ vs. $V_f\bar{\sigma}_f$. Eq. (22) states that

$$\log V_f\bar{\sigma}_f = \log \sigma_m + \log(k(1-V_f)) \tag{25}$$

The vector connecting the matrix curve and the constructed $V_f\bar{\sigma}_f$-curve at a given $\dot{\varepsilon}_c$ (e.q. the vector FE in fig. 2) therefore has the magnitude $(\log(k(1-V_f)), 0)$. Hence, the value of k is easily calculated, and from eq. 24 the relative magnitude of the matrix contribution can be assessed.

In case the additive matrix term cannot be neglected, eq. (23) should be applied. At the given composite creep rate $\dot{\varepsilon}_c$, the point on the (true) composite curve is found by displacing the point on the $V_f\bar{\sigma}_f$-curve by $\log\left(\frac{1+k}{k}\right)$ along the log σ-axis.

3. DISCUSSION

The present analysis of the steady-state creep of discontinuous fibre composites with partly creeping fibres is a more general version of the previous analysis (BBL) on which we imposed the condition of rigid fibres. The case including creeping fibres is complicated by the fact, that the creep properties at a given strain rate, $\dot{\varepsilon}_c$, depend both on the creep properties of the fibres at this strain rate and on the creep properties of the matrix at a higher strain rate. This means that the composite creep curve no longer is given by a simple displacement of the matrix curve. The principle of corresponding points is, however, maintained, but the magnitude of the displacement vector is varying from point to point.

Previous predictions (Mileiko 1970, Kelly and Street 1972) of the composite creep strength have been made from matrix creep data at the same strain rate as that of the composite. As in the case of composites with rigid fibres this approach only gives a correct prediction at very low composite strain rates where the corresponding matrix strain rate is still in the power law range. At higher strain rates, the composite creep strength will always be overestimated.

The present, more correct, analysis has been made under the assumption that the matrix creep curve can be described by a power law at intermediate stresses and an exponential law at high stresses. However, the analysis places no restriction on the form of the fibre creep curve.

Since the shape of the $\dot{\varepsilon}_c$ vs. σ_c curve in the range with creeping fibres is determined by the shape of both the matrix and the fibre curves it is not possible to give a detailed general description of the composite creep curve. We shall, however, make some remarks on the example sketched in fig. 2, which shows a matrix curve and a fibre curve typical of practical composites ($n_f > n_m$). The important observation is that the stress exponent at, say, E is much higher than the stress exponent of the matrix at the same strain rate. The reason for this is partly that the corresponding matrix data are in the exponential range, and partly that the fibres (with $n_f > n_m$ where n_f is taken at D' and n_m at A) are creeping; this additional cause further promotes large values of n_c. This effect can be described in terms of the effective aspect ratio ρ_{eff} which is a measure of the rigid part of the partly creeping fibre. For $n_f > n_m$ the matrix curve and the fibre curve approach each other; this means that ρ_{eff} decreases for increasing applied stress and this in turn has the effect of making n_c larger than n_m. It is therefore clear that very high values of n_c can be expected for a typical composite ($n_f > n_m$) with partly creeping fibres.

In fig. 1 we considered the case where the fibres begin to creep with an increase in the applied stress. Kelly and Street (1972) pointed out that for the case $n_m > n_f$ the fibres at some value of the stress would stop behaving as creeping fibres for a stress increase (the fibres would actually not stop creeping completely, but their creep rate would everywhere be less than that of the matrix, and they would effectively behave as rigid fibres). This situation is also included in the present analysis.

In summarizing, we expect that experiments on fibre composites often will show n_c - values (significantly) higher than those of the pure matrix. The limited range of experimentally obtainable stresses and strain rates can present difficulties for a correct interpretation of a (fairly short) curving part of a composite creep curve (see fig. 2). On the other hand, the local value of n_c can be a valuable support to an identification of the actual region of creep law.

Finally, we should like to point out that we have not made any changes in the theories, which we have reassessed in this and our previous report (BBL). All the points we have called attention to are actually inherent in the original theories, but were not taken to their logical conclusion. McLean (1972) even discussed amplification factors for the shear stress and the shear strain rate, but only on the basis of a power creep law.

A more correct interpretation of the models is obtained through our analysis, so that an experimental assessment of the models can be made on a fair basis. This has already (BBL) proved of value in explaining the unexpectedly high stress exponents for composites in certain stress ranges. More experiments are, however, needed before an unambiguous assessment of the models can be made.

4. SUMMARY AND CONCLUSIONS

The analysis of the creep properties of discontinuous fibre composites with non-creeping fibres (BBL) has been extended to cover the case of composites with partly creeping fibres.

We have shown that the composite creep curve can be obtained from the matrix and the fibre creep curves by simple vector operations in a log $\dot{\varepsilon}$ vs. log σ diagram.

For a matrix creep law $\sigma = f(\dot{\varepsilon})$, the composite creep law becomes

$$\frac{2\,\sigma_c}{\rho_{eff} V_f \left(2 - \frac{\rho_{eff}}{\rho}\right)} = f\left(\frac{\dot{\varepsilon}_c \rho_{eff}}{3\,\sqrt{\varepsilon a}}\right)$$

where ρ_{eff} is the effective aspect ratio. For $\rho_{eff} = \rho$ this equation degenerates to that previously found for composites with non-creeping fibres.

The composite creep properties are governed by those of both the matrix and the fibres, and it is shown that for practical composites ($n_f > n_m$) very high values of the exponent n_c can be expected.

The composite creep law is derived under the assumption that the additive matrix term is negligible. An easy method of checking this assumption is presented, and it is also shown that the additive matrix term can be included if it is not negligible.

We finally note, that the composite creep curve of course could be determined analytically on basis of the equations given in this report. The calculations involved would, however, be rather lengthy. From the computational viewpoint, the easy graphical determination presented in this report is therefore a significant advantage.

APPENDIX

In order to rearrange eq. (15), we introduce the local slope of the matrix creep curve, $n_m = \frac{d \log \dot{\epsilon}}{d \log \sigma}$, around $\sigma = f\left(\frac{\dot{\epsilon}_c \rho_{eff}}{3\sqrt{e}\alpha}\right)$. We then obtain the following relation:

$$\log\left[f\left(\frac{\dot{\epsilon}_c \rho_{eff}}{3\sqrt{e}\alpha}\right)\right] - \log\left[f\left(\frac{\dot{\epsilon}_c \rho_{eff}}{3\ e\alpha}\right)\right] = \frac{1}{n_m} \log \sqrt{e} \tag{A1}$$

or

$$\frac{f\left(\frac{\dot{\epsilon}_c \rho_{eff}}{3\sqrt{e}\alpha}\right)}{f\left(\frac{\dot{\epsilon}\ \rho_{eff}}{3\ e\alpha}\right)} = e^{\frac{1}{2n_m}} \tag{A2}$$

If eq. (A2) is introduced into eq. (15) we find:

$$\frac{2\sigma_c}{V_f \rho_{eff}} = \left(\frac{\rho_{eff}}{\rho} + 2e^{-\frac{1}{2n_m}}\left(1 - \frac{\rho_{eff}}{\rho}\right)\right) f\left(\frac{\dot{\epsilon}_c \rho_{eff}}{3\sqrt{e}\alpha}\right) \tag{A3}$$

Eq. (A3) can of course be used instead of the approximative eq. (16), but the introduction of the additional parameter n_m which has to be measured at each individual point, renders the determination of the composite curve more complicated.

The following table, which gives the ratio

$$\frac{2 - \frac{\rho_{eff}}{\rho}}{\frac{\rho_{eff}}{\rho} + 2 \cdot e^{-\frac{1}{2n_m}} \left(1 - \frac{\rho_{eff}}{\rho}\right)}$$

for different values of $\frac{\rho_{eff}}{\rho}$ and n_m, shows that in most cases the difference between the approximative approach of eq. (16) and the exact approach of eq. (A3) is so small, that eq. (16) can be safely applied.

$\frac{\rho_{eff}}{\rho}$ =	1	0.8	0.6	0.4	0.2
n_m = 3	1	1.054	1.096	1.130	1.158
5	1	1.033	1.058	1.077	1.092
10	1	1.017	1.029	1.038	1.045

LIST OF SYMBOLS

a	vector component
b	vector component
d	diameter of fibre
h	thickness of zone of constant shear strain rate
k	numerical constant
ℓ	length of fibre
n	stress exponent in creep law (subscripts c,f, and m refer to composite, fibre, and matrix, respectively)
q	numerical constant
V_f	volume fraction of fibres
$\bar{V}$	displacement vector
$\bar{V}^*$	displacement vector
z	length coordinate along fibre; $z = 0$ at fibre midpoint
z_c	half-length of creeping part of fibre
α	geometrical parameter
$\dot{\gamma}$	shear strain rate in matrix
$\dot{\varepsilon}$	tensile strain rate of unreinforced matrix
$\dot{\varepsilon}_c$	tensile strain rate of composite
$\dot{\varepsilon}_f$	tensile strain rate of fibre
$\dot{\varepsilon}_o$	constant in power creep law
$\dot{\varepsilon}_o'$	constant in exponential creep law
ρ	aspect ratio ($= \ell/d$)
ρ_{eff}	effective aspect ratio ($=(\ell-2z_c)/d$)
σ	tensile stress in unreinforced matrix
σ_m	tensile stress in reinforced matrix
σ_c	tensile stress in composite
σ_f	tensile stress in fibre
$\bar{\sigma}_f$	average tensile stress in fibre
$\sigma_{f,max}$	maximum tensile stress in fibre
σ_o	constant in power creep law
σ_o'	constant in exponential creep law
τ	shear stress

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